2008 American Control Conference
Westin Seattle Hotel, Seattle, Washington, USA
June 11-13, 2008

A Hierarchical Model Predictive Control Framework for Autonomous Ground Vehicles

P. Falcone, F. Borrelli, H. E. Tseng, J. Asgari, D. Hrovat

Abstract— **A hierarchical framework based on Model Predictive Control (MPC) for autonomous vehicles is presented. We formulate a predictive control problem in order to best follow a given path by controlling the front steering angle while fulfilling various physical and design constraints.**

We start from the low-level active steering-controller presented in [3], [9] and integrate it with a high level trajectory planner. At both levels MPC design is used. At the high-level, a trajectory is computed on-line, in a receding horizon fashion, based on a simplified point-mass vehicle model. At the low-level a MPC controller computes the vehicle inputs in order to best follow the desired trajectory based on detailed nonlinear vehicle model.

This article presents the approach, the method for implementing it, and successful preliminary simulative results on slippery roads at high entry speed.

I. Introduction

In Model Predictive Control (MPC) a model of the plant is used to *predict* the future evolution of the system [19]. Based on this prediction, at each time step t a performance index is optimized under operating constraints with respect to a sequence of future input moves in order to best follow a given trajectory. The first of such optimal moves is the *control* action applied to the plant at time t. At time $t+1$, a new optimization is solved over a shifted prediction horizon.

Parallel advances in theory and computing systems have enlarged the range of applications where real-time MPC can be applied [2], [4], [14], [15], [24]. Yet, for a wide class of "fast" applications the computational burden of Nonlinear MPC is still a serious barrier for its implementation. As an example, in [9] we have implemented a nonlinear MPC controller on a passenger vehicle for an Active Front Steering (AFS) system at 20 Hz, by using the state of the art of optimization solvers and rapid prototyping systems. We have shown that its real time execution is limited to low vehicle speed, because of its computational complexity. Nevertheless, the capability of handling constraints in a systematic way makes MPC a very attractive control technique, especially for applications where the process is required to work *in wide operating regions* and close to the boundary of the set of admissible states and inputs.

The work presented in this paper is the continuation of a study on the application of MPC techniques to vehicle dynamics control problems. In our first work [3] we investigated the potentiality of nonlinear MPC in solving an autonomous path following problem via AFS. The resulting Nonlinear MPC (NMPC) controller showed good performance but the computational complexity limited its implementation to low vehicle speed. In order to decrease the computational complexity, in [9], [10] we presented a LTV MPC approach to the path following via AFS problem. Experimental results [9] demonstrated the capability of the controller to stabilize the vehicle up to 72 Kph in a double lane change manoeuvre on slippery (snow covered) surfaces. In [8], [12], [13] additional control variables are considered. In particular we allow independent braking at the four wheels and active differentials (i.e., independent tractive torques at the four wheels). The interested reader can refer to the paper [9] for a detailed review of alternative existing approaches to autonomous vehicle dynamics control.

In all the aforementioned literature we assume that the trajectory is known over a finite horizon and computed by simple geometrical considerations in order to avoid given obstacles. In this paper we start from the low-level controller presented in [3], [9] and integrate it with a high level trajectory planner. At both levels MPC design is used. At the high-level, a trajectory is computed on-line, in a receding horizon fashion, based on a simplified point-mass vehicle model by means of nonlinear MPC. At the low-level a Linear-Time-Varying (LTV) MPC controller [9] computes the vehicle inputs in order to best follow the desired trajectory based on a more detailed nonlinear vehicle model.

The approach is systematic, model-based and can be generalized to any number of vehicle control inputs. In this article, the scheme is implemented for an active-steering problem and simulation are performed at high speeds on icy roads. Preliminary simulative results are presented, discussed and compared with the scheme presented in [9] which lacks of the higher-level planning algorithm. Lastly, we show how additional constraints introduced in [9] for stabilizing the vehicle are not need with the approach presented in this paper under the same driving conditions.

The paper is structured as follows. In Section II, we present a generic hierarchical architecture of guidance and navigation control systems for autonomous vehicles and explain the architecture considered in this paper. In Section III we describe the vehicle models used for trajectory planning and low level control. Sections IV and V present the formulations of the trajectory planning and the low level control problems, respectively. Section VI closes the paper by presenting the considered scenario and the simulations

P. Falcone is with the Department of Signals and Systems, Chalmers University of Technology, 412 96 Göteborg, Sweden, E-mail: falcone@unisannio.it

F. Borrelli is with the Department of Mechanical Engineering, University of California, Berkeley, CA 94720-1740, USA, E-mail: fborrelli@me.berkeley.edu

J. Asgari, E. H. Tseng and D. Hrovat are with Ford Research Laboratories, Dearborn, MI 48124, USA, E-mail: {jasgari,htseng,dhrovat}@ford.com

results.

