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Applications of a Model Matching Framework in Switched Systems*

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Abstract— **In this paper we build on our previous work to consider some applications of a prototypical model matching framework where the various mappings involved are systems that switch arbitrarily among n given linear time-invariant (LTI) systems. The design interest was placed on minimizing the worst case $\ell_\infty-$induced norm of this model matching system over all possible switchings. It was shown that the model matching problem can be solved via linear programming (LP) with an arbitrary a-priori precision. To illustrate the efficiency of the proposed architecture, we consider herein three related applications: sensitivity minimization of switched systems, linear parameter-varying (LPV) control, and cooperative control of dynamic agents. These applications share a common feature that information switching about the system dynamics results in a switched closed-loop system and thus generate the necessity to address transient system stability and performance optimization. A numerical example is included to demonstrate the difference of achievable performance under different switchings.**

Keywords: $\ell_\infty-$induced optimality, ℓ_1 optimal, worst case switching

I. INTRODUCTION

There has been an increasing interest in switched and hybrid systems, due to their great flexibility and less conservativeness in modeling and control of logic-based systems, event-driven systems, parameter-varying systems, and so on; for details, see the survey papers [1], [2], [3] and the book [4]. Loosely speaking, switched systems can be classified into two classes: systems under controlled switching and systems under uncontrolled switching. In the first case the interest is placed on synthesizing a switching signal or even a corresponding controller to stabilize the switched systems. The other is on the analysis of system stability and synthesis of a feedback controller with given switching signals, including arbitrary switching, slow switching, and switching according to Markov chains. In the case of arbitrary switching, the well-known result is that the uniform exponential stability of switched linear systems under arbitrary switching is equivalent to the existence of a common Lyapunov function for its constituent linear time-invariant (LTI) systems [3]. However, such a common Lyapunov function may neither be strictly convex nor continuously differentiable, which makes the analysis of stability under arbitrary switching very challenging, let alone the synthesis of optimal control laws.

In the class of systems under arbitrary switching, we introduced a special class $\mathcal{S}_n$ of output switching systems and adopted the worst case $\ell_\infty-$induced norm as the performance criterion in [5]. In particular, we proposed and analyzed a prototypical model matching problem where the various mappings involved are systems that switch arbitrarily among n given stable LTI systems. The design interest was placed on minimizing the worst case $\ell_\infty-$induced norm of this model matching system over all possible switchings. Research along this line is motivated by control over networks and reliable control design, such as control with limited bandwidth, control over packet-dropping networks, and control subject to sensor/actuator failures. In these applications, the available information about the system dynamics is time-varying, resulting in a switched closed-loop system. Hence it is important to address transient system stability and performance optimization due to information switching.

There are several classes of problems which relate to the model matching formulation in [5]. To further illustrate the efficiency of the proposed framework, we introduce three related applications: sensitivity minimization of switched systems, linear parameter-varying (LPV) control, and cooperative control of dynamic agents. These applications can be formulated exactly in the model matching framework or solved similarly by the proposed algorithms in [5], as demonstrated in this paper.

The paper is organized as follows. In Section II, we introduce the preliminaries and review the model matching results in [5]; in Section III we cast the sensitivity minimization problem of switched systems in the proposed framework; in Section IV and Section V, we deal with applications to LPV control and cooperative control of dynamic agents respectively; in Section VI we conclude.

The notation in the paper is as follows: $\|x\|_\infty := \sup_k |x(k)|$ is the ℓ_∞ norm of a real valued sequence $x = \{x(k)\}_{k=0}^\infty$. For a vector-valued signal $x = [x_1, x_2, \ldots, x_n]'$, $\|x\|_\infty := \max_i \|x_i\|_\infty$ and for multi-input multi-output (MIMO) systems $T = \{T_{ij}\}$, $\|T\|_1 := \max_i \sum_j \|T_{ij}\|_1$ where T_{ij} is the ith input to jth output map; $\|T\| := \sup_{x\neq 0} \frac{\|Tx\|_\infty}{\|x\|_\infty}$ is the ℓ_∞-induced norm of a possibly time-varying and/or nonlinear system T (note that $\|T\| = \|T\|_1$ if T is LTI); T is stable if $\|T\| < \infty$; $\mathcal{L}_{TV}$ denotes all stable

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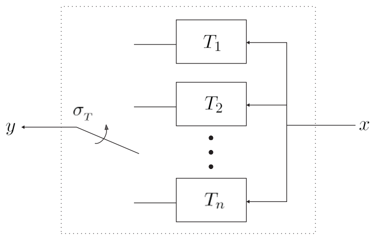

