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Stability-Assured Robust Adaptive Control of Semi-active Suspension Systems

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Abstract— **The paper is concerned with a stable robust adaptive scheme for sem-iactive control of suspension system installed with MR damper, which can deal with uncertainties in both models of MR damper and suspension mechanism. The proposed scheme consists of two main adaptive controllers: One is an adaptive inverse control for compensating the nonlinear hysteresis dynamics of the MR damper, which can be realized by identifying a forward model of MR damper or by directly adjusting an inverse model of MR damper. The other is an adaptive reference control which gives the desired damping force to match the seat dynamics to a specified reference dynamics, which can also be designed by taking into account the passivity of the MR damper. The stability of the total system including the two adaptive controllers is discussed and its stability condition is explored. Validity of the proposed algorithm is also examined in simulation studies.**

I. Introduction

Magnetorheological (MR) damper is a promising semi-active device in areas of vibration isolation for suspension systems and civil structures. The viscosity of MR fluid is controllable depending on input voltage or current. The MR damper provides better performance over a wide bandwidth of road excitation compared to passive damping methods, while consuming less power than active damping schemes. Since the MR damper inherently has uncertain nonlinear hysteresis dynamics, its modeling is an important issue in realization of semi-active vibration isolation control. In this paper, a stable robust adaptive control approach which can deal with uncertainties in both modeling of MR damper and suspension mechanism is presented. The proposed approach consists of two main adaptive controllers: one is an adaptive inverse control for compensating the nonlinear hysteresis dynamics of the MR damper, and the other is an adaptive reference control which gives the desired damping force to match the seat dynamics to a specified reference dynamics, which can also be designed by taking into account the dissipativity of the MR damper. Since two adaptive controllers are mutually coupled, a main issue in this paper is to consider the stability of the total system.

The adaptive inverse controller is realized by identifying a forward model of the MR damper or by directly adjusting the inverse model of MR damper without identification of the forward model. Many efforts have been devoted to construction of forward models of MR damper from static and dynamic points of view [1][2][3][4]. The Bouc-Wen model and its variations are typical models which can express the hysteresis dynamics explicitly [1][2], and Hammerstein class of nonlinear model was also investigated [5]. However, they include too many parameters in nonlinear forms to identify them in an on-line manner. Alternative modeling is based on the LuGre friction model [6] which was originally developed to describe nonlinear friction phenomena [7]. It has a rather simple structure and the number of model parameters can also be reduced, however, it is not adequate for real-time design of an inverse controller by using the obtained forward model. We have obtained the new MR damper model by modifying the LuGre model and given an analytical method for adaptive inverse controller design [8][9]. However, since some adjusted parameters appear in the denominator of the adaptive inverse controller, the stability of the total system is only assured in restricted conditions. In this paper, we also give an inverse modeling to directly adjust the inverse controller without using the forward model, and thus by taking into account the inverse modeling error we proposed a robust adaptive algorithm assuring the stability of the total system in a more general case.

The adaptive reference control gives the desired damping force to match the seat dynamics to a desirable reference dynamics even in the presence of uncertainties in the suspension mechnism, unlike previous works based on deterministic control schemes, for instance, the clipped-optimal control [10][11], LQ control [12], gain-scheduled control [14], and H_∞ control [15][16][17]. Although an adaptive skyhook approach was also proposed [18] to deal with uncertainty of the suspension system, it cannot be applied directly to the semi-active control case using MR damper. Hence, the adaptive algorithm is modified to assure the stability of the total semi-active control system. Finally the validity of the proposed control scheme is discussed in numerical simulation.

II. Fully Robust Adaptive Control Algorithm

Fig. 1 illustrates a simple suspension system installed with MR damper between the car chassis and the wheel assembly, which is modified from our previous one [9]. The dynamic is expressed by

$$M_s\ddot{x}_s + C_s(\dot{x}_s - \dot{x}_u) + K_s(x_s - x_u) = -F_{MR}(\dot{x}, v) \quad (1)$$

$$M_u\ddot{x}_u + C_s(\dot{x}_u - \dot{x}_s) + K_s(x_u - x_s) + K_t(x_u - x_r) = F_{MR}(\dot{x}, v) \quad (2)$$

$$x \equiv x_s - x_u \quad (3)$$

where x is the relative displacement between the car chassis and the wheel assembly; M_s is the sprung mass, which represents the car chassis, M_u is the unsprung mass, which represents the wheel assembly; C_s and K_s are damping and

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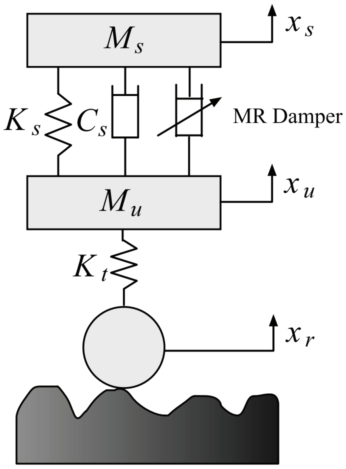

