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On Optimal Signal Reconstruction over Switching Networks*

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Abstract— **In this paper we consider signal reconstruction over networks where the communication channel can be modeled as an input switching system (e.g., wireless communication). In particular, we formulate the design problem as a prototypical model matching problem where the various mappings involved belong to a class of input switching systems. The design interest is placed on minimizing the worst case performance of this model matching system over all possible switchings with either ℓ_1-induced norm or $\mathcal{H}_2$ norm as the performance criterion. This minimization is performed over all stable receivers Q in the class of input switching systems. For the particular set-up at hand and in the case of matched switching, two convergent sequences to the optimal solution from above and below respectively are formulated in terms of linear programs and quadratic programs respectively for the ℓ_1-induced and $\mathcal{H}_2$ norm optimizations. An approximate solution with any given precision is possible by finite truncation. Also, it is shown that the optimal receiver Q need not depend on the switching sequence in the cases of partially matched switching and unmatched switching, and that it can be obtained as a linear time-invariant (LTI) solution to an associated ℓ_1-induced or $\mathcal{H}_2$ norm optimization.**

Keywords: signal reconstruction, ℓ_1-induced optimality, $\mathcal{H}_2$ optimal, worst case switching

I. INTRODUCTION

In recent years, there has been an increasing interest in communication and control over wireless networks, due to their low installation/maintenance costs, great physical mobitlity, ease of replacement and upgrading, and so on [1]. This research is also motivated by a wide class of potential applications, such as automated highway systems [2], environmental monitoring and motion monitoring [3], home automation [4], unmanned aerial vehicles [5], wireless and mobile data networks [6]. In these applications, wireless communication plays a key role in supporting information exchange between people or devices. Although there has been a fast development in wireless technology recently, many technical challenges exist and must be solved to enable future wireless applications. These challenges extend across all aspects of the system design, such as energy constraint, finite bandwidth, random variations of wireless channels, security, and cross-layer protocol [7].

Note that most of the works in the literature deal with wireless communication in stochastic formulations, such as channel estimation [8], [9] and data reconstruction (estimation) [7], [9], [10], [11]. In these formulations, the usual performance criterion adopted is the probability of error decision or mean square error (distortion) under various assumptions on noise model, fading nature and side information of the communication channel. In general it is a hard problem to optimize the performance criterion and determine the corresponding channel capacity and optimal transmission scheme exactly, due to the unpredictable nature of wireless channels.

Our goal, in this paper, is to consider a class of communication channels which can be modeled as input switching systems in Section II and present a worst case optimization approach of signal reconstruction. Consider a flat fading channel with n_t transmit antennas and n_r receive antennas, which can be characterized by a discrete-time baseband model as

$$y = Hx + w \tag{1}$$

where $x \in \mathbb{C}^{n_t}$ is the transmitted signal, $y \in \mathbb{C}^{n_r}$ is the channel output, $w \in \mathbb{C}^{n_r}$ is the channel noise, and $H \in \mathbb{C}^{n_r \times n_t}$ is the channel gain matrix[1]. In particular we assume that the channel gain matrix at time k denoted by $H(k)$ is a complex matrix and takes values in a finite set $\{H_1, \cdots, H_n\}$ independent of $H(l), \forall\, l \neq k$, where $\{H_m\}_{m=1}^n$ can be viewed as n quantization levels of the channel gain matrix H or n most possible channel gains identified via some classical methods [9]. In this case the channel model in (1) can be equivalently formulated in Figure 1 with n parallel virtual channels $\{H_m\}_{m=1}^n$, where the switching signal σ indicates the channel status and the transmitted signal at each time can only be connected to one virtual channel of $\{H_m\}_{m=1}^n$.

Based on the input switching model of wireless channels, we formulate the signal reconstruction problem over wireless networks as a model matching problem in the class of input switching systems. The interest is placed on minimizing the worst case performance of this model matching system over all possible switchings with either ℓ_1-induced norm or $\mathcal{H}_2$ norm as the performance criterion. This minimization is performed over all stable receivers Q in the class of input switching systems. For the particular set-up at hand and in

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[1] $\mathbb{C}^m$ denotes a complex vector of dimension m and $\mathbb{C}^{k\times l}$ denotes a complex matrix of k rows and l columns.

