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Unifying Geometric Approach to Real-time Formation Control

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***Abstract*— We consider a class of problems in formation control. This class comprises of the so called radar deception, rigid formation keeping and formation reconfiguration problems. An intrinsic geometric formulation of the associated constraints unifies the three problems. It is the first time such a generalization has been presented. The constraints can include nonholonomic constraints and actuator limitations. Deriving the constrained dynamics describing the motion eliminates the need for nonlinear programming making the approach amenable to real time motion planning. The constrained dynamics along with the motion planning algorithm that generates reference trajectories online and in real-time, are validated for the formation keeping problem using simulations.**

I. INTRODUCTION

Cooperating multi-agent systems have received increased attention in the recent past and have applications in exploration and mapping, search and rescue, surveillance, cooperative manipulation, automated highways and network centric warfare. Autonomous, distributed and real-time control is an important if not imperative feature for the successful implementation of such multi-agent systems, often involving some sort of formation control [1], [2]. Our interest in this paper is limited to formation control problems in cooperating multi-agent systems. There are two main approaches seen in the literature on formation control. One approach is to formulate it as a constrained optimization problem while the other approach is to formulate it in the framework of a tracking control problem. The most limiting characteristic of the former approach is the computational complexity [3], while in the latter it is that the reference trajectories might be dynamically infeasible for the individual agents to track. Dynamic constraints that limit the maneuverability of single agents will have a pronounced effect in limiting the maneuverability of a multi-agent system accomplishing a prescribed group behavior. Surprisingly though, this critical aspect of dynamic feasibility has been ignored in most approaches to formation control, with [4], [5] being exceptions. However the approach in [4] is to solve a constrained optimization problem using nonlinear programming and [5] captures dynamic constraints only to the extent that the designed reference trajectories will be smooth. This paper advocates a change in paradigm to formation control by addressing both the key issues of dynamic feasibility and computational complexity.

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The work presented in this paper generalizes and gives intrinsic meaning to results we have given in [6]. Although a lot of research has been done on each of the formation control problems of formation flying [7], [8], box pushing [9], [10], scouting [11], [12], formation reconfiguration [13], [14], moving into formation [15], [16] and radar deception [17], [18], we are unaware of any motion planning work that unifies these problems or gives solutions that would work for all or most of them. The generalization based on the intrinsic geometry of the constraints presented in this paper allows the unification of a subclass of problems in formation control and will encompass all the problems listed above.

A Class of Problems in Formation Control

We look at the following three general problems in formation control, each of which involves coordinated motion planning of multi-agents to achieve a team goal in the presence of configuration and dynamic constraints.

- Radar deception problem
- Rigid formation keeping
- Formation reconfiguration

The first problem, which we shall call the radar deception problem, serves as a motivating example in formation control involving a unique constraint on the system configuration. Here a team of fixed winged UAVs cooperate to deceive a ground radar network into seeing a spurious phantom track in its radar space. Each UAV engaging a radar it is assigned to has the capability to intercept, introduce a time delay and re-transmit the radar's transmitted pulses thereby making the radar detect a target at a false range. The challenge is to deceive the entire radar network into seeing a single coherent phantom track. This involves all the extended lines of sight, from the radars to the UAVs engaging them, intersecting at a common point and tracing a path in space. This is a constraint on the system configuration space. The second problem we consider, rigid formation keeping, requires the relative distances of all the agents in the system to be fixed which is again a constraint on the configuration space. Rigid formation keeping can in general be too restrictive for an environment with obstacles and therefore formation reconfiguration, the third problem we consider, becomes important.

Motion planning for the above three problems require satisfying constraints on the configuration of the multi-agent system while also satisfying constraints on the dynamics of the individual agents. At a minimum, constraints on individual agent dynamics will come through limitations

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on actuator capabilities or operation constraints. Dynamic constraints can also often include nonholonomic constraints, for example when the multi-agents are wheeled robots. We show for the first time, that the multi-agent motion planning for the above three problems are intrinsically geometric problems in the configuration space-time and can be expressed in a unifying manner. By deriving the constrained dynamics of the multi-agent system, we in effect embed the configuration and dynamic constraints of formation control in to the real-time design of reference trajectories. The proposed formulation makes actuator/operating constraints transparent in the constrained dynamics and a simple control strategy ensures these actuator constraints are satisfied explicitly.

The main contributions made in this paper are: (1) Unifying formulation of constrained dynamics for a class of problems in formation control; (2) Deriving these constrained dynamics eliminates the need for nonlinear programming, making the approach amenable to real time motion planning; (3) Deriving these constrained dynamics intrinsically and hence valid in any choice of frame; (4) Explicit consideration of actuator/operating constraints to address dynamic feasibility in formation control.

