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Output Estimator Based Fault Detection for a Class of Nonlinear Systems with Unknown Inputs

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Abstract— **Fault detection problem is studied using output estimator design rather than observer design for a class of nonlinear systems with unknown inputs. In order to carry out the output estimator design, an input-output relation is derived from the original state space model, which removes the effect of the unknown inputs on fault detection completely. Based on the input-output relation, a necessary and sufficient condition is derived for the existence of an output estimator which is invariant to the unknown inputs. An output estimator is designed based on the input-output relation for the purpose of fault detection, and a novel fault detection strategy using output estimator design is proposed which neither has the relative degree restriction, nor requires the system under consideration to be detectable. The efficacy of the proposed fault diagnosis strategy is tested on a single-link flexible robot manipulator model thorough computer simulations.**

I. INTRODUCTION

Unknown inputs which could account for uncertainties and/or unknown disturbances present a challenge to model based fault diagnosis. To deal with various types of unknown inputs, observer design based strategy has been developed to remove their effect completely by designing fault diagnosis schemes that are invariant to the unknown inputs. Some unknown input observer (UIO) and sliding mode observer (SMO) based schemes adopt this strategy. For example, UIO based schemes can be found in [1], [2], [3], while SMO based ones can be found in [9]–[14].

One limitation of observer design based strategy is that it often requires restrictive matching conditions. For example, conventional UIOs or SMOs require the relative degrees from the known systems inputs and/or the unknown inputs to the system outputs to be no larger than one [1], [2], [3], [9]–[14]. Higher order sliding mode observers, though removes the relative degree limitation, require the system under consideration has or can be put into a triangular system form [15]. For systems that do not satisfy the relaxed matching conditions in [15], it is not clear whether observer based fault diagnosis schemes can be designed. Another obvious requirement of observer design based strategy is that the system under consideration has to be detectable.

When the system under consideration does not satisfy the relaxed matching in [15] or is not even detectable, the observer design based fault diagnosis strategy is either extremely difficult or impossible. In order to develop a new fault diagnosis strategy that requires less restrictive matching conditions or even when system is not detectable, a new strategy which uses output estimator design and does not rely on observer design was employed in [16], [17], [18], [19], [20], [21]. In [16], robust output estimators were designed based on the state space model and system decomposition to deal with unmatched unknown inputs. Unlike in [16], the output estimators designed in [17], [18], [19] were based input-output relations using the recently developed high order sliding mode differentiators. Although the adaptive output estimators in [20], [21] were also based on input-output relations, they took a very different approach from those in [17], [18], [19], which can avoid the use of higher order derivatives of the outputs. However, unknown inputs were not considered in [20], [21].

The purpose of this paper is to employ the approach in [20], [21] to develop a novel output estimator based fault detection strategy, which does not have the relative degree restriction, is applicable to systems that are not detectable, and is capable of removing the effect of unknown inputs completely.

The remainder of the paper is arranged as follows: In Section 2, a system model is introduced and the problem of interest is formulated. In Section 3, an input-output relation is established. In Section 4, an output estimator is designed based on the input-output relation for the purpose of fault detection. In Section 5, a fault detection scheme is proposed based on the output estimator. The proposed fault detection scheme is tested on a single-link flexible robot manipulator model in Section 6 and simulation results are provided. Finally, concluding remarks are made in the last section.

II. SYSTEM AND PROBLEM FORMULATION

Consider MIMO nonlinear systems described as below

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Ff(y(t)) + Bu(t) + Dd(t) \\ y(t) &= Cx(t) \end{aligned} \tag{1}$$

where $x(t)$, $y(t)$, $u(t)$, and $d(t)$ are the system state vector, output vector, known input vector, and unknown input vector respectively, and $x(t) \in R^n$, $y(t) = (y_1(t) \ \cdots \ y_p(t))^T$, $u(t) = (u_1(t) \ \cdots \ u_m(t))^T$, $d(t) = (d_1(t) \ \cdots \ d_\gamma(t))^T$, $f(y(t)) = (f_1(y(t)) \ \cdots \ f_q(y(t)))^T$ is a vector of nonlinear functions.

- Assumption A1: The system matrices A, B, C, D, and F are known.
- Assumption A2: $f_j(y(t)), j = 1, \cdots, q$ are known.

The following problem is formulated.

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Output Estimator Based Fault Detection Problem:
Under the condition that assumptions A1 and A2 are satisfied, design a fault diagnosis scheme for system (1) using output estimator design such that it can detect faults regardless of the presence of unknown inputs.