Fig. 1. Suspension system with MR damper

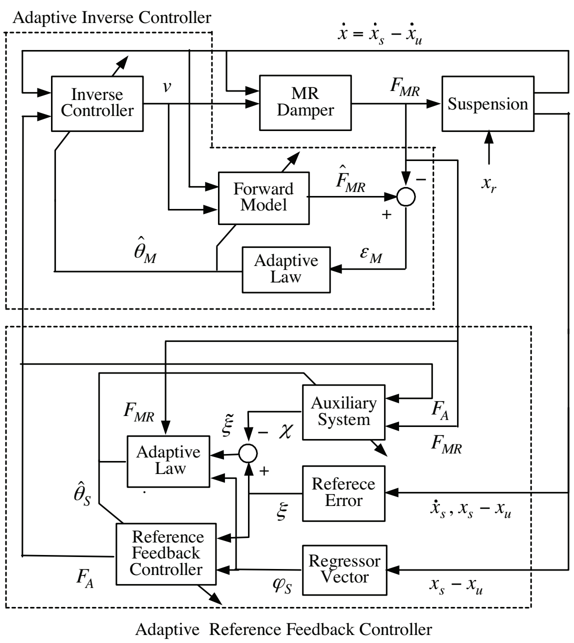

Fig. 2. Proposed fully adaptive semi-active control scheme based on *forward* modeling

stiffness of the uncontrolled suspension system, respectively; K_t serves to model the compressibility of the pneumatic tyre. x_s and x_u are the displacements of the sprung and unsprung mass, respectively; x_r is the road displacement input; $F_{MR}(\dot{x}, v)$ is the damping force supplied by the MR damper.

Fig. 2 and Fig. 3 show schematic diagrams of the proposed fully adaptive semiactive control for the suspension system. The adaptive algorithm consists of two controllers: one is an adaptive inverse controller which can give required input voltage v to MR damper so that the damping force F_{MR} be equal to specified command damping force F_A. If the adaptive inverse controller is designed so that the linearization from F_A to F_{MR} can be attained, that is, $F_A = F_{MR}$, we can realize almost active control performance. For construction of the inverse controller, the forward model of MR damper

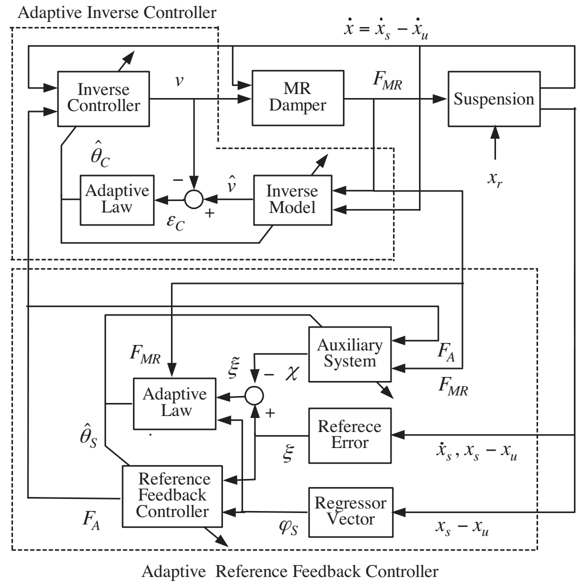

Fig. 3. Proposed fully adaptive semi-active control scheme based on *inverse* modeling

is identified and then the input voltage to MR damper is calculated as shown in Fig. 2. Fig. 3 gives an alternative scheme in which the inverse controller is directly updated without identification of MR damper. The adaptive reference feedback control can match the chassis dynamic response to a desired reference dynamics even when the suspension system involves parametric uncertainty in M_s, C_s and K_s. Since the MR damper is actually a nonlinear semi-active device, it is difficult to make it work as an active device, and it needs very fine and complicated tuning of both the adaptive inverse controller and adaptive reference controller.

III. Adaptive Damper Control

A. Adaptive Damper Control via Forward Modeling

MR damper is a semi-active device in which the viscosity of the fluid is controllable by the input voltage or current. A variety of approaches have been taken to modeling of the nonlinear hysteresis behavior of the MR damper. Compared to the Bouc-Wen model [1][2], the LuGre model has simpler structure and smaller number of parameters needed for expression of its behavior [6]. The LuGre model can also be modified so that a necessary input voltage can be analytically calculated to produce the specified command damping force F_A [8].

The damping force F_{MR} is expressed by:

$$F_{MR} = \sigma_a z + \sigma_0 z v + \sigma_1 \dot{z} + \sigma_2 \dot{x} + \sigma_b \dot{x} v \quad (4)$$

$$\dot{z} = \dot{x} - a_0 |\dot{x}| z \quad (5)$$

where z is an internal state variable [m], $\dot{x}$ velocity of structure attached with MR damper [m/s], σ_0 stiffness of z influenced by v [N/(m·V)], σ_1 damping coefficient of $z(t)$ [N·s/m], σ_2 viscous damping coefficient [N·s/m], σ_a stiffness of $z(t)$ [N/m], σ_b viscous damping coefficient influenced by $v(t)$ [N·s/(m·V)], and a_0 constant value [V/N].

Substituting (5) into (4) gives the nonlinear input-output relation as:

$$F_{MR} = \sigma_a z + \sigma_0 z v - \sigma_1 a_0 |\dot{x}| z + (\sigma_1 + \sigma_2)\dot{x} + \sigma_b \dot{x} v = \theta_M^T \varphi_M \quad (6)$$

where $\theta_M = (\sigma_a,\ \sigma_0,\ \sigma_1 a_0,\ \sigma_1+\sigma_2,\ \sigma_b)^T = (\theta_1,\ \theta_2,\ \theta_3,\ \theta_4,\ \theta_5)^T$, and $\varphi_M = (z,\ zv,\ -|\dot{x}|z,\ \dot{x},\ \dot{x}v)^T$.

