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EWMA Run-to-Run Controllers with Gain Updating: Stability and Sensitivity Analysis

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***Abstract*— Exponentially weighted moving average (EWMA) controllers are the most commonly used run-to-run controllers in semiconductor manufacturing. Based on a linear model, an EWMA controller is usually implemented in a way that the process gain is kept as the off-line estimate and the intercept term is updated using an EWMA filter at each run. However, in practice, there are many applications that an EWMA controller is implemented in a way that the process gain is updated in a run-to-run manner while the intercept is kept as the off-line estimate. Although the stability and sensitivity of EWMA controllers with intercept updating has been well known, there is no analysis result on the stability and sensitivity of EWMA controllers with gain updating. In this paper, we analyze the behavior of an EWMA controller with gain updating and compare it to that of an EWMA controller with intercept updating. Both stationary and drifting processes are considered, the expression of the process output are derived and the output variances for stochastic processes are evaluated. In addition, simulation examples are given to illustrate the analysis results.**

keywords: exponentially weighted moving average, stability, sensitivity, asymptotic variance.

I. INTRODUCTION

Due to their simplicity and robustness, exponentially weighted moving average (EWMA) controllers are the most commonly used run-to-run feedback controllers in semiconductor manufacturing processes [1], [2], [3], [4]. In general, a linear regression model of a process is obtained based on off-line experiments or historical data. During online estimation and control, one of the model parameters (i.e., intercept or gain) is updated using EWMA statistics based on newly observed process data, then the recipe for the next run is calculated based on the updated process model.

Consider a single input single output (SISO) linear static model

$$y[n] = \alpha + \beta u[n] + \epsilon[n] \tag{1}$$

where α and β are model parameters, $\epsilon[n]$ is the process disturbance sequence, $u[n]$ is the input to the process at the beginning of run n, and $y[n]$ is the output of the process at the end of run n. Because there is no process dynamics involved in Eqn. (1), the minimum variance controller to achieve a desired target T is a deadbeat controller, i.e.,

$$u = \frac{T - \hat{\alpha}}{\hat{\beta}} \tag{2}$$

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where $\hat{\alpha}$ and $\hat{\beta}$ are the estimates of α and β. Notice that either $\hat{\alpha}$ or $\hat{\beta}$ can be updated using EWMA statistics. An EWMA controller with intercept updating, denoted as EWMA-I controller, is formulated as the following

$$u[n+1] = \frac{T - a[n]}{b} \tag{3}$$

$$a[n] = \hat{\alpha}_{n+1|n} = \omega(y[n] - bu[n]) + (1-\omega)a[n-1] \tag{4}$$

where b is the off-line estimate of β, $a[n]$ is the on-line estimate of α that is updated after each run, and ω is the EWMA weighting. For EWMA-I controllers, Ingolfsson and Sachs [5] analyzed its stability and sensitivity. Partially due to its stability, together with its simplicity and robustness, EWMA-I controllers have gained wide applications in the semiconductor manufacturing industry.

However, in many cases an EWMA controller is implemented in a way that the process gain, instead of the intercept, is updated using EWMA statistics in a run-to-run manner. The examples for EWMA controllers with gain updating include many processes where processing rate is updated, such as chemical mechanical polishing processes (CMP) [6], [7], [8], chemical vapor deposition processes (CVD) [9], and etch processes [10], [11] where polishing rate, deposition rate and etch rate are updated respectively. For an EWMA controller with gain updating (denoted as EWMA-G controller), the control law and the state estimation equations are

$$u[n+1] = \frac{T - a}{b[n]} \tag{5}$$

$$b[n] = \hat{\beta}_{n+1|n} = \omega\frac{y[n] - a}{u[n]} + (1-\omega)b[n-1] \tag{6}$$

where a is the off-line estimate of α and $b[n]$ is the on-line estimate of β. Although EWMA-G controllers have been applied to many processes, to the best of the authors knowledge, there has not been any results on their stability and sensitivity analysis. In this work, we analyze the behavior of an EWMA-G controller, and compare it with that of an EWMA-I controller [5]. We also use simulation examples to illustrate the relevance of the analysis.

