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Discrete Time Lyapunov Stable Staggered Estimator MIMO Adaptive Interference Cancelation with Experimental Verification

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***Abstract*— In this paper we develop a closed-loop discrete-time interference cancelation algorithm. The novel features of this algorithm are its ability to deal with multiple channels being affected by interferences with different frequency spectrums. Also we provide a proof of Lyapunov stability of closed loop system and asymptotically perfect interference cancelation for a class of interference signals. Furthermore we introduce a new approach for updating the estimator through the use of staggered estimate. The goal of staggered estimation is to minimize the total number of a estimates / calculations done within a time period while ensuring that there is no estimator aliasing. Finally the proposed algorithm is implemented on a TMS320C6713 DSP Kit and experimental verification obtained.**

I. INTRODUCTION

Traditionally notch filters have been used to eliminate sinusoidal interferences in an information bearing signal. A fixed notch filter can eliminate the noise when its distribution is centered exactly at the frequency for which the filter is designed [1]. If the frequency of interference signal is unknown or if it drifts slowly then an adaptive notch filter is required [2]. Adaptive notch filters can be implemented using a feed forward or a feed back structure [3]. While the feed forward structure alleviates stability concerns, it requires a measurement of the interference or a signal correlated with the interference [2, p.24]. In the feedback structure direct measurement of the disturbance is not required but stability is not guaranteed and needs to be demonstrated.

In this paper we consider the problem of simultaneous multiple channel interference cancelation via a multi variable feedback control law.The Lyapunov stability of the closed loop system is demonstrated by using the Lyapunov function candidate whose difference is shown to be non positive. Asymptotically perfect interference cancelation for a class of disturbances is demonstrated through the use of lemma on maximally monotonically increasing subsequences.

From the algorithm implementation point of view computation of the parameter estimate at each step entails the use of a sophisticated and therefore expensive digital signal processor . In order to make use of less capable and therefore less expensive components for implementation, it is some time desirable not to update the parameter estimates at each time step but to allow a number of time step before an update is made. This scheme can however lead to estimator aliasing in which although the parameter error is growing periodically it is zero at the update instants. To circumvent this problem we propose staggered estimator updating in this paper.

The contents of the paper are as follows. In section II we describe controller model. In section III we formulate the closed-loop system. The adaptive algorithm with regular estimator updating is described in section IV. In section V we discuss an algorithm with staggered estimator updating and provide a complete proof of Lyapunov stability of the closed-loop system and asymptotically perfect disturbance rejection for bounded time invariant interference. Computer simulations of the proposed algorithm are used to reject reject tonal and amplitude modulated interferences in section VI. Finally, in section VII we implement the proposed algorithm on a DSP kit and obtain experimental verification of its workability.

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II. ADAPTIVE CONTROLLER ARCHITECTURE

The structure of the proposed control scheme is depicted in Figure 1, where w, u, z represent the interference , the control

Fig. 1. Adaptive Controller Architecture

and the performance respectively. Let the instantaneously linear controller $G_{\rm c}(k)$ be represented by the MIMO ARMA model

$$u(k) = -\sum_{j=1}^{n_c} \hat{\Gamma}_j(k)u(k-j) + \sum_{j=1}^{n_c} \hat{\Lambda}_j(k)z(k-j), \qquad (2.1)$$

for all $k \geq 0$, where $u \in \mathbb{R}^m$, $z \in \mathbb{R}^l$ and n_c is order of the instantaneously linear controller $G_{\rm c}(k)$. Also the controller parameter matrices $\hat{\Gamma}_1, \hat{\Gamma}_2 \ldots, \hat{\Gamma}_{n_c} \in \mathbb{R}^{m\times m}$ and $\hat{\Lambda}_1, \hat{\Lambda}_2 \ldots, \hat{\Lambda}_{n_c} \in \mathbb{R}^{m\times l}$ are determined by a yet unspecified controller parameter estimator. Next, define the control horizon vector

$$U(k) \quad \triangleq \quad \begin{bmatrix} u(k-1) \\ \vdots \\ u(k-n_c) \end{bmatrix} \in \mathbb{R}^{q_1}, \qquad (2.2)$$

