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Adaptive Fuzzy Control of Unknown Nonlinear Systems with Actuator Failures for Robust Output Tracking

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Abstract— **In this paper, an adaptive fuzzy approach is proposed to deal with robust output tracking of unknown nonlinear systems with actuator failures. The actuator failures under consideration can be both of lock-in-place and loss of effectiveness. By incorporating fuzzy logic approximation, adaptive algorithm and attenuation technique to our design, a fault tolerant control law is developed to guarantee desired output tracking of the controlled system to the given reference model as well as the closed-loop stability, despite there are unknown actuator failures and large uncertainties in the system. A numerical simulation example illustrates the effectiveness of the proposed control approach.**

I. INTRODUCTION

With the development of modern industry, lots of systems are becoming more and more advanced and complicated. However, this also makes it more possible that some faults may occur in these systems. As actuator faults may cause undesired system behavior and sometimes lead to instability or even catastrophic accidents, it is very important to develop fault tolerant control (FTC) approaches that would achieve control objective in spite of actuator faults. Adaptive control has been used widely in linear systems to accommodate actuator faults. For example, lock-in-place (stuck at an unknown value) failures were compensated based on matching conditions in [1], loss of effectiveness was studied in [2]-[4] in the framework of linear matrix inequality and multiple simultaneous actuator failures were dealt with in [5]-[6] using Multiple Models, Switching, and Tuning method.

However, most real-life systems are nonlinear in nature. So, nonlinear adaptive FTC has attracted much attention. Tao *et al* also developed adaptive control for nonlinear systems with actuator failures in [1]. And [7] extended its results to multiple-input-multiple-output systems based virtual grouping. However, only feedback linearizable and parametric-strict-feedback systems were considered. Boskovic considered multiple simultaneous actuator failures for known nonlinear systems in [8]. [9] proposed fault accommodation approach for systems with Lipschitz nonlinearities. However, the related results have not appeared in nonlinear systems whose nonlinearities are completely unknown.

With the development of artificial intelligence, fuzzy logic and neural network have been introduced into fault detection and accommodation. Polycarpou presented a general framework for constructing automated fault diagnosis and accommodation architectures using on-line approximators in [10], he furthered his research in [11] based on a neural network to fault tolerant control of nonlinear flight control systems. [12]-[15] provided FTC shemes using fuzzy logic systems (FLSs) or neural networks (NNS), however these results were obtained on the basis of fault detection and diagnosis (FDD) mechanism. Since [16] proved that FLSs are universal approximators and [17] gave the stable adaptive fuzzy control design for unknown nonlinear systems, many researchers studied nonlinear systems using adaptive fuzzy systems to approximate the unknown functions. Though [18] presented fault tolerant adaptive fuzzy control for a turbine engine, there is few results on adaptive fuzzy control of systems with unknown nonlinearities and external disturbances to accommodate actuator faults without FDD.

This paper studies fault tolerant tracking control for unknown nonlinear systems with external disturbances against actuator faults. The main properties compared with the existing results are that: first, a novel adaptive fuzzy FTC scheme without FDD is prosed, so the undesired system behavior caused by false or omitted alarm of FDD mechanism is avoided; second, external disturbance is attenuated effectively to achieve robustness besides the fault tolerant ability of the closed-loop system; third, our design can broaden the tolerable fault set by allowing any combination of lock-in-place and loss of effectiveness happen only if the system is still controllable. The difficulty in the design is that the system functions are not continuous because of the occurred faults, so piecewise analysis is used.

The rest of this paper is organized as follows. Section II formulates the problem first. Section III introduces the proposed adaptive fuzzy fault tolerant control scheme. In Section IV, a numerical simulation example illustrates the effectiveness of the control method. Finally, Section V concludes this paper.

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II. Problem Formulation:

Consider the following nonlinear plant

$$\begin{aligned} \dot{x}_n &= f(x) + \sum_{i=1}^{m} g_i(x)u_i + d \\ y &= x_1 \end{aligned} \tag{1}$$

where $x = (x_1, x_2, \cdots, x_n)^T = (x_1, \dot{x}_1, \cdots, x_1^{(n-1)})^T \in U \subseteq R^n$ is the state vector, U is a compact set in R^n, $y \in R$ is the output, and $u_i \in R$ is the control input which may fail during operation, $i = 1, 2, \cdots, m$, $m \geq 2$ provides some redundancy of the actuator so that when some of them are failed, the remaining part can still drive the system stable and achieve acceptable performance. This is reasonable because for some practical systems, the control surfaces can be divided into several individually actuated segments. For example, the aileron segments of an aircraft provide some redundancy needed for failure compensation. $f(x) \in R$ and $g_i(x) \in R$ are unknown nonlinear smooth functions. $d \in R$ denotes the external disturbance which is unknown but bounded. So we cannot get the model of the controlled plant.