II. Proposed algorithm to formation control

The proposed approach is on real-time reference trajectory generation as apposed to formation tracking. These reference trajectories are then to be simultaneously used as the input for the formation agents' relative state tracking control law. Explicitly incorporating the dynamic model, including all dynamic constraints of the agents, in the design of the reference trajectories will ensure zero tracking error in the relative state tracking control stage, at least theoretically. We say at least theoretically, since this is with idealized assumptions of zero model uncertainty and zero disturbance. We propose higher level control for the design of these reference trajectories where the essential dynamic constraints are captured but through a simplified dynamic model. For example, the dynamic capabilities of a four wheeled robot having many degrees of freedom and controls can be captured approximately but reasonably well through the much simpler uni-cycle model. The Uni-cycle model essentially captures the no slip condition of the wheeled robot while appropriate constraints on its higher level controls of "speed" and "steer" can effectively capture the wheeled robot's actuator and dynamic capabilities. This is the reason why a lot of studies on wheeled robots or even UAVs employ the uni-cycle model to represent the agent dynamics. The accuracy with which the dynamic models of the individual agents are captured in the design of the reference trajectories will determine the degree of tracking error in the tracking control stage and ultimately in the degree of the error in formation. In actual implementation, model uncertainty and disturbances will be accounted through feedback in the tracking controllers.

For distributed control of the multi-agent system having N agents, the problem is decoupled into N sub problems. From a geometric control point of view, this means the configuration and dynamic constraints defining the formation control problem can be separated into N geometrically similar sets of constraints. In the radar deception problem the phantom and each UAV makes up a separate subsystem. In rigid formation keeping and formation reconfiguration, each agent and a unique point on the virtual structure (VS) defining the formation is treated as a separate subsystem.

Next constrained dynamics are developed for the subsystem which is the basis to this motion planning algorithm. Constrained dynamics are formulated intrinsically to make it applicable to the class of problems considered and is presented in detail in the next section. Control functions are identified for the constrained dynamics of the subsystem such that consensus between all the subsystems can be achieved. A control law that would satisfy the dynamic constraints representing actuator and operating limitations is identified. A control law that would optimize the team goal is developed for the subsystem next. A simple switching control strategy is proposed based on these two control laws. When actuator and operating constraints of all the subsystems are satisfied, the control law that optimizes the team goal is implemented on all the constrained subsystems. If actuator or operating constraints of even one of the subsystems are violated then the control law that satisfies the actuator constraints is implemented. Synchronized and global communication is proposed for the control architecture of the motion planning algorithm. For the implementation of the switching control strategy, all that needs to be communicated amongst all the agents in the team is which controller to be used and for how long. The constrained dynamics takes care of the equality constraints while the switching control strategy takes care of the inequality constraints corresponding to actuator/operating limitations and the approach is amenable to real-time control. The admittedly strong assumption of synchronized communication is the weakest link in the proposed algorithm. For details on this motion planning algorithm see [6].

III. Geometric Formulation of Constrained Motion

We refer the reader to [19], [20] for the differential geometric ideas and notation used in this section. Consider a multi-agent system $\mathcal{A}$ constrained to satisfy holonomic and nonholonomic constraints. Q is the configuration manifold of the system and TQ, T^*Q its tangent and cotangent bundles respectively. A trajectory of the system $\mathcal{A}$ is a curve on Q, $\gamma : [a,b] \mapsto Q$, whose tangent vector on Q along γ we denote by γ'.

A. Constrained Kinematics

A map $\mathcal{C} : Q \mapsto \mathbf{0} \in \mathcal{R}^m$ captures the configuration constraints (holonomic) on Q. $\mathcal{M} = \mathcal{C}^{-1}(\mathbf{0}) = \{q \in Q \mid \mathcal{C}(q) = \mathbf{0}\}$ is an embedded submanifold of Q ($\mathcal{M} \subset Q$) and is the true configuration manifold of the constrained system $\mathcal{A}$. The differential of the map $\mathcal{C}$, denoted $d\mathcal{C}$, is a codistribution that annihilates the entire tangent space $T_q\mathcal{M}$ for every $q \in \mathcal{M}$ and uniquely identifies $T\mathcal{M}$.