II. HIERARCHICAL FRAMEWORK FOR AUTONOMOUS GUIDANCE

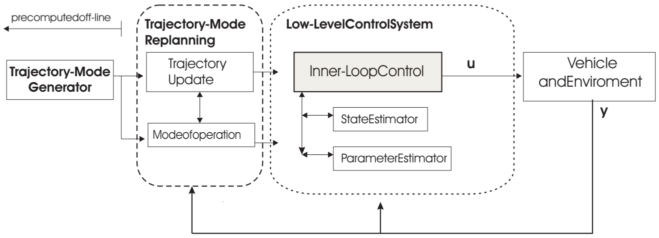


Fig. 1. Simplified Architecture for Fully-Autonomous Vehicle Guidance System

The architecture in Figure 1 describes the main elements of an autonomous vehicle guidance system and it is composed of four modules: the trajectory/mode generator, the trajectory/mode replanning, the low-level control system, and the vehicle and the environmental model. The trajectory/mode planning module pre-computes off-line the vehicle trajectory together with the timing and conditions for operation mode change. In the aerospace field, examples of operation mode selection include aeroshell parachute deployment or heatshield release, in the automotive filed this could include switching between two or more types of energy source (i.e., gas, electricity, hydrogen) or (in a very futuristic scenario) morphing between different vehicle shapes.

The trajectory and the mode of operation computed off-line can be recomputed on-line during the drive by the trajectory/mode replanning module based on current measurements, at fixed points or on the occurrence of certain events (such as tracking errors exceeding certain bounds, hardware failure, excessive wind, the presence of a pop-up obstacle).

The low-level control system commands the vehicle actuators such as front and rear steering angles, four brakes, engine torque, active differential and active suspensions based on sensor measurements, states and parameters estimations and reference commands coming from the trajectory/mode replanning module. Such reference commands can include lateral and longitudinal positions, pitch, yaw and roll rates. The low-level control system objective is to keep the vehicle as close as possible to the currently planned trajectory despite measurement noise, unmodeled dynamics, parameteric uncertainties and sudden changes on vehicle and road conditions which are not (or not yet) taken into account by the trajectory replanner. In particular, when a vehicle is operating near its stability limit, these additional noises, disturbances and uncertainties must be considered, possibly through detecting the vehicle's internal state, and compensated for. For example, if rear tires saturates, a skillful driver would switch his/her steering input from the usual steering command for trajectory following to a counter-steering one for stabilizing the vehicle.

We remark that the scheme in Figure 1 is an oversimplified scheme and that additional hierarchical levels could be present both in the trajectory/mode replanning module and in the low-level control system module. The union of the first three modules is often referred to as Guidance and Navigation Control System (GNC system).

Typically the trajectory replanner and the low-level control system modules do not share the same information on environment and vehicle. For instance, the replanning algorithms can use information coming from cameras or radars which may not be used at the lower level. Also, typically, the frequency at which the trajectory replanning module is executed is lower than the one of the lower level control system. The design of both modules makes use of vehicle and environment models with different levels of detail. The fidelity of the dynamical model used for the design of the two modules is dictated, among many factors, by a performance/computational resource compromise and in the literature there is no accepted standard on this. One of the possible control paradigms for the two modules consists in using a high-fidelity vehicle model for designing the lower level controller while the trajectory planner relies on a rougher/less detailed dynamical model of the vehicle. Clearly, the higher the fidelity of the models used at the higher level is, the easier the job for the lower level control algorithm becomes.

Studies on GNC algorithms vary in *(i)* the focus (trajectory replanner and/or the low-level control system) *(ii)* the type of vehicle dynamical model used, *(iii)* the type of control design used, and *(iv)* inputs and sensors choice. In [17] the trajectory replanner module is based on a receding horizon control design. The planning problem is formulated as a constrained optimization problem minimizing a weighted sum of arrival time, steering and acceleration control efforts. The vehicle model is a simple rear-centered kinematic model with acceleration, speed, steering, steering rate and rollover constraints. The lower level control module uses two separated PIDs to control longitudinal and lateral dynamics. The longitudinal controller acts on throttle and brakes while the lateral controls on the steering angle.

The GNC architecture in [22] is similar to [17]. The trajectory planning task is posed as a constrained optimization problem. The cost function penalizes obstacles collision, distance from the pre-computed offline trajectory and the lateral offset from the current trajectory. At the lower level, a PI controller acts on brakes and throttle to control the longitudinal dynamics. A simple nonlinear controller, instead, is used to control the lateral dynamics through the steering angle. Details on the vehicle dynamical model used in [22] are not disclosed. In [21] a scheme similar to the one in [17] is used to design a GNC systems for for a flight control application.