Let the identified parameter vector $\hat{\theta}_M$ be denoted by $\hat{\theta}_M = (\hat{\theta}_1,\ \hat{\theta}_2,\ \hat{\theta}_3,\ \hat{\theta}_4,\ \hat{\theta}_5)^T$. Since the internal state z of the MR damper model cannot be measured, the regressor vector φ_M should be replaced with its estimate $\hat{\varphi}_M$ as:

$$\hat{\varphi}_M = (\hat{z},\ \hat{z}v,\ -|\dot{x}|\hat{z},\ \dot{x},\ \dot{x}v)^T \quad (7)$$

where the estimate $\hat{z}$ is given later by using the updated model parameters. The output of the identification model is now described as:

$$\hat{F}_{MR} = \hat{\theta}_M^T \hat{\varphi}_M \quad (8)$$

By using the damping force estimation error defined by $\varepsilon_M = \hat{F}_{MR} - F_{MR}$, and the identified parameter $\hat{a}_0$, the estimate $\hat{z}$ of the internal state can be calculated as:

$$\dot{\hat{z}} = \dot{x} - \hat{a}_0 |\dot{x}| \hat{z} - L\varepsilon_M \quad (9)$$

where L is an observer gain such that $0 \le L \le 1/\hat{\sigma}_{1\max}$, and the upper bound is decided by the stability of the adaptive observer [9].

To assure the stability of the adaptive identification algorithm, introduce the normalizing signal as $N_M = \left(c_1 + \hat{\varphi}_M^T \hat{\varphi}_M\right)^{1/2}$, where $c_1 > 0$. By dividing the signals and errors by N_M as $\varphi_{M_N} = \varphi_M / N_M$, $\hat{\varphi}_{M_N} = \hat{\varphi}_M / N_M$ and $\varepsilon_{M_N} = \hat{F}_{MR_N} - F_{MR_N}$, where $F_{MR_N} = F_{MR}/N_M$ and $\hat{F}_{MR_N} = \hat{\theta}_M^T \hat{\varphi}_{M_N}$, the adaptive law for updating the model parameters is given as:

$$\dot{\hat{\theta}}_M = -\Gamma_M \hat{\varphi}_{M_N} \varepsilon_{M_N} - \sigma_M \Gamma_M \hat{\theta}_M \quad (10)$$

where $\tilde{\theta}_M = \hat{\theta}_M - \theta_M$. Γ_M is a positive-definite matrix and σ_M is a positive design constant. For practical implementation, Γ_M is chosen constant. Thus, the physical model parameters can be calculated from the relation (6).

The role of the adaptive inverse controller shown in Fig. 2 is to decide the control input voltage v to the MR damper so that the actual damping force F_{MR} may coincide with the specified command damping force F_A, even in the presence of uncertainty in the MR damper model. The input voltage giving F_A can be analytically calculated from the identified forward model of MR damper (6). Actually using the identified model parameters, the input voltage v is obtained from (6) as:

$$\rho = \hat{\sigma}_0 \hat{z} + \hat{\sigma}_b \dot{x} \quad (11)$$

$$d_\rho = \begin{cases} \rho & \text{for} \quad \rho < -\delta, \quad \delta < \rho \\ \delta\,\mathrm{sgn}(\rho) & \text{for} \quad -\delta \le \rho \le \delta \end{cases} \quad (12)$$

$$v_A = \frac{F_A - \{\hat{\sigma}_a \hat{z} - \hat{\sigma}_1 \hat{a}_0 |\dot{x}| \hat{z} + (\hat{\sigma}_1 + \hat{\sigma}_2)\dot{x} - L\varepsilon_M\}}{d_\rho} \quad (13)$$

$$v = \begin{cases} 0 & \text{for} \quad v_A \le 0 \\ v_A & \text{for} \quad 0 < v_A \le V_{\max} \\ V_{\max} & \text{for} \quad V_{\max} < v_A \end{cases} \quad (14)$$

where F_A is the specified command damping force. v is assumed to be fixed near $\rho = 0$ to avoid division by zero. Due to the semi-active nature of the MR damper, F_{MR} may not fully match the desired control force F_A.

B. Adaptive Damper Control via Inverse Modeling

In the previous section, the inverse controller is obtained analytically from the estimated parameters of the forward model of MR damper. However, as expressed in (14), some adjustable parameters appear in the denominator of the inverse controller and so zero-division should be avoided. Therefore, we consider a linearly parameterized *inverse* model, as shown in Fig. 3. Since the damper force F_{MR} is given as a function of the velocity $\dot{x}$, input voltage v and internal state z as shown in (6), its inverse model for the input voltage v can be expressed as a function of $\dot{x}$, z and F_{MR}. Hence, an approximate inverse model, which is expressed by a linearly parameterized polynomial function, is considered as:

$$v = \sum_{j=0}^{n} \sum_{i=0}^{m} h_{i+(m+1)k+1} |\dot{x}|^i |z|^j F_{MR}\,\mathrm{sgn}(\dot{x}) + \delta_C \quad (15)$$

where δ_C is an approximation error term, and is assumed to have an unknown constant bound $|\delta_C| \le \Delta_C$. The inverse model has three inputs of $\dot{x}$, z and F_{MR}, and one output of v. z is an internal state of the MR damper, which can be calculated as given previously by:

$$\dot{z} = \dot{x} - a_0 |\dot{x}| z \quad (16)$$

where a nominal value of a_0 is assumed to be known via the forward modeling. In simulation, an inverse model with $m = 4$ and $n = 1$ is adopted.