In this work, the stability and sensitivity analysis is limited to linear processes. We will analyze the performance of an EWMA-G controller applied to both stationary and drifting processes. The process output $y[n]$ is expressed in closed-form whenever possible, and the stability condition that guarantees the convergence of $y[n]$ is derived. For stochastic processes, we further evaluate the variance of the output sequence.

II. Linear Stationary Processes

In this section we analyze the stability of an EWMA-G controller when applied to stationary first-order processes. Both deterministic and stochastic processes are considered. Simulation examples are given to illustrate the obtained analysis results.

A. A Deterministic First-order Process

First, we consider a deterministic first-order process, i.e.,

$$y[n] = \alpha + \beta u[n] \tag{7}$$

This is a simplified case of a real process but it provides the basic procedure for stability analysis of more realistic cases. Plugging Eqn. (5) into Eqn. (6), we have

$$b[n] = (1 - \frac{T-\alpha}{T-a}\omega)b[n-1] + \omega\beta \tag{8}$$

The dynamic system described by Eqn. (8) has a pole located at $1 - \frac{T-\alpha}{T-a}\omega$. Therefore, as long as $|1 - \frac{T-\alpha}{T-a}\omega| < 1$, the system is stable and the sequence of $b[n]$ will converge to a steady-state value, denoted by b_∞. Consequently, the output sequence $y[n]$ is stable, and will converge to a steady-state value $\alpha + \frac{\beta}{b_\infty}(T-a)$. Notice that the stability condition, $|\theta| < 1$, is equivalent to

$$0 < \frac{T-\alpha}{T-a}\omega < 2 \tag{9}$$

Next, we show that if the stability condition is satisfied, the output of the process will converge to the target without bias. Let $\theta \equiv 1 - \frac{T-\alpha}{T-a}\omega$, from Eqn. (8) we obtain

$$b[n-1] = \theta^{n-1}b[0] + \frac{1-\theta^{n-1}}{1-\theta}\omega\beta \tag{10}$$

Notice that for run n, we have

$$y[n] = \alpha + \beta u[n] \tag{11}$$

$$u[n] = \frac{T-a}{b[n-1]} \tag{12}$$

After some algebraic manipulation, we obtain

$$y[n] = T - \frac{\theta^{n-1}(T-\alpha)\left(\frac{b[0](T-\alpha)}{\beta(T-a)} - 1\right)}{1 + \theta^{n-1}\left(\frac{b[0](T-\alpha)}{\beta(T-a)} - 1\right)} \tag{13}$$

From Eqn. (13), it is clear that

$$\lim_{k\to\infty} y[n] = T \qquad \text{iff} \qquad |\theta| < 1 \tag{14}$$

It is worth to note the similarity between stability regions of an EWMA-G controller and an EWMA-I controller, i.e., both stability regions can be described by the following formula

$$0 < \left(\frac{\text{model pramater}}{\text{off-line estimate}}\right) \bullet (\text{EWMA weighting}) < 2 \tag{15}$$

for an EWMA-I controller, the model parameter is β, while for an EWMA-G controller, the model parameter is $(T-\alpha)$.

Despite the similarity in their stability regions, the dynamic behaviors of EWMA-G and EWMA-I controllers are different when the controllers are unstable. When both controllers are unstable, although both estimated states, i.e., $a[n]$ for EWMA-I and $b[n]$ for EWMA-G, will approach infinity, the corresponding inputs and outputs behave differently. Fig. 1 shows the dynamic behavior of the unstable EWMA controllers. We observe that the process output converges to the process intercept α for the EWMA-G controller, while the process output approaches infinity for the EWMA-I controller. The unstable dynamic behaviors of both controllers are summarized in Table I.