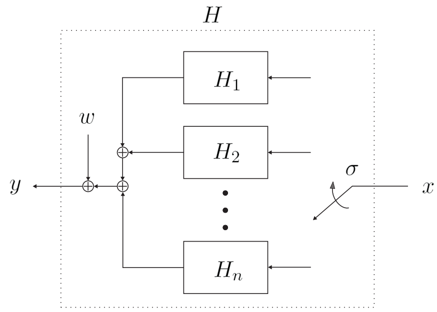

Fig. 1. Equivalent formulation of a flat fading channel H

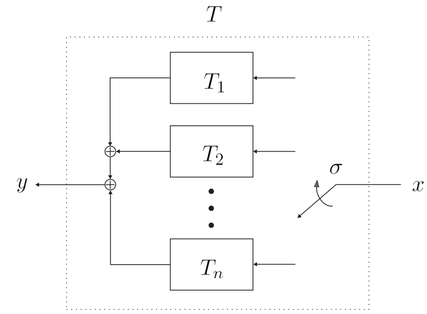

Fig. 2. Structure of system $T \in \mathcal{T}_n$

the case of matched switching, two convergent sequences to the optimal solution from above and below respectively are formulated in terms of linear programs and quadratic programs respectively for the ℓ_1-induced and $\mathcal{H}_2$ norm optimizations. An approximate solution with any given precision is possible by finite truncation. Also, it is shown that the optimal receiver Q need not depend on the switching sequence in the cases of partially matched switching and unmatched switching, and that it can be obtained as a linear time-invariant (LTI) solution to an associated ℓ_1-induced or $\mathcal{H}_2$ norm optimization.

The paper is organized as follows. In Section II, we introduce the preliminaries and formulate the design problem; in Section III, we deal with three kinds of switchings: matched switching, partially matched switching, and unmatched switching; formulate two linear programming problems converging to the optimal solution respectively in the matched case with an arbitrary precision, and propose two dual ℓ_1-induced norm optimization problems which solve the rest two cases; we also show that the final solutions in the rest two cases can be determined through solving standard ℓ_1 optimization problems; in Section IV, we consider the model matching problem with $\mathcal{H}_2$ norm as the performance criterion, and produce similar results to those in Section III; in Section V we conclude.

The notation in the paper is as follows: For a matrix A with real entries, $A^{'}$ denotes its transpose and the Frobenius norm of matrix A is $\|A\|_F = \sqrt{\mathrm{tr}(AA')}$. $\|x\|_1 := \sum_k |x(k)|$ is the ℓ_1 norm of a real valued sequence $x = \{x(k)\}_{k=0}^{\infty}$. For a vector-valued signal $x = [x_1, x_2, \ldots, x_n]'$, the ℓ_1 norm of x is defined as $\|x\|_1 := \sum_{m=1}^{n} \|x_m\|_1$. For a multi-input multi-output (MIMO) system T (which may be time-varying or nonlinear), $\|T\| = \sup_{x\neq 0} \frac{\|Tx\|_1}{\|x\|_1}$ is the ℓ_1-induced norm. T is stable if $\|T\| < \infty$; Λ^k denotes the $k-$step delay operator while Λ^{-k} the $k-$step advance operator (note $\Lambda^{-k}\Lambda^k = I$); $\mathcal{L}_{TV}$ denotes all stable causal time-varying systems while $\mathcal{L}_{TI}$ denotes its subset of time-invariant systems. If T is an LTI system having unit impulse response $\{T(k)\}_{k=0}^{\infty}$, $T^{'}$ denotes the LTI system $\sum_{k=0}^{\infty} T(k)^{'} \lambda^k$, $[\mathcal{R}_T]_t$ denotes the (block) row matrix $[T(t) \cdots T(0)]$, and the ℓ_1 norm of T is defined as

$$\|T\|_1 = \sup_t \max_m \sum_k |\left([\mathcal{R}_T]_t\right)_{mk}|$$

where $([\mathcal{R}_T]_t)_{mk}$ is the (m,k)th (scalar) entry of $[\mathcal{R}_T]_t$. For two integers m and k, $m \vee k$ denote $\max\{m,k\}$ and $m \wedge k$ denotes $\min\{m,k\}$. For a random variable x, $\mathcal{E}(x)$ denotes the expectation. $\mathbb{Z}^+$ denotes the set of nonnegative integers. For a linear time-varying (LTV) system P with a lower (block) triangular representation [12]

$$\begin{bmatrix} P(0,0) & & & \\ P(1,0) & P(1,1) & & \\ P(2,0) & P(2,1) & P(2,2) & \\ \vdots & \ddots & \ddots & \ddots \end{bmatrix} \tag{2}$$

where $P(t,r)$ denotes the linear mapping from the input at time r to the output at time t, let $[\mathcal{C}_P]_t^k$ denote the truncated t^{th} (block) column in the infinite matrix representation of P, i.e.,

$$[\mathcal{C}_P]_t^k = \begin{bmatrix} P(t,t) \\ P(t+1,t) \\ \vdots \\ P(k,t) \end{bmatrix}.$$