For system (1), if $rank(CD) \neq rank(D)$, conventional SMOs or UIOs in [1], [2], [3], [9]–[14] can no longer be designed to be invariant to the unknown inputs. It is not clear how to design high order sliding mode observers if the relaxed matching conditions in [15] are not matched. Moreover, if system (1) is not detectable, no observers can be designed to estimate all the states asymptotically. Because of all these difficulties in observer design, the idea of observer based fault diagnosis is abandoned in this paper. Instead, a novel fault detection strategy using output estimator design is proposed. As will be shown later in this paper, it is the idea of designing output estimators rather than state observers that leads to an elegant solution to the formulated Output Estimator Based Fault Detection Problem.

III. AN INPUT-OUTPUT RELATION

It is found that trying to estimate all the outputs directly from the MIMO system given by (1) without observer design is very difficult. This observation motivates us to use the idea proposed in [21], that is, to transform the difficult MIMO output estimator design problem into several simpler MISO output estimator design problems through decomposing (1) into a group of MISO systems, as will be shown in the sequel.

Let $C = (C_1^T \ \cdots \ C_p^T)^T$, $B = (B_1 \ \cdots \ B_m)$, $F = (F_1 \ \cdots \ F_q)$, and $D = (D_1 \ \cdots \ D_\gamma)$. It is obvious that a MIMO system given by (1) can be decomposed into p MISO systems, where for $1 \leq j \leq p$, the jth MISO system is of the following form

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Ff(y(t)) + Bu(t) + Dd(t), \\ y_j(t) &= C_j x(t). \end{aligned} \tag{2}$$

Because (C_j, A) is not necessarily detectable even when (C, A) is, it is not appropriate to design any observer for the MISO system defined by (2). Therefore, the outputs will not be estimated through observer design in this paper.

In order to be able to estimate the outputs without designing observers for the MISO systems defined by (2), the input-output relation of $u(t), d(t)$ and $y_j(t)$ described by the following transfer function will be used.

$$\begin{aligned} y_j(t) &= \sum_{l=1}^{m} G_{jl}(s) u_l(t) \\ &+ \sum_{k=1}^{q} G_{jk}(s) f_k(y(t)) + \sum_{r=1}^{\gamma} G_{jr}(s) d_r(t) \end{aligned} \tag{3}$$

where for $1 \leq j \leq p$, $1 \leq l \leq m$, $1 \leq k \leq q$, and $1 \leq r \leq \gamma$,

$$\begin{aligned} G_{jl}(s) &= C_j(sI - A)^{-1} B_l \\ &= \frac{b_{jl,n-1}s^{n-1} + \cdots + b_{jl,1}s + b_{jl,0}}{s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0}, \end{aligned} \tag{4}$$

$$\begin{aligned} G_{jk}(s) &= C_j(sI - A)^{-1} F_k \\ &= \frac{f_{jk,n-1}s^{n-1} + \cdots + f_{jk,1}s + f_{jk,0}}{s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0}, \end{aligned} \tag{5}$$

$$\begin{aligned} G_{jr}(s) &= C_j(sI - A)^{-1} D_r \\ &= \frac{d_{jr,n-1}s^{n-1} + \cdots + d_{jr,1}s + d_{jr,0}}{s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0}, \end{aligned} \tag{6}$$

and $s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0 = det(sI - A)$.

For convenience, define $a(s) = s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0$, $b_{jl}(s) = b_{jl,n-1}s^{n-1} + \cdots + b_{jl,1}s + b_{jl,0}$, $f_{jk}(s) = f_{jk,n-1}s^{n-1} + \cdots + f_{jk,1}s + f_{jk,0}$, and $d_{jr}(s) = d_{jr,n-1}s^{n-1} + \cdots + d_{jr,1}s + d_{jr,0}$.

In the remaining part of this paper, the dependence of variables on time t will not be shown explicitly for the sake of simplicity, for example, $u_j(t)$ will be simply written as u_j. For each $1 \leq j \leq p$, based on (3), (4), and (5), and inspired by [22] and [23], the following state space realization can be given for (3).

$$\begin{aligned} \dot{x}_j &= \bar{A}x_j - ay_j + b_{j1}u_1 + \cdots + b_{jm}u_m \\ &\quad + f_{j1}f_1(y) + \cdots + f_{jq}f_q(y) \\ &\quad + d_{j1}d_1 + \cdots + d_{j\gamma}d_\gamma \\ y_j &= x_{j,1} \end{aligned} \tag{7}$$

where $x_j = (x_{j,1} \ \cdots \ x_{j,n})^T$ and

$$\begin{aligned} \bar{A} &= \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \ddots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 & 1 \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix}, \quad a = \begin{bmatrix} a_{n-1} \\ \vdots \\ a_0 \end{bmatrix}, \\ b_{jl} &= \begin{bmatrix} b_{jl,n-1} \\ \vdots \\ b_{jl,0} \end{bmatrix}, \ 1 \leq l \leq m, \\ f_{jk} &= \begin{bmatrix} f_{jk,n-1} \\ \vdots \\ f_{jk,0} \end{bmatrix}, \ 1 \leq k \leq q, \\ d_{jr} &= \begin{bmatrix} d_{jr,n-1} \\ \vdots \\ d_{jr,0} \end{bmatrix}, \ 1 \leq r \leq \gamma. \end{aligned} \tag{8}$$