The inverse model is also expressed in vector form as:

$$v = \theta_C^T \varphi_C + \delta_C \quad (17)$$

where:

$$\theta_C = \begin{bmatrix} h_1 & h_2 & \dots & h_{(n+1)(m+1)} \end{bmatrix}^T \quad (18)$$

$$\varphi_C = \big[F_{MR}\mathrm{sgn}(\dot{x}) \quad |\dot{x}|F_{MR}\mathrm{sgn}(\dot{x}) \quad \dots \quad |z|F_{MR}\mathrm{sgn}(\dot{x}) \quad |\dot{x}||z|F_{MR}\mathrm{sgn}(\dot{x}) \quad \dots \quad |\dot{x}|^m |z|^n F_{MR}\mathrm{sgn}(\dot{x})\big]^T \quad (19)$$

Then the identified model is expressed as:

$$\hat{v} = \hat{\theta}_C^T \varphi_C + \mu \quad (20)$$

The identification error ε_C is defined as:

$$\varepsilon_C = \hat{v} - v \quad (21)$$

and μ is a robustifying term given as:

$$\mu = -\hat{\Delta}_C \eta_C \tanh((a+bt)\varepsilon_C) \quad (22)$$

with $\eta_C > 1$ and $a, b > 0$. To assure the stability of the adaptive identification algorithm, introduce the normalizing signal as $N_C = \left(c_2 + \varphi_C^T \varphi_C\right)^{1/2}$, where $c_2 > 0$. By dividing the signals and errors by N_C as $\varphi_{C_N} = \varphi_C / N_C$, and $\varepsilon_{C_N} = \hat{v}_N - v_N$, where $v_N = v/N_C$ and $\hat{v}_N = \hat{\theta}_C^T \hat{\varphi}_{C_N} + \mu_N$, and $\mu_N =$

$-\hat{\Delta}_C\eta_C\tanh((a+bt)\varepsilon_{C_N})$, the adaptive laws for updating the model parameters are given as:

$$\dot{\hat{\theta}}_C = -\Gamma_C\varphi_{C_N}\varepsilon_{C_N} - \sigma_C\Gamma_C\hat{\theta}_C \tag{23}$$

$$\dot{\hat{\Delta}}_C = \gamma_{\Delta_C}|\varepsilon_{C_N}| - \sigma_{\Delta_C}\gamma_{\Delta_C}\hat{\Delta}_C \tag{24}$$

where $\tilde{\theta}_C = \hat{\theta}_C - \theta_C$ and $\tilde{\Delta}_C = \hat{\Delta}_C - \Delta_C$. Γ_C is a positive definite matrix, γ_{Δ_C}, σ_C and σ_{Δ_C} are positive design constants. For practical implementation, Γ_C and γ_{Δ_C} are chosen constant.

Fig. 3 describes the adaptive damper control via inverse modeling. The control input voltage v is given as:

$$v_A = \hat{\theta}_C^T\varphi_A \tag{25}$$

$$v = \begin{cases} 0 & \text{for } v_A \le 0 \\ v_A & \text{for } 0 < v_A \le V_{\max} \\ V_{\max} & \text{for } V_{\max} < v_A \end{cases} \tag{26}$$

where:

$$\varphi_A = \big[F_A\mathrm{sgn}(\dot{x}) \quad |\dot{x}|F_A\mathrm{sgn}(\dot{x}) \quad \dots \quad |z|F_A\mathrm{sgn}(\dot{x}) \quad |\dot{x}||z|F_A\mathrm{sgn}(\dot{x}) \quad \dots \quad |\dot{x}|^m|z|^nF_A\mathrm{sgn}(\dot{x})\big]^T \tag{27}$$

Again due to the semi-active nature of the MR damper, F_{MR} may not fully match the desired control force F_A.

IV. Adaptive Reference Feedback Control

The role of the adaptive reference feedback controller is to provide a desired damper force F_A to the adaptive inverse controller so that the car chassis dynamics can match the reference dynamics. The desired damper force is decided by the skyhook approach in the case when the mass and spring constants are both unknown. Following the adaptive scheme [18], the desired reference dynamics is specified by:

$$\ddot{x}_s + 2\zeta\omega\dot{x}_s + \omega^2(x_s - x_u) = 0 \tag{28}$$

where ω is the natural frequency, and ζ is a damping constant. Then, the control error ξ is given by:

$$\xi = \dot{x}_s + (s+2\zeta\omega)^{-1}\omega^2(x_s - x_u) \tag{29}$$

Taking the derivative of (29):

$$\begin{aligned}\dot{\xi} &= \ddot{x}_s + (s+2\zeta\omega)^{-1}\omega^2 s(x_s - x_u) \\ &= -\frac{C_s}{M_s}(\dot{x}_s - \dot{x}_u) - \frac{K_s}{M_s}(x_s - x_u) \\ &\quad -\frac{1}{M_s}F_{MR} + (s+2\zeta\omega)^{-1}\omega^2 s(x_s - x_u) \\ &= -\theta_{S,1}^T\varphi_{S,1} - \theta_{S,2}F_{MR} + (s+2\zeta\omega)^{-1}\omega^2 s(x_s - x_u)\end{aligned} \tag{30}$$

where:

$$\theta_{S,1} = \left[\tfrac{C_s}{M_s} \quad \tfrac{K_s}{M_s}\right]^T \tag{31}$$

$$\varphi_{S,1} = \left[\dot{x}_s - \dot{x}_u \quad x_s - x_u\right]^T \tag{32}$$

$$\theta_{S,2} = \frac{1}{M_s} \tag{33}$$

Then the adaptive control law is given as:

$$F_A = \frac{1}{\hat{\theta}_{S,2}}\left(\kappa\xi - \hat{\theta}_{S,1}^T\varphi_{S,1} + (s+2\zeta\omega)^{-1}\omega^2 s(x_s - x_u)\right) \tag{34}$$

where $\kappa > 0$ is a design constant, and $\hat{\theta}_{S,1}$ and $\hat{\theta}_{S,2}$ are corresponding parameter estimates. Due to parametric uncertainties and the semi-active nature of the MR damper, F_{MR} cannot match F_A as given in (34). To develop stable adaptive control laws in the presense of this semi-active constraint, it is necessary to define an auxiliary signal χ as:

$$\dot{\chi} = -\kappa\chi + \hat{\theta}_{S,2}(F_A - F_{MR}) \tag{35}$$

A modified error signal is given by:

$$\tilde{\xi} = \xi - \chi \tag{36}$$

Employing this modified error signal, the adjustable parameters $\hat{\theta}_{S,1}$ and $\hat{\theta}_{S,2}$ are updated by:

$$\dot{\hat{\theta}}_{S,1} = -\dot{\tilde{\theta}}_{S,1} = -\Gamma_{S,1}\varphi_{S,1}\tilde{\xi} - \sigma_{S,1}\Gamma_{S,1}\hat{\theta}_{S,1} \tag{37}$$

$$\dot{\hat{\theta}}_{S,2} = -\dot{\tilde{\theta}}_{S,2} = -\gamma_{S,2}\tilde{\xi}F_{MR} - \sigma_{S,2}\gamma_{S,2}\hat{\theta}_{S,2} \tag{38}$$

where $\tilde{\theta}_{S,1} = \hat{\theta}_{S,1} - \theta_{S,1}$ and $\tilde{\theta}_{S,2} = \hat{\theta}_{S,2} - \theta_{S,2}$. $\Gamma_{S,1}$ is a positive-definite matrix, $\gamma_{S,2} > 0$, $\sigma_{S,1}$ and $\sigma_{S,2}$ are positive design constants. For practical implementation, $\Gamma_{S,1}$ and $\gamma_{S,2}$ are chosen constant.

The main stability result for the adaptive reference feedback controller is presented as follows. Consider a candidate of the Lyapunov function as:

$$V_S = \frac{1}{2}\tilde{\xi}^2 + \frac{1}{2}\tilde{\theta}_{S,1}^T\Gamma_{S,1}^{-1}\tilde{\theta}_{S,1} + \frac{1}{2\gamma_{S,2}}\tilde{\theta}_{S,2}^2 \tag{39}$$

Taking the time-derivative of V_S and using the control law (34):

$$\begin{aligned}\dot{V}_S &= \tilde{\xi}\dot{\tilde{\xi}} + \tilde{\theta}_{S,1}^T\Gamma_{S,1}^{-1}\dot{\tilde{\theta}}_{S,1} + \frac{1}{\gamma_{S,2}}\tilde{\theta}_{S,2}\dot{\tilde{\theta}}_{S,2} \\ &= \tilde{\xi}\left(\dot{\xi} - \dot{\chi}\right) + \tilde{\theta}_{S,1}^T\Gamma_{S,1}^{-1}\dot{\tilde{\theta}}_{S,1} + \frac{1}{\gamma_{S,2}}\tilde{\theta}_{S,2}\dot{\tilde{\theta}}_{S,2} \\ &= \tilde{\xi}\big(-\theta_{S,1}^T\varphi_{S,1} - \theta_{S,2}F_{MR} + (s+2\zeta\omega)^{-1}\omega^2 s(x_s - x_u) \\ &\quad + \kappa\chi - \hat{\theta}_{S,2}(F_A - F_{MR})\big) + \tilde{\theta}_{S,1}^T\Gamma_{S,1}^{-1}\dot{\tilde{\theta}}_{S,1} + \frac{1}{\gamma_{S,2}}\tilde{\theta}_{S,2}\dot{\tilde{\theta}}_{S,2} \\ &= -\kappa\tilde{\xi}^2 + \tilde{\theta}_{S,1}^T\varphi_{S,1}\tilde{\xi} + \tilde{\theta}_{S,2}\tilde{\xi}F_{MR} + \tilde{\theta}_{S,1}^T\Gamma_{S,1}^{-1}\dot{\tilde{\theta}}_{S,1} \\ &\quad + \frac{1}{\gamma_{S,2}}\tilde{\theta}_{S,2}\dot{\tilde{\theta}}_{S,2}\end{aligned} \tag{40}$$

Now using the adaptive laws (37) and (38):

$$\begin{aligned}\dot{V}_S &= -\kappa\tilde{\xi}^2 - \sigma_{S,1}\tilde{\theta}_{S,1}^T\hat{\theta}_{S,1} - \sigma_{S,2}\tilde{\theta}_{S,2}\hat{\theta}_{S,2} \\ &\le -\kappa\tilde{\xi}^2 - \frac{\sigma_{S,1}}{2}\tilde{\theta}_{S,1}^T\tilde{\theta}_{S,1} - \frac{\sigma_{S,2}}{2}\tilde{\theta}_{S,2}^2 + \frac{\sigma_{S,1}}{2}\theta_{S,1}^T\theta_{S,1} \\ &\quad + \frac{\sigma_{S,2}}{2}\theta_{S,2}^2\end{aligned} \tag{41}$$

Thus:

$$\dot{V}_S \le -c_S V_S + \lambda_S \tag{42}$$

where:

$$c_S = \min\left\{2\kappa, \frac{\sigma_{S,1}}{\lambda_{\max}(\Gamma_{S,1}^{-1})}, \gamma_{S,2}\sigma_{S,2}\right\} \tag{43}$$

$$\lambda_S = \frac{\sigma_{S,1}}{2}\theta_{S,1}^T\theta_{S,1} + \frac{\sigma_{S,2}}{2}\theta_{S,2}^2 \tag{44}$$

Since κ, $\sigma_{S,1}$ and $\sigma_{S,2}$ are positive design constants, $\lambda_S/c_S > 0$ and the following result is obtained:

$$0 \leq V_S(t) \leq \lambda_S/c_S + (V_S(0) - \lambda_S/c_S)\, e^{-c_S t} \tag{45}$$