Also, $[\mathcal{M}_P]_t^k$ will denote the lower (block) triangular matrix

$$\begin{bmatrix} P(t,t) & & & \\ P(t+1,t) & P(t+1,t+1) & & \\ \vdots & \ddots & \ddots & \\ P(k,t) & \ldots & \ldots & P(k,k) \end{bmatrix},$$

which is the truncated input-output mapping of P over time $[t,\, k]$. In the definitions of $[\mathcal{C}_P]_t^k$ and $[\mathcal{M}_P]_t^k$, we assume implicitly that $k \geq t$, which is also true in the rest of this paper. The ℓ_1-induced norm of LTV system P with a lower (block) triangular representation (2) is defined as is

$$\begin{aligned} \|P\| &= \sup_t \sup_k \|[\mathcal{M}_P]_t^k\| \\ &= \sup_t \sup_k \|[\mathcal{C}_P]_t^k\| \\ &= \sup_t \sup_k \max_{m_2} \sum_{m_1} \left| \left([\mathcal{C}_P]_t^k\right)_{m_1 m_2} \right|. \end{aligned}$$

II. PRELIMINARIES AND PROBLEM DEFINITION

In this section we introduce a class $\mathcal{T}_n$ of input switching systems. Each switched system T in $\mathcal{T}_n$ is associated with $(\{T_m\}_{m=1}^n, \sigma)$ as shown in Figure 2, where $\{T_m\}_{m=1}^n$ is a set of LTI systems and $\sigma : \mathbb{Z}^+ \to \{1, \cdots, n\}$ is a switching

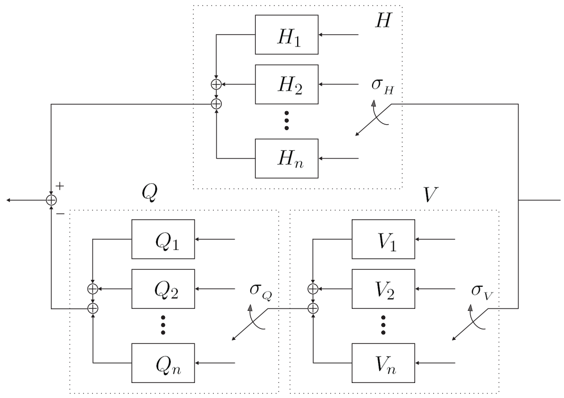

Fig. 3. Connection structure of a switching system $H_\sigma - Q_\sigma V_\sigma$

signal. For each LTI system T_m, let $\{T_m(k)\}_{k=0}^{\infty}$ denote its unit impulse response. We assume that $T_m(k)$ has real entries $\forall m\,\forall k$, which can also model communication channels with complex entries in (1), as shown in [13]. Given an input $x = \{x(t)\}_{t=0}^{\infty}$ and a switching trajectory $\{\sigma(t)\}_{t=0}^{\infty}$ of σ, the output $y = \{y(t)\}_{t=0}^{\infty}$ of T is defined as

$$\begin{aligned} y(t) &= (Tx)(t) \\ &= \sum_{k=0}^{t} T_{\sigma(t-k)}(k)x(t-k), \end{aligned}$$

where $T_{\sigma(t-k)}(k) = T_m(k)$ if $\sigma(t-k) = m \in \{1, \cdots, n\}$. This means that $y(t)$ depends on the switching signal up to time t, i.e., $\{\sigma(k)\}_{k=0}^{t}$. It also shows that T is a causal mapping in time from $(x, \sigma) \to y$. For a specific trajectory $\{\sigma(t)\}_{t=0}^{\infty}$ of the switching signal σ, let T_σ denote the LTV system with a lower (block) triangular representation

$$\begin{bmatrix} T_{\sigma(0)}(0) & & & \\ T_{\sigma(0)}(1) & T_{\sigma(1)}(0) & & \\ T_{\sigma(0)}(2) & T_{\sigma(1)}(1) & T_{\sigma(2)}(0) & \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix}.$$

Given this notation, the problem of interest is shown in Figure 3, where H, V, and Q in $\mathcal{T}_n$ are associated with $(\{H_m\}_{m=1}^n, \sigma_H)$, $(\{V_m\}_{m=1}^n, \sigma_V)$, and $(\{Q_m\}_{m=1}^n, \sigma_Q)$ respectively. For simplicity of notation, let σ denote the triplet of switching signals $(\sigma_H, \sigma_V, \sigma_Q)$ in the following. The precise design problem can be stated as follows.

Find a switched system Q in $\mathcal{T}_n$ solving

$$\nu(\Xi) := \inf_{\{Q_m\}_{m=1}^n} \mu(Q, \Xi) \tag{3}$$

where $\mu(Q, \Xi) := \sup_{\sigma \in \Xi} \|H - QV\|$, and Ξ denotes the set of admissible switching trajectories.