The failure model under consideration for fault tolerant control of the system (1) is

$$u_i^F = \rho_i u_i, \quad \rho_i \in [\underline{\rho}_i, \bar{\rho}_i], \quad i \in \{1, 2, \cdots, m\} \tag{2}$$

and

$$u_j^F = \bar{u}_j, \quad j \in \{1, 2, \cdots, m\} \tag{3}$$

In this failure model, (2) describes the fault of loss of effectiveness, where ρ_i denotes the percentage of the remaining effective part of the corresponding actuator, $0 < \underline{\rho}_i \leq 1$, $0 < \bar{\rho}_i \leq 1$, are the lower and upper bounds of ρ_i respectively, $\underline{\rho}_i \leq \bar{\rho}_i$. If $\underline{\rho}_i = \bar{\rho}_i = 1$, there is no failure occurred, i.e. the actuator is normal. For (2), $\rho_i = 0$ which means the complete loss of effectiveness is not considered since it is included in (3) for $\bar{u}_j = 0$. (3) describes the lock-in-place (stuck at an unknown value) failure. If (3) has occurred, the actuator must completely lose the effectiveness, thus the control input has no impact on the controlled system, but the actuator failure will bring some disturbances if $\bar{u}_j \neq 0$. Actuators can be failed as (2) or (3), also, both (2) and (3) may occur during operation.

The reference model is:

$$\begin{aligned} \dot{x}_m &= A_m x_m + B_m r \\ y_m &= C_m x_m \end{aligned} \tag{4}$$

where

$$\begin{aligned} A_m &= \begin{bmatrix} 0 & 1 & 0 \cdots 0 & 0 \\ 0 & 0 & 1 \cdots 0 & 0 \\ & & \cdots \cdots & & \\ 0 & 0 \cdots 0 & & 0 & 1 \\ -k_n & -k_{n-1} & \cdots & -k_2 & -k_1 \end{bmatrix} \in R^{n \times n}, \\ B_m &= [0 \cdots \cdots 0\ 1]^T \in R^{n \times 1} \\ C_m &= [1\ 0 \cdots \cdots 0\ 0] \in R^{1 \times n} \end{aligned} \tag{5}$$

$k_1, \cdots, k_n$ are constants designed such that $s^n + k_1 s^{(n-1)} + \cdots + k_{(n-1)} s + k_n$ is a Hurwitz polynomial.

The objective for actuator fault tolerant control is to use feedback control design for the plant (1) with the actuator failures (2) or/and (3), to guarantee that all signals in the closed-loop system are bounded and robust tracking the output of the given reference model y_m. Here the "robust tracking" refers to attenuate the fuzzy approximation error and the external disturbance to a prescribed level η. In order to accomplish this task, the following basic assumption for the actuator fault tolerant control problem is needed.

Assumption 1: The plant (1) is so constructed that for any p actuators fail as (3), $0 \leq p \leq m-1$, and all the other(s) may lose effectiveness as (2), the remaining effective part of the actuators can still achieve the desired control objective.

Remark 1: From Assumption 1 we can see that, if only failure (2) occurred in the system, it allows no redundancy of the actuator; but as long as failure (3) may occur there must be some redundant actuators for fault-tolerant. Here both (2) and (3) are taken into account, so we make $m \geq 2$.

III. Adaptive Fuzzy Fault Tolerant Control

In this section, the design of the fault tolerant control using adaptive fuzzy approach for plant (1) with actuator failures is presented. Inspired by [1], a specific structure is considered in order to obtain the closed-loop stability and the robust output tracking when the system model and the actuator failures are all uncertain, here we respect to the nonlinear system under control is unknown and with disturbance; the failure style (which actuator has failed and as which failure model), the failure value (ρ_i and $\bar{u}_j$), and the failure time (at which the failure occurred) are all unknown.