In [23], an explicit MPC scheme has been applied at the lower level control to allocate four wheel slips in order to get a desired yaw moment. The steering angle is not controlled.

In this paper we present an approach to GNC based on MPC for the trajectory replanning module and for the low-level control system module. In particular we consider a path following scenario, where a vehicle has to autonomously follow a desired path by controlling the front steering angle. At the high level, an MPC scheme, based on an oversimplified vehicle, model replans the path by taking into account the limitation imposed by the estimated road friction coefficient. At the low-level, an MPC active steering controller tries to best follow the computed desired path by using a more detailed vehicle model.

Compared to the lower level control algorithms presented in the aforementioned literature, our approach *(i)* is model based and uses the vehicle model (1) and the highly nonlinear tire model (see Section III-A), *(ii)* includes constraints on inputs and states in the control design, *(iii)* is systematic and multivariable and can accommodate new actuators and higher fidelity models. Moreover we have experimentally validated the low-level controller presented in this paper with a dSPACE$^{\mathrm{TM}}$ AutoBox$^{\mathrm{TM}}$ system which is a standard rapid prototyping system used in automotive industries [5].

III. Modeling

Next we introduce the vehicle models used for predictive active steering control design and trajectory planning. In particular, in Section III-A a nonlinear sixth order vehicle model which captures the most important nonlinearities associated to lateral and yaw stabilization of the vehicle is briefly described. The model is based on a nonlinear Pacejka tire model. In Section III-B a simple point-mass vehicle model is presented.

A. Bicycle model

A "bicycle model" [18] is used to model the dynamics of the car under the following

Assumption 1: At front and rear axles, the left and right wheels are lumped in a single wheel.

Figure 2 depicts a diagram of the vehicle model under the Assumption 1. The following notation is used: $\dot{y}$ and $\dot{x}$ are the lateral and longitudinal vehicle velocities in the body frame, respectively, ψ is the vehicle orientation (yaw angle) in the inertial frame, $\dot{\psi}$ is the yaw rate, Y and X are the lateral and longitudinal position of the vehicle in the inertial frame, respectively, δ_f is the front steering angle, α_f is front tire slip angle (i.e., the angle between the tire velocity vector and the tire longitudinal axis), F_{c_f} and F_{l_f} are the front lateral (or cornering) and longitudinal tire forces, respectively. By replacing the second subscript f with r, the variables for the rear axis are defined similarly .

Under the Assumption 1, the planar motion of the vehicle in an inertial frame, subject to lateral, longitudinal and yaw dynamics, is described through a sixth order nonlinear model. The detailed model equations are not reported here and can be found in [3], [9], [11]. The nonlinear dynamics can be compactly written as follows:

$$\frac{d\xi(t)}{dt} = f\left(\xi(t), u(t)\right), \tag{1}$$

where she state and input vectors are $\xi = [\dot{y},\ \dot{x},\ \psi,\ \dot{\psi},\ Y,\ X]'$ and $u = \delta_f$, respectively.

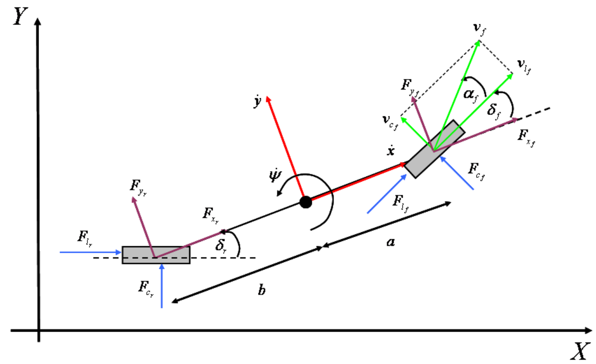


Fig. 2. The simplified vehicle "bicycle model".

Remark 1: In the bicycle model presented in [3], [9], [11] and used in this paper, the normal forces at the four wheels are assumed constant. This assumption stems from the observation that on low friction surfaces, due to limited lateral and longitudinal accelerations, load transfers can be neglected.

The function f in (1) is a complex nonlinear function of states and input. The most relevant sources of nonlinearities are the cornering and longitudinal tire forces F_c and F_l, respectively (see Figure 2). We use a Pacejka tire model [1] to compute the tire forces. This is a semi-empirical nonlinear model that takes into consideration the interaction between the longitudinal force and the cornering force in combined braking and steering. A thorough presentation of the Pacejka tire model can be found in [3], [9].