Fig. 1. Structure of system $T \in \mathcal{S}_n$

causal time varying systems while $\mathcal{L}_{TI}$ denotes its subset of time invariant systems (which can be identified by the space ℓ_1.) For a linear time-varying (LTV) system G with a lower (block) triangular representation [7]

$$\begin{bmatrix} G(0,0) & & & \\ G(1,0) & G(1,1) & & \\ G(2,0) & G(2,1) & G(2,2) & \\ \vdots & \ddots & \ddots & \ddots \end{bmatrix},$$

$[\mathcal{R}_G]_t$ denotes the causal part of the t^{th} (block) row, i.e., $[\mathcal{R}_G]_t = [G(t,0) \cdots G(t,t)]$, and the $\ell_\infty-$induced norm of G is defined as

$$\|G\| = \sup_t \max_j \sum_k |[G(t,0) \cdots G(t,t)]_{jk}|$$

where $(G(t,0) \cdots G(t,t))_{jk}$ is the (j,k)th (scalar) element of the (block) row matrix $(G(t,0) \cdots G(t,t))$. $\text{diag}_i(A_i)$ denotes a block diagonal matrix with sub-matrices $A_1, \cdots, A_l$ on its diagonal and $\text{cat}_i(x_i)$ denotes the vector $[x_1', \cdots, x_l']'$. For two integers i and j, $i \vee j$ denote $\max\{i,j\}$ and $i \wedge j$ denotes $\min\{i,j\}$.

II. PRELIMINARIES AND MODEL MATCHING RESULTS

In [5] we introduced a class $\mathcal{S}_n$ of output switching systems. Each switched system T in $\mathcal{S}_n$ is associated with a corresponding switching signal $\sigma_T = \{\sigma_T(t)\}_{t=0}^\infty$ and n stable LTI components $\{T_i\}_{i=1}^n$, as shown in Figure 1. The switching signal σ_T takes values in a finite set $\{s_1, \cdots, s_n\}$ for any time $t\,(\geq 0)$. For each component T_i, let $\{T_i(\tau)\}_{\tau=0}^\infty$ denote its unit impulse response. Given an input $x = \{x(t)\}_{t=0}^\infty$ and a switching trajectory $\sigma_T = \{\sigma_T(t)\}_{t=0}^\infty$, the output $y = \{y(t)\}_{t=0}^\infty$ of T is defined as

$$y(t) = (Tx)(t) = \sum_{k=0}^t T_{\sigma_T(t)}(k)x(t-k),$$

where $T_{\sigma_T(t)}(k) = T_i(k)$ if $\sigma_T(t) = s_i$.

For a specific trajectory $\{\sigma_T(t)\}_{t=0}^\infty$ of the switching signal σ_T, let T_{σ_T} denote the LTV system with a lower (block) triangular representation

$$T_{\sigma_T} = \begin{bmatrix} T(0,0) & & & \\ T(1,0) & T(1,1) & & \\ T(2,0) & T(2,1) & T(2,2) & \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix}$$

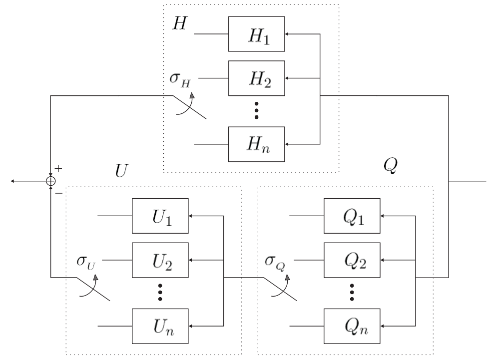

Fig. 2. Connection structure of a switched system $H_\sigma - Q_\sigma V_\sigma$

where $T(t,\tau)$ denotes the linear mapping from the input at time τ to the output at time t, and $T(t,\tau) = T_i(t-\tau)$, if $\sigma_T(t) = s_i$ for some $i \in \{1, \ldots, n\}$.

The problem of interest is shown in Figure 2. Given two switched systems H and U in $\mathcal{S}_n$ which are associated with the sets $\{H_i\}_{i=1}^n$, $\{U_i\}_{i=1}^n$ of LTI components and the corresponding switching signals σ_H, σ_U respectively, we consider the following model matching problem:

Find a switched system $Q \in \mathcal{S}_n$ associated with the set $\{Q_i\}_{i=1}^n$ of LTI components solving

$$\nu(\Xi) := \inf_{\{Q_i\}_{i=1}^n} \mu(Q, \Xi) \tag{1}$$

where $\mu(Q,\Xi) = \sup_{\sigma \in \Xi} \|H - UQ\|$, $\sigma = (\sigma_H, \sigma_U, \sigma_Q)$, and Ξ denotes the set of admissible switching trajectories.