the performance horizon vector

$$Z(k) \triangleq \begin{bmatrix} z(k-1) \\ \vdots \\ z(k-n_c) \end{bmatrix} \in \mathbb{R}^{q_2}, \tag{2.3}$$

the regressor

$$\phi(k) \triangleq \begin{bmatrix} U(k) \\ Z(k) \end{bmatrix} \in \mathbb{R}^{q_3}, \tag{2.4}$$

and the controller parameter matrix

$$\hat{\Theta}(k) \triangleq [\ -\hat{\Gamma}_1(k) \quad \cdots \quad -\hat{\Gamma}_{n_c}(k) \quad \hat{\Lambda}_1(k) \quad \cdots \quad \hat{\Lambda}_{n_c}(k)] \tag{2.5}$$

where $\hat{\Theta}(k) \in \mathbb{R}^{m \times q_3}$,$q_1 = n_c m$, $q_2 = n_c l$ and $q_3 = q_1 + q_2$. With this notation, (2.1) can be written as

$$u(k) = \phi(k)\hat{\Theta}(k). \tag{2.6}$$

To vectorize the matrix $\hat{\Theta}$ we use the Kronecker product identity [4, p. 245]

$$\text{vec } (I\phi(k)\hat{\Theta}(k)) = (\phi^T \otimes I)\text{vec } \hat{\Theta}(k), \tag{2.7}$$

to obtain

$$u(k) = \psi^T(k)\hat{\theta}(k), \tag{2.8}$$

where

$$\psi^T(k) \triangleq \phi(k) \otimes I, \ \in \mathbb{R}^{q_4 \times l} \tag{2.9}$$

is the regressor matrix and

$$\hat{\theta}(k) \triangleq \text{vec } \hat{\Theta}, \ \in \mathbb{R}^{q_4}$$

is the estimated controller parameter vector. Also, $q_4 \triangleq mq_3$.

III. CLOSED-LOOP SYSTEM

From Fig. 1 the closed-loop performance is given by

$$z(k) = w(k) - u(k) \tag{3.1}$$

From (2.8)

$$z(k) = w(k) - \psi^T(k)\hat{\theta}(k) \tag{3.2}$$

Assumption 3.1: The disturbance $w(k)$ is generated by the free response of a Lyapunov stable LTI system of order $n_w \leq n_c$.

Remark 3.1: Assumption 3.1 is satisfied by the linear combinations of sinusoidal and step signals.

Assumption 3.1 implies that $\exists\ \Theta^* \in \mathbb{R}^{m \times q_3}$ such that $w(k) = \psi^T(k)$ vec Θ^*, where

$$\theta^* \triangleq \text{vec } \Theta^* \in \mathbb{R}^{q_4}$$

From (3.2)

$$\begin{aligned} z(k) &= \psi^T(k) \text{ vec } \Theta^* - \psi^T(k)\hat{\theta}(k) \\ &= \psi^T(k)\theta^* - \psi^T(k)\hat{\theta}(k) \\ &= \psi^T(k)[\theta^* - \hat{\theta}(k)] \end{aligned} \tag{3.3}$$

Define the parameter error vector

$$\tilde{\theta} \triangleq \theta^* - \hat{\theta}(k), \tag{3.4}$$

then

$$z(k) = \psi^T(k)\tilde{\theta}(k). \tag{3.5}$$

IV. ADAPTIVE ALGORITHM AND ITS STABILITY ANALYSIS WITH REGULAR ESTIMATOR UPDATING

A. Adaptive Algorithm

The adaptive feedback mechanism in Figure 1 consists of an instantaneously linear controller $G_c(k)$ given by (2.6) and a parameter update law that modifies the controller parameters at each time step k.