First we rewrite system (1) in its equivalent matrix form:

$$\begin{aligned} \dot{x} &= Ax + B(f(x) + g^T(x)u + d) \\ y &= Cx \end{aligned} \tag{6}$$

where $g(x) = (g_1(x), g_2(x), \cdots, g_m(x))^T$, $u = (u_1, u_2, \cdots, u_m)^T$,

$$A = \begin{bmatrix} 0\,1\,0 \cdots 0\,0 \\ 0\,0\,1 \cdots 0\,0 \\ \cdots \cdots \\ 0\,0 \cdots 0\,0\,1 \\ 0\,0 \cdots 0\,0\,0 \end{bmatrix} \in R^{n \times n}, \quad B = B_m, \quad C = C_m \tag{7}$$

Consider the actuator failures described as (2) and (3), the actual control input vector u can be expressed as

$$u = \rho \upsilon(t) + \sigma(\bar{u} - \rho \upsilon(t)) \tag{8}$$

where $\upsilon(t) = [\upsilon_1(t), \upsilon_2(t), \cdots, \upsilon_m(t)]$ is the applied control to be designed, and

$$\begin{aligned} \rho &= diag\{\rho_1, \rho_2, \cdots, \rho_m\}, \\ \sigma &= diag\{\sigma_1, \sigma_2, \cdots, \sigma_m\}, \\ \sigma_j &= \begin{cases} 1 & if\ the\ jth\ actuator\ fails\ as\ (3) \\ 0 & otherwise \end{cases} \\ \bar{u} &= [\bar{u}_1, \bar{u}_2, \cdots, \bar{u}_m]^T \end{aligned} \tag{9}$$

Note that $\rho \upsilon(t)$ includes the case where the actuator has no failure when the corresponding ρ_i, $i \in \{1, 2, \cdots, m\}$, equals to 1.

With the actual input (8) in the presence of actuator failures (2) and/or (3), the plant (6) can be rewritten as

$$\begin{aligned} \dot{x} &= Ax + B(f(x) + g^T(x)\sigma\bar{u} + g^T(x)(I-\sigma)\rho\upsilon + d), \\ y &= Cx \end{aligned} \tag{10}$$

where $I = diag\{1, \cdots, 1\}_{m\times m}$. Here a specific proportional actuation structure is used,

$$\upsilon_i(t) = b_i\upsilon_0(t) \quad i = 1, \cdots, m \tag{11}$$

where b_i is a nonzero constant, $\upsilon_0(t)$ is the control signal needs to be designed.

With the proportional actuation structure (11), (10) is equivalent to the following form

$$\begin{aligned} \dot{x} = Ax + B(f(x) &+ \sum_{j=j_1,\cdots,j_{p_2}} g_j(x)\bar{u}_j \\ &+ \sum_{j\neq j_1,\cdots,j_{p_2}} g_j(x)\rho_j b_j\upsilon_0 + d), \\ y = Cx & \end{aligned} \tag{12}$$

where p_2 is the total number of the actuators which fail as (3), $\sum_{j\neq j_1,\cdots,j_{p_2}} g_j(x)\rho_j b_j \neq 0$ from Assumption 1 to accomplish the control task. For this condition to be always true, Assumption 2 is set

Assumption 2: $sign(g_j(x))$ is known.

Without loss of generality, the sign of b_j is made the same as the sign of $g_j(x)$, so $\sum_{j\neq j_1,\cdots,j_{p_2}} g_j(x)\rho_j b_j > 0$ holds.

Let

$$\begin{aligned} \bar{f}(x) &= f(x) + \sum_{j=j_1\cdots j_{p_2}} g_j(x)\bar{u}_j \\ \bar{g}(x) &= \sum_{j\neq j_1\cdots j_{p_2}} g_j(x)\rho_j b_j \end{aligned} \tag{13}$$

Then the formation (12) can be described as

$$\begin{aligned} \dot{x} &= Ax + B(\bar{f}(x) + \bar{g}(x)\upsilon_0 + d), \\ y &= Cx \end{aligned} \tag{14}$$

Assumption 3: Assume x is available.