TABLE I

Comparison of EWMA controllers: unstable region

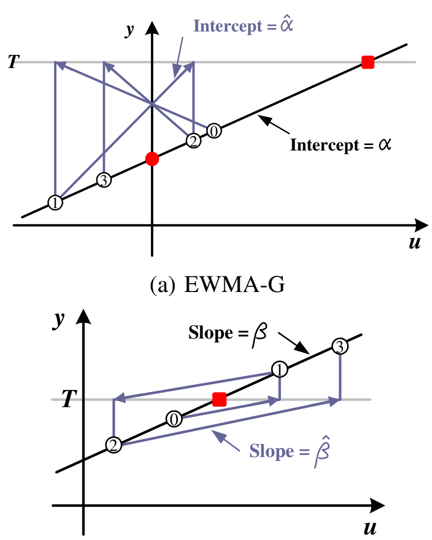
	EWMA-G	EWMA-I
unstable region	$\frac{T-\alpha}{T-a}\omega < 0$ or $\frac{T-\alpha}{T-a}\omega > 2$	$\frac{\beta}{b}\omega < 0$ or $\frac{\beta}{b}\omega > 2$
state	$\lim_{n\to\infty} b[n] = \infty$	$\lim_{n\to\infty} a[n] = \infty$
input	$\lim_{n\to\infty} u[n] = 0$	$\lim_{n\to\infty} u[n] = \infty$
output	$\lim_{n\to\infty} y[n] = \alpha$	$\lim_{n\to\infty} y[n] = \infty$

The differences in Table I are important because they show that unlike an unstable EWMA-I controller whose output approaches infinity, the output controlled by an unstable EWMA-G controller is still bounded, approaching the process intercept. This property is desirable, because an EWMA-G controller can be tuned more aggressively if needed due to the bounded response.

B. A First-Order Process with Noise

Because a real manufacturing process is always subject to various (unpredictable) disturbances, in this subsection we consider a first-order process with random noise term $\epsilon[n]$, which represents the measurement error and other unpredictable sources of variations.

If the noise term $\epsilon[n]$ is added to the process output directly, where $\epsilon[n]$ is a white noise sequence with variance



(b) EWMA-I

Fig. 1. Dynamic behavior of unstable EWMA controllers

σ^2, it will result in a time-varying dynamic system for $b[n]$ updating, as shown below:

$$b[n] = (\theta + \frac{\omega}{T-\alpha}\epsilon[n])b[n-1] + \omega\beta \tag{16}$$

which makes it quite cumbersome to derive a closed-form expression for the process output. To address this difficulty, we add the noise term to the process gain β, i.e., the state to be estimated,

$$y[n] = \alpha + (\beta + \epsilon[n])u[n] \tag{17}$$

which is equivalent to

$$y[n] = \alpha + \beta \cdot u[n] + \epsilon[n]u[n] \tag{18}$$

It is straightforward to verify that for a stationary process $\epsilon[n]u[n]$ is also a white noise sequence, but with a different variance $(u[n])^2\sigma^2$ (we will show later that the average input, $E\left(u[n]\right)$, approaches a steady-state value if the controller is stable). Therefore, adding a noise term to the process gain is equivalent to adding a scaled noise term to the process output which can significantly simplify the analysis. From Eqn. (17), the process gain updating becomes

$$b[n] = \theta b[n-1] + \omega\beta + \omega\epsilon[n] \tag{19}$$

and the process output is

$$\begin{aligned} y[n] = \ & T + \frac{(T-\alpha)\left(\epsilon[n] - (1-\theta)\xi\right)}{\beta + (1-\theta)\xi + \theta^{n-1}\left(\frac{T-\alpha}{T-a}b[0] - \beta\right)} \\ & - \frac{\theta^{n-1}(T-\alpha)\left(\frac{T-\alpha}{T-a}b[0] - \beta\right)}{\beta + (1-\theta)\xi + \theta^{n-1}\left(\frac{T-\alpha}{T-a}b[0] - \beta\right)} \end{aligned} \tag{20}$$