To obtain the parameter update law for $\hat{\theta}$, we define the a priori performance as

$$\hat{z}(k) \triangleq \psi^T\tilde{\theta}(k+1) \tag{4.1}$$

and the cost function

$$\begin{aligned} J(k, \hat{\theta}(k+1)) &\triangleq \frac{1}{2}\hat{z}^T(k)\hat{z}(k) \\ &= \frac{1}{2}[\psi^T(k)\tilde{\theta}(k+1)]^T[\psi^T(k)\tilde{\theta}(k+1)] \end{aligned} \tag{4.2}$$

which is quadratic in the *a priori* performance $\psi^{\mathrm{T}}(k)\tilde{\theta}(k+1)$. Then we use a recursive least squares estimate of $\hat{\theta}(k+1)$ to minimize (4.2); for details see, for example, [5]. The recursive least square estimate of $\hat{\theta}(k+1)$ is given by

$$\hat{\theta}(k+1) = \hat{\theta}(k) - \mathcal{P}(k+1)\psi(k)z(k), \tag{4.3}$$

$$\mathcal{P}(k+1) = \mathcal{P}(k) - G(k)\psi^{\mathrm{T}}(k)\mathcal{P}(k), \tag{4.4}$$

where

$$G(k) = \mathcal{P}(k)\psi(k)\left[I + \psi^{\mathrm{T}}(k)\mathcal{P}(k)\psi(k)\right]^{-1}$$

Using (3.4), (4.3) and (4.4) the closed-loop error system is given by

$$\tilde{\theta}(k+1) = \tilde{\theta}(k) - \mathcal{P}(k+1)\psi(k)\psi^{\mathrm{T}}(k)\tilde{\theta}(k), \tag{4.5}$$

$$\mathcal{P}(k+1) = \mathcal{P}(k) - G(k)\psi^{\mathrm{T}}(k)\mathcal{P}(k). \tag{4.6}$$

where $\mathcal{P}(0) > 0$.

B. Stability Analysis

The Lyapunov stability of every equilibrium of the system (3.5),(4.5) and (4.6) is demonstrated in [6].

V. ADAPTIVE ALGORITHM WITH STAGGERED ESTIMATOR UPDATING AND STABILITY ANALYSIS

In this section we consider the case where the estimator is updated less frequently i.e, it is not updated at each time step. In subsection (V-A), first we consider the same adaptive algorithm described in section (IV) except that the parameter estimates remains unchanged for n steps. We then explain the problem encountered by adopting this methodology. Finally we suggest staggered estimator updating as a means of allowing more time for estimator computations while avoiding the problem of estimator aliasing.

A. *Adaptive Algorithm*

The adaptive feedback mechanism in Figure 1 consists of an instantaneously linear controller $G_{\mathrm{c}}(jn)$ given by (2.6) and a parameter update law that modifies the controller parameters at time step jn. Consider the control law (2.8) under the constraint

$$\hat{\theta}(jn) = \hat{\theta}(jn-1) =, \cdots, = \hat{\theta}(jn-n+1), \tag{5.1}$$

where $j = 1, 2, 3, \ldots$. The gain estimate is not updated for n steps. To obtain the parameter update law for $\hat{\theta}$, we define the a priori performance as

$$\hat{z}(jn) \triangleq \psi^T \tilde{\theta}(jn+1) \tag{5.2}$$

and the cost function

$$J = \frac{1}{2}[\psi^T(jn)\tilde{\theta}(jn+1)]^T[\psi^T(jn)\tilde{\theta}(jn+1)] \tag{5.3}$$

which is quadratic in the *a priori* performance $\psi^{\mathrm{T}}(jn)\tilde{\theta}(jn)$. Then we use a recursive least square estimate of $\hat{\theta}(jn+1)$ to minimize (5.3); for details see,for example, [5], [7] and [8]. The recursive least square estimate of $\hat{\theta}(jn+1)$ is given by

$$\hat{\theta}(jn+1) = \hat{\theta}(jn) - \mathcal{P}(jn+1)\psi(jn)z(jn), \tag{5.4}$$

$$\mathcal{P}(jn+1) = \mathcal{P}(jn) - G(jn)\psi^{\mathrm{T}}(jn)\mathcal{P}(jn), \tag{5.5}$$

where

$$G(jn) = \mathcal{P}(jn)\psi(jn)\left[I + \psi^{\mathrm{T}}(jn)\mathcal{P}(jn)\psi(jn)\right]^{-1}$$