Here Q depends only on the switching signal σ_Q explicitly, and its dependence on switching signals σ_H, σ_V is explored in the following sections through the specified relations among σ_H, σ_V and σ_Q, which affect the explicit formulation of Ξ (for details, see Section III) and hence the achievable reconstruction performance. The switching signal σ_Q is included in the supremizing parameter σ because the focus of this paper is on the case of uncontrolled switching, as usually happens in related applications. Given a system $Q \in \mathcal{T}_n$, according to our previous notation $\mu(Q)$ can be also expressed as

$$\mu(Q) = \sup_{\sigma \in \Xi} \|H_\sigma - Q_\sigma V_\sigma\|,$$

where in an abuse of notation $H_\sigma, V_\sigma, Q_\sigma$ refer to H_{σ_H}, V_{σ_V}, Q_{σ_Q} respectively since each time-varying system $T\,(= H, V, \text{or}\, Q)$ depends only (causally in time) on the corresponding switching signal σ_T in σ.

In accordance with the notation in the control literature, herein we use system V to model the channel dynamics and system H to indicate the dynamics of a reference model. Both H and V can be memoryless gain matrices or finite impulse response (FIR) filters. If each LTI component of system V is a FIR filter, the effects of transmitted signal $x(k)$ on the channel output will depends only on the channel status $\sigma(k)$ via the channel gains $\{V(k,k), V(k+1,k), \cdots, \}$. If $H_m = \lambda^L\,(L \geq 0)$ for all $m \in \{1, \cdots, n\}$, the model matching problem (3) is equivalent to finding an optimal receiver with $L-$step delays.

The model matching setup depicted in Figure 3 can encompass the signal reconstruction problem in wireless communication as indicated in Section I where channel variations and noise are present.

III. Signal Reconstruction over ℓ_1

In this section we consider the model matching problem (3) over ℓ_1, i.e., we adopt the worst case ℓ_1-induced norm as the performance criterion. Note that for different relations among the switching signals σ_H, σ_V and σ_Q, the set Ξ of admissible switching trajectories is different and hence the optimal performance is different. To proceed, we first introduce the following definitions.

For two switched systems $T, \tilde{T} \in \mathcal{T}_n$, $\sigma_T = \sigma_{\tilde{T}}$ if $\sigma_T(t) = \sigma_{\tilde{T}}(t)$, $\forall t \geq 0$; σ_T and $\sigma_{\tilde{T}}$ are independent if $\forall t \geq 0$, $(\sigma_T(t), \sigma_{\tilde{T}}(t))$ can assume any value in the set $\{(j,k) \,|\, j \in \{1, \cdots, n\}, k \in \{1, \cdots, n\}\}$. The distance between σ_T and $\sigma_{\tilde{T}}$ is

$$d(\sigma_T, \sigma_{\tilde{T}}) = \sum_{t=0}^{\infty} 1_{\left\{\sigma_T(t) \neq \sigma_{\tilde{T}}(t)\right\}},$$

where $1_A = 1$ if statement A is true and $1_A = 0$ otherwise. Then $d(\sigma_T, \sigma_{\tilde{T}})$ denotes the number of mismatches between σ_T and $\sigma_{\tilde{T}}$.

A. Matched switching

In this section we consider the case of matched switching where $\sigma_H = \sigma_V = \sigma_Q$, i.e., the set of admissible switching trajectories is

$$\Xi_1 = \{\sigma \,|\, \forall t \geq 0,\ \sigma_H(t) = \sigma_V(t) = \sigma_Q(t) \in \{1, \cdots, n\}\}.$$

Let ν_1 denote the corresponding system performance of (3), i.e., $\nu_1 = \nu(\Xi_1)$. As is well-known, system stability and performance optimization under arbitrary switching are very difficult to handle. Although the whole system under arbitrary switching here is guaranteed to be stable, there is no simple algorithm that can provide a precise solution to the worst case model matching problem (3). Instead, we will formulate two sequences of linear programs with increasing complexity, the solutions to which provide a convergent sequence to the optimal solution from below and a convergent sequence to the optimal solution from above, as stated in Theorem 3.1.