where $\xi = \sum_{i=1}^{n-1}\epsilon[i]\theta^{n-1-i}$. The second term in Eqn. (20) shows that a new noise term $\epsilon[n]$ is added at each run, which indicates that the sequence $y[n]$ does not converge. However, because $E(\epsilon[i]) = 0$, it is clear that if $|\theta| < 1$,

$$E\left(\lim_{n\to\infty} y[n]\right) = T \tag{21}$$

For a stochastic process, it is desirable to derive the output variance when the process is stable, and we apply the Delta method [12] to calculate the variance of $y[n]$. The variance of $y[n]$ as a function of $\boldsymbol{\epsilon}$ is given below

$$\begin{aligned} &\mathrm{Var}\left(y[n]\right) = \\ &\sigma^2\left(\frac{(T-\alpha)^2(1-\theta)^2\beta^2}{(\beta+\theta^{n-1}B)^4} \cdot \frac{(1-\theta^{2n-2})}{(1-\theta^2)} + \frac{(T-\alpha)^2}{(\beta+\theta^{n-1}B)^2}\right) \end{aligned} \tag{22}$$

where $B = \left(\frac{T-\alpha}{T-a}b[0] - \beta\right)$. For a stable EWMA-G controller,

$$\lim_{n\to\infty} \mathrm{Var}(y[n]) = \sigma^2\frac{(T-\alpha)^2}{\beta^2}\left(1 + \frac{1-\theta}{1+\theta}\right) \tag{23}$$

Now we compare the output variance generated from an EWMA-G controller with that generated from an EWMA-I controller. In [5], for a first-order stochastic process controlled by a stable EWMA-I controller, the output variance is

$$\lim_{n\to\infty} \mathrm{Var}\left(y[n]\right) = \sigma^2\left(1 + \frac{1-\theta}{1+\theta}\right) \tag{24}$$

Comparing Eqn. (23) to Eqn. (24), we see that the extra term $\frac{(T-\alpha)^2}{\beta^2}$ in Eqn. (23) is the only difference between them. Recall that if the EWMA-G controller is stable, the mean of the process output will approach the target, which indicates that when $n \to \infty$,

$$\begin{aligned} E\left(\lim_{n\to\infty} u[n]\right) &= \frac{E\left(\lim_{n\to\infty} y[n]\right) - \alpha}{\beta} \\ &= \frac{T-\alpha}{\beta} \end{aligned} \tag{25}$$

Based on Eqn. (18), the variance of the equivalent noise added to the process output is $E^2(u[n])\sigma^2$, which is exactly $\frac{(T-\alpha)^2}{\beta^2}\sigma^2$. Therefore, for a stationary stochastic first-order process, when $n \to \infty$, the variance of the output controlled by an EWMA-G controller is the same as that controlled by an EWMA-I controller.

C. Simulation Examples

In this subsection, we use simulation examples to illustrate the results obtained in Subsections II-A and II-B. The linear process model used in the simulation examples is:

$$y[n] = 0.056 + 0.049 \cdot u[n] \tag{26}$$

The initial estimates for the intercept and gain are $a_0 = 0.013$ and $b_0 = 0.011$. For an EWMA-G controller, the estimate of α is kept at 0.013; for an EWMA-I controller, the estimate of β is kept at 0.011. In Fig. 2 we compare the dynamic behaviors of two unstable EWMA controllers when applied to a stochastic linear process, where where the output target is 0, $\omega = 0.6$ and $\sigma = 0.02$. Fig. 2 shows that when the controllers are unstable, although both estimated parameters approach infinity, the output controlled by the EWMA-G controller approaches process intercept α while the output controlled by the EWMA-I controller approaches infinity. In Fig. 3, the ratio of the asymptotic variance of the output and the noise variance, i.e. $Var(y)/\sigma^2$, is plotted as a function of the EWMA weighting ω. The solid lines are generated using Eqn. (23) and the points shown on the figure are obtained through simulation with 5000 runs. It is clear that they agree well with each other.