Then, if $\bar{f}(x)$ and $\bar{g}(x)$ are known, and there is no external disturbance come into the system, the control signal

$$\upsilon_0^i = \frac{1}{\bar{g}(x)}(-\bar{f}(x) - k^T x + r) \tag{15}$$

with $k = (k_n \cdots k_1)^T$ makes the system (14) meet that

$$\dot{x} = Ax - Bk^T x + Br = A_m x + B_m r \tag{16}$$

which exactly matches the reference model if $x(0) = x_m(0)$. Let $e = x - x_m$, then from (4) and (16), one can get

$$\dot{e} = A_m e \tag{17}$$

From (5) we see that the matrix A_m is defined stable, so there exit symmetric positive definite matrices P and Q such that

$$A_m^T P + PA_m \leq -Q \tag{18}$$

is satisfied. Then from the Lyapnov stability theory, it can be seen that $\lim_{t\to\infty} e = 0$, thus $\lim_{t\to\infty} y = y_m$. So, the conclusion can be drawn that if the nonlinear functions $\bar{f}(x)$ and $\bar{g}(x)$ are known, and there is no disturbance, with the control law (15), the output of system (1) will asymptotically track the output of the reference model (4).

But the problem is that the nonlinear functions $\bar{f}(x)$ and $\bar{g}(x)$ are unknown, thus the ideal control law (15) can not be applied directly. Since FLS are universal approximators, they can be used to approximate the unknown functions.

Lemma 1: For any given real continuous function $F(x)$, on a compact set $U \subseteq R^n$, there exits an FLS of the following form that can uniformly approximate $F(x)$ over U to arbitrary accuracy.

$$Y(x) = \theta^T\xi(x)$$

where $\theta = (\theta_1, \theta_2, \cdots, \theta_M)^T$ is the estimate parameter vector, and $\xi(x) = (\xi_1(x), \xi_2(x), \cdots, \xi_M(x))^T$ is the vector of fuzzy basis functions, M is the number of fuzzy rules. One can refer to [16] and [17] for more details.

We make the fuzzy approximation of $\bar{f}(x)$ and $\bar{g}(x)$ as

$$\begin{aligned} \hat{\bar{f}}(x) &= \theta_f^T\xi(x) \\ \hat{\bar{g}}(x) &= \theta_g^T\xi(x) \end{aligned}$$

where estimate parameter vectors θ_f and θ_g can be adjusted by the corresponding adaptive laws respectively. Define the optimal parameters as

$$\begin{aligned} \theta_f^* &= \arg\min\left[\sup\left|\hat{\bar{f}}(x) - \bar{f}(x)\right|\right] \\ \theta_g^* &= \arg\min\left[\sup\left|\hat{\bar{g}}(x) - \bar{g}(x)\right|\right] \end{aligned}$$

The minimum approximation error of FLS is

$$\begin{aligned} \omega_e &= (\hat{\bar{f}}(x|\theta_f^*) - \bar{f}(x)) + (\hat{\bar{g}}(x|\theta_g^*) - \bar{g}(x))\upsilon_0 \\ &= (\theta_f^{*T}\xi(x) - \bar{f}(x)) + (\theta_g^{*T}\xi(x) - \bar{g}(x))\upsilon_0 \end{aligned} \tag{19}$$

The estimate parameter errors are

$$\begin{aligned} \tilde{\theta}_f &= \theta_f - \theta_f^* \\ \tilde{\theta}_g &= \theta_g - \theta_g^* \end{aligned} \tag{20}$$

The estimate of the ideal control law (15) is obtained as

$$\hat{\upsilon}_0^i = \frac{1}{\hat{\bar{g}}(x)}(-\hat{\bar{f}}(x) - k^T x + r) = \frac{1}{\theta_g^T\xi(x)}(-\theta_f^T\xi(x) - k^T x + r) \tag{21}$$

And note that the disturbance should be taken into account when design the applied control law. Inspired by [19], we employ extra control signal u_a to attenuate the external disturbance d and the fuzzy logic approximation error ω_e in (19).

$$u_a = -\frac{1}{r}B^T Pe \tag{22}$$

where r is a positive scalar value satisfies the following Riccati-like equation

$$A_m^T P + PA_m + Q - \frac{2}{r}PBB^T P + \frac{1}{\eta^2}PBB^T P = 0 \tag{23}$$

η is the prescribed attenuation level.Then the applied control signal is designed as

$$\upsilon_0 = \frac{1}{\theta_g^T\xi(x)}(-\theta_f^T\xi(x) - k^T x + u_a + r) \tag{24}$$

We design the adaptive laws as follows:

$$\dot{\theta}_f = \gamma_1 e^T PB\xi(x)$$
$$\dot{\theta}_{gi} = \begin{cases} \gamma_2 e^T PB\xi_i(x)\upsilon_0 & if\ \theta_{gi} > \delta\ or \\ & (\theta_{gi} = \delta\ and\ e^T PB\xi_i(x)\upsilon_0 > 0) \\ 0 & otherwise \end{cases} \tag{25}$$

where γ_1 and γ_2 are the adaptive gain, and δ is a chosen small positive constant. The project algorithm of the adaptive law is used to keep $\hat{\bar{g}}(x)$ off an neighborhood of zero, so that the proposed control law will be nonsingular. Here we think that the estimate parameters θ_f and θ_g are within certain bound by the adjustment of (25) to clarify the presentation. The adaptive fuzzy control scheme has the following properties.