Towards this goal, define for each $i\,(\geq 0)$

$$\begin{aligned}\bar{\nu}^i &= \inf_{\substack{\{Q_m(l)\}_{m=1}^n=0\\ \forall l>i}} \sup_t \sup_k \sup_{\sigma\in\Xi_1} \left\|[\mathcal{M}_{H_\sigma-Q_\sigma V_\sigma}]_t^k\right\|,\\ \underline{\nu}^i &= \inf_{\{Q_m\}_{m=1}^n} \sup_t \sup_{\sigma\in\Xi_1} \left\|[\mathcal{M}_{H_\sigma-Q_\sigma V_\sigma}]_t^i\right\|.\end{aligned} \tag{4}$$

By the definitions of $\bar{\nu}^i$ and $\underline{\nu}^i$, it is easy to show that

$$\bar{\nu}^i \geq \bar{\nu}^{i+1} \geq \nu^*, \quad \text{and} \quad \underline{\nu}^i \leq \underline{\nu}^{i+1} \leq \nu^*,\ \forall i$$

where

$$\nu^* = \inf_{\{Q_m\}_{m=1}^n} \sup_t \sup_k \sup_{\sigma\in\Xi_1} \left\|[\mathcal{M}_{H_\sigma-Q_\sigma V_\sigma}]_t^k\right\|.$$

First, we need to prove the following lemma.

Lemma 3.1: $\nu_1 = \nu^*$.

Proof of Lemma: For details, see [18].

Next, we show that

Lemma 3.2: $\left\{\sup_{\sigma\in\Xi_1} \left\|[\mathcal{C}_{H_\sigma-Q_\sigma V_\sigma}]_j^k\right\|\right\}_{j=0}^k$ is a decreasing sequence in j for a given integer k.

Proof: For details, see [18].

By Lemma 3.2, it follows that

$$\sup_{\sigma\in\Xi_1} \left\|[\mathcal{M}_{H_\sigma-Q_\sigma V_\sigma}]_t^k\right\| = \sup_{\sigma\in\Xi_1} \left\|[\mathcal{C}_{H_\sigma-Q_\sigma V_\sigma}]_t^k\right\|,$$

hence the formulations of $\bar{\nu}^i$ and $\underline{\nu}^i$ can be simplified as

$$\begin{aligned}\bar{\nu}^i &= \inf_{\substack{\{Q_m(l)\}_{m=1}^n=0\\ \forall l>i}} \sup_j \sup_{\sigma\in\Xi_1} \left\|[\mathcal{C}_{H_\sigma-Q_\sigma V_\sigma}]_0^j\right\|,\\ \underline{\nu}^i &= \inf_{\{Q_m\}_{m=1}^n} \sup_{\sigma\in\Xi_1} \left\|[\mathcal{C}_{H_\sigma-Q_\sigma V_\sigma}]_0^i\right\|.\end{aligned}$$

Let $\Phi^i = \begin{bmatrix} \Phi_0^i \\ \vdots \\ \Phi_i^i \end{bmatrix}$ denote $[\mathcal{C}_{H_\sigma-Q_\sigma V_\sigma}]_0^i$, then for each $k \in \{0,\cdots,i\}$

$$\Phi_k^i \;=\; H_{\sigma(0)}(k) - \sum_{m=0}^{k} Q_{\sigma(m)}(k-m)V_{\sigma(0)}(m).$$

Note that for a given $Q \in \mathcal{S}_n$, Φ^i depends on both $\sigma(i)$ and $\{\sigma(t)\}_{t=0}^{i-1}$, and hence $\sup_{\sigma\in\Xi_1}\left\|\Phi^i\right\|$ is a linear programming optimization over all the n^{i+1} trajectories of $\{\sigma(t)\}_{t=0}^i$. This will be in general intractable since there is no explicit finite bound on i and the number of constraints n^{i+1} will increase exponentially to ∞ as $i \to \infty$. Fortunately, in most wireless networks the channel dynamics V can be chosen to be an FIR filter of certain order $\tau\,(\geq 0)$. More generally, since $\{V_i\}_{i=1}^n$ are in $\mathcal{L}_{TI}$, they can always be approximated by FIR filters, and thus V itself can be approximated as an FIR mapping in $\mathcal{T}_n$. In this case the complexity is manageable, as we indicate in what follows.

1) The FIR V case: If there exists a finite τ such that $\{V_m(k)\}_{m=1}^n = 0$ for all $k > \tau$, then for each $k \in \{0,\cdots,i\}$

$$\Phi_k^i = H_{\sigma(0)}(k) - \sum_{m=0}^{k\wedge\tau} Q_{\sigma(m)}(k-m)V_{\sigma(0)}(m), \tag{5}$$

and thus Φ^i will depend on at most $\tau+1$ switching instants, i.e., $\{\sigma(t)\}_{t=0}^{i\wedge\tau}$. This means that we only need to concentrate on the switching sequences over the time period $[0, i\wedge\tau]$, i.e., at most $n^{\tau+1}$ switching sequences. In this case, we formulate two sequences of linear programs determining $\underline{\nu}^i$ and $\bar{\nu}^i$ respectively as follows.