A distribution Δ on Q captures the nonholonomic constraints on Q. There is a unique annihilating codistribution

$\Delta^{\perp} = \{\boldsymbol{\alpha} \in T^*Q \mid \boldsymbol{\alpha}(\mathbf{v}) = 0;\ \forall \mathbf{v} \in \Delta\}$ on Q associated with Δ (Here we have made an abuse of notation by denoting the distribution as well as the set of vector fields taking their values in the distribution by the same symbol since it should be clear from the context which we mean). Let $\{\mathbf{e}_1, \cdots, \mathbf{e}_{rank(\Delta)}\}$ be a basis for the distribution Δ. Then $\gamma' = v^i \mathbf{e}_i$ is its equivalent control system form associated with the nonholonomic constraints and the nonholonomic constraints alone. Here, and in the rest of this paper, we use the Einstein summation convention. For the three formation control problems we consider, this equivalent control form represents the individual agent kinematics (that need not satisfy the configuration constraints) and we propose to capture actuator/operating limitations of the individual agents through inequality constraints on the functions v^i.

$\Omega = \{\boldsymbol{\alpha} \in T^*Q \mid \boldsymbol{\alpha} \in d\mathcal{C},\ \boldsymbol{\alpha} \in \Delta^{\perp}\}$ is the intersection of the codistributions $d\mathcal{C}$ and $\Delta^{\perp}$. A trajectory will satisfy both the holonomic and nonholonomic constraints of $\mathcal{A}$ *iff* its associated γ' along the curve γ is annihilated by Ω. i.e. $\Omega(\gamma') = \mathbf{0}$. There exists a unique distribution on Q, call it the constrained distribution $\mathcal{D} = \{\mathbf{v} \in T_qQ,\ \forall q \in Q \mid \Omega(\mathbf{v}) = \mathbf{0}\}$, associated with the annihilating codistribution Ω.

Feasibility: The trajectory γ satisfies the holonomic and nonholonomic constraints of $\mathcal{A}$ *iff* γ' is in the distribution $\mathcal{D}$. Hence for the existence of feasible trajectories for $\mathcal{A}$, the distribution $\mathcal{D}$ has to be non-empty. This condition is given in terms of an algebraic rank condition on the matrix representation of the annihilating codistribution Ω in [21].

If $\{\mathbf{X}_1, \cdots, \mathbf{X}_{rank(\mathcal{D})}\}$ is a basis for the distribution $\mathcal{D}$, then $\gamma' = u^i \mathbf{X}_i$ describes the equivalent kinematic control system of the constrained system $\mathcal{A}$. In general it will not be possible to find a relationship between u^i and actuator/operating constraints and we turn to the dynamics of the constrained system.

B. Constrained Dynamics

For the local coordinates $q = (q^1, \cdots, q^n)$ in Q, let $\boldsymbol{\partial_q} = (\partial_{q^1}, \cdots, \partial_{q^n})$ be its coordinate frame of vector fields and $\mathbf{dq} = (dq^1, \cdots, dq^n)$ its associated dual frame of covector fields (i.e. $dq^i(\partial_{q^j}) = \delta^i_j$). Also consider the frame of vector fields $\mathbf{e} = (\mathbf{e}_1, \cdots, \mathbf{e}_{rank(\Delta)}, \cdots, \mathbf{e}_n)$ where $\{\mathbf{e}_1, \cdots, \mathbf{e}_{rank(\Delta)}\}$ forms a basis for Δ and $\{\mathbf{e}_{rank(\Delta)+1}, \cdots, \mathbf{e}_n\}$ forms a basis for $\Delta^{\perp}$. The frame $\mathbf{e}$ has the associated frame of covector fields $\boldsymbol{\sigma} = (\sigma^1, \cdots, \sigma^n)$ on Q (i.e. $\mathbf{e}_i(\sigma^j) = \delta^i_j$). The frame $\mathbf{e}$ is locally a coordinate frame *iff* $[\mathbf{e}_i, \mathbf{e}_j] = 0,\ \forall i, j$ in which case we can always find local coordinates $p = (p^1, \cdots, p^n)$ such that $\mathbf{e}_j = \partial_{p^j}$ and $\sigma^j = dp^j$. i.e. locally each σ^j will be exact. Here $[\mathbf{e}_i, \mathbf{e}_j]$ is the Lie bracket between the vector fields $\mathbf{e}_i, \mathbf{e}_j$. For the three problems considered, the choice of $\mathbf{e}$ will be such that it will not be a coordinate frame.