It is important to note that (7), which is observable, is *not* the same as (2), which might not be observable. It is also crucial to know that the outputs for both (7) and (2) are the same. Therefore, (7) can always be used to estimate y_j regardless of whether (2) is observable or not. This implies that it is not necessary to require the original system (1) to be observable for the purpose of output estimation.

For each $1 \leq j \leq p$, in order to estimate y_j, it is required to first derive the state estimate for (7). To do so, $u_l, 1 \leq$

$l \leq m$, y_j, $f_k(y), 1 \leq k \leq q$, and $d_r, 1 \leq r \leq \gamma$ are filtered by the following $n-$dimensional filters:

$$\dot{\lambda}_l = A_0\lambda_l + e_n u_l, \quad 1 \leq l \leq m, \tag{9}$$

$$\dot{\eta}_j = A_0\eta_j + e_n y_j, \tag{10}$$

$$\dot{\nu}_k = A_0\nu_k + e_n f_k(y), 1 \leq k \leq q, \tag{11}$$

and

$$\dot{\phi}_r = A_0\phi_r + e_n d_r, 1 \leq r \leq \gamma, \tag{12}$$

where $A_0 = \bar{A} - K(1\ 0\ \cdots\ 0)$ and $K = (K_1 \cdots K_n)^T$ is chosen such that A_0 is Hurwitz, and for any $1 \leq i \leq n$, $e_i = (e_{i,1}, \cdots, e_{i,n})^T \in R^n$ is defined by $e_{i,i} = 1$ and $e_{i,j} = 0$ for $j \neq i$.

After some matrix manipulations, it can be shown that

$$\begin{aligned} a(A_0)e_n &= a - K, \\ b_{jl}(A_0)e_n &= b_{jl}, \quad 1 \leq l \leq m, \\ f_{jk}(A_0)e_n &= f_{jk}, \quad 1 \leq k \leq q, \\ d_{jr}(A_0)e_n &= d_{jr}, \quad 1 \leq r \leq \gamma, \end{aligned} \tag{13}$$

where $a(A_0)$, $b_{jl}(A_0), 1 \leq l \leq m$, $f_{jk}(A_0), 1 \leq l \leq q$, and $d_{jr}(A_0), 1 \leq r \leq \gamma$ are matrix polynomials with $a(s)$, $b_{jl}(s), 1 \leq l \leq m$, $f_{jk}(s), 1 \leq k \leq q$, and $d_{jr}(s), 1 \leq r \leq \gamma$ being defined earlier.

Now the estimate for x_j is formed as

$$\begin{aligned} \hat{x}_j &= \sum_{l=1}^{m} b_{jl}(A_0)\lambda_l + \sum_{k=1}^{q} f_{jk}(A_0)\nu_k \\ &\quad -a(A_0)\eta_j + \sum_{r=1}^{\gamma} d_{jr}(A_0)\phi_r \end{aligned} \tag{14}$$

Using (7) and (9)-(14), it can be verified that the estimation error $\varepsilon_j = (\varepsilon_{j,1}, \varepsilon_{j,2}, \cdots, \varepsilon_{j,n})^T = x_j - \hat{x}_j$ satisfies $\dot{\varepsilon}_j = A_0\varepsilon_j$. Denote $\xi_{ji} = A_0^i\eta_j, 0 \leq i \leq n-1$, $\xi_{jn} = -A_0^n\eta_j$, $\upsilon_{li} = A_0^i\lambda_l, 0 \leq i \leq n-1, 1 \leq l \leq m$, $\varphi_{ki} = A_0^i\nu_k, 0 \leq i \leq n-1, 1 \leq k \leq q$, and $\omega_{ri} = A_0^i\phi_r, 0 \leq i \leq n-1, 1 \leq r \leq \gamma$, then (14) can be rewritten as

$$\begin{aligned} x_j &= \xi_{jn} - \sum_{i=0}^{n-1} a_i\xi_{ji} + \sum_{l=1}^{m}\sum_{i=0}^{n-1} b_{jl,i}\upsilon_{li} \\ &+ \sum_{k=1}^{q}\sum_{i=0}^{n-1} f_{jk,i}\varphi_{ki} + \sum_{r=1}^{\gamma}\sum_{i=0}^{n-1} d_{jr,i}\omega_{ri} + \varepsilon_j \end{aligned} \tag{15}$$