For an arbitrarily given i, Φ^i, and hence $\underline{\nu}^i$ depend only on $\{Q_m(k)\}_{m=1}^n,\ k=0,\cdots i$. The corresponding optimization problem is:

$$\underline{\nu}^i = \inf_{\{Q_m(k)\}_{m=1}^n,\ k=0,\cdots i} \gamma_i \tag{$\underline{\mathrm{LP}}_i$}$$

$$\left\|\begin{bmatrix} \Phi_0^i \\ \vdots \\ \Phi_i^i \end{bmatrix}\right\| \leq \gamma_i,\ \forall\,\sigma(t)\in\{1,\cdots,n\}, t=0,\cdots,i\wedge\tau,$$

where $\{\Phi_k^i\}_{k=0}^i$ are provided in (5). Note that the number of constraints in ($\underline{\mathrm{LP}}_i$) is at most $n^{\tau+1}$ independent of the truncation level i and related optimizing parameters are $\{Q_m(k)\}_{m=1}^n$, $k=0,\cdots,i$ with a total number of $n(i+1)$, which means that problem ($\underline{\mathrm{LP}}_i$) can be solved in polynomial time for any i.

If $\{Q_m(k)\}_{m=1}^n = 0,\ \forall\, k > i$, there are $\tau+i+1$ terms of Φ^j depending on the nonzero optimizing parameters $\{Q_m(k)\}_{m=1}^n,\ k \leq i$, i.e., $\Phi_0^j,\cdots,\Phi_{\tau+i}^j$ when $j \geq \tau+i$. Then the upper bound $\bar{\nu}^i$ is determined as:

$$\begin{aligned}\bar{\nu}^i &= \inf_{\{Q_m(k)\}_{m=1}^n=0,\ \forall\, k>i} \bar{\gamma}_i\\ \sup_j &\left\|\Phi^j\right\| \leq \bar{\gamma}_i,\ \forall\,\sigma(t)\in\{1,\cdots,n\}, t=0,\cdots,\tau\end{aligned}$$

where $\Phi_k^j = H_{\sigma(0)}(k),\ k > \tau+i$

$$\text{and } \Phi_k^j = H_{\sigma(0)}(k) - \sum_{m=0\vee(k-i)}^{k\wedge\tau} Q_{\sigma(m)}(k-m)V_{\sigma(0)}(m),$$

if $k \leq \tau+i$. For a given $j\,(>\tau+i)$, define

$$\begin{aligned}b_{\sigma(0)}(m_1) &= \sum_{k=\tau+i+1}^{\infty} \sum_{m_1=1}^{n_z} \left|\left(H_{\sigma(0)}(k)\right)_{m_1 m_2}\right|,\\ b_{\sigma(0)} &= [b_{\sigma(0)}(1)\cdots b_{\sigma(0)}(n_w)],\end{aligned}$$

where n_z, n_w are the dimensions of the output and input of the time-varying system H respectively. Then the equivalent formulation to determine $\bar{\nu}^i$ is

$$\bar{\nu}^i = \inf_{\{Q_m(k)\}_{m=1}^n,\ k=0,\cdots,i} \bar{\gamma}_i \tag{$\overline{\mathrm{LP}}_i$}$$

$$\left\|\begin{bmatrix} \Phi_0^j \\ \vdots \\ \Phi_{r+i}^j \\ b_{\sigma(0)} \end{bmatrix}\right\| \leq \bar{\gamma}_i,\ \forall\,\sigma(t)\in\{1,\cdots,n\}, t=0,\cdots,\tau.$$

Note that problem ($\overline{\text{LP}}_i$) can also be solved in polynomial time for any i since there are at most n^{r+1} constraints independent of the truncation level i and $n(i+1)$ optimizing parameters $\{Q_m(k)\}_{m=1}^n$, $k = 0, \cdots, i$. The solution to problem ($\overline{\text{LP}}_i$) is independent of j as long as $j > \tau+i$, since $\Phi_{\tau+i}^j, \cdots, \Phi_0^j$ are the only terms relating to the optimizing parameters $\{Q_m(k)\}_{m=1}^n$, $k = 0, \cdots, i$. The other constants are summed up as vector $b_{\sigma(0)}$, and the optimization is considered over all n^{r+1} switching sequences, independent of $\sigma(0)$.

Following the proofs of Theorem 6.1 and 6.2 in [17], we can show that

$$\bar{\nu}^i \searrow \nu_1, \quad \underline{\nu}^i \nearrow \nu_1, \text{ as } i \to \infty.$$

Summarizing we have

Theorem 3.1: For matched switching and V FIR of order τ, the model matching problem (3) is solved by the two sequences of linear programs in ($\underline{\text{LP}}_i$) and ($\overline{\text{LP}}_i$).