B. Point-Mass Vehicle Model

The point-mass model introduced in this section describes in a simple way the motion of a vehicle along a prescribed path. We assume that the path is defined in the inertial frame XY through a sequence of pairs $(X(t), Y(t)),\ t \geq 0$. We make use of the the following simplifying assumptions:

Assumption 2: The vehicle exactly follow the prescribed path $(X(t), Y(t)),\ t \geq 0$.

Assumption 3: The vehicle acceleration is bounded by the constant μg, where μ is the road friction coefficient and g is the gravitational acceleration.

Under the Assumptions 2 and 3 the motion of the vehicle along the path is described by the following system of constrained differential equations:

$$\dot{v}_x(t) = a_x(t), \tag{2a}$$
$$\dot{v}_y(t) = v_x^2(t)c(t) \tag{2b}$$

where v_x and v_y are the velocities tangential and normal to the path, a_x is the longitudinal vehicle acceleration, $c(t)$ is the path curvature at time t and is computed as follows:

$$c(t) = \frac{\dot{X}(t)\ddot{Y}(t) - \dot{Y}(t)\ddot{X}(t)}{\left(\dot{X}^2(t) + \dot{Y}^2(t)\right)^{\frac{3}{2}}}. \tag{3}$$

According to the Assumption 3 the system (2)-(3) is subject to the following constraint:

$$\sqrt{a_x^2(t) + \dot{v}_x^4(t)c^2(t)} \le \mu g,\ \forall t \ge 0. \tag{4}$$

Remark 2: The constraint (4) on the longitudinal and lateral accelerations stems from the limitation of the maximum force transmission to the road surface [16].

IV. Receding Horizon Trajectory Replanning

Next we show how the trajectory replanning task can be formulated as a receding horizon problem. We discretize the model (2) and equation (3) with the sampling time T. We assume that at each sampling time t a desired trajectory is provided over a finite time horizon of N steps, with $N \in \mathbb{Z}^+$, in terms of a sequence of pairs (X_i^r, Y_i^r), with $i = t+1, \ldots, t+N$. Consider the following cost function:

$$\begin{aligned} &J_{repl}(\mathcal{A}_t,\ \mathcal{X}_t,\ \mathcal{Y}_t,\ v_{des},\ \mathcal{X}_t^r,\ \mathcal{Y}_t^r) \\ &= \sum_{i=t+1}^{t+N} \alpha_1 \left[(X_i - X_i^r)^2 + (Y_i - Y_i^r)^2 \right] \\ &+ \sum_{i=t}^{t+N-1} \left[\alpha_2 \left(\dot{v}_{x_{i,t}} - v_{des} \right)^2 + \alpha_3 a_{x_{i,t}}^2 \right], \end{aligned} \tag{5}$$

where $\mathcal{A}_t = [a_{x_{t,t}}, \ldots, a_{x_{t,t+N-1}}]$, $\mathcal{X}_t = [X_{t+1}, \ldots, X_{t+N}]$, $\mathcal{Y}_t = [Y_{t+1}, \ldots, Y_{t+N}]$, v_{des} is the desired vehicle velocity, $\mathcal{X}_t^r = [X_{t+1}^r, \ldots, X_{t+N}^r]$ and $\mathcal{Y}_t^r = [Y_{t+1}^r, \ldots, Y_{t+N}^r]$.

In the cost function (5) the first summand penalizes the deviation of the replanned path (X_i, Y_i) from the desired path (X_i^r, Y_i^r), $i = t, \ldots, t+N$, the second summand penalizes the deviation of the vehicle longitudinal velocity from the desired velocity v_{des}, while the third summand penalizes the longitudinal acceleration. These three summands are weighted through the tuning parameters α_1, α_2 and α_3.

At each time $t = kTN_t,\ k \in \mathbb{Z}^+$, we solve the following optimization problem:

$$\begin{aligned} \min_{\mathcal{A}_t,\ \mathcal{X}_t, \mathcal{Y}_t} \quad & J_{repl}(\mathcal{A}_t,\ \mathcal{X}_t,\ \mathcal{Y}_t,\ v_{des}, \mathcal{X}_t^r, \mathcal{Y}_t^r) \\ \text{subj. to} \quad & \dot{v}_{x_{k+1,t}} = \dot{v}_{x_{k,t}} + T a_{x_{k,t}}, & \text{(6a)} \\ & \sqrt{a_{x_{k,t}}^2 + \dot{v}_{x_{k,t}}^4 c_{k,t}^2} \le \mu g, & \text{(6b)} \\ & \|a_{x_{k,t}}\| \le a_{max}, & \text{(6c)} \\ & \qquad k = t+1, \ldots, N. \\ & v_{x_{t,t}} = \dot{x}(t), & \text{(6d)} \end{aligned}$$

where a_{max} is the maximum longitudinal acceleration.