III. A Deterministic Drifting Process

Because of tool wearing, material deposition and other factors, slow drifting is a common attribute to many semiconductor manufacturing processes. It is shown in [5] that for a drifting process controlled by an EWMA-I controller, if the stability condition is satisfied, the closed-loop process is stable with a steady-state bias in the output, and the bias is proportional to the ratio of the drift slope and the EWMA weighting. In this subsection, the output of a deterministic drifting process controlled by an EWMA-G controller is derived, and two cases are considered. In the first case, the

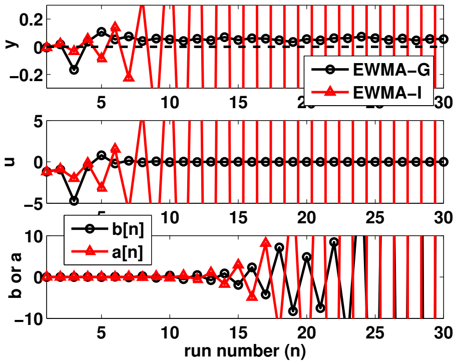


Fig. 2. Stochastic first-order processes: unstable EWMA controllers

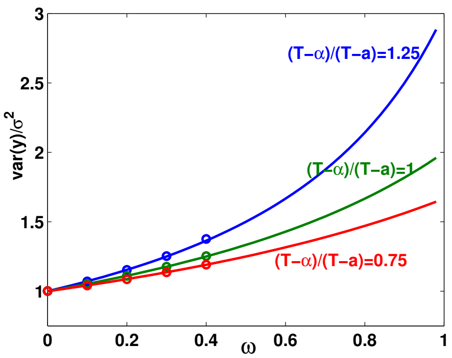


Fig. 3. The asymptotic variance as a function of ω for a first-order stationary process

drifting term is added to the process gain; while in the second case, the drifting term is added to the output, or the intercept equivalently.

A. Adding the drifting term to the process gain

In this subsection the drifting term is added to the process gain, β, and the process model becomes:

$$y[n] = \alpha + (\beta + n\delta)u[n] \tag{27}$$

Consequently the update of $b[n]$ becomes

$$b[n] = \theta b[n-1] + \omega\beta + n\omega\delta \tag{28}$$

After some algebraic manipulation we obtain

$$y[n] = T + (T-\alpha)\frac{\frac{\delta}{(1-\theta)} - \frac{T-\alpha}{T-a}\theta^{n-1}\left(b[0] - \frac{\omega\beta}{(1-\theta)} + \frac{\omega\delta\theta}{(1-\theta)^2}\right)}{\beta + n\delta - \frac{\delta}{(1-\theta)} + \frac{T-\alpha}{T-a}\theta^{n-1}\left(b[0] - \frac{\omega\beta}{(1-\theta)} + \frac{\omega\delta\theta}{(1-\theta)^2}\right)} \tag{29}$$

Assuming that $|\theta| < 1$, when $n \to \infty$, the second term in the above equation approaches zero as $n\delta \to \infty$. Therefore,

$$\lim_{n\to\infty} y[n] = T \tag{30}$$

which is different from EWMA-I controller.

In order to examine whether an EWMA-I controller can stabilize a drifting process described by Eqn. (27), we substitute Eqns. (27) and (3) into Eqn. (4) and obtain

$$a[n] = (1-\frac{\beta}{b}\omega-\frac{n\delta}{b}\omega)a[n-1]+\alpha\omega+(\frac{\beta+n\delta}{b}-1)\omega T \tag{31}$$

Denoting $\theta' \equiv 1 - \frac{\beta}{b}\omega$, the above equation becomes

$$a[n] = (\theta' - \frac{n\delta}{b}\omega)a[n-1] + \alpha\omega + (\frac{\beta+n\delta}{b} - 1)\omega T \tag{32}$$