It can be checked that all the ξ, υ, and φ signals and their derivatives are explicitly available:

$$\begin{aligned} \xi_{jn} &= -A_0^n\eta_j, \quad \dot{\xi}_{jn} = A_0\xi_{jn} + ky_j, \\ \xi_{ji} &= A_0^i\eta_j, \quad \dot{\xi}_{ji} = A_0\xi_{ji} + e_{n-i}y_j, \\ \upsilon_{li} &= A_0^i\lambda_l, \quad \dot{\upsilon}_{li} = A_0\upsilon_{li} + e_{n-i}u_l, \\ \varphi_{ki} &= A_0^i\nu_k, \quad \dot{\varphi}_{ki} = A_0\varphi_{ki} + e_{n-i}f_k(y), \end{aligned} \tag{16}$$

where $0 \leq i \leq n-1, 1 \leq l \leq m, 1 \leq k \leq q$.

However, because the unknown input vector d is not available, none of the ω signals and their derivatives are computable. Therefore, the estimate given by (14) can not be applied directly.

For each $1 \leq j \leq p$, under a no fault scenario, it follows from (7) and (15) that

$$\begin{aligned} \dot{y}_j &= \xi_{jn,2} - (\xi_{j(2)} + e_1^T y_j)a \\ &+ \sum_{l=1}^{m}(\upsilon_{l(2)} + e_1^T u_l)b_{jl} + \sum_{k=1}^{q}(\varphi_{k(2)} + e_1^T f_k(y))f_{jk} \\ &+ \sum_{r=1}^{\gamma}(\omega_{r(2)} + e_1^T d_r)d_{jr} + \varepsilon_{j,2} \\ &= \xi_{jn,2} - (\xi_{j(2)} + e_1^T y_j)a \\ &+ \sum_{l=1}^{m} b_{jl}^T(\upsilon_{l(2)} + e_1^T u_l)^T + \sum_{k=1}^{q} f_{jk}^T(\varphi_{k(2)} + e_1^T f_k(y))^T \\ &+ \sum_{r=1}^{\gamma} d_{jr}^T(\omega_{r(2)} + e_1^T d_r)^T + \varepsilon_{j,2} \end{aligned} \tag{17}$$

where $\xi_{jn}^T = (\xi_{jn,1}, \xi_{jn,2}, \cdots, \xi_{jn,n})$, $\xi_{(2)} = (\xi_{j(n-1),2}, \cdots, \xi_{j0,2})$, $\upsilon_{l(2)} = (\upsilon_{l(n-1),2}, \cdots, \upsilon_{l0,2})$, $\varphi_{k(2)} = (\varphi_{k(n-1),2}, \cdots, \varphi_{k0,2})$, and $\omega_{r(2)} = (\omega_{k(n-1),2}, \cdots, \omega_{k0,2})$.

Let's define vectors $\xi_{n,2} = (\xi_{1n,2}, \cdots, \xi_{pn,2})^T$ and $\varepsilon_2 = (\varepsilon_{1,2}, \cdots, \varepsilon_{p,2})^T$ and several matrices $M_y = ((\xi_{1(2)} + e_1^T y_1)^T, \cdots, (\xi_{p(2)} + e_1^T y_p)^T)^T$, $M_{u,l} = (b_{1l}, \cdots, b_{pl})^T$, $M_{f,k} = (f_{1k}, \cdots, f_{pk})^T$, and $M_{d,r} = (d_{1r}, \cdots, d_{pr})^T$, then, it follows from (17) that

$$\begin{aligned} \dot{y} &= \xi_{n,2} - M_y a + \sum_{l=1}^{m} M_{u,l}(\upsilon_{l(2)} + e_1^T u_l)^T \\ &+ \sum_{k=1}^{q} M_{f,k}(\varphi_{k(2)} + e_1^T f_k(y))^T \\ &+ \sum_{r=1}^{\gamma} M_{d,r}(\omega_{r(2)} + e_1^T d_r)^T + \varepsilon_2 \end{aligned} \tag{18}$$

(18) reveals the effect of the unknown inputs on the outputs explicitly through the term $\sum_{r=1}^{\gamma} M_{d,r}(\omega_{r(2)} + e_1^T d_r)^T$. In (18), $\omega_{r(2)} + e_1^T d_r$ is unknown because d is unknown. However, the matrix $M_{d,r}$ is known because it depends only on the known system matrices A, C, D. Because $M_{d,r}, 1 \leq r \leq \gamma$ are all known, one can eliminate the effect of the unknown inputs by choosing a nonzero matrix T_0 of full row rank such that $T_0 M_{d,r} = 0$ for all $1 \leq r \leq \gamma$.