Using (3.4), (5.4) and (5.5) the closed-loop error system is given by

$$\tilde{\theta}(jn+1) = \tilde{\theta}(jn) - \mathcal{P}(jn+1)\psi(jn)\psi^{\mathrm{T}}(jn)\tilde{\theta}(jn), \tag{5.6}$$

$$\mathcal{P}(jn+1) = \mathcal{P}(jn) - G(jn)\psi^{\mathrm{T}}(jn)\mathcal{P}(jn). \tag{5.7}$$

where $\mathcal{P}(0) > 0$, and

$$\hat{\theta}(jn) = \hat{\theta}(jn-1) =, \cdots, = \hat{\theta}(jn+n-1),$$
$$\mathcal{P}(jn) = \mathcal{P}(jn-1) =, \cdots, = \mathcal{P}(jn+n-1).$$

B. *Stability Analysis*

To demonstrate that every equilibrium of the system (5.2), (5.6) and (5.7) is Lyapunov stable we require the following lemma

Lemma 5.1: Define

$$V_{\mathcal{P}}(\mathcal{P}) \triangleq \operatorname{tr} \mathcal{P}^2, \tag{5.8}$$

$$\triangle V_{\mathcal{P}}(jn) \triangleq \operatorname{tr}\left[\mathcal{P}^2(jn+1) - \mathcal{P}^2(jn)\right], \tag{5.9}$$

$$V_{\tilde{\theta}}(\tilde{\theta}, \mathcal{P}) \triangleq \tilde{\theta}^{\mathrm{T}}\mathcal{P}^{-1}\tilde{\theta}, \tag{5.10}$$

and

$$\triangle V_{\tilde{\theta}}(jn) \triangleq \tilde{\theta}^{\mathrm{T}}(jn+1)\mathcal{P}^{-1}(jn+1)\tilde{\theta}(jn+1) - \tilde{\theta}^{\mathrm{T}}(jn)\mathcal{P}^{-1}(jn)\tilde{\theta}(jn). \tag{5.11}$$

Then,

$$\triangle V_{\mathcal{P}}(jn) \leq 0, \tag{5.12}$$

$$\triangle V_{\tilde{\theta}}(jn) = -\tilde{\theta}^{\mathrm{T}}(jn)\psi(jn)\left[I + \psi^{\mathrm{T}}(jn)\mathcal{P}(jn)\psi(jn)\right]^{-1}\psi^{\mathrm{T}}\tilde{\theta} \tag{5.13}$$

$$\leq \frac{-\|z(jn)\|_2^2}{1 + \gamma\sum_{i=0}^{n_c}\left[\|u(jn-i)\|_2^2 + \|z(jn-i)\|_2^2\right]}, \tag{5.14}$$

where

$$\gamma \triangleq \lambda_{\max}\left[\mathcal{P}(0)\right], \tag{5.15}$$

Furthermore,

$$\lim_{k\to\infty} \triangle V_{\tilde{\theta}}(jn) = 0, \tag{5.16}$$

and $\lim_{k\to\infty}\tilde{\theta}(jn)$ and $\lim_{k\to\infty}\mathcal{P}(jn)$ exist.

Proof. The results (5.12), (5.13), (5.16), and the convergence of $\{\tilde{\theta}(jn)\}_{k=0}^{\infty}$ and $\{\mathcal{P}(jn)\}_{k=0}^{\infty}$ follow from standard properties of recursive least square, see [8, p. 60], [9, p. 22], [10, p. 58] and [11, p. 202]. From (5.12) follows that

$$\mathcal{P}(jn) \leq \mathcal{P}(jn-1).$$

Now we demonstrate the asymptotic convergence of the performance to zero. From (2.9) it follows that

$$\psi^T(jn)\psi(jn) = \phi^T(jn)\phi(jn) \otimes I. \tag{5.17}$$

where $I \in \mathbb{R}^{q_4\times q_4}$. Equation (5.17) follows that

$$\|\psi(jn)\|_2 = \|\phi(jn)\|_2, \tag{5.18}$$

Using (5.18) in (2.4) we have

$$\psi^T(jn)\psi(jn) = \sum_{i=1}^{n_c}\left[\|u(jn-i)\|_2^2 + \|z(jn-i)\|_2^2\right]. \tag{5.19}$$