Theorem 1: With the designed controller (24) and the adaptive laws (25), the controlled unknown nonlinear system (1) with external disturbance and unknown actuator failures (2) and/or (3) will achieve the control objective that all closed-loop signals are bounded, and the output robustly tracks the output of the reference model (4) with the following H^∞ performance:

$$\begin{aligned}\int_{t_{j-1}}^{t_j} e^T Qe\, dt \le & e^T(t_{j-1}^+)Pe(t_{j-1}^+) + \tfrac{1}{\gamma_1}\tilde{\theta}_f^T(t_{j-1}^+)\tilde{\theta}_f(t_{j-1}^+) + \\ & \tfrac{1}{\gamma_2}\tilde{\theta}_g^T(t_{j-1}^+)\tilde{\theta}_g(t_{j-1}^+) + \eta^2 \int_{t_{j-1}}^{t_j} \omega^T \omega dt\end{aligned} \tag{26}$$

where $\omega = (\omega_e - d) \in L_2(t_{j-1}, t_j)$, (t_{j-1}, t_j) is the interval where the actuator failure pattern is unchanged.

Proof: Take the control (24) into the system formulation (14), and by the reference model (4), we can get the following error equation

$$\begin{aligned}\dot{e} &= Ax + B(\bar{f}(x) - \hat{\bar{f}}(x) + \bar{g}(x)\upsilon_0 - \hat{\bar{g}}(x)\upsilon_0 + d - k^T x + u_a + \\ &\quad r) - A_m x_m - B_m r \\ &= Ax - Bk^T x + Br - B[(\theta_f - \theta_f^*)^T \xi(x) + (\theta_g - \theta_g^*)^T \xi(x) \\ &\quad \upsilon_0 + (\theta_f^{*T}\xi(x) - \bar{f}(x)) + (\theta_g^{*T}\xi(x) - \bar{g}(x))\upsilon_0 - u_a - d] \\ &\quad - A_m x_m - B_m r \\ &= A_m e - B(\tilde{\theta}_f^T \xi(x) + \tilde{\theta}_g^T \xi(x)\upsilon_0 - u_a - \omega)\end{aligned} \tag{27}$$

Suppose that one or more than one actuators fail at time instant t_j, $j = 1, 2, \cdots, q$, $1 \le q \le m-1$ if all faults are of the form (3), $1 \le q \le m$ otherwise. And at time $t \in (t_{j-1}, t_j)$, there are $p_1 (0 \le p_1 \le m)$ actuators fail as (2), and $p_2 (0 \le p_2 \le m-1)$ actuators fail as (3). Define Lyapunov function on the interval (t_{j-1}, t_j), as

$$V_{j-1} = \frac{1}{2} e^T Pe + \frac{1}{2\gamma_1}\tilde{\theta}_f^T \tilde{\theta}_f + \frac{1}{2\gamma_2}\tilde{\theta}_g^T \tilde{\theta}_g \tag{28}$$

It's derivative along error equation (27) with the control signal (24) is

$$\begin{aligned}\dot{V}_{j-1} &= \tfrac{1}{2}\left(\dot{e}^T Pe + e^T P\dot{e}\right) + \tfrac{1}{\gamma_1}\tilde{\theta}_f^T \dot{\tilde{\theta}}_f + \tfrac{1}{\gamma_2}\tilde{\theta}_g^T \dot{\tilde{\theta}}_g \\ &= \tfrac{1}{2}[e^T A_m^T Pe - (\tilde{\theta}_f^T \xi(x) + \tilde{\theta}_g^T \xi(x)\upsilon_0 - u_a - \omega)B^T Pe + \\ &\quad e^T PA_m e - e^T PB(\tilde{\theta}_f^T \xi(x) + \tilde{\theta}_g^T \xi(x)\upsilon_0 - u_a - \omega)] + \\ &\quad \tfrac{1}{\gamma_1}\tilde{\theta}_f^T \dot{\tilde{\theta}}_f + \tfrac{1}{\gamma_2}\tilde{\theta}_g^T \dot{\tilde{\theta}}_g\end{aligned} \tag{29}$$