For a vector $\mathbf{x} = X^j \mathbf{e}_j$ and a vector field $\mathbf{v} = v^k \mathbf{e}_k$, the covariant derivative of $\mathbf{v}$ with respect to $\mathbf{x}$ is;

$$\nabla_{\mathbf{x}} \mathbf{v} = \mathbf{e}_i \{dv^i + v^k \omega^i_k\}(\mathbf{x}) \tag{1}$$

where the connection coefficients ω^i_{jk} and the connection 1-forms ω^i_k are defined by $\nabla_{\mathbf{e}_j} \mathbf{e}_k := \mathbf{e}_i \omega^i_{jk},\ \omega^i_k := \omega^i_{jk} \sigma^j$ and where we have used the fact that $\mathbf{x}(v^k) = dv^k(\mathbf{x})$.

Let $\mathbb{G}$ be the Riemannian metric on Q specified by the kinetic energy of the system $\mathcal{A}$. The Levi-Civita connection $\overset{\mathbb{G}}{\nabla}$ is the unique affine connection associated with $(Q, \mathbb{G})$, satisfying $\overset{\mathbb{G}}{\nabla} = 0$ and $\overset{\mathbb{G}}{\nabla}_{\mathbf{x}}\ \mathbf{y} - \overset{\mathbb{G}}{\nabla}_{\mathbf{y}}\ \mathbf{x} = [\mathbf{x}, \mathbf{y}],\ \forall \mathbf{x}, \mathbf{y}$. The connection coefficients of the Levi-Civita connection, called Christoffel symbols, are given in the coordinates q by;

$$\overset{\mathbb{G}}{\Gamma^i_{jk}} = \frac{1}{2} \mathbb{G}^{ir} \left(\frac{\partial \mathbb{G}_{jr}}{\partial q^k} + \frac{\partial \mathbb{G}_{kr}}{\partial q^j} - \frac{\partial \mathbb{G}_{jk}}{\partial q^r} \right)$$

where $\mathbb{G}^{ij}$ are defined by $\mathbb{G}_{ij} \mathbb{G}^{jk} = \delta^k_i$.

For a force represented by the one-form $F(t, \gamma'(t)) \in T^*Q$, a curve $\gamma : [a, b] \mapsto Q$ satisfies the Lagrange-d'Alembert principle and is a solution of the constrained system $\mathcal{A}$ *iff*;

$$\begin{aligned} \nabla_{\gamma'(t)} \gamma'(t) &= \lambda(t) + Y(\gamma(t)) \\ P'(\gamma'(t)) &= \mathbf{0} \end{aligned}$$

where λ is in $\mathcal{D}^{\perp}$, the $\mathbb{G}$ orthogonal compliment to $\mathcal{D}$, Y is the vector field associated with the one-form F given by $Y = \mathbb{G}^{\sharp}(F)$, $\mathbb{G}^{\sharp} : T^*Q \mapsto TQ$ is the isomorphism associated with the metric $\mathbb{G}$ mapping covector fields to vector fields, and $P' : TQ \mapsto TQ$ is the $\mathbb{G}$ orthogonal projection map onto $\mathcal{D}^{\perp}$. Taking the covariant derivative of $P'(\gamma'(t))$ leads us to another affine connection, the constrained affine connection $\overset{\mathcal{D}}{\nabla}$ given by;

$$\overset{\mathcal{D}}{\nabla}_{\gamma'(t)}\ \gamma'(t) = \nabla_{\gamma'(t)} \gamma'(t) + (\nabla_{\gamma'(t)} P')(\gamma'(t))$$

A property of $\overset{\mathcal{D}}{\nabla}$ is that it *restricts* to $\mathcal{D}$ meaning that $\overset{\mathcal{D}}{\nabla}_{X_1}\ X_2 \in \mathcal{D}$ for every $X_2 \in \mathcal{D}$. In practice however, computation of $\overset{\mathcal{D}}{\nabla}$ can be quite troublesome and for computational convenience we instead consider the constrained connection given in [22];

$$\overset{A}{\nabla}_{\gamma'(t)}\ \gamma'(t) = \nabla_{\gamma'(t)} \gamma'(t) + A^{-1} \left(\left(\nabla_{\gamma'(t)} A P' \right) \left(\gamma'(t) \right) \right)$$

where A can be *any* invertible matrix. Usually one would choose A to cancel out the denominator terms of P' that would cause computational problems in the covariant differentiation of P'. It is shown in [22] (along with a proof) that this connection $\overset{A}{\nabla}$ too restricts to $\mathcal{D}$ and hence serves just as well as $\overset{\mathcal{D}}{\nabla}$ in determining the constrained equations of motion as long as $\gamma'(t_0) \in \mathcal{D}$.