Eqn. (32) describes a time-varying dynamic system for $a[n]$, which has a pole located at $(\theta' - \frac{n\delta}{b}\omega)$. Because $\lim_{n\to\infty}(\theta' - \frac{n\delta}{b}\omega) = \infty$, the system pole will be outside of unit circle as n increases no matter what value θ' is set to. Consequently,

$$\lim_{n\to\infty} a[n] = \infty \tag{33}$$

$$\lim_{n\to\infty} u[n] = \frac{T - a[n-1]}{b} = \infty$$

$$\lim_{n\to\infty} y[n] = \alpha + (\beta + n\delta)u[n] = \infty \tag{34}$$

The above analysis shows that if the process gain has a deterministic drift, an EWMA-I controller cannot stabilize the process, no matter what tuning parameter is used. This is shown in Fig. 4 where the process gain has a drift with slope $\delta = 0.04$. However, if an EWMA-G controller is applied, the process can be stabilized without steady-state bias if $0 < \frac{T-\alpha}{T-a}\omega < 2$, otherwise, the output will approach the intercept α if the controller is tuned to be unstable. Both stable ($\omega = 0.15$) and unstable ($\omega = 0.6$) cases are shown in Fig. 5.

B. Adding the drifting term to the intercept

In this subsection we consider the case where the drifting term is added to the process output, or intercept equivalently,

$$y[n] = \alpha + \beta u[n] + n\delta \tag{35}$$

The update of $b[n]$ based on Eqn. (35) becomes

$$b[n] = \left(\theta + \frac{\omega n\delta}{T-a}\right)b[n-1] + \omega\beta \tag{36}$$

which describes a time-varying dynamic system with the system pole located at $\theta + \frac{\omega n\delta}{T-a}$. Similar to Eqn. (32), the system pole will be outside of unit circle when $n \to \infty$, no matter what value θ is set to.

Eqn. (36) indicates that as $n \to \infty$, the absolute value of $b[n]$ will approach infinity. Consequently, the input $u[n]$ will approach 0, and output will approach infinity, i.e.

$$\lim_{n\to\infty} u[n] = \lim_{n\to\infty}\frac{T-a}{b[n]} = 0 \tag{37}$$

$$\lim_{n\to\infty} y[n] = \lim_{n\to\infty}(\alpha + \beta u[n] + n\delta) = \infty \tag{38}$$

Eqns. (36) to (38) show that if the intercept term has a deterministic drift, an EWMA-G controller will not be able to stabilize the process, no matter what EWMA weighting is applied, which is illustrated in Fig. 6.

In [5] it is shown that if the intercept term has a deterministic drift, an EWMA-I controller can stabilize the process, if and only if $0 < \frac{\beta}{b}\omega < 2$, with the steady-state bias $y - T = \frac{b\delta}{\beta\omega}$, which is illustrated in Fig. 7.