It follows from (3.1), (5.14) and (5.19) that

$$\begin{aligned}\triangle V_{\tilde{\theta}}(jn) &\leq \frac{-\|z(jn)\|_2^2}{1 + \gamma\sum_{i=1}^{n_c}\left[\|u(jn-i)\|_2^2 + \|z(jn-i)\|_2^2\right]}\\ &\leq \frac{-\|z(jn)\|_2^2}{1 + \gamma\sum_{i=1}^{n_c}\left[\|u(jn-i)\|_2^2 + \|z(jn-i)\|_2^2\right] + M(jn)},\end{aligned} \tag{5.20}$$

where

$$\gamma = \lambda_{\max}\left[\mathcal{P}(0)\right] > \lambda_{\max}\left[\mathcal{P}(jn)\right], \tag{5.21}$$

and

$$M(jk) = \gamma\|z(jn)\|_2^2 + \gamma\|u(jn)\|_2^2. \tag{5.22}$$

From (3.1) it follows that

$$\|u(jn)\| \leq \|w(jn)\| + \|z(jn)\| \tag{5.23}$$

From (5.20), (5.22) and (5.23), we have

$$\triangle V_{\tilde{\theta}}(jn) \leq \frac{-\|z(jn)\|_2^2}{1 + \gamma\sum_{i=0}^{n_c}\left[\|w(jn-i)\|_2^2 + 2\|z(jn-i)\|_2^2\right]}$$

From Assumption 3.1 it follows that $w(jn)$ is bounded therefore $\exists\ c \in \mathbb{R}^+$ such that $\sum_{i=0}^{n_c} \|w(jn-i)\|_2^2 < c$. Consequently

$$\begin{aligned}\triangle V_{\tilde{\theta}}(jn) &\le \frac{-\|z(jn)\|_2^2}{1+\gamma c + 2\gamma \sum_{i=0}^{n_c} \|z(jn-i)\|_2^2} \\ &= \frac{-\|z(jn)\|_2^2}{c_1 + c_2 \sum_{i=0}^{n_c} \|z(jn-i)\|_2^2},\end{aligned}$$

where $c_1 \triangleq 1+\gamma c$ and $c_2 \triangleq 2\gamma$. Now suppose that $\{\|z(jn)\|\}_{k=0}^{\infty}$ is unbounded. Then it follows from Lemma 1.1 that there exist $c_3 > 0$ and $c_4 > 0$ such that the maximal monotonically increasing subsequence $\{\|z(jn_i)\|\}_{i=0}^{\infty}$ satisfies

$$c_1 + c_2 \sum_{i=0}^{n_c} \|z(jn-i)\|_2^2 < c_3 + c_4 \|z(jn)\|_2^2$$

which implies that

$$\triangle V_{\tilde{\theta}}(jn_i) \le \frac{-\|z(jn_i)\|_2^2}{c_3 + c_4 \|z(jn_i)\|_2^2}$$

for all $i = 1,\ 2,\ \cdots$. Furthermore, since by (5.16) $\lim_{i\to\infty} \triangle V_{\tilde{\theta}}(jn_i) = 0$, it follows that

$$\lim_{i\to\infty} \frac{-\|z(jn_i)\|_2^2}{c_3 + c_4 \|z(jn_i)\|_2^2} = 0$$

Therefore the maximally monotonically increasing subsequence $z(jn_i) \to 0$ as $k \to \infty$, which is a contradiction. Hence $\{\|z(jn)\|\}_{k=0}^{\infty}$ is bounded and it follows from Lemma. (1.1) that there exist $c_5 > 0$ and $c_6 > 0$ such that, for all $k \ge 0$,

$$\triangle V_{\tilde{\theta}}(n) \le \frac{-\|z(jn)\|_2^2}{c_5 + c_6 \|z(jn)\|_2^2}$$

Now using (5.16) we have

$$\lim_{i\to\infty} \frac{-\|z(jn)\|_2^2}{c_5 + c_6 \|z(jn)\|_2^2} = 0 \tag{5.24}$$

Therefore, $z(jn) \to 0$ as $k \to \infty$. □

Remark 5.1: Although Lemma 5.1 ensures convergence of the performance to zero only at the time instants when the parameter estimate is updated.