By defining a new output vector as $\bar{y} = T_0 y$, it follows from (18) that

$$\begin{aligned} \dot{\bar{y}} &= T_0\xi_{n,2} - T_0 M_y a + \sum_{l=1}^{m} T_0 M_{u,l}(\upsilon_{l(2)} + e_1^T u_l)^T \\ &+ \sum_{k=1}^{q} T_0 M_{f,k}(\varphi_{k(2)} + e_1^T f_k(y))^T \\ &+ T_0\varepsilon_2 \end{aligned} \tag{19}$$

On the right hand side of (19), only ε_2 is not available but approaches zero exponentially. This observation together with the definition of $\bar{y}$ makes (19) ready for output estimator for $\bar{y}$. Because (19) only involves the system outputs and the known system inputs, it is called an input-output relation in this paper.

Before one can design an estimator for $\bar{y}$, one has to guarantee the existence of $\bar{y}$, that is, the existence of a nonzero matrix T_0.

Let us define

$$\begin{aligned}(sI-A)^{-1} &= \frac{1}{a(s)}(R_0 s^{n-1} \\ &+ R_1 s^{n-2} + \cdots + R_{n-2}s + R_{n-1}). \qquad (20)\end{aligned}$$

A necessary and sufficient condition for the existence of a nonzero matrix T_0 is given in the following theorem.

Theorem 1: Under Assumptions A1 and A2, a nonzero matrix T_0 of full row rank exists such that $T_0 M_{d,r} = 0$ for all $1 \leq r \leq \gamma$ if and only if $rank(C(R_0 D \cdots R_{n-2}D\ R_{n-1}D)) < p$.

Proof: (20) together with the definition of $G_{jr}(s)$ implies that

$$\begin{aligned}G_{jr}(s) &= C_j(sI-A)^{-1}D_r \\ &= \frac{1}{a(s)}(C_jR_0D_rs^{n-1} + C_jR_1D_rs^{n-2} \\ &+ \cdots + C_jR_{n-2}D_rs + C_jR_{n-1}D_r) \qquad (21)\end{aligned}$$

By comparing this expression of $G_{jr}(s)$ with the second equality in (6), it is easy to see that

$$d_{jr}^T = C_j(R_0D_r \ \cdots \ R_{n-2}D_r\ R_{n-1}D_r). \qquad (22)$$

Based on (22) and the definition of $M_{d,r}$, it is easy to get

$$M_{d,r} = C(R_0D_r \ \cdots \ R_{n-2}D_r\ R_{n-1}D_r) \qquad (23)$$

The above equation implies that

$$(M_{d,1} \ \cdots \ M_{d,\gamma}) = C(R_0D \ \cdots \ R_{n-2}D\ R_{n-1}D)T_{trans}, \qquad (24)$$

where T_{trans} is an invertible matrix.

It is obvious that $T_0M_{d,r} = 0$ for all $1 \leq r \leq \gamma$ is equivalent to $T_0C(R_0D \ \cdots \ R_{n-2}D\ R_{n-1}D) = 0$. Since the matrix $C(R_0D \ \cdots \ R_{n-2}D\ R_{n-1}D)$ has p rows, it follows that a nonzero T_0 exists such that $T_0C(R_0D \ \cdots \ R_{n-2}D\ R_{n-1}D) = 0$ if and only if $rank(C(R_0D \ \cdots \ R_{n-2}D\ R_{n-1}D)) < p$. This completes the proof. ¶

IV. FAULT DETECTION BASED ON OUTPUT ESTIMATOR DESIGN

In this section, a fault detection scheme will be proposed using output estimator design based on the input-output relation given by (19).

Suppose a nonzero matrix T_0 exists and is obtained, then, based on (19), an estimator for $\bar{y}$ can be given as follows.

$$\begin{aligned}\dot{\hat{\bar{y}}} &= -c_M(\hat{\bar{y}} - \bar{y}) + T_0\xi_{n,2} - T_0M_ya \\ &+ \sum_{l=1}^{m} T_0M_{u,l}(\upsilon_{l(2)} + e_1^Tu_l)^T \\ &+ \sum_{k=1}^{q} T_0M_{f,k}(\varphi_{k(2)} + e_1^Tf_k(y))^T \qquad (25)\end{aligned}$$

where c_M is a positive definite diagonal design matrix.

The following result is obtained.