Remark 3: We highlight that, for the sake of readability, in the equation (6b) the dependency of $c_{k,t}$ on $X_k, X_{k-1}, X_{k-2}, Y_k, Y_{k-1}, Y_{k-2}$ is dropped, i.e., $c_{k,t}$ is used instead of $c_{k,t}(X_k, X_{k-1}, X_{k-2}, Y_k, Y_{k-1}, Y_{k-2})$.

Once a solution $\mathcal{A}_t^*$, $\mathcal{X}_t^*$, $\mathcal{Y}_t^*$ of problem (6) has been found, the first N_t samples, with $N_t \in \mathbb{Z}^+$ and $N \ge N_t$, of the sequences $\mathcal{X}_t^*$ and $\mathcal{Y}_t^*$ are sent to the low-level control as reference trajectory. At time $t + N_t T$ the problem (6) is solved over a shifted horizon based on new measurements of vehicle longitudinal speed and lateral and longitudinal positions in the inertial frame.

V. Receding Horizon Active-Steering Controller

In this section we design a low level AFS controller, in order to follow a path defined by desired references for the heading angle ψ and the lateral position Y. The control objective is to minimize the vehicle position and orientation errors by varying the front steering angle δ_f.

In this paper, in order to solve the path following problem stated above, we adopt the approach in [9]. This is a low complexity MPC algorithm based on successive on line linearizations of a nonlinear vehicle model. In particular at each time step a linear approximation of the bicycle model (1) is computed. Based on this linear model approximations, a Quadratic Programming (QP) problem is formulated and solved in receding horizon in order to compute the front steering angle minimizing the deviation from a desired path over a future finite horizon. We refer to the MPC approach in [9] summarized above as Linear Time Varying (LTV) MPC.

Remark 4: In [9] it is shown that performance enhancement and vehicle stability at high speed are achieved by adding a constraint on tire slip angle to the LTV MPC formulation. This is a non standard state and input constraint included in order to forbid the system from entering a strongly nonlinear and possibly unstable region of the tire characteristic.

VI. Presentation and Discussion of Results

Next we present two approaches to the considered path following via AFS problem.

The first approach is based on the trajectory replanning algorithm presented in Section IV and the low level LTV MPC AFS control presented in Section V. In particular we assume that the trajectory replanning algorithm is executed every $N_t T$ seconds. Moreover we assume that the replanning frequency is lower than the low level AFS control frequency (i.e., $N_t T \gg T_s$, where $1/T_s$ is the low level AFS control frequency) and $N_t T \ge H_p T_s$. At each time $t = kN_tT$, $k \ge 0$ a replanned trajectory $(\mathcal{X}_t, \mathcal{Y}_t)$, is computed starting from desired vehicle velocity v_{des} and path $(\mathcal{X}_t^r,\ \mathcal{Y}_t^r)$, the current vehicle longitudinal speed and the vehicle lateral and longitudinal positions in the inertial frame at times t, $t-1$ and $t-2$. The replanned trajectory is fed into the low level LTV MPC AFS control, presented in Section V, until time $t+(N_t-1)T$. At time $t+N_tT$ the trajectory is replanned starting from new states measurements.

The union of the trajectory replanning algorithm and the low level AFS control will be referred to as Approach 1 in the following.

Remark 5: We observe that the result of the trajectory replanning is a replanned trajectory in terms of longitudinal and lateral positions in the inertial frame. In the low level AFS control, instead, the desired trajectory is defined in terms of heading angle ψ and the lateral distance Y. We point out that, given the replanned trajectory in terms of longitudinal $\mathcal{X}_t$ and lateral $\mathcal{Y}_t$ positions in the inertial frame, the corresponding vehicle orientation can be computed through numerical derivative of the signal $\mathcal{Y}_t^*$ with respect to $\mathcal{X}_t^*$.

The second approach is based on the use of the low level AFS LTV MPC control with constraints on the tire slip angles (see Remark 4) only. In particular the desired trajectory $(\mathcal{X}_t^r,\ \mathcal{Y}_t^r)$, $t \geq 0$ is fed into the AFS LTV MPC controller. In the following this approach will be referred to as Approach 2.

The two approaches have been implemented to perform a sequence of double lane changes at different entry speeds on slippery surfaces (snow or ice).