C. Estimator Aliasing

Estimator aliasing refers to the inability of the estimator to adapt controller parameters in spite of a periodically growing performance variable whose period ia an integral multiple of the estimator update period. Therefore, even though Lemma 5.1 ensures convergence of the performance to zero at the time instants when the parameter estimate is updated estimator aliasing can still lead to a periodic and performance vector. The mechanism of estimator updating suggests the use a staggered updating i.e., the time between two consecutive gain updates be varied instead of updating after a fixed interval. We idea is presented formally in the lemma below.

Lemma 5.2: Let n be a positive integer and let

$$\begin{aligned}\tau(j+1) &= \tau(j)+n+1 \qquad \text{if } j \text{ is odd} \\ \tau(j+1) &= \tau(j)+n \qquad \text{if } j \text{ is even.}\end{aligned}$$

where $j = 1,2,\ldots$ and $\tau(1) = n$. Furthermore let $k = 0,1,2,\ldots$. Then there does not exist an integer $T > 1$ such that for all $k = \tau(j)$

$$\sin\left(\frac{2\pi k}{T} + \phi\right) = 0, \tag{5.25}$$

where $0 \le \phi < 2\pi$.

Proof. To satisfy (5.25) for all $k = \tau(j)$, $T/2$ must divide

$$n, 2n+1, 3n+1, 4n+2, 5n+2, 6n+3, \ldots$$

Therefore T can not be an integer. □

Using Lemma 5.2 and Lemma 5.1 we state the following lemma.

Lemma 5.3: Define the gain update

$$\begin{aligned}\hat{\theta}(\tau(j+1)) &= \hat{\theta}(\tau(j)) \\ &\quad + \mathcal{P}(\tau(j)+1)\psi(\tau(j))\psi^T(\tau(j))\hat{\theta}(\tau(j)), \quad (5.26) \\ \mathcal{P}(\tau(j)+1) &= \mathcal{P}(\tau(j)) \\ &\quad - \mathcal{P}(\tau(j))\psi(\tau(j))L(\tau(j))\psi^{\mathrm{T}}(\tau(j))\mathcal{P}(\tau(j)), \\ \hat{\theta}(\tau(j)-1) &= \hat{\theta}(\tau(j)-2) =, \ldots, = \hat{\theta}(\tau(j-1)+1), \\ \mathcal{P}(\tau(j)-1) &= \mathcal{P}(\tau(j)-2) =, \ldots, = \mathcal{P}(\tau(j-1)+1), \\ \mathcal{P}(0) &> 0 \quad (5.27)\end{aligned}$$

and

$$\tau(j+1) = \tau(j)+n+1 \qquad \text{if } j \text{ is odd} \tag{5.28}$$
$$\tau(j+1) = \tau(j)+n \qquad \text{if } j \text{ is even,} \tag{5.29}$$

where

$$L(\tau(j)) = \left[I_n + \psi^{\mathrm{T}}(\tau(j))\mathcal{P}(\tau(j))\psi(\tau(j))\right]^{-1}, \tag{5.30}$$

also $j = 1,2,\ldots$ and $\tau(1) = n$. Then all parameters of the closed-loop system (3.1),(5.26)-(5.29) remain bounded, and $z(k) \to 0$ as $k \to \infty$.

Proof. From Lemma 5.1 with $jn+1$ replaced by $\tau(j)$ we have $x(\tau(j)) \to 0$ as $j \to \infty$. Now we use Lemma 5.2 to conclude that $z(k) \to 0$ as $k \to \infty$. □

VI. COMPUTER SIMULATION

Example 6.1: In this example the interference is a single tone at $0.8\ rad/sec$, and the controller is of the order 4. A plot if the performance 'z' vs time is shown in Fig. 2. A bode plot of the controller after the controller parameters have converged is shown in Fig. 3. Note that the controller has high gain at the interference frequency of $0.8\ rad/sec$. In this regard our proposed method can be viewed as an adaptive internal model control (IMC) scheme.

Example 6.2: In this example, two interferences are given at two different channels. The first channel is affected by a single tone interference at $(0.2\ rad/sec$, while on the second channel the interference frequency is $0.9\ rad/sec$. Plots of the closed-loop performance 'z' vs time for two channels are shown in Fig. 4 and Fig. 5 respectively. In this example the order of controller is $n_c = 4$.