Note that for different relations among the switching signals σ_H, σ_U and σ_Q, the set Ξ of admissible switching trajectories of σ is different, and hence the optimal performance $\nu(\Xi)$ is different. To proceed, we first introduce the following definitions. For two switched systems T and $\tilde{T}$ in $\mathcal{S}_n$, $\sigma_T = \sigma_{\tilde{T}}$ if $\sigma_T(t) = \sigma_{\tilde{T}}(t)$, $\forall t \geq 0$; σ_T and $\sigma_{\tilde{T}}$ are independent if $\forall t\,(\geq 0)$, $(\sigma_T(t), \sigma_{\tilde{T}}(t))$ can assume any value in the set $\{(j,k) | j \in \{1, \cdots, n\}, k \in \{1, \cdots, n\}\}$. The distance between σ_T and $\sigma_{\tilde{T}}$ is $d(\sigma_T, \sigma_{\tilde{T}}) = \sum_{t=0}^\infty 1_{\{\sigma_T(t) \neq \sigma_{\tilde{T}}(t)\}}$, where $1_A = 1$ if statement A is true and $1_A = 0$ otherwise. Thus $d(\sigma_T, \sigma_{\tilde{T}})$ denotes the number of mismatches between σ_T and $\sigma_{\tilde{T}}$.

We dealt with two kinds of switching, namely partially matched switching and matched switching in [5], where the corresponding set of admissible switching trajectories is $\Xi_1 = \left\{ \sigma \,\middle|\, \begin{array}{l} \forall t \geq 0, \sigma_H(t) = \sigma_U(t) \in \{1, \cdots, n\} \\ \forall t \geq 0,\, \sigma_Q(t) \in \{1 \cdots, n\} \\ d(\sigma_H, \sigma_Q) = 1 \end{array} \right\}$ and $\Xi_2 = \{\sigma \,|\, \forall t \geq 0, \sigma_H(t) = \sigma_U(t) = \sigma_Q(t) \in \{s_1, \cdots, s_n\}\}$ respectively. The main result there about the model matching problem is as follows:

Theorem 2.1: 1) Partially matched switching: for any $\epsilon > 0$ and $\bar{Q}$ in $\mathcal{L}_{TI}$ with $\max_{1 \leq i \leq n} \left\|H_i - U_i\bar{Q}\right\|_1 \leq \nu_1 + \epsilon$, where $\nu_1 := \inf_{Z \in \mathcal{L}_{TI}} \max_{1 \leq i \leq n} \|H_i - U_i Z\|_1$, it holds that $\mu(\bar{Q}, \Xi_1) \leq \nu_1 + \epsilon$, and thus $\nu(\Xi_1) = \nu_1$.

2) Matched switching: The model matching problem (1) is solvable in the case of matched switching when U is

a FIR filter, and an approximate solution with any given precision can be determined by solving two sequences of linear programming problems.

It is shown in [6] that similar results hold in the case of totally unmatched switching where the set of admissible switching trajectories is

$$\Xi_3 = \{\sigma \,|\, \sigma_G, \sigma_H \text{ and } \sigma_R \text{ are independent}\}.$$

Furthermore, if the $\mathcal{H}_2$ norm is adopted as the performance criterion instead of the $\ell_\infty-$induced norm, the results in three cases of switchings still hold except that the ℓ_1 optimization formulations are replaced by $\mathcal{H}_2$ optimization formulations; for details, see Chapter 5 in [6].

There are several classes of problems which relate to the model matching formulation under consideration. In particular, we introduce three applications: sensitivity minimization of switched systems, LPV control, and cooperative control of dynamic agents in Section III, Section IV, and Section V respectively.

III. SENSITIVITY MINIMIZATION

Consider a plant P in $\mathcal{S}_n$ associated with the set of possible plants $\{P_1, \ldots, P_n\}$ where each P_i is stable in $\mathcal{L}_{TI}$. Similarly, let W in $\mathcal{S}_n$ be associated with $\{W_1, \ldots, W_n\}$ where each W_i is a stable weight in $\mathcal{L}_{TI}$. We are interested in controllers K that minimize a weighted sensitivity mapping over all possible switchings. Specifically, the problem of interest is to find a K that depends causally on the switching sequence $\sigma = \{\sigma(t)\}_{t=0}^{\infty}$ with $u(t) = (K(\sigma)y)(t)$, where[1] $K(\sigma)$ is linear time-varying for a given σ.