Thus, the modified error $\tilde{\bar{\xi}}$ and the parameter estimation errors $\tilde{\theta}_{S,1}$ and $\tilde{\theta}_{S,2}$ are uniformly bounded and converge to a small neighborhood of the origin. The assumption that $M_s > 0$ and some type of parameter projection method is necessary to ensure that $\hat{\theta}_{S,2}$ does not approach zero. From (35), it is clear that if $F_{MR} = F_A$ then $\chi \to 0$, and therefore the control error ξ also converges to a small neigborhood of the origin. This result is summarized as:

Result 1 *Assume $M_s > 0$ and $\kappa > 0$ are satisfied. Then the control law (34), along with the adaptive laws (37) and (38), guarantee that the modified error signal $\tilde{\bar{\xi}}$ and the parameter estimation errors $\tilde{\theta}_{S,1}$ and $\tilde{\theta}_{S,2}$ remain bounded and converge to a small neighborhood of the origin. Furthermore, if $F_{MR} = F_A$, then the control error ξ also converges to a small neighborhood of the origin.*

V. Stability Analysis

A. Total Adaptive System with Forward Modeling

We discuss the stability of the integrated total system consisting of the MR damper and structure with the two adaptive controllers using forward modeling, as shown in Fig. 2. To investigate the stability, we assume that the parameter a_0 is known. From this assumption, the internal state z is available and the regressor vector φ_{M_N} can be employed in (10). Let a candidate Lyapunov function to the total system with forward modeling be denoted by:

$$V_M = V_S + \frac{1}{2}\tilde{\theta}_M^T \Gamma_M^{-1} \tilde{\theta}_M \tag{46}$$

Taking the time-derivative of V_M and using Result 1, along with the adaptive law (10), leads to:

$$\begin{aligned}
\dot{V}_M &= \dot{V}_S + \tilde{\theta}_M^T \Gamma_M^{-1} \dot{\tilde{\theta}}_M \\
&= \dot{V}_S - \tilde{\theta}_M^T \varphi_{M_N} \varepsilon_{M_N} - \sigma_M \tilde{\theta}_M^T \hat{\theta}_M \\
&= \dot{V}_S - \varepsilon_{M_N}^2 - \sigma_M \tilde{\theta}_M^T \hat{\theta}_M \\
&\leq \dot{V}_S - \frac{\sigma_M}{2}\tilde{\theta}_M^T \tilde{\theta}_M + \frac{\sigma_M}{2}\theta_M^T \theta_M
\end{aligned} \tag{47}$$

Therefore:

$$\dot{V}_M \leq -c_M V_M + \lambda_M \tag{48}$$

where:

$$c_M = \min\left\{ c_S, \frac{\sigma_M}{\lambda_{\max}(\Gamma_M^{-1})} \right\} \tag{49}$$

$$\lambda_M = \lambda_S + \frac{\sigma_M}{2}\theta_M^T \theta_M \tag{50}$$

As $\lambda_M/c_M > 0$, the following result is obtained:

$$0 \leq V_M(t) \leq \lambda_M/c_M + (V_M(0) - \lambda_M/c_M)\, e^{-c_M t} \tag{51}$$

Thus, all the error signals associated with the total system, along with the parameter estimation error $\tilde{\theta}_M$, are uniformly bounded and converge to a small neighborhood of the origin. Furthermore, if $F_{MR} = F_A$, then the control error ξ also converges to a small neighborhood of the origin. This result is summarized as:

Result 2 *Assume $M_s > 0$ and $\kappa > 0$ are satisfied, and a_0 is known. Then the control law (34), along with the adaptive laws (10), (37) and (38) guarantee that all error signals remain bounded and converge to a small neighborhood of the origin. Furthermore, if $F_{MR} = F_A$, then the control error ξ also converges to a small neighborhood of the origin.*

B. Total Adaptive System with Inverse Modeling

This section discusses the stability of the integrated total system consisting of the MR damper and structure with the two adaptive controllers using inverse modeling, as shown in Fig. 3. To investigate the stability, we assume that the parameter a_0 is known. From this assumption, the internal state z is available and the regressor vector φ_{M_N} can be employed in (10). Let a candidate Lyapunov function to the total system with inverse modeling be denoted by:

$$V_C = V_S + \frac{1}{2}\tilde{\theta}_C^T \Gamma_C^{-1} \tilde{\theta}_C + \frac{1}{2\gamma_{\Delta_C}}\tilde{\Delta}_C^2 \tag{52}$$

Taking the time-derivative of V_C and using Result 1, along with the control laws (23) and (24), leads to:

$$\begin{aligned}
\dot{V}_C &= \dot{V}_S + \tilde{\theta}_C^T \Gamma_C^{-1} \dot{\tilde{\theta}}_C + \frac{1}{\gamma_{\Delta_C}}\tilde{\Delta}_C \dot{\tilde{\Delta}}_C \\
&= \dot{V}_S - \tilde{\theta}_C^T \varphi_{C_N} \varepsilon_{C_N} + \tilde{\Delta}_C |\varepsilon_{C_N}| - \sigma_C \tilde{\theta}_C^T \hat{\theta}_C - \sigma_{\Delta_C} \tilde{\Delta}_C \hat{\Delta}_C \\
&= \dot{V}_S - \varepsilon_{C_N}^2 + \mu_N \varepsilon_{C_N} - \delta_C \varepsilon_{C_N} + \tilde{\Delta}_C |\varepsilon_{C_N}| \\
&\quad - \sigma_C \tilde{\theta}_C^T \hat{\theta}_C - \sigma_{\Delta_C} \tilde{\Delta}_C \hat{\Delta}_C \\
&\leq \dot{V}_S + \hat{\Delta}_C \left(1 - \eta_C \tanh((a+bt)|\varepsilon_{C_N}|)\right) |\varepsilon_{C_N}| \\
&\quad - \frac{\sigma_C}{2}\tilde{\theta}_C^T \tilde{\theta}_C - \frac{\sigma_{\Delta_C}}{2}\tilde{\Delta}_C^2 + \frac{\sigma_C}{2}\theta_C^T \theta_C + \frac{\sigma_{\Delta_C}}{2}\Delta_C^2
\end{aligned} \tag{53}$$