A curve $\gamma : [a, b] \mapsto Q$ is a solution of the constrained system $\mathcal{A}$ *iff* $\gamma'(t_0) \in \mathcal{D}$ and γ satisfies;

$$\overset{A}{\nabla}_{\gamma'(t)}\ \gamma'(t) = P \left(Y(\gamma(t)) \right)$$

where $Y = \mathbb{G}^{\sharp}(F)$ and $P : TQ \mapsto TQ$ is the $\mathbb{G}$ orthogonal projection map onto $\mathcal{D}$.

Let $\gamma' = \dot{q}^k \partial_{q^k} = v^k \mathbf{e}_k$ and using (1) we have;

$$\begin{aligned} \overset{A}{\nabla}_{\gamma'}\ \gamma' &= \mathbf{e}_k \left(dv^k + v^j \omega^k_j \right)(\gamma') &&= \partial_{q^k} \left(d\dot{q}^k + \dot{q}^j \Gamma^k_j \right)(\gamma') \\ &= \mathbf{e}_k \left(\dot{v}^k + v^j \omega^k_j (v^r \mathbf{e}_r) \right) &&= \partial_{q^k} \left(\ddot{q}^k + \dot{q}^j \Gamma^k_j (\dot{q}^r \partial_{q^r}) \right) \end{aligned}$$

Consider a type $(1,1)$ tensor $\mathcal{P}$ with components $\mathcal{P}^i_j$. The components of the covariant derivative of $\mathcal{P}$ with respect to $\mathbf{x}$, $\nabla_{\mathbf{X}}\mathcal{P}$, in the coordinate frame $\boldsymbol{\partial_q}$ are;

$$(\nabla_{\mathbf{x}}\mathcal{P})^i_j = \frac{\partial \mathcal{P}^i_j}{\partial q^k}X^k + \Gamma^i_{kr}\mathcal{P}^r_j X^k - \Gamma^r_{kj}\mathcal{P}^i_r X^k \tag{2}$$

where the connection coefficients Γ^i_{jk} are defined by $\nabla_{\partial_{q^j}}\partial_{q^k} := \partial_{q^j}\Gamma^i_{jk}$. The connection coefficients of $\overset{A}{\nabla}$ in the coordinate frame $\boldsymbol{\partial_q}$ are computed using (2) as;

$$\overset{A}{\Gamma}{}^i_{jk} = \overset{\mathbb{G}}{\Gamma}{}^i_{jk} + (A^{-1})^i_r \frac{\partial (AP')^r_j}{\partial q^k} + (A^{-1})^i_r \ \overset{\mathbb{G}}{\Gamma}{}^r_{km} \ (AP')^m_j$$
$$- (A^{-1})^i_r \ \overset{\mathbb{G}}{\Gamma}{}^m_{kj} \ (AP')^r_m$$

Since for the three problems considered, the frame $\mathbf{e}$ will not be a coordinate frame, we need to transform the connection 1-forms $\overset{A}{\Gamma}{}^j_k$ from the basis $\boldsymbol{\partial_q}$ to the basis $\mathbf{e}$ to compute the 1-forms ω^j_k. Define $\nabla\mathbf{e}_j(\mathbf{e}_i) := \nabla_{\mathbf{e}_j}\mathbf{e}_i = \mathbf{e}_k\omega^k_{ij}$. This can also be written in terms of a vector valued 1-form as $\mathbf{e}_k \otimes \omega^k_{rj}\sigma^r(\mathbf{e}_i) = \mathbf{e}_k\omega^k_{ij}$. Since $\omega^k_j := \omega^k_{rj}\sigma^r$ we have $\nabla\mathbf{e}_j = \mathbf{e}_k \otimes \omega^k_j$ and hence $\overset{A}{\nabla}\ \mathbf{e} = \mathbf{e}\omega$ where $\omega := (\omega^k_j)$ is the $n \times n$ matrix of connection 1-forms. Since $\overset{A}{\nabla}$ is well defined, independent of basis, we have compatible $\overset{A}{\nabla}\ \mathbf{e} = \mathbf{e}\omega$ and $\overset{A}{\nabla}\ \boldsymbol{\partial_q} = \boldsymbol{\partial_q}\ \overset{A}{\Gamma}$ where $\overset{A}{\Gamma} := (\ \overset{A}{\Gamma}{}^k_j)$. Let $\mathbf{e} = \boldsymbol{\partial_q}\mathcal{S}$ be the change of basis where $\mathbf{e}_i = \partial_{q^j}\mathcal{S}^j_i$ and $\mathcal{S}$ is the non-singular matrix whose $(i,j)th$ element is $\mathcal{S}^i_j$. Then, $\overset{A}{\nabla}\ \mathbf{e} = \overset{A}{\nabla}\ (\boldsymbol{\partial_q}\mathcal{S}) = (\overset{A}{\nabla}\ \boldsymbol{\partial_q})\mathcal{S} + \boldsymbol{\partial_q}d\mathcal{S} = \boldsymbol{\partial_q}\ \overset{A}{\Gamma}\ \mathcal{S} + \boldsymbol{\partial_q}d\mathcal{S} = \mathbf{e}\omega = \boldsymbol{\partial_q}\mathcal{S}\omega$. We must then have,