By the definition of u_a in (22), (29) can be rewritten as

$$\begin{aligned}\dot{V}_{j-1} &= \tfrac{1}{2} e^T (A_m^T P + PA_m - \tfrac{2}{r} PBB^T P)e + e^T PB\omega - e^T PB\tilde{\theta}_f^T \\ &\quad \xi(x) - e^T PB\tilde{\theta}_g^T \xi(x)\upsilon_0 + \tfrac{1}{\gamma_1}\tilde{\theta}_f^T \dot{\tilde{\theta}}_f + \tfrac{1}{\gamma_2}\tilde{\theta}_g^T \dot{\tilde{\theta}}_g \\ &= -\tfrac{1}{2} e^T Qe - \tfrac{1}{2\eta^2} e^T PBB^T Pe + e^T PB\omega - \varepsilon \\ &= -\tfrac{1}{2} e^T Qe - \tfrac{1}{2}(\tfrac{1}{\eta} e^T PB - \eta\omega)^2 + \tfrac{1}{2}\eta^2\omega^2 - \varepsilon \\ &\le -\tfrac{1}{2} e^T Qe + \tfrac{1}{2}\eta^2\omega^2 - \varepsilon\end{aligned} \tag{30}$$

Where the Riccati-like equation (23), the adaptive laws (25) and the fact that $\dot{\tilde{\theta}}_f = \dot{\theta}_f$, $\dot{\tilde{\theta}}_g = \dot{\theta}_g$ are considered. ε is defined as follows: $\varepsilon = 0$, if for all i, $\theta_{gi} > \delta$ or $\theta_{gi} = \delta\ and\ \xi^T PB\zeta_i\upsilon_0 > 0$; $\varepsilon = \sum_i \tilde{\theta}_{gi}\xi^T PB\zeta_i\upsilon_0$, otherwise. It can be proved easily that $\varepsilon \ge 0$, then the following inequality is obtained

$$\dot{V}_{j-1} \le -\frac{1}{2} e^T Qe + \frac{1}{2}\eta^2\omega^2 \tag{31}$$

Integrating both sides of (31) from t_{j-1} to t_j, and by the definition of the Lyapunov function (28), the H^∞ performance (26) can be obtained, which means that the tracking error e is bounded. Because θ_f and θ_g are bounded with adaptive laws (25), then all closed-loop signals are bounded. Thus Theorem 1 has been proven.

Remark 2: If θ_f and θ_g are bounded with (25), the following parameter Projection algorithm can be implied to modify the adaptive laws

$$\dot{\theta}_f = \begin{cases} \gamma_1 e^T PB\xi(x) & if\ \|\theta_f\| < M_f\ or \\ & (\|\theta_f\| = M_f\ and\ e^T PB\theta_f^T \xi(x) < 0) \\ P_f[\cdot] & if\ \|\theta_f\| = M_f\ and\ e^T PB\theta_f^T \xi(x) > 0 \end{cases}$$
$$\dot{\theta}_g = \begin{cases} \gamma_2 e^T PB\xi(x)\upsilon_0 & if\ \|\theta_g\| < M_g\ or (\|\theta_g\| = M_g\ and \\ & e^T PB\theta_g^T \xi(x)\upsilon_0 < 0) \\ P_g[\cdot] & if\ \|\theta_g\| = M_g\ and\ e^T PB\theta_g^T \xi(x)\upsilon_0 > 0 \end{cases}$$

where M_f and M_g are the boundary of θ_f and θ_g respectively, and

$$P_f[\cdot] = \gamma_1 e^T PB\xi(x) - \theta_f \frac{\gamma_1 e^T PB\theta_f^T \xi(x)}{\|\theta_f\|^2}$$

$$P_g[\cdot] = \gamma_2 e^T PB\xi(x)\upsilon_0 - \theta_g \frac{\gamma_2 e^T PB\theta_g^T \xi(x)\upsilon_0}{\|\theta_g\|^2}$$

then θ_f and θ_g can be constrained to be bounded.