We point out that the difference between Approaches 1 and 2 resides in the reference trajectory generation. In particular, in Approach 1 the reference trajectory $(\mathcal{X}_i, \mathcal{Y}_i)$, $i = t, \ldots, NT$ is provided to the low level AFS control system by the trajectory planning presented in Section IV. In Approach 2 the desired reference trajectory $(\mathcal{X}_i^r,\ \mathcal{Y}_i^r)$, $i = t, \ldots, NT$ is provided directly to the low level AFS controller.

In Figure 3 the simulation results of the Approach 1 are presented when a double lane change is performed on a snowy surface ($\mu = 0.3$) at 70 Kph. The following parameters have been used for the trajectory replanning algorithm:

- *Sampling time*: $T = 0.3$ s
- *Replanning horizons*: $N = 15$, $N_t = 8$;
- *Weights*: $\alpha_1 = 1$, $\alpha_2 = 0$ and $\alpha_3 = 0$;
- *Maximum longitudinal acceleration*: $a_{max} = 0$.

Since in the considered simulation scenario the vehicle is coasting while performing the double lane change, in the replanning algorithm *(i)* the deviation from the desired vehicle velocity has not been penalized *(ii)* the vehicle longitudinal acceleration is constrained to zero.

The following tuning parameters have been used for the low level AFS control:

- *Sampling time*: $T_s = 0.05$ s
- *weights on tracking errors*: $Q_\psi = 5$, $Q_Y = 300$, $Q_{ij} = 0$ for $i \neq j$;
- *weights on input rates*: $R = 1$;
- *weights on input*: $S = 1$;
- *horizons*: $H_p = 17$, $H_c = 1$;
- *bounds on tire slip angles*: $\pm\infty$.

In Figure 4 the simulation results of the Approach 2 are presented in the same scenario as Approach 1. The following tuning parameters have been used:

- *weights on tracking errors*: $Q_\psi = 5$, $Q_Y = 100$, $Q_{ij} = 0$ for $i \neq j$;
- *weights on input rates*: $R = 10$;
- *weights on input*: $S = 1$;
- *horizons*: $H_p = 30$, $H_c = 1$;
- *bounds on tire slip angles*: ± 6 deg.

The Root Mean Squared (RMS) and maximum tracking errors are summarized in Table I.

By comparing the RMS and maximum tracking errors reported in Table I we observe similar performance of the two approaches in the considered scenario. Nevertheless, we observe that the prediction horizon in the low level control of Approach 2 is almost twice the prediction horizon of low level control in Approach 1. This significant reduction of the low level controller complexity is allowed by the usage of the upper level trajectory replanning which accounts for vehicle stability limits in generating the reference trajectory. We also observe that the stabilizing constraints on the tire slip angles (see Remark 4) are not necessary in the low level control in Approach 1. In fact, in Approach 1, the vehicle is not required to operate at the limits of stability in order to track the replanned trajectory. By comparing the tuning parameters of the low level controls in the two approaches, we finally observe that in Approach 2 the low level controller has been detuned compared to low level control in Approach 1. In particular we observe that the weight on the lateral position tracking error in Approach 2 is significantly lower compared to Approach 1. Furthermore the input rate in Approach 2 is more penalized compared to Approach 1.

Controller	ψ_{rms} [deg]	Y_{rms} [m]	ψ_{max} [deg]	Y_{max} [m]
Approach 1	$2.94 \cdot 10^2$	$4.46 \cdot 10^1$	8.20	1.23
Approach 2	$2.83 \cdot 10^2$	$5.30 \cdot 10^1$	8.38	1.42

TABLE I

RMS AND MAXIMUM TRACKING ERRORS OF APPROACHES 1 AND 2 WHEN A DOUBLE LANE CHANGE MANOEUVRE IS PERFORMED ON SNOW AT 70 KPH.

REFERENCES

[1] E. Bakker, L. Nyborg, and H. B. Pacejka. Tyre modeling for use in vehicle dynamics studies. *SAE paper # 870421*, 1987.

[2] F. Borrelli, A. Bemporad, M. Fodor, and D. Hrovat. An MPC/hybrid system approach to traction control. *IEEE Trans. Control Systems Technology*, 14(3):541–552, May 2006.

[3] F. Borrelli, P. Falcone, T. Keviczky, J. Asgari, and D. Hrovat. MPC-based approach to active steering for autonomous vehicle systems. *Int. J. Vehicle Autonomous Systems*, 3(2/3/4):265–291, 2005.

[4] F. Borrelli, T. Keviczky, G. J. Balas, G. Stewart, K. Fregene, and D. Godbole. Hybrid decentralized control of large scale systems. In *Hybrid Systems: Computation and Control*, Lecture Notes in Computer Science. Springer Verlag, March 2005.

[5] dSPACE GmbH. dSPACE Autobox, 2006.