$$\omega = \mathcal{S}^{-1}\ \overset{A}{\Gamma}\ \mathcal{S} + \mathcal{S}^{-1}d\mathcal{S} \tag{3}$$

which is the transformation rule for the matrix of connection 1-forms. Notice that $\overset{A}{\Gamma}$ does not transform as would the components of a tensor since $\overset{A}{\Gamma}$ is in fact not a tensor.

Since $\mathbf{e} = \boldsymbol{\partial_q}\mathcal{S}$ we have $\boldsymbol{\sigma} = \mathcal{S}^{-1}\boldsymbol{dq}$. Let $\boldsymbol{\alpha}$ be a 1-form and $\boldsymbol{\alpha} = a^k dq^k = b^k\sigma^k$. This can be written as $\boldsymbol{\alpha} = \mathbf{a}\boldsymbol{dq} = \mathbf{b}\boldsymbol{\sigma} = \mathbf{b}\mathcal{S}^{-1}\boldsymbol{dq}$ and we have $\mathbf{a} = \mathbf{b}\mathcal{S}^{-1}$ and hence $\mathbf{b} = \mathbf{a}\mathcal{S}$ which is the transformation rule for 1-forms. This will be required in the actual computations of (3).

The significance of deriving constrained dynamics in the $\mathbf{e}$ frame is that we then have the equations of motion of the constrained system in the functions v^i which also capture the actuator/operating constraints of the individual agents. Note that in the above constrained dynamics, $v^{rank(\mathcal{D})}, \cdots, v^n$ will be identically zero since $\mathbf{e}_{rank(\mathcal{D})}, \cdots, \mathbf{e}_n \in \mathcal{D}^{\perp}$, to satisfy the nonholonomic constraints.

IV. Rigid Formation Keeping and Formation Reconfiguration

Consider N agents restricted to the plane making up a virtual structure (VS) with an arbitrary point O_c (the centroid of the VS at time t_0 for example). An orthogonal local coordinate frame B is assumed fixed to the VS at O_c and let $(b_{i,1}, b_{i,2})$ denote the place holder for the ith agent in this B frame. When $b_{i,1}, b_{i,2}$ are constant the VS will be rigid and when $b_{i,1}, b_{i,2}$ are time varying the VS too will be time varying making the formalism applicable to both the rigid formation keeping and the formation changing problems. Let (x, y) be local coordinates of O_c with respect to an inertial frame I and ϕ the orientation of the B frame with respect to I. Suppose (x_i, y_i) describes the position and θ_i the orientation of an ith agent with respect to the frame I. Similarly suppose (x, y, θ) describes the position and orientation of a virtual agent at O_c. Consider the ith subsystem made up of the ith agent, the virtual agent at O_c and the B frame. This subsystem has the structure of a manifold Q with local coordinates $q_i = (x, y, \theta, \phi, b_{i,1}, b_{i,2}, x_i, y_i, \theta_i)$. Configuration constraints on Q are;

$$\begin{aligned} x_i - x - b_{i,1}\cos\phi + b_{i,2}\sin\phi = 0 \\ y_i - y - b_{i,1}\sin\phi - b_{i,2}\cos\phi = 0 \end{aligned} \tag{4}$$

Suppose the dynamics of an agent (including the virtual agent at O_c) include the nonholonomic constraints of a unicycle;

$$\begin{aligned} \dot{x}\sin\theta - \dot{y}\cos\theta = 0 \\ \dot{x}_i\sin\theta_i - \dot{y}_i\cos\theta_i = 0 \end{aligned} \tag{5}$$