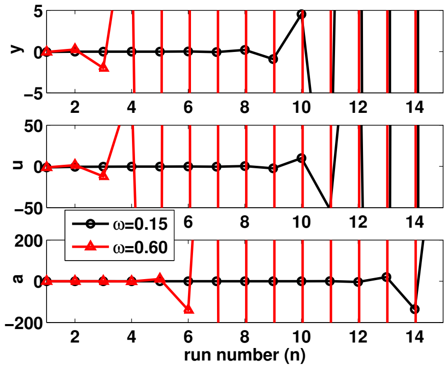


Fig. 4. Process with drifting gain controlled by EWMA-I controllers

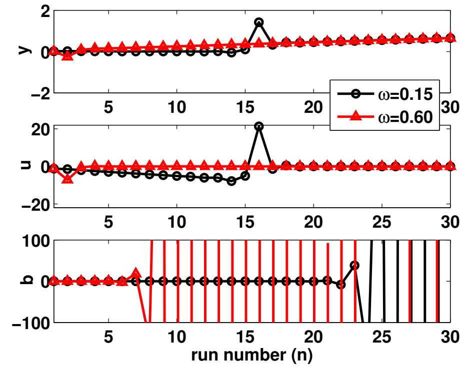


Fig. 6. Process with drifting intercept controlled by EWMA-G controllers

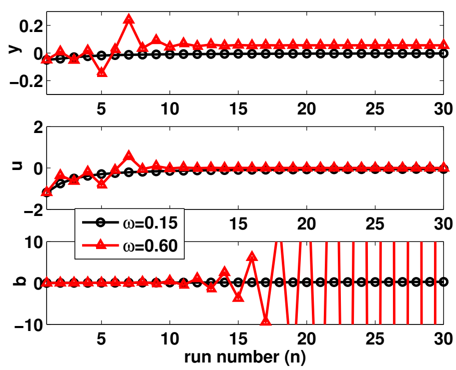


Fig. 5. Process with drifting gain controlled by EWMA-G controllers

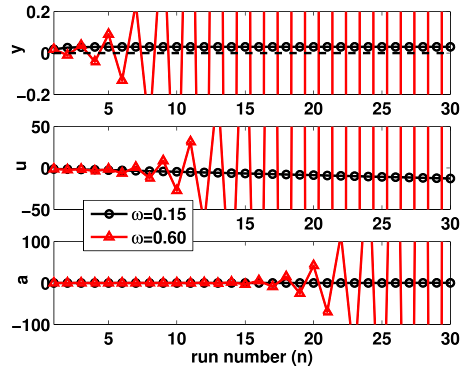


Fig. 7. Process with drifting intercept controlled by EWMA-I controllers

The analysis results in this section show that if a process has a deterministic drift, a corresponding EWMA controller which updates the drifting term is required to stabilized the process. The results are summarized in Table II.

TABLE II

COMPARISON OF EWMA CONTROLLERS: DRIFTING PROCESSES

	EWMA-G	EWMA-I
drifting gain	$y[\infty] = T$ if $\lvert\theta\rvert < 1$ $y[\infty] = \alpha$ if $\lvert\theta\rvert \geq 1$	$y[\infty] = \infty$ if $\lvert\theta'\rvert < 1$ $y[\infty] = \infty$ if $\lvert\theta'\rvert \geq 1$
drifting intercept	$y[\infty] = \infty$ if $\lvert\theta\rvert < 1$ $y[\infty] = \infty$ if $\lvert\theta\rvert \geq 1$	$y[\infty] = T - \frac{\delta}{1-\theta'}$ if $\lvert\theta'\rvert < 1$ $y[\infty] = \infty$ if $\lvert\theta'\rvert \geq 1$

IV. A Drifting Process with Measurement Noise

In this section we consider a process model that includes both a random error term and a drifting term, as described in the following equation.

$$y[n] = \alpha + (\beta + n\delta)u[n] + \epsilon[n] \tag{39}$$

Here we only consider the case with drifting gain because it has been shown that an EWMA-G controller cannot stabilize a process with a drifting intercept. In addition, the error term $\epsilon[n]$ is added to the output (or intercept equivalently) instead of the process gain. This is because if the noise term is added to the process gain, the effect of the noise will become negligible compared to the drifting term $n\delta$ when n is large.

Plugging Eqns. (39) and (5) into Eqn. (6), the update of the process gain becomes

$$b[n] = \left(\theta + \frac{\epsilon[n]\omega}{(T-a)}\right) b[n-1] + \omega\beta + n\omega\delta \tag{40}$$

Defining $\varepsilon[n] \equiv \frac{\epsilon[n]\omega}{(T-a)}$, and $\theta_n \equiv \theta + \varepsilon[n]$, we can obtain

$$b[n-1] = b[0]\prod_{i=1}^{n-1}\theta_i + \beta\omega\left(1 + \sum_{i=1}^{n-2}\prod_{j=i+1}^{n-1}\theta_j\right) + \delta\omega\left((n-1) + \sum_{i=1}^{n-2}\left(i\prod_{j=i+1}^{n-1}\theta_j\right)\right) \tag{41}$$