Notice that the condition:

$$1 - \eta_C \tanh\left((a+bt)|\varepsilon_{C_N}|\right) \leq 0 \tag{54}$$

is satisfied when:

$$|\varepsilon_{C_N}| \geq \nu_C = \frac{1}{a+bt}\ln\left(\frac{\eta_C + 1}{\eta_C - 1}\right), \quad \eta_C > 1 \tag{55}$$

As $t \to \infty$ and $b > 0$, the region defined by ν_C goes to zero, and thus the condition (54) is satisfied as $t \to \infty$. Therefore:

$$\begin{aligned}
\dot{V}_C &\leq \dot{V}_S - \frac{\sigma_C}{2}\tilde{\theta}_C^T \tilde{\theta}_C - \frac{\sigma_{\Delta_C}}{2}\tilde{\Delta}_C^2 + \frac{\sigma_C}{2}\theta_C^T \theta_C + \frac{\sigma_{\Delta_C}}{2}\Delta_C^2 \\
&\leq -c_C V_C + \lambda_C
\end{aligned} \tag{56}$$

where:

$$c_C = \min\left\{ c_S, \frac{\sigma_C}{\lambda_{\max}(\Gamma_C^{-1})}, \gamma_{\Delta_C}\sigma_{\Delta_C} \right\} \tag{57}$$

$$\lambda_C = \lambda_S + \frac{\sigma_C}{2}\theta_C^T \theta_C + \frac{\sigma_{\Delta_C}}{2}\Delta_C^2 \tag{58}$$

As $\lambda_C/c_C > 0$, the following result is obtained:

$$0 \leq V_C(t) \leq \lambda_C/c_C + (V_C(0) - \lambda_C/c_C)\, e^{-c_C t} \tag{59}$$

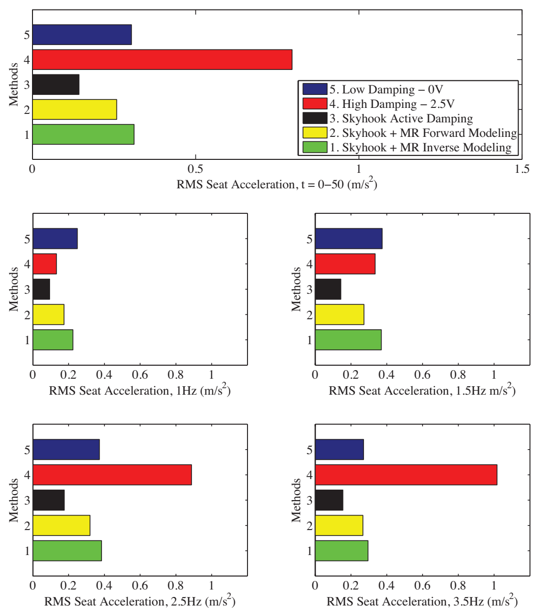

Fig. 4. Comparison of RMS seat acceleration for the entire simulation, and divided into frequency ranges.

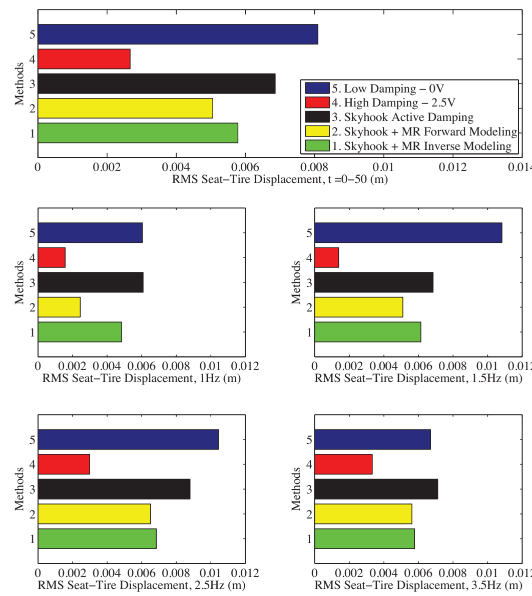

Fig. 5. Comparison of RMS seat-tire displacement for the entire simulation, and divided into different frequency ranges.

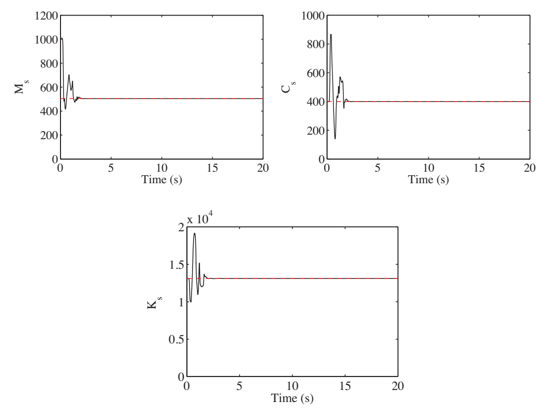

Fig. 6. Convergence of suspension parameter estimates.