Let $S = (I - PK)^{-1}$ denote the sensitivity mapping; we are interested in

$$\nu = \inf_K \sup_\sigma \left\| \begin{bmatrix} S(\sigma) \\ W_\sigma K(\sigma) S(\sigma) \end{bmatrix} \right\|.$$

We employ the set of all stabilizing K as $K = -Q(I - PQ)^{-1}$ where Q is a system depending causally on σ and its action $(e, \sigma) \rightarrow u$ given as $u(t) = (Q(\sigma)e)(t)$ with $Q(\sigma) \in \mathcal{L}_{TV}$ for every σ, as shown in Figure 3. Then the design problem transforms to

$$\nu = \inf_Q \sup_\sigma \left\| \begin{bmatrix} I - P_\sigma Q(\sigma) \\ W_\sigma Q(\sigma) \end{bmatrix} \right\|.$$

In this application we restrict Q to be in $\mathcal{S}_n$, i.e., its action is $v(t) = (Q_{\sigma(t)}e)(t)$ where $Q_{\sigma(t)} \in \{Q_1, \ldots, Q_n\}$ with $Q_i \in \mathcal{L}_{TI}$ for $i = 1, \ldots, n$. In this case the problem becomes

$$\nu_{\mathcal{S}_n} = \inf_{\{Q_i\}_{i=1}^n} \sup_\sigma \|H_\sigma - U_\sigma Q_\sigma\|, \tag{2}$$

where $H = \begin{bmatrix} I \\ 0 \end{bmatrix}$ and $U = \begin{bmatrix} P \\ W \end{bmatrix}$ are in $\mathcal{S}_n$. Hence the sensitivity minimization problem can be formulated as a model matching problem in (2).

In the case of matched switching, problem (2) can be solved by the convergent algorithm in Section III-B of [5] when both P and W are FIR filters. Since H is a constant system, there is no difference between partially matched switching and unmatched switching in this case. In the case of partially matched switching, the result in Theorem 2.1 still holds. Such mismatches between switching signals can happen for example, if the estimation part (I) of the controller K in Figure 3 is collocated with plant P, while part II is implemented remotely and connected to plant P via networks.

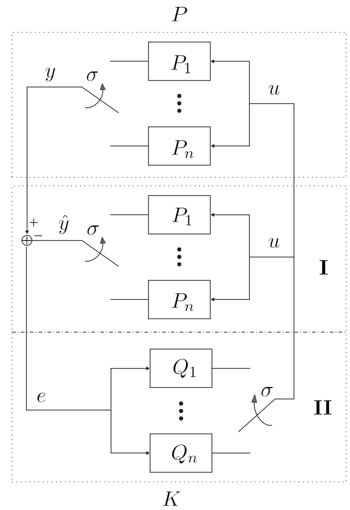

Fig. 3. Sensitivity minimization

IV. LPV CONTROL

Consider an LPV system described by

$$\begin{aligned} x(t+1) &= A(\sigma(t))x(t) + B_1 w(t) + B_2(\sigma(t))u(t) \\ z(t) &= C_1(\sigma(t))x(t) + D_{11}(\sigma(t))w(t) + D_{12}(\sigma(t))u(t) \\ y(t) &= C_2 x(t) \end{aligned}$$

where $\sigma(t) \in \{s_1, \ldots, s_n\}$, $\forall t$. We further assume that $B_2(\sigma(t))$ has a (right) inverse $B_2^{-1}(\sigma(t))$ for all possible values of $\sigma(t)$ and C_2 has a (left) inverse. This allows for the existence of state feedback and filter gains $F(\sigma(t))$ and $L(\sigma(t))$, respectively, such that

$$\begin{aligned} A(\sigma(t)) - B_2(\sigma(t))F(\sigma(t)) = 0 \\ A(\sigma(t)) - L(\sigma(t))C_2 = 0 \end{aligned} \tag{3}$$

for all $\sigma(t)$. For example, that would be the case when there is at least one control input for each state and the state is also measured, i.e., $C_2 = I$.

Employing an observer based parametrization of all stabilizing controllers K of the form $u(t) = (K(\sigma)y)(t)$, where $K(\sigma)$ depends causally on σ and is a linear time-varying system for any given σ, the system performance from w to z can be expressed as

$$\mu(Q, \Xi) = \sup_{\sigma \in \Xi} \|H_\sigma - U_\sigma Q_\sigma V_\sigma\| \tag{4}$$

[1] K depends causally on σ, and is written as $K(\sigma)$ to emphasize this dependence. Similar notations hold for $S(\sigma)$ and $Q(\sigma)$.

with

$$H_\sigma = \begin{bmatrix} X_0(0) & & & & \\ X_1(1) & X_0(1) & & & \\ X_2(2) & X_1(2) & X_0(2) & & \\ 0 & X_2(3) & X_1(3) & X_0(3) & \\ \vdots & \ddots & \ddots & \ddots & \ddots \end{bmatrix},$$

$$-U_\sigma = \begin{bmatrix} Y_0(0) & & & \\ Y_1(1) & Y_0(1) & & \\ 0 & Y_1(2) & Y_0(2) & \\ \vdots & \ddots & \ddots & \ddots \end{bmatrix},$$

$$V_\sigma = \begin{bmatrix} 0 & & & \\ C_2B_1 & 0 & & \\ 0 & C_2B_1 & 0 & \\ \vdots & \ddots & \ddots & \ddots \end{bmatrix},$$

where

$$\begin{aligned} &X_0(t) = D_{11}(\sigma(t)), \quad X_1(t) = C_1(\sigma(t))B_1, \\ &X_2(t) = [C_1(\sigma(t)) - D_{12}(\sigma(t))F(\sigma(t))]A(\sigma(t-1))B_1, \\ &Y_0(t) = D_{12}(\sigma(t)), \\ &Y_1(t) = [C_1(\sigma(t)) - D_{12}(\sigma(t))F(\sigma(t))]B_2(\sigma(t-1)). \end{aligned}$$