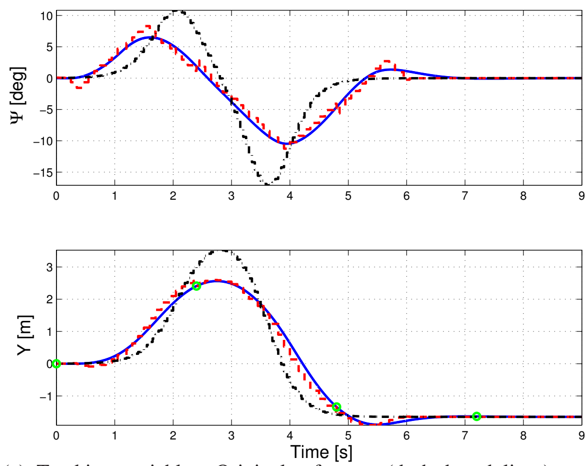
[6] P. Falcone. *Nonlinear Model Predictive Control for Autonomous Vehicles*. PhD thesis, Universitá del Sannio, Dipartimento di Ingegneria, Piazza Roma 21, 82100, Benevento, Italy, June 2007.

[7] P. Falcone, F. Borrelli, J. Asgari, H. E. Tseng, and D. Hrovat. Linear time varying model predictive control and its application to active steering systems: Stability analisys and experimental validation. *Technical Report TR414, Universitá del Sannio (*available at http://www.grace.ing.unisannio.it/home/pfalcone*)*, 2006.

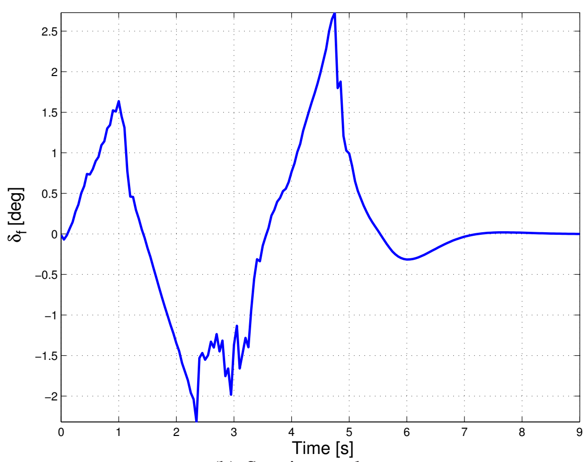
[8] P. Falcone, F. Borrelli, J. Asgari, H. E. Tseng, and D. Hrovat. A model predictive control approach for combined braking and steering in autonomous vehicles. *Submitted to the 15th Mediterranean Conference on Control and Automation (*available at http://www.grace.ing.unisannio.it/home/pfalcone*)*, 2007.

[9] P. Falcone, F. Borrelli, J. Asgari, H. E. Tseng, and D. Hrovat. Predictive active steering control for autonomous vehicle systems. *IEEE Trans. on Control System Technology*, 15(3), 2007.

[10] P. Falcone, F. Borrelli, J. Asgari, H. Eric Tseng, and D. Hrovat. A real-time model predictive control approach for autonomous active steering. In *Nonlinear Model Predictive Control for Fast Systems, Grenoble, France*, 2006.

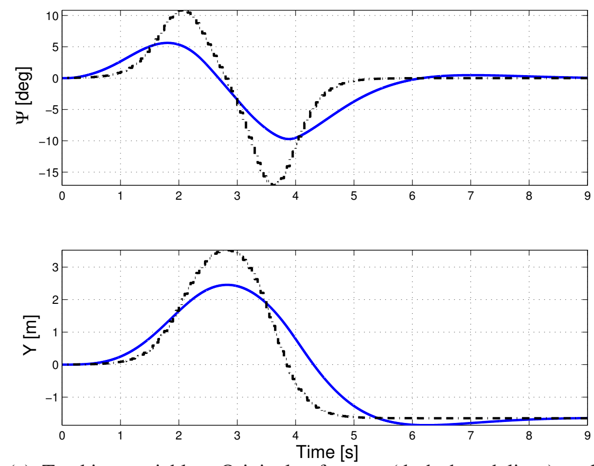


(a) Tracking variables. Original reference (dash-dotted lines), replanned reference (dashed lines) and actual (solid lines) signals. The circles highlight the replanning time instants.