B. Partially matched switching

In this subsection we consider the case where $\sigma_H = \sigma_V$ and there are some mismatches (at least one) between σ_H and σ_Q. First, define the auxiliary optimization problem

$$\nu_2 := \inf_{Z \in \mathcal{L}_{TI}} \max_{1 \le m \le n} \|H_m - ZV_m\|, \tag{6}$$

which equals

$$\inf_{Z \in \mathcal{L}_{TI}} \|[H_1 \cdots H_n] - Z[V_1 \cdots V_n]\|,$$

and is a standard (LTI) ℓ_1-induced model matching problem. The solution to (6) provides a solution to the worst-case model matching problem (3) in $\mathcal{T}_n$ if there is one mismatch between σ_G and σ_R, i.e., the set of admissible switching trajectories is

$$\Xi_2 = \left\{ \sigma \left| \begin{array}{l} \forall t \ge 0, \sigma_H(t) = \sigma_V(t) \in \{1, \cdots, n\} \\ \forall t \ge 0, \sigma_Q(t) \in \{1 \cdots, n\} \\ d(\sigma_H, \sigma_Q) = 1 \end{array} \right. \right\},$$

as one of our main results indicates :

Theorem 3.2: For any $\epsilon > 0$ and $\bar{Q}$ in $\mathcal{L}_{TI}$ with

$$\max_{1 \le m \le n} \left\|H_m - \bar{Q}V_m\right\| \le \nu_2 + \epsilon,$$

it holds that

$$\mu(\bar{Q}, \Xi_2) \le \nu_2 + \epsilon,$$

and thus $\nu(\Xi_2) = \nu_2$.

Proof: For details, see [18].

Theorem 3.2 shows that in the case of partially matched switching, there is no need to "switch" the receiver Q and no performance loss if we choose Q as an LTI system, which is the solution to an auxiliary ℓ_1-induced norm optimization problem.

Note that the proof obviously goes through for $d(\sigma_H, \sigma_Q) > 1$. So, in the general case that σ_H, σ_Q are independent,i.e., the set of admissible switching trajectories is

$$\Xi_{2g} = \left\{ \sigma \left| \begin{array}{l} \forall t \ge 0, \sigma_H(t) = \sigma_V(t) \in \{1, \cdots, n\} \\ \sigma_H \text{ and } \sigma_Q \text{ are independent} \end{array} \right. \right\},$$

we have the following result:

Corollary 3.1: For any $\epsilon > 0$ and $\bar{Q}$ in $\mathcal{L}_{TI}$ with

$$\max_{1 \le m \le n} \left\|H_m - \bar{Q}V_m\right\| \le \nu_2 + \epsilon,$$

it holds that

$$\mu(\bar{Q}, \Xi_{2g}) \le \nu_2 + \epsilon,$$

and thus $\nu(\Xi_{2g}) = \nu_2$.

C. Unmatched switching

Last, we consider the case where σ_H, σ_V and σ_Q are independent from each other, i.e., the set of admissible switching trajectories is

$$\Xi_3 = \{\sigma \,|\, \sigma_H, \sigma_V \text{ and } \sigma_Q \text{ are independent}\}.$$

Define the following ℓ_1-induced model matching problem

$$\nu_3 := \inf_{Z \in \mathcal{L}_{TI}} \max_{\substack{1 \le m \le n \\ 1 \le k \le n}} \|H_m - ZV_k\|. \tag{7}$$

The main result in the case of unmatched switching is

Proposition 3.1: For any $\epsilon > 0$ and $\tilde{Q}$ in $\mathcal{L}_{TI}$ with

$$\max_{\substack{1 \le m \le n \\ 1 \le k \le n}} \left\|H_m - \tilde{Q}V_k\right\| \le \nu_3 + \epsilon,$$

it holds that

$$\mu(\tilde{Q}, \Xi_3) \le \nu_3 + \epsilon,$$

and thus $\nu(\Xi_3) = \nu_3$.

Proof: For details, see [18].

D. Dual formulation

In Section III-B and III-C we dealt with partially matched switching and unmatched switching separately. The final solutions depend on the solution to a ℓ_1-induced model matching problem

$$\nu_o = \inf_{Z \in \mathcal{L}_{TI}} \|T_1 - ZT_2\|, \tag{8}$$

where $T_1 \in \mathcal{L}_{TI}$ and $T_2 \in \mathcal{L}_{TI}$. In this subsection, we show that the optimization problem in (8) can be related to a standard ℓ_1 model matching problem, the solution to which also provides a solution to (8).