Note that the formulation of $\mu(Q)$ in (4) is not exactly the same as that in the model matching problem (1) since $[\mathcal{R}_{H_\sigma}]_t$ and $[\mathcal{R}_{U_\sigma}]_t$ here depend on $\{\sigma(t), \sigma(t-1)\}$ and there is an additional LTI system V_σ. However, there is no difficulty in adopting the proposed methods to solve $\inf_Q \mu(Q, \Xi)$, where $\mu(Q, \Xi)$ is provided in (4).

Define $\tilde{Q}_\sigma = Q_\sigma V_\sigma$, then $\tilde{Q}_{\sigma(t)}(i) = Q_{\sigma(t)}(i-1)C_2B_1$ for $i = 1, \cdots, t$ and $\tilde{Q}_{\sigma(t)}(0) = 0$. In this case U_σ is an FIR filter with $r = 1$, and Lemma 3.2 in [5] still holds resulting in the following result

$$\sup_{\sigma\in\Xi}\sup_{j\le i}\left\|[\mathcal{R}_{H_\sigma - U_\sigma\tilde{Q}_\sigma}]_j\right\| = \sup_{\sigma\in\Xi}\left\|[\mathcal{R}_{H_\sigma - U_\sigma\tilde{Q}_\sigma}]_i\right\|. \quad (5)$$

Let $\Phi^i = [\Phi^i_i, \Phi^i_{i-1}, \cdots, \Phi^i_0]$ denote $[\mathcal{R}_{H_\sigma - U_\sigma\tilde{Q}_\sigma}]_i$, then

$$\begin{aligned} \Phi^i_0 &= X_0(i), \qquad \Phi^i_1 = X_1(i) - Y_0(i)Q_{\sigma(i)}(0)C_2B_1, \\ \Phi^i_2 &= X_2(i) - Y_1(i)Q_{\sigma(i-1)}(0)C_2B_1 \\ &\qquad - Y_0(i)Q_{\sigma(i)}(1)C_2B_1, \\ \Phi^i_k &= -Y_1(i)Q_{\sigma(i-1)}(k-2)C_2B_1 \\ &\qquad - Y_0(i)Q_{\sigma(i)}(k-1)C_2B_1, \ k = 3, 4, \cdots, i. \end{aligned} \quad (6)$$

which means that given Q, Φ^i depends only on the switching sequence $\{\sigma(i-1), \sigma(i)\}$. By Equality (5), it follows that the formulations of two sequences determining $\{\bar{\nu}^i\}$ and $\underline{\nu}^i$ in [5] still hold with n^2 norm inequality constraints and Φ^i provided in (6).

This shows that it is not important whether $H_{\sigma(t)}$ and $U_{\sigma(t)}$ depend only on $\sigma(t)$ or not in the case of matched switching. What really matters in the complexity of optimization problems in Theorem 2.1 is the number of norm inequality constraints, which is determined by the number of time steps that the switching signal σ is involved in $[\mathcal{R}_{H_\sigma}]_t - [\mathcal{R}_{U_\sigma}]_t[\mathcal{M}_{Q_\sigma}]_t$ for each t. If the number of related time steps is finite for all t, similar convergent algorithms to those in Section III-B of [5] hold, which is valuable in relaxing the conditions (3).

In the case of partially matched switching, similar result to Theorem 2.1 holds if there are at least two mismatches between σ_H and σ_Q, i.e., $d(\sigma_H, \sigma_Q) \ge 2$. The relevant problem to solve is the following optimization

$$\inf_{\bar{Q}} \max_{\substack{1\le i\le n \\ 1\le j\le n}} \left\|H_{ij} - U_{ij}\bar{Q}V\right\|_1,$$

where $\bar{Q}$ belongs to $\mathcal{L}_{TI}$, $\hat{V}(\lambda) = C_2B_1\lambda$, and

$$\begin{aligned} \hat{H}_{ij}(\lambda) &= D_{11}(s_i) + C_1(s_i)B_1\lambda \\ &\qquad + [C_1(s_i) - D_{12}(s_i)F(s_i)]A(s_j)B_1\lambda^2, \\ \hat{U}_{ij}(\lambda) &= D_{12}(s_i) + [C_1(s_i) - D_{12}(s_i)F(s_i)]B_2(s_j)\lambda. \end{aligned}$$