Notice that because the noise sequence $\epsilon[n]$ is white, i.e., $E(\epsilon[i]\epsilon[j]) = 0$ if $i \neq j$, the mean of $b[n-1]$ is

$$E\left(b[n-1]\right) = \frac{\omega\beta}{1-\theta} + \frac{\omega n\delta}{1-\theta} - \frac{\omega\delta}{(1-\theta)^2} + \theta^{n-1}\left(b[0] - \frac{\omega\beta}{(1-\theta)} + \frac{\omega\delta\theta}{(1-\theta)^2}\right) \tag{42}$$

which is the same as the $b[n-1]$ derived for a deterministic drifting case. Plugging the deadbeat controller Eqn. (5) and

Eqn. (42) into Eqn. (39), we obtain

$$y[n] = \alpha + (\beta + n\delta)\frac{T-a}{b[n-1]} + e[n] \quad (43)$$

$$E(y[n]) = \alpha(\beta + n\delta)\frac{T-a}{E(b[n-1])} = T \quad (44)$$

which is illustrated in Fig. 8. Because there is no steady-

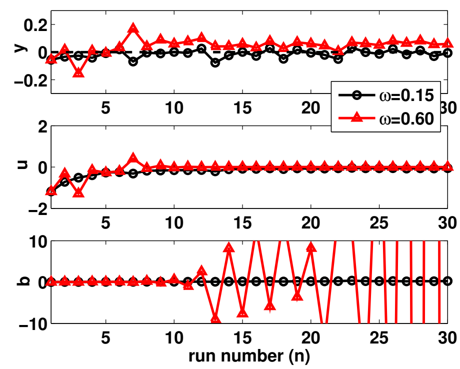


Fig. 8. Stochastic process with a drifting gain controlled by EWMA-G controllers

state bias if the EWMA-G controller is stable, the output variance should be similar to the output variance derived for a stationary process. In Fig. 9, we plot the ratio of the asymptotic output variance and the noise variance, $Var(y)/\sigma^2$ as a function of ω. The solid line is obtained through Eqn. (23) and the points are obtained through simulation data with different drifting slopes. Again, 5000 runs of data are used to estimate the output variance. From Fig. 9 we see that the output variance is independent of drifting slope because the output variances obtained from different drifting slopes completely overlap each other, which agrees well with Eqn. (23).

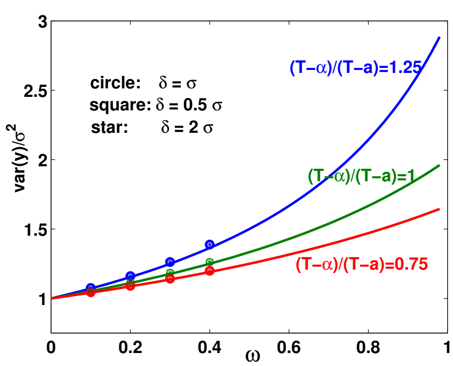


Fig. 9. The asymptotic variance as a function of ω for a first-order drifting process

V. Conclusions

In this work, we analyzed the stability and sensitivity of an EWMA controller with gain updating. It is found that when applied to a linear process, the stability region of an EWMA-G controller is the same as that of an EWMA-I controller. In addition, for a stochastic linear process, the asymptotic output variance controlled by an EWMA-G controller is the same as that from an EWMA-I controller. However, In the case where the controllers are unstable, an EWMA-G controller is different from an EWMA-I controller. In the unstable case, the output controlled by an EWMA-G controller will converge to the process intercept and stay bounded while that of an EWMA-I controller will approach infinity. Due to this reason, when controlling a stationary process, an EWMA-G is preferred.

For a linear drifting process, it is found that an EWMA controller that updates the drifting term is needed to stabilize the process. In other words, if the process gain is drifting, an EWMA-G controller is required to keep the output stable; if the intercept is drifting, an EWMA-I controller is required to keep the output stable. Otherwise, no matter what tuning parameter is chosen, the EWMA controller will not be able to stabilize the process. Another difference is that, for a linear process with a drifting gain, an EWMA-G controller can stabilize the process without steady-state bias, while for a linear process with a drifting intercept, an EWMA-I controller can stabilize the process but with a steady-state bias.

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