Lemma 3.3: Suppose that G is a solution to the standard ℓ_1 optimization problem

$$\nu_d = \inf_{X \in \ell_1} \left\|T_1^{'} - T_2^{'}X\right\|_1, \tag{9}$$

then $Z = G^{'}$ is a solution to the ℓ_1-induced model matching problem in (8).

Proof: For details, see [18].

IV. $\mathcal{H}_2$ OPTIMAL SIGNAL RECONSTRUCTION

In this section we consider the model matching problem (3) and adopt the $\mathcal{H}_2$ norm as the performance criterion, i.e., the design problem is to find a system $Q \in \mathcal{T}_n$ to solve[2]

$$\begin{aligned} \nu(\Xi) &= \inf_{\{Q_m\}_{m=1}^n} \mu(Q,\Xi) \\ \text{where} \quad \mu(Q,\Xi) &= \sup_{\sigma\in\Xi} \|H_\sigma - Q_\sigma V_\sigma\|_2 . \end{aligned} \tag{10}$$

First, we introduce the following notation. For an LTV system P ith a lower (block) triangular representation (2), we define the $\mathcal{H}_2$ norm of system P as

$$\begin{aligned} \|P\|_2 &= \sqrt{\sup_t \sup_k \operatorname{tr}\left\{[\mathcal{C}_P]_t^k \left([\mathcal{C}_P]_t^k\right)'\right\}}, \\ &= \sup_t \sup_k \left\|[\mathcal{C}_P]_t^k\right\|_F , \end{aligned}$$

the square of which can be interpreted as the worst case trace of the output variance when there is only a white noise input at time t with identity spectral density, i.e.,

$$\|P\|_2^2 = \sup_t \sum_{k=0}^{\infty} \operatorname{tr} \mathcal{E}(z(k)z(k)'),$$

where

$$\begin{aligned} &\mathcal{E}(w(t)w(t)') = I, \quad w(k) = 0 \ \forall k \neq t, \\ &\quad \text{and } z(k) = (Pw)(k). \end{aligned}$$

Following the proof procedure in Section III, we can produce similar results here to those in the case of ℓ_1-induced norm optimization. Our main results in this section are stated in the following without proof due to space limit. In the cases of partially matched switching and unmatched switching, we have the following result:

Theorem 4.1: 1) Partially matched switching: for any $\epsilon > 0$ and $\bar{Q}$ in $\mathcal{L}_{TI}$ with

$$\max_{1\leq m\leq n} \left\|H_m - \bar{Q}V_m\right\|_2 \leq \nu_4 + \epsilon,$$

where

$$\nu_4 := \inf_{Z\in\mathcal{L}_{TI}} \max_{1\leq m\leq n} \|H_m - ZV_m\|_2 ,$$

it holds that

$$\mu(\bar{Q},\Xi_2) \leq \nu_4 + \epsilon,$$

and thus $\nu(\Xi_2) = \nu_4$.

2) Unmatched switching: for any $\epsilon > 0$ and $\tilde{Q}$ in $\mathcal{L}_{TI}$ with

$$\nu_5 \leq \max_{\substack{1\leq m\leq n \\ 1\leq k\leq n}} \left\|H_m - \tilde{Q}V_k\right\|_2 \leq \nu_5 + \epsilon,$$

where

$$\nu_5 := \inf_{Z\in\mathcal{L}_{TI}} \max_{\substack{1\leq m\leq n \\ 1\leq k\leq n}} \|H_m - ZV_k\|_2 ,$$

it holds that $\mu(\tilde{Q},\Xi_3) \leq \nu_5 + \epsilon$, and thus $\nu(\Xi_3) = \nu_5$.

In the case of matched switching, the model matching problem (10) is solved by the two sequences of quadratic programs similar to those in ($\underline{\mathrm{LP}}_i$) and ($\overline{\mathrm{LP}}_i$) if V is a FIR filter of finite order τ.

[2]To differentiate from the ℓ_1-induced norm, the subscript 2 is included as $\|\cdot\|_2$ when $\mathcal{H}_2$ norm criterion is adopted.

V. CONCLUSIONS

In this paper we considered signal reconstruction over networks where the communication channel can be modeled as an input switching system in $\mathcal{T}_n$. The design problem was transformed into a model matching problem in $\mathcal{T}_n$ with the worst case ℓ_1-induced norm or $\mathcal{H}_2$ norm as the performance criterion. We dealt with three kinds of switchings, namely matched switching, partially matched switching, and unmatched switching. We showed that the design problem in the first case can be solved by two sequences of linear/quadratic programs which converges to the optimal solution from above and below respectively, and that the rest two cases can be solved exactly by solving a ℓ_1-induced or $\mathcal{H}_2$ norm optimization problem.

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