Example 6.3: In this example we again consider the two channels interference problem with the first channel be affected

Fig. 2. Performance for the interference at 0.2 rad/sec [Example 6.1]

Fig. 3. Bode plot of controller obtained after converges of parameter with frequency at 0.8 rad/sec [Example 6.1]

by a tone at frequency 5Hz and the second channel be affected by Amplitude modulated wave with carrier frequency is 100Hz and modulating signal at 10Hz . The controller order used for this example was $n_c = 8$. A plot of closed-loop performance 'z' vs time is shown in Fig. 6.

VII. HARDWARE IMPLEMENTATION

A. Experimental Setup

The controller was implemented on a real time TMS320C6713 DSP Kit at a sampling rate of 8000 Hz. The interference signals were generated via a external signal generator and the output of the DSP Kit was sent to a PC via the audio card where the performance variable 'z' was recorded using the Windows audio recorder.

B. Experiment 1

In first experiment the interference was a tone at 5 kHz. The plot of closed-loop 'z' vs time are shown in Fig.7. Initially, the interference is allowed to effect the performance and then the adaptive algorithm was switched on via a DIP switch on the DSP kit. The adaptive algorithm then adapted the controller parameters to reject the tonal disturbance.

Fig. 4. Performance for the interference at 0.2 rad/sec [Example 6.2]

Fig. 5. Performance for the interference at 0.9 rad/sec [Example 6.2]

C. Experiment 2

In this experiment two interference channels are considered. Channel one being effected by a tonal disturbance of 5 kHz and channel 2 by an Amplitude modulated wave at 1 kHz. The plots of closed-loop 'z' vs time are shown in Fig. 8 and Fig. 9.

VIII. CONCLUSIONS

A. Conclusions

In this paper we proposed an algorithm for multiple channel interference cancelation via a multi-variable feedback control law and infrequent estimator updating. The theoretical foundation for the proposed algorithm was established via a comprehensive proof of Lyapunov stability of the closed-loop system. Effectiveness of the algorithm as an interference canceler was demonstrated via simulations and experiments.

APPENDIX

Lemma 1.1: Let $\{\alpha(k)\}_{k=0}^{\infty}$ be a sequence of positive scalars. Let N be a positive integer, let $g_1 > 0$, $g_2 > 0$, and

Fig. 6. Performance for the AM interference at frequency 10Hz [Example 6.3]

Fig. 7. Performance for the Tone interference at frequency 5kHz [Experiment:1]

define

$$L(k) \triangleq g_1 + g_2 \sum_{j=0}^{N} \alpha(k-j). \tag{1.1}$$

Also, define the maximal monotonically increasing subsequence $\{\alpha(k_i)\}_{i=0}^{\infty}$ such that $\alpha(k) < \alpha(k_i)$ for all $k < k_i$. Then the following statement hold

1) If $\{\alpha(k)\}_{k=0}^{\infty}$ is bounded, then there exist $g_3 > 0$, $g_4 > 0$ such that, for all $k \geq 0$

$$L(k) \leq g_3 + g_4 \alpha(k). \tag{1.2}$$

2) If $\{\alpha(k)\}_{k=0}^{\infty}$ is unbounded then there exist $g_3 > 0$, $g_4 > 0$ such that for all $i = 1, 2, \ldots$ the maximal monotonically increasing subsequence $\{\alpha(k_i)\}_{i=0}^{\infty}$, satisfies

$$L(k_i) \leq g_3 + g_4 \alpha(k_i). \tag{1.3}$$

Proof. If $\{\alpha(k)\}_{k=0}^{\infty}$ is bounded, then (1.2) is satisfied with $g_3 = g_1 + (N+1) g_2 \sup_{k\geq 0} \alpha(k)$ and $g_4 > 0$. Now ,suppose that $\{\alpha(k)\}_{k=0}^{\infty}$ is unbounded, then (1.3) is satisfied with $g_3 = g_1$ and $g_4 = N + 1$. $\square$

Fig. 8. Performance for the Tone interference at frequency 5kHz [Experiment:2]

Fig. 9. Performance for the AM interference at frequency 1KHz [Experiment:2]

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