V. COOPERATIVE CONTROL

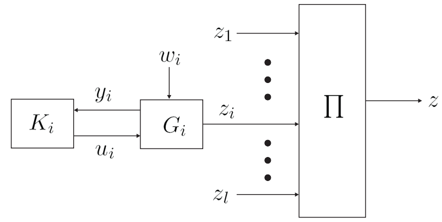

Fig. 4. Connection structure of a mixing system with l agents

Consider a network of l dynamic agents whose dynamics are described by the following state-space equations:

$$G_i: \begin{cases} x_i^+ = A_ix_i + B_{wi}w_i + B_iu_i \\ z_i = C_{zi}x_i + D_{zwi}w_i + D_{zui}u_i \qquad i = 1, \cdots, l \\ y_i = C_ix_i + D_{ywi}w_i \end{cases}$$

The connection among the l agents is shown in Figure 4, where the local regulated variables $\{z_i\}_{i=1}^l$ enter a linear mixing system Π to form variable z as a common goal, i.e., $z = \Pi\,\mathrm{cat}_i(z_i)$. To make our design problem simpler, we assume that system Π is a constant matrix and hence z can be formulated as $z = C_zx + D_{zw}w + D_{zu}u$, where $x = \mathrm{cat}_i(x_i)$, $w = \mathrm{cat}_i(w_i)$, $u = \mathrm{cat}_i(u_i)$. When Π has dynamics, the results are similar as long as Π is stable. Dynamics of the overall system with l agents are formulated more compactly as

$$\begin{aligned} x^+ &= Ax + B_ww + Bu \\ z &= C_zx + D_{zw}w + D_{zu}u \\ y &= Cx + D_{yw}w \end{aligned}$$

where $A = \mathrm{diag}_i(A_i)$ and $y = \mathrm{cat}_i(y_i)$; similar expressions hold for B_w, B, C, D_{yw}.

We assume that (A_i, B_i) is stabilizable and (A_i, C_i) is detectable for $i = 1, \cdots, l$, which means that there exist $F = \mathrm{diag}_i(F_i)$ and $L = \mathrm{diag}_i(L_i)$ to stabilize the system. Based on this, we propose a distributed controller as follows. Each local controller K_i is defined as the classic observer-based controller as shown in Figure 5, where v^r_{ij} is the received information from other agent j. Based on

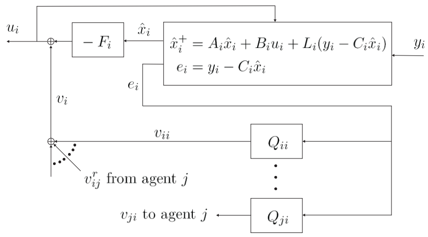

Fig. 5. Structure of subcontroller K_i

the received information v_{ij}^r and local measurement y_i, each local controller K_i computes a state estimate $\hat{x}_i$, thus a local estimation residual $e_i := y_i - C_i\hat{x}_i$ and control action $u_i = -F_i\hat{x}_i + v_i$, where $v_i = Q_{ii}e_i + \sum_{j\neq i} v_{ij}^r$. The communication information to agent $j\ (\neq i)$ is $v_{ji} = Q_{ji}e_i$.

Due to limited communication resources such as bandwidth and communication power, only limited information exchange among the agents is allowed resulting in structured interconnections. The process of transmitting information among the agents and generating exchange information based on local available information can be modeled abstractly as information flow. Information flow topology (IFT) refers to the way in which a group of dynamic agents exchange local information directly or indirectly, and reflects the structured interconnections among the agents. In general, IFT can be conveniently described by directed graphs. Let $R(t)$ denote the adjacency matrix of a directed graph describing the structured interconnection among the agents at time t, defined as follows:

$$r_{ij}(t) = \begin{cases} 1, & \text{if agent } i \text{ receives information from } j \text{ at } t \\ 0, & \text{otherwise} \end{cases}$$

where $r_{ij}(t)$ is the (i,j)th entry of matrix $R(t)$. In particular, we assume $r_{ii}(t) = 1\ \forall i, \forall t$, which means that each agent i can use its local information.

For simplicity, we assume that if there exists communication from node j to node i, then the transmission is ideal, i.e. $v_{ij}^r = v_{ij}$; otherwise there is no information available from agent j, i.e. $v_{ij}^r = 0$. Therefore the received information from agent j can be expressed as $v_{ij}^r(t) = r_{ij}(t)v_{ij}(t)$. Then $v(t)$ equals

$$\begin{bmatrix} Q_{11} & \cdots & r_{1l}(t)Q_{1l} \\ \vdots & \ddots & \vdots \\ r_{l1}(t)Q_{l1} & \cdots & Q_{ll} \end{bmatrix} e(t),$$

which is equivalently formulated as

$$\begin{bmatrix} \mathrm{diag}_j(r_{j1}(t)I_{p_j}), \cdots, \mathrm{diag}_j(r_{jl}(t)I_{p_j}) \end{bmatrix} \mathrm{diag}_j(\bar{Q}_j)e(t).$$