Thus, all the error signals associated with the total system, along with the parameter estimation errors $\tilde{\theta}_C$ and $\tilde{\Delta}_C$, are uniformly bounded and converge to a small neighborhood of the origin. Furthermore, if $F_{MR} = F_A$, then the control error ξ also converges to a small neighborhood of the origin. This result is summarized as:

Result 3 *Assume $M_s > 0$ and $\kappa > 0$ are satisfied, and a_0 is known. Then the control law (34), along with the adaptive laws (23), (24), (37) and (38) guarantee that all error signals remain bounded and converge to a small neighborhood of the origin. Furthermore, if $F_{MR} = F_A$, then the control error ξ also converges to a small neighborhood of the origin.*

VI. Simulation Results

Consider a suspension system shown in Fig. 1, where the parameters are set as $M_s = 504.5$ [kg], $M_u = 62$ [kg], $C_s = 400$ [Ns/m], $K_s = 1.31 \times 10^4$ [N/m] and $K_t = 2.52 \times 10^5$ [N/m]. The parameters of the MR damper are specified as: $\sigma_0 = 4.0 \times 10^4$ [N/mV], $\sigma_1 = 2.0 \times 10^2$ [Ns/m], $\sigma_2 = 1.0 \times 10^2$ [Ns/m], $\sigma_a = 1.5 \times 10^4$ [N/m], $\sigma_b = 2.5 \times 10^3$ [Ns/(mV)], $a_0 = 1.9 \times 10^2$, which are all unknown. An upper limit of input voltage to the MR damper is set at 2.5[V], so v varies between 0 to 2.5[V]. The base of the dynamic system in Fig. 1 is excited by the road surface, which is given by a random signal sequence with a frequency range of 0-3.5 Hz. To analyze the effectiveness of each control schemes for various frequency ranges, the road excitation was designed so that the bandwidth increases every ten seconds from 1Hz, 1.5Hz, 2.5Hz to 3.5Hz. The following schemes are compared: (1) Passive low damping with 0 [V] fixed, (2) Passive high damping with 2.5 [V] fixed, (3) Active skyhook-based scheme, (4) Skyhook control with forward modeling based scheme (Proposed) and (5) Skyhook control with inverse modeling based scheme (Proposed). In simulation, an inverse model with $m = 4$ and $n = 1$ is adopted.

Next, the results of the various control algorithms are presented. The damping results are compared by the following criterions: (1) the RMS seat acceleration in Fig. 4, and (2) the RMS positional deflection of the seat and the tire in Fig. 5. It is noted that an apparent trade off exists between minimizing seat acceleration and seat-tire deflection. The passive low damping produces a small damping force and therefore is suited for higher level of frequencies. The passive high damping provides the stiffest damping, and performs better during the low frequency ranges. The trade off between low and high damping can clearly be seen as the bandwidth of the road excitation is increased. The active control meanwhile provides the best performance regardless of the level of excitation. The semi-active forward and inverse modeling schemes also perform better overall than

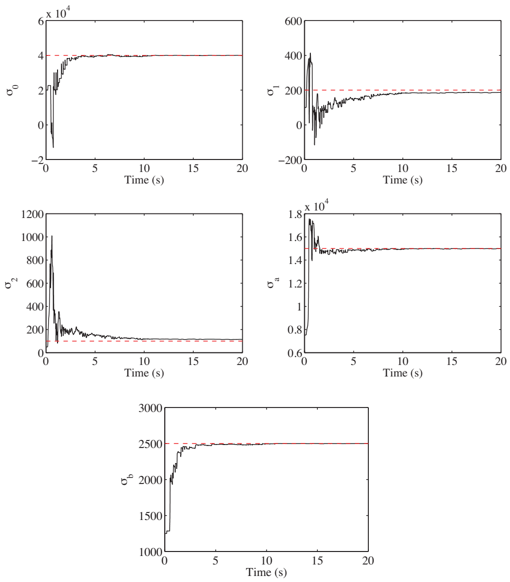

Fig. 7. Convergence of MR parameter estimates for skyhook control with forward modeling.

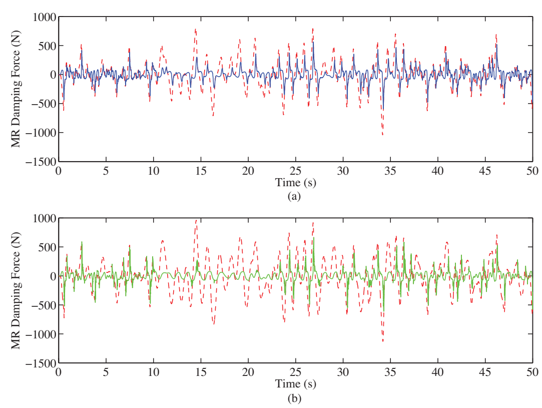

Fig. 8. Plot of F_A (red, dashed) and F_{MR} versus time for (a) skyhook control with forward modeling (blue, solid) and (b) skyhook control with inverse modeling (green, solid).

the fixed damping, as it is able to adjust the stiffness to account for the road excitation. It is clear that the suspension parameters can be identified very rapidly and accurately, as shown in Fig. 6. The convergence of the feedforward modeling parameters are shown in Fig. 7. A comparison of the active and semi-active damping force is given in Fig. 8. The auxiliary function introduced in this research guarantees that the adaptive algorithm remains robust despite this semi-active constraint.

VII. Conclusion

We have presented the fully adaptive semiactive control algorithm which consists of the adaptive inverse controller compensating for nonlinear hysteresis dynamics of MR damper, and the adaptive reference controller matching the seat response to a reference dynamics even if the mass and spring constants are unknown. The forward modeling or inverse modeling scheme was introduced for realizing the adaptive inverse controller. The stability conditions for the total system consisting of two adaptive controllers have been clarified, and the effectiveness of the proposed scheme has been validated in numerical simulation.

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