Theorem 2: Under Assumptions A1 and A2, if there is no fault in the control system, then $lim_{t\to\infty}\|\hat{\bar{y}} - \bar{y}\| = 0$.

Proof: Define $e_y = \hat{\bar{y}} - \bar{y}$, it follows from (19) and (25) that

$$\dot{e}_y = -c_Me_y - T_0\varepsilon_2 \qquad (26)$$

Note that ε_2 approaches zero exponentially and c_M is a positive definite diagonal matrix, the conclusion of theorem follows from (26) immediately. ¶

Theorem 2 serves as a foundation for fault detection. If a residual is defined as $r = \|\hat{\bar{y}} - \bar{y}\|$ and there is no fault in presence, then, according to Theorem 2, r will tend to zero. Hence, a fault is declared if it becomes nonzero. To be specific, a fault detection scheme is proposed as follows.

1) Solve equations (25) to obtain $\hat{\bar{y}}$.
2) Compute the residual r.
3) Choose a threshold ϵ.
4) Compare the residual r with the threshold ϵ. If it exceeds its corresponding threshold, faults are detected.

Theoretically speaking, the threshold, that is, ϵ could be chosen arbitrarily small. However, in practical situations, because other unconsidered uncertainties may exist, too small ϵ may lead to too many false alarms. On the other hand, too large ϵ may increase the missed detections. Trade-off has to be made on the choice of a suitable threshold.

V. AN EXAMPLE AND SIMULATION RESULTS

In this section, a single-link flexible robot manipulator model in [24] is used to test the proposed fault detection scheme. The single-link flexible robot manipulator model in [24] takes the following form

$$\begin{aligned}J_1\ddot{q}_1 + F_1\dot{q}_1 + K(q_1 - \frac{q_2}{N}) + mgdcosq_1 &= 0 \\ J_2\ddot{q}_2 + F_2\dot{q}_2 - \frac{K}{N}(q_1 - \frac{q_2}{N}) &= K_ti \\ LDi + Ri + K_b\dot{q}_2 &= u \qquad (27)\end{aligned}$$

According to [24], after suitable change of variables, (27) can be made into the following form

$$\begin{aligned}\dot{x}_1 &= x_2 + \theta_1x_1 \\ \dot{x}_2 &= x_3 + \theta_2x_1 + \theta_3cosx_1 \\ \dot{x}_3 &= x_4 + \theta_4x_1 + \theta_5cosx_1 \\ \dot{x}_4 &= x_5 + \theta_6x_1 + \theta_7cosx_1 \\ \dot{x}_5 &= \theta_8cosx_1 + b_0u \qquad (28)\end{aligned}$$

Due to lack of space, the physical meanings of all variables and notations in this section are omitted but can be found in [24]. Assume that all parameters are known.

Since this model does not have any unknown input, to test the proposed scheme, the nonlinear function $cosx_1$ in $\theta_5 cosx_1$ and $\theta_7 cosx_1$ is treated as an unknown input, and thus one has $D = (0\ 0\ \theta_5\ \theta_7\ 0)^T$. Assume that $y_1 = x_1$ and $y_2 = x_5$, (28) can be rewritten as

$$\begin{aligned} \dot{x} &= Ax + Bu + Fcosy_1 + Dd(t), \\ y &= Cx, \end{aligned} \tag{29}$$

where $d(t) = cosx_1$, $F = (0\ \theta_3\ 0\ \ 0\ \theta_8)^T$, and

$$A = \begin{pmatrix} \theta_1 & 1 & 0 & 0 & 0 \\ \theta_2 & 0 & 1 & 0 & 0 \\ \theta_4 & 0 & 0 & 1 & 0 \\ \theta_6 & 0 & 0 & 0 & 1 \end{pmatrix}, \ C = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}.$$

Obviously, one has $CD = 0$, which means conventional SMOs and UIOs can not be designed to remove the effect the unknown input. However, it is easy to see that (29) takes the form (1). Therefore, the fault detection scheme developed can be readily applied to the single-link flexible robot manipulator model.

Simulations are done based on (29). The design parameters are chosen as $K = (17.5\ 120\ 402.5\ 659\ 420)^T$, c_M(which is one dimensional for this example) is chosen as 2. Three fault cases are considered.

- Case A The actuator is stuck at a constant, that is, $u = 0$ for $t > 20s$.
- Case B The sensor for x_5 has a scaling error, that is, $y_2(t) = 0.9x_5(t)$ for $t > 6s$, where $x_5(t)$ is the real output, and $y_2(t)$ is the measurement provided by the sensor.
- Case C The actuator has a slow changing scaling error (an incipient fault), that is, $u(t) = (1 - 0.1(1 - e^{-0.005t}))u^*(t)$ for $t > 20s$.