Here I_{p_j} is the identity matrix with order p_j equal to the dimension of u_j, and $\bar{Q}_j = \begin{bmatrix} Q_{1j} \\ \vdots \\ Q_{lj} \end{bmatrix}$. Define

$$\Gamma(t) = \begin{bmatrix} \mathrm{diag}_j(r_{j1}(t)I_{p_j}), \cdots, \mathrm{diag}_j(r_{jl}(t)I_{p_j}) \end{bmatrix},$$
$$Q_d = \mathrm{diag}_j(\bar{Q}_j),$$

then $v(t) = \Gamma(t)Q_d e(t)$. Hence the regulated output

$$z = Hw - Uv = (H - U\Gamma Q_d V)w,$$

where H, U, V are provided in [7].

Let $\{R_m\}_{m=1}^n$ denote n IFTs under consideration and $\sigma(t)$ the status of IFT at time t, i.e., $R(t) = R_m$ if $\sigma(t) = s_m$. Since the time-varying system $\Gamma(t)$ is on a one-to-one correspondence with $R(t)$ and hence with $\sigma(t)$, it can be written as Γ_σ to emphasize its dependence on the switching signal σ. Note that Γ_σ assumes n possible values $\{\Gamma_m\}_{m=1}^n$, i.e., $\Gamma_{\sigma(t)}(t) = \Gamma_m$ if $\sigma(t) = s_m$. Define $\tilde{Q}_\sigma = \Gamma_\sigma Q_d$, then the design problem is to solve

$$\inf_Q \sup_\sigma \left\| H - U\tilde{Q}_\sigma V \right\|. \tag{7}$$

Here H and U are LTI systems, which means that in the model matching framework (1), $H_1 = H_2 = \cdots = H_n$ and $U_1 = U_2 = \cdots = U_n$. What is special about problem (7) is that $\tilde{Q}_\sigma$ is switching between n structured components $\{\tilde{Q}_m\}_{m=1}^n$ determined by the n information flow topologies $\{R_m\}_{m=1}^n$. These topologies generate a structure in $\tilde{Q}_\sigma$ which at each time t is determined by the relation $\tilde{Q}_{\sigma(t)} = \Gamma_{\sigma(t)}(t)Q_d$, i.e., $\tilde{Q}_m = \Gamma_m Q_d$.

A. Three-agent case

Consider a network of three agents ($l = 3$), each with the dynamics described by

$$x_i^+ = \begin{bmatrix} -1.0090 & 1.5823 \\ 0.9080 & -0.9791 \end{bmatrix} x_i + \begin{bmatrix} 1.0079 \\ 0.1585 \end{bmatrix} w_i + \begin{bmatrix} -0.5869 \\ 1.5741 \end{bmatrix} u_i$$
$$y_i = \begin{bmatrix} -0.5166 & 1.2278 \end{bmatrix} x_i + 1.5839 w_i.$$

The system matrices were randomly generated and led to eigenvalues of the open-loop system $\{-2.1928,\ 0.2047\}$, i.e., it is unstable. The design objective is to minimize the system performance of the mapping from $w \to z$, where z is described by

$$z = \begin{bmatrix} -2.0890 & 1.0501 & -1.3718 & 0.5891 & -0.4913 & -0.7354 \\ 2.9495 & -0.7672 & -1.2677 & 1.8426 & -2.1776 & -1.7794 \\ 1.3561 & -0.2577 & -0.8949 & 1.3480 & 0.2370 & 0.4480 \end{bmatrix} x + \begin{bmatrix} 0.5812 & -0.4175 & 0.2176 \\ 0.8566 & -0.2058 & 1.6843 \\ -0.2663 & -0.1743 & 0.1195 \end{bmatrix} u.$$

Consider the case where the IFTs are described by the adjacency matrices

$$R_1 = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}, \quad R_2 = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix},$$

resulting in two structured systems

$$\tilde{Q}_1 = \begin{bmatrix} Q_{11} & 0 & Q_{13} \\ Q_{21} & Q_{22} & 0 \\ 0 & Q_{32} & Q_{33} \end{bmatrix}, \tilde{Q}_2 = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ 0 & Q_{22} & Q_{23} \\ Q_{31} & 0 & Q_{33} \end{bmatrix}.$$

TABLE I

SYSTEM PERFORMANCE $\inf_Q \sup_\sigma \|w \to z\|$ V.S. INFORMATION FLOW TOPOLOGY

	Fixed information flow topology			Arbitrary switching among	
	R_1	R_2	R_3	R_1 and R_2	R_1, R_2, and R_3
$\ell_\infty-$induced norm	18.4622	18.4105	24.3997	21.5333	24.3997