These nonholonomic equations define the following equivalent control system on Q;

$$\begin{aligned} \dot{x} &= v\cos\theta & \dot{x}_i &= v_i\cos\theta_i \\ \dot{y} &= v\sin\theta & \dot{y}_i &= v_i\sin\theta_i \\ \dot{\theta} &= w & \dot{\theta}_i &= w_i \end{aligned} \tag{6}$$

where dynamic constraints due to actuator/operating limitations are explicitly captured through constraints on the kinematic controls (v, w, v_i, w_i) of the above equivalent control system;

$$\begin{aligned} v^{min} \le v_i, v \ &\le v^{max} \\ -\varphi v_i \le w_i \ &\le \varphi v_i \\ -\varphi v \le w \ &\le \varphi v \end{aligned} \tag{7}$$

where $\varphi = \frac{w^{max}}{v^{max}}$. We assume that the dynamics of the agent and the virtual agent are captured reasonably well through (6) and (7). As a preliminary step we will only consider explicit actuator constraints involving the velocity coefficients v, w, v_i, w_i while acknowledging the importance of actuator constraints involving the the time derivatives of these ("acceleration" terms at a minimum).

V. Radar Deception Problem

We consider the radar deception problem that is restricted to the plane. Suppose there are N-UAVs engaging N- stationary radars and also suppose that we assign an imaginary UAV to mimic the motion of the phantom to make the phantom track realistic. The multi-agent system is decoupled into N-subsystems corresponding to the N radar-UAV pairs. Each subsystem (N of them) now only has two UAVs, one representing the phantom and the other the UAV engaging the radar. The configuration space of the ith subsystem has the structure of a manifold Q, and we assign the local coordinates $q_i = (x, y, \theta, x_i, y_i, \theta_i)$. Here, (x, y, θ) gives the position and orientation of the phantom UAV while (x, y, θ) gives that of the UAV engaging the ith radar at $(\bar{x}_i, \bar{y}_i)$. Again we assume the dynamics of a UAV can be captured reasonably well through constraints of a uni-cycle. The nonholonomic constraints, its equivalent control form and actuator constraints of this subsystem are identical to (5), (6) and (7) respectively. The requirement that the UAV has to be in-line with its corresponding radar and the phantom gives rise to the configuration constraint;

$$(x - \bar{x}_i)(y_i - \bar{y}_i) - (y - \bar{y}_i)(x_i - \bar{x}_i) = 0 \tag{8}$$

VI. Example: Rigid formation keeping

The constrained dynamics along with the proposed algorithm is verified in this section for the rigid formation keeping problem. For a rigid formation the place holders $b_{i,1}, b_{i,2}$ in the B frame will be constants. The manifold Q representing the ith subsystem will have local coordinates $q_i = \{x, y, \theta, x_i, y_i, \theta_i, \phi\}$ where $\boldsymbol{\partial_q} = \{\partial_x, \partial_y, \partial_\theta, \partial_{x_i}, \partial_{y_i}, \partial_{\theta_i}, \partial_\phi\}$ is the coordinate basis for T_qQ and $\boldsymbol{dq} = \{dx, dy, d\theta, dx_i, dy_i, d\theta_i, d\phi\}$ its dual basis for T_q^*Q. The Riemannian metric corresponding to the kinetic energy of the system is $\mathbb{G} = m(dx \otimes dx + dy \otimes dy) + J d\theta \otimes \theta + m_i(dx_i \otimes dx_i + dy_i \otimes dy_i) + J_i d\theta_i \otimes \theta_i + \tilde{J} d\phi \otimes d\phi$ where (m_i, J_i) are mass and inertia of the ith agent, (m, J) the fictitious mass and inertia of the virtual agent and $\tilde{J}$ the fictitious inertia of the formation about O_c. Without loss of generality, these are assumed to be of unit magnitude for ease of symbolic computations.

$$\begin{aligned}
\Delta^\perp :\ & \alpha_1 = \sin\theta dx - \cos\theta dy \\
& \alpha_2 = \sin\theta_i dx_i - \cos\theta_i dy_i \\
d\mathcal{C} :\ & \beta_1 = dx - dx_i - (b_{i,1}\sin\phi + b_{i,2}\cos\phi)d\phi \\
& \beta_2 = dy - dy_i + (b_{i,1}\cos\phi - b_{i,2}\sin\phi)d\phi \\
\Omega :\ & \Delta^\perp \oplus d\mathcal{C}
\end{aligned}$$