Remark 3: Actuators fail at time $t = t_j$, $j = 1, 2, \cdots, q$, then there may be a finite jumping in the optimal parameters θ_f^* and θ_g^*, so $\tilde{\theta}_f$ and $\tilde{\theta}_g$ will change their values to form the new Lyapunov function V_j on the next time interval (t_j, t_{j+1}). Consequently, the H^∞ boundary of the output tracking will jump infinitely. If after some actuator failures, the the boundary of the output tracking error will jump to a level that not be satisfying any more, then we can tune the attenuation level η to further decrease the boundary of the output tracking error. This can be reasonable because computer is used to achieve the control task, so the prescribed attenuation level η can be tuned on-line. For example, if $|e| \le m_e$ is requested after the initial transient adjustment. If after some time t, $|e| > m_e$, then a higher attenuation level η will be adopted to recover the requested output tracking performance if it is possible. So the proposed control scheme can guarantee the tracking performance as well as the closed-loop stability of the controlled system with actuator failures.

IV. Simulation Example

In this section, the presented adaptive fuzzy fault tolerant control law is applied to a nonlinear system with external disturbance and actuator faults described as (2) and (3).

Example: We consider the nonlinear system which can be written as the following form with redundant actuators and external disturbance.

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= \frac{5\sin x_1 - 0.02x_2^2\cos x_1 \sin x_1}{3-0.2\cos x_1\cos x_2} + \frac{\cos^2 x_1}{3-0.2\cos x_1\cos x_2}u_1 + \\ &\quad \frac{2\cos^2 x_1}{3-0.2\cos x_1 \cos x_2}u_2 + d \end{aligned} \tag{32}$$

where the actuator of u_1 is stuck at 2 at $t = 4s$, i.e. $u_1^F = 2$ when $t \geq 4$, while the actuator of u_2 loses 80% effectiveness at $t = 13s$ that is $u_2^F = 0.2u_2$ when $t \geq 13$ in simulation. The situation of the failure is already very severe. The external disturbance d is assumed to be a square wave with the amplitude 0.05 and the period 2π. The reference model is as follows:

$$\begin{pmatrix} \dot{x}_{m1} \\ \dot{x}_{m2} \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -1 & -2 \end{pmatrix} \begin{pmatrix} x_{m1} \\ x_{m2} \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} r$$

where $x_m = (x_{m1},\ x_{m2})^T = (\pi/30sin(t),\ \pi/30cos(t))$, $r = \pi/15cos(t)$. Choose $k_1 = 2$, $k_2 = 1$, and $Q = diag(10,10)$. Then by solving Riccati-like equation (23) one can obtain

$$P = \begin{bmatrix} 15 & 5 \\ 5 & 5 \end{bmatrix}$$

$\gamma_1 = 0.1$, and $\gamma_2 = 0.01$ for adaptive adjusting. We define seven fuzzy sets over each axis, which label as $F_i^1, \cdots, F_i^7$, $i = 1,2$, respectively. The fuzzy membership functions are

$$\begin{aligned} \mu_{F_i^1}(x_i) &= \frac{1}{1+\exp(5\times(x_i+0.6))} \\ \mu_{F_i^2}(x_i) &= \exp(-(x_i+0.4)^2) \\ \mu_{F_i^3}(x_i) &= \exp(-(x_i+0.2)^2) \\ \mu_{F_i^4}(x_i) &= \exp(-x_i^2) \\ \mu_{F_i^5}(x_i) &= \exp(-(x_i-0.2)^2) \\ \mu_{F_i^6}(x_i) &= \exp(-(x_i-0.4)^2) \\ \mu_{F_i^7}(x_i) &= \frac{1}{1+\exp(-5\times(x_i-0.6))} \end{aligned}$$

So we have $M = 7\times 7 = 49$ rules for each fuzzy logic system. The initial values are selected as $\theta_f(0) = 0$, $\theta_g(0) = 0.2I_{49\times 1}$, and $x(0) = (0.2,\ 0.2)^T$. Two cases are simulated: in one case, η is 0.1 for all the time; while in the other case, η is changed from 0.1 to 0.05 when the condition $100\int_0^t e^T e\, dt > 2.1$ which is equivalent to $|e| > m_e$ is met due to the severe actuator failures. The simulation results of the both cases for $0 \leq t \leq 40$ are shown in Fig.1 - Fig.4