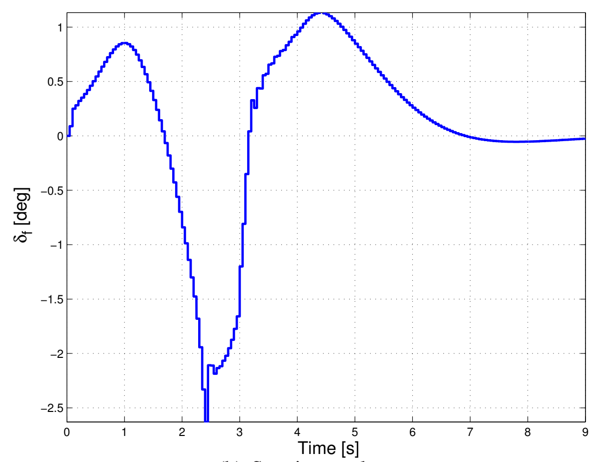


(b) Steering angle.

Fig. 3. Simulation results of Approach 1 at 70 Kph on a snow covered road.



(a) Tracking variables. Original reference (dash-dotted lines) and actual (solid lines) signals.



(b) Steering angle.

Fig. 4. Simulation results of Approach 2 at 70 Kph on a snow covered road.

[11] P. Falcone, F. Borrelli, J. Asgari, H. Eric Tseng, and D. Hrovat. Predictive active steering control for autonomous vehicle systems. Technical report, Universitá del Sannio. Dipartimento di Ingegneria., January 2007. http://www.grace.ing.unisannio.it/publication/416.

[12] P. Falcone, F. Borrelli, H. E. Tseng, J. Asgari, and D. Hrovat. Integrated braking and steering model predictive control approach in autonomous vehicles. *Fift IFAC Symposium on Advances of Automotive Control*, 2007.

[13] P. Falcone, F. Borrelli M. Tufo, H. E. Tseng, and J. Asgari. Predictive autonomous vehicles: A linear time varying model predictive control approach. *46^{th} Conference on Decision and Control*, 2007.

[14] H. J. Ferrau, H. G. Bock, and Moritz Diehl. An online active set strategy for fast parametric quadtratic programming in mpc applications. *IFAC Workshop on Nonlinear Model Predictive Control for Fast Systems, plenary talk*, 2006.

[15] T. Keviczky and G. J. Balas. Flight test of a receding horizon controller for autonomous uav guidance. In *Proc. American Contr. Conf.*, 2005.

[16] U. Kiencke and L. Nielsen. *Automotive Control Systems*. Springer, 2000.

[17] T. B. Foote L. B. Cremean, J. H. Gillula, G. H. Hines, D. Kogan, K. L. Kriechbaum, J. C. Lamb, J. Leibs, L. Lindzey, C.E. Rasmussen, A.D. Stewart, J.W. Burdick, and R.M. Murray. Alice: An information-rich autonomous vehicle for high-speed desert navigation. In *Submitted to Journal of Field Robotics.*, 2006.

[18] D. L. Margolis and J. Asgari. Multipurpose models of vehicle dynamics for controller design. *SAE Technical Papers*, 1991.

[19] D.Q. Mayne, J.B. Rawlings, C.V. Rao, and P.O.M. Scokaert. Constrained model predictive control: Stability and optimality. *Automatica*, 36(6):789–814, June 2000.

[20] E. Ono, S. Hosoe, H. D. Tuan, and S. Doi. Bifurcation in vehicle dynamics and robust front wheel steering control. *IEEE Trans. on Control System Technology*, 6(3):412–420, May 1998.

[21] J. Hauser R. M Murray, A. Jadbabaie, M. B. Miliam, N. Petit, W. B. Dunbar, and R. Franz. Online control customization via optimization-based control. In T. Samad and G. Balas, editors, *Software-Enabled Control: Information Technology for Dynamical Systems*. IEEE Press, 2003.

[22] M. Montemerlo S. Thrun, H. Dahlkamp, D. Stavens, A. Aron, J. Diebel, P. Fong, J. Gale, M. Halpenny, G. Hoffmann, K. Lau, C. Oakley, M. Palatucci, V. Pratt, P. Stang, S. Strohband, C. Dupont, L.-E. Jendrossek, C. Koelen, C. Markey, C. Rummel, J. van Niekerk, E. Jensen, P. Alessandrini, G. Bradski, B. Davies, S. Ettinger, A. Kaehler, A. Nefian, and P. Mahoney. Stanley, the robot that won the darpa grand challenge. In *Journal of Field Robotics. Accepted for publication.*, 2006.

[23] P. Tøndel and T. A. Johansen. Control allocation for yaw stabilization in automotive vehicles using multiparametric nonlinear programming. In *Proc. American Contr. Conf.*, 2005.

[24] V. M. Zavala, C. D. Laird, and L. T. Biegler. Fast solvers and rigorous models: Can both be accomodated in nmpc? *IFAC Workshop on Nonlinear Model Predictive Control for Fast Systems, plenary talk*, 2006.