In both cases, each agent receives the information from another agent directly, in addition to its own local information. For each fixed information flow topology, the optimal $\ell_\infty-$induced performance can be determined via solving the standard ℓ_1 optimization, with results as follows:

$$\gamma_1^o \simeq 18.4622, \quad \gamma_2^o \simeq 18.4105,$$

where $\gamma_m^o = \inf_Q \left\| H - U\tilde{Q}_m V \right\|$. In these problems U and V are selected as FIR filters of order $r = 2$. If arbitrary switching of IFTs between R_1 and R_2 is allowed, it means that $\tilde{Q}_\sigma$ switches between two ($n = 2$) structured systems $\tilde{Q}_1$ and $\tilde{Q}_2$. Then by solving the sequence of linear programming problems in [5], we can determine the worst case $\ell_\infty-$induced performance, approximately equal to 21.5333, which is greater than $\gamma_1^o \vee \gamma_2^o \simeq 18.4622$.

When no information exchange between the agents occurs, the IFT is described by the adjacency matrix

$$R_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad \text{and } \tilde{Q}_3 = \begin{bmatrix} Q_{11} & 0 & 0 \\ 0 & Q_{22} & 0 \\ 0 & 0 & Q_{33} \end{bmatrix}.$$

The corresponding optimal system performance is obtained as $\gamma_3^o \simeq 24.3997$. If arbitrary switching among R_1, R_2, and R_3 is allowed, the optimal system performance equals 13.1368 approximately. The numerical results are summarized in Table I.

B. Remarks

Given Q, if $\sigma(t) = m, \forall t \geq 0$, it follows that $\left\| H - U\tilde{Q}_\sigma V \right\| = \left\| H - U\tilde{Q}_m V \right\|$, and hence

$$\sup_\sigma \left\| H - U\tilde{Q}_\sigma V \right\| \geq \max_{1 \leq m \leq n} \left\| H - U\tilde{Q}_m V \right\|.$$

Then it holds that

$$\begin{aligned} \inf_Q \sup_\sigma \left\| H - U\tilde{Q}_\sigma V \right\| &\geq \inf_Q \max_{1 \leq m \leq n} \left\| H - U\tilde{Q}_m V \right\| \\ &\geq \max_{1 \leq m \leq n} \inf_Q \left\| H - U\tilde{Q}_m V \right\|, \end{aligned}$$

$$\text{i.e.} \quad \inf_Q \sup_\sigma \left\| H - U\tilde{Q}_\sigma V \right\| \geq \max_{1 \leq m \leq n} \gamma_m^o. \tag{8}$$

The meaning of (8) is that the best overall (transient) system performance under arbitrary switching among n components can not be better than the worst (steady) system performance without switching.

If there are structure constraints on $\{\tilde{Q}_m\}_{m=1}^n$ due to information constraints $\{\Gamma_m\}_{m=1}^n$, the inequality relation can be strict, as demonstrated by the case of arbitrary switching among $\{R_1, R_2\}$ in the example of Section V-A. However, there are some cases where the inequality relation in (8) is in fact equality. For example, if there exists an information flow topology $R_{\bar{m}} \in \{R_m\}_{m=1}^n$ such that $R_{\bar{m}} \preceq R_m$[2], $\forall m$, it follows that $\inf_Q \sup_\sigma \left\| H - U\tilde{Q}_\sigma V \right\| = \gamma_{\bar{m}}^o$. This is so, because if we select Q such that $Q_{jk} = 0$ when the (j,k)th entry of $R_{\bar{m}}$ is 0, then it follows that $\Gamma_m Q_d = \Gamma_{\bar{m}} Q_d = \tilde{Q}_{\bar{m}}, \forall m$, and the closed-loop mapping from $w \to z$ is independent of the information flow topology at any time t, i.e., $H - U\tilde{Q}_\sigma V = H - U\tilde{Q}_{\bar{m}} V$, $\forall \sigma$. Thus this exhibits a selection for Q which achieves equality in (8), and is verified by the case of arbitrary switching among $\{R_1, R_2, R_3\}$ in the example of Section V-A. In this example, $R_3 \preceq R_1$ and $R_3 \preceq R_2$, thus the best achievable performance under arbitrary switching equals that of the fixed topology R_3.

VI. CONCLUSION

In this paper we considered applications of a model matching framework to sensitivity minimization of switched systems, LPV control, and cooperative control of dynamic agents. We showed that these design problems can be solved by the proposed LP algorithms. A numerical example was also included to demonstrate their efficiency. Applications to sensor/actuator failures, optimal estimation of switched systems, and other switched systems are currently pursued by the authors.

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[2] $R \preceq \bar{R}$ means that the (j,k)th entry of R is less than or equal to the (j,k)th entry of $\bar{R}$, $\forall j, \forall k$.