We can see from the results that with the proposed control scheme, the controlled system can be stable and achieve robust output tracking performance. However, some severe actuator failures may cause obvious tracking error between the output of the system and the reference model (see Fig.1

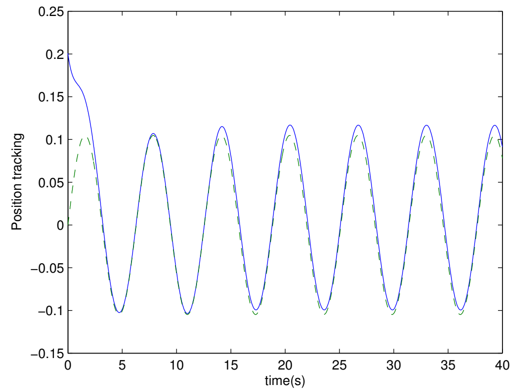

Fig. 1. The output tracking with unvaried η

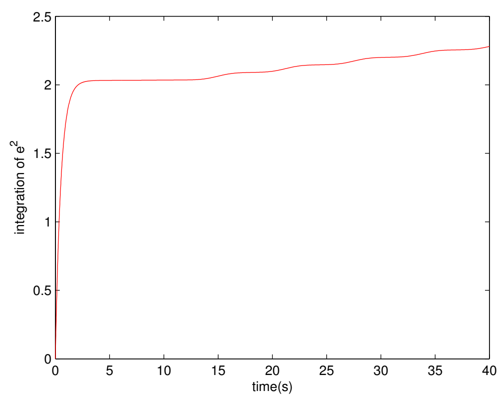

Fig. 2. $100\int_0^t e^T e\, dt$ with unvaried η

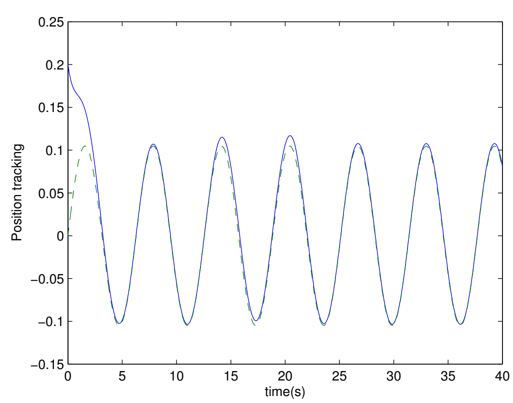

Fig. 3. The output tracking with variable η

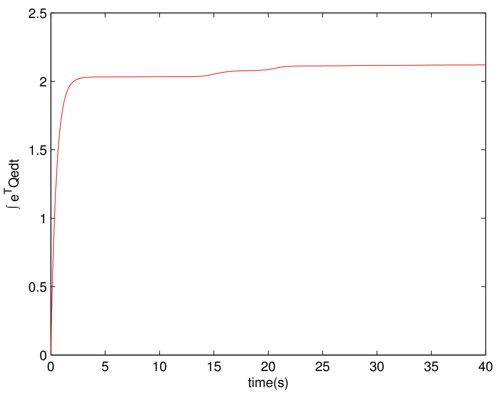

Fig. 4. $100\int_0^t e^T e\, dt$ with variable η

and Fig.2). If the attenuation level can be changed higher on-line when the output tracking performance is not satisfying, the tracking error can be decreased effectively to get almost perfect output tracking (see Fig.3 and Fig.4).

V. Conclusion

This paper studies fault tolerant control problem for unknown nonlinear systems with actuator failures and external disturbance. With the developed adaptive fuzzy control law, the closed-loop system can be guaranteed stable and the output tracks the reference model output robustly. This is achieved by an adaptive control law with fuzzy logic systems approximating the unknown system functions and actuator failures together, and attenuation technique to attenuate the influence of both fuzzy logic approximation error and external disturbance on the tracking error to a prescribed level, the adaptive laws adjusting estimate parameters in the fuzzy logic approximators are obtained on the basis of Lyapunov stability theory. Note that the proposed adaptive fuzzy fault tolerant control approach need not resort to the fault detection and isolation mechanism by adaptively adjust the control law, thus the undesired system behavior caused by false alarm or omitted alarm can be avoided. And the fault under consideration can be lock-in-place, loss of effectiveness or both of them, so the types of actuator fault that can be tolerant have been broadened even in unknown nonlinear system with external disturbance.The simulation results show the effectiveness of the proposed control scheme though the fault occurred is severe.

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