The results for Case A and Case B are presented in Figure 1 and Figure 2, respectively. After the presence of an actuator fault at $t = 20s$, the residual in Figure 1 exceeds the threshold 0.05 at $t = 20.02s$. After the presence of a scaling sensor fault at $t = 6s$, the first residual in Figure 2 exceeds the threshold 0.05 at $t = 6.01s$. Both figures show that abrupt faults can be detected correctly very quickly.

The results for Case C are presented in Figure 3, where the two straight lines represent two thresholds, that is, 0.05 and 0.01, respectively. For threshold 0.05, the fault detection time is $t = 17.76s$, which is much longer than the abrupt faults. For threshold 0.01, the fault detection time is $t = 0.05s$, which is quite fast.

In summary, for the considered example, the proposed fault detection scheme is able to detect both abrupt and incipient faults successfully. Moreover, it is observed that a larger threshold may be chosen for abrupt faults and a smaller threshold should be used in order to detect the faults in a timely manner for incipient faults.

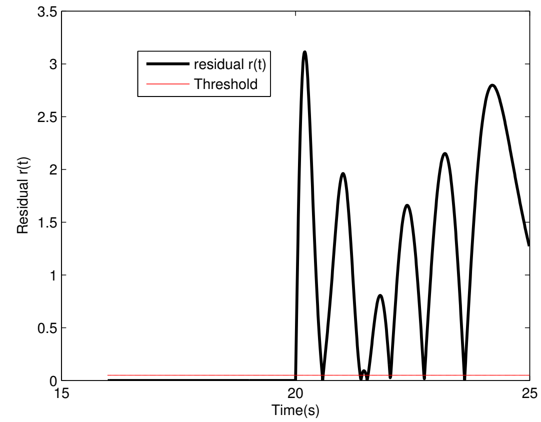

Fig. 1. Detection of abrupt actuator faults

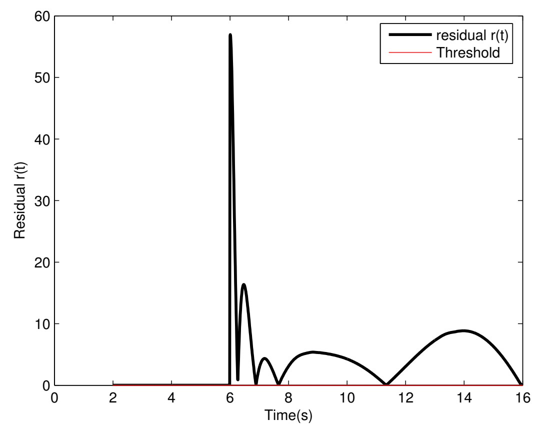

Fig. 2. Detection of abrupt sensor faults

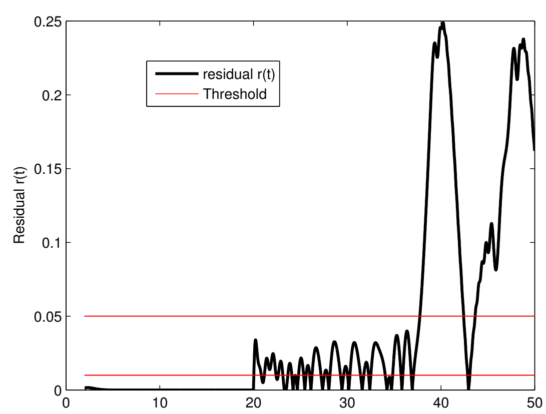

Fig. 3. Detection of incipient actuator faults

VI. CONCLUSIONS AND FUTURE WORK

A. Conclusions

In this paper, fault detection problem in a class of nonlinear systems with unknown inputs was solved using output estimator design. A necessary and sufficient condition for the existence of the fault detection output estimator was derived. Simulation results on a single-link flexible robot manipulator model showed the proposed fault detection scheme was able to detect faults correctly.

B. Future Works

Because both observer design and output estimator design can be used to eliminate the effect of unknown inputs under different conditions, comparisons between the two design strategies are needed to ascertain the strength and weakness of each strategy in the future.

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REFERENCES

[1] M. Saif and Y. Guan "A new approach to robust fault detection and identification," *IEEE Transactions on Aerospace and Electronic Systems,* vol. 29, pp. 685-695, 1993.

[2] J. Chen, R.J. Patton and Hong-Yue Zhang, "Design of unknown input observer and robust fault detection filters," *International Journal of Control,* vol. 63, pp. 85-105, 1996.

[3] Y. Xiong and M. Saif, "A novel design for robust fault diagnostic observers,"*Proceedings of the IEEE CDC,* Florida, 1998, pp. 592-597.

[4] H. Yang and M. Saif, "State observation, failure detection and isolation(FDI) in bilinear systems," *International Journal of Control,* vol. 67, pp. 901-920, 1997.

[5] R. Rajamani and A. Ganguli, "Sensor fault diagnosis for a class of nonlinear systems using linear matrix inequalities," *International Journal of Control,* vol.77, No.10, 920-930,2004.

[6] W. Chen and M. Saif, " Unknown Input Observer Design for a Class of Nonlinear Systems: an LMI Approach," *Proceedings of the ACC,* Minneapolis, Minnesota, USA, June 2006, pp. 834 - 838.

[7] R. Seliger and P.M. Frank, "Fault diagnosis by disturbance decoupled nonlinear observers,"*Proceedings of the IEEE CDC,* Brighton, England, 1991, pp. 2248-2253.

[8] R. Seliger and P.M. Frank, "Robust component fault detection and isolation in nonlinear dynamic systems using nonlinear unknown input observers," *Proceedings of the IFAC/IMACS Symposium SAFEPROCESS'91,* Baden-Baden, Germany,1991, pp.313-318.

[9] H. Yang and M. Saif, "Nonlinear adaptive observer design for fault detection," *Proceedings of the ACC,* Seattle, 1995, pp. 1136-1139.

[10] C. Edwards, S.K. Spurgeon, and R.J. Patton, " Sliding mode observers for fault detection and isolation," *Automatica,* vol. 36, No 4, pp.541-553, 2000.

[11] Y. Xiong and M. Saif, "Robust and nonlinear fault diagnosis using sliding mode observers,"*Proceedings of the IEEE CDC,* 2001, pp. 567-572.

[12] C.P. Tan and C. Edwards, "Sliding mode observers for reconstruction of simultaneously actuator and sensor faults,"*Proceedings of the IEEE CDC,* Maui, Hawaii USA, 2003, pp. 1455-1460.

[13] T. Floquet, J. P. Barbot, W. Perruquetti and M. Djemai"On the robust fault detection via a sliding mode observer," *International Journal of Control,* vol. 77, pp. 622-629, 2004.

[14] W. Chen and M. Saif, "Actuator fault isolation and estimation for uncertain nonlinear systems, " *Proceedings of IEEE SMC,* Hawaii, 2005, pp. 2560-2565.

[15] T. Floquet and J.P. Barbot, "A sliding mode approach of unknown input observers for linear systems," *Proceedings of the IEEE CDC,* Bahamas, 2004, pp. 1724-1729.

[16] W. Chen and M. Saif, "Output estimator based fault detection, isolation and estimation for systems with unmatched unknown inputs ," *IEEE Joint CCA/CACSD/ISIC*, Munich, Germany, 2006, pp. 1753 - 1758.

[17] W. Chen and M. Saif, "High-order sliding-mode differentiator based actuator fault diagnosis for linear systems with arbitrary relative degree and unmatched Unknown Inputs,"*Proceedings of the IEEE CDC*, San Diego, CA, 2006, pp. 1153-1158.

[18] W. Chen and M. Saif, "Actuator fault diagnosis for uncertain linear systems using a high order sliding mode robust differentiator (HOSMRD)," *International Journal of Robust and Nonlinear Control*, Vol. 18, No.4-5, pp. 413-426, 2008.

[19] W. Chen and M. Saif, "Actuator fault diagnosis in affine nonlinear systems with unknown inputs," *Proceedings of the IEEE CDC*, New Orleans, Louisiana USA, 2007.

[20] W. Chen and M. Saif, "Adaptive actuator fault Detection, isolation and accommodation (FDIA) in uncertain systems," *International Journal of Control*, Vol. 80, pp. 45-63, 2007.

[21] W. Chen and M. Saif, "Adaptive sensor fault detection and isolation in uncertain MIMO linear systems," *Proceedings of the ACC*, New York, USA, 2007, pp. 3240-3245.

[22] G. Kreisselmeier, "Adaptive observers with exponential rate of convergence," *IEEE Trans. On Automatic Control*, Vol. 22, pp. 2-8, 1977.

[23] M. Krstic, I. Kanellakopoulos, and P.V. Kokotovic, "Nonlinear design of adaptive controllers for linear systems," *IEEE Trans. On Automatic Control*, Vol. 39, pp. 738-752, 1994.

[24] I. Kanellakopoulos, P.V. Kokotovic, and A.S. Morse, "Adaptive output-feedback control of a class of noninear systems," *Proceedings of the CDC*, Brighton, England, 1991, pp. 1082-1087.