The constrained distribution $\mathcal{D}$ associated with the annihilating codistribution Ω is spanned by: $\mathbf{x}_1 = \frac{h_i \cos\theta}{\sin(\theta_i - \theta)}\partial_x + \frac{h_i \sin\theta}{\sin(\theta_i - \theta)}\partial_y + \frac{h \cos\theta_i}{\sin(\theta_i - \theta)}\partial_{x_i} + \frac{h \sin\theta_i}{\sin(\theta_i - \theta)}\partial_{y_i} + \partial_\phi$; $\mathbf{x}_2 = \partial_\theta$; $\mathbf{x}_3 = \partial_{\theta_i}$ and $\mathcal{D}^\perp$ is spanned by: $\mathbf{x}_4 = \mathbb{G}^\sharp(\alpha_1) = \sin\theta\partial_x - \cos\theta\partial_y$; $\mathbf{x}_5 = \mathbb{G}^\sharp(\alpha_2) = \sin\theta_i\partial_{x_i} - \cos\theta_i\partial_{y_i}$; $\mathbf{x}_6 = \mathbb{G}^\sharp(\beta_1) = \partial_x - \partial_{x_i} - (b_{i,1}\sin\phi + b_{i,2}\cos\phi)\partial_\phi$; $\mathbf{x}_7 = \mathbb{G}^\sharp(\beta_2) = \partial_y - \partial_{y_i} + (b_{i,1}\cos\phi - b_{i,2}\sin\phi)\partial_\phi$. where $h = b_{i,1}\cos(\theta - \phi) + b_{i,2}\sin(\theta - \phi)$ and $h_i = b_{i,1}\cos(\theta_i - \phi) + b_{i,2}\sin(\theta_i - \phi)$.

Let $\mathbf{x} = \boldsymbol{\partial_q}\mathcal{R}$ be the change of basis where $\mathbf{x}_i = \partial_{q^j}\mathcal{R}_i^j$ and $\mathcal{R}_j^i$ is the $(i,j)th$ element of $\mathcal{R}$. The projection map $P' : TQ \to TQ$ has the matrix representation $\left[P'\right]_{\mathbf{x}} = \begin{bmatrix} [\mathbf{0}]_{3\times3} & [\mathbf{0}]_{3\times4} \\ [\mathbf{0}]_{4\times3} & [\mathbf{I}]_{4\times4} \end{bmatrix}$ and $\left[P'\right]_{\boldsymbol{\partial_q}} = \mathcal{R}\left[P'\right]_{\mathbf{x}}\mathcal{R}^{-1}$ in the two basis $\mathbf{x}$ and $\boldsymbol{\partial_q}$. P' in the basis $\boldsymbol{\partial_q}$ has $a = h^2 + h_i^2 + \sin^2(\theta_i - \theta)$ appearing as a common denominator and we choose $A = a\left[\mathbf{I}\right]$ and use AP' instead of P' to compute the connection coefficients $\overset{A}{\Gamma}$.

Since the kinetic metric $\mathbb{G}$ is constant $\overset{\mathbb{G}}{\Gamma}{}^k_{ij} = 0,\ \forall i, j, k$ and A is diagonal, we have $\overset{A}{\Gamma}{}^i_{jk} = \frac{1}{a}\frac{\partial (AP')^i_j}{\partial q^k}$. However we are interested in deriving the constrained dynamics in the frame $\mathbf{e}$ given by the change of basis $\mathbf{e} = \boldsymbol{\partial_q}\mathcal{S}$ where $\mathcal{S} = \begin{bmatrix} \cos\theta & 0 & 0 & 0 & 0 & -\sin\theta & 0 \\ \sin\theta & 0 & 0 & 0 & 0 & \cos\theta & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \cos\theta_i & 0 & 0 & 0 & -\sin\theta_i \\ 0 & 0 & \sin\theta_i & 0 & 0 & 0 & \cos\theta_i \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}$

Associated with the frame $\mathbf{e} = \{\mathbf{e}_v, \mathbf{e}_w, \mathbf{e}_{v_i}, \mathbf{e}_{w_i}, \mathbf{e}_u, \mathbf{e}_z, \mathbf{e}_{z_i}\}$ is its dual frame $\boldsymbol{\sigma} = \{\sigma^v, \sigma^w, \sigma^{v_i}, \sigma^{w_i}, \sigma^u, \sigma^z, \sigma^{z_i}\}$ and the tangent vector field on Q associated to a system trajectory is given by $\gamma' = \dot{x}\partial_x + \dot{y}\partial_y + \dot{\theta}\partial_\theta + \dot{x}_i\partial_{x_i} + \dot{y}_i\partial_{y_i} + \dot{\theta}_i\partial_{\theta_i} + \dot{\phi}\partial_\phi = v\mathbf{e}_v + w\mathbf{e}_w + v_i\mathbf{e}_{v_i} + w_i\mathbf{e}_{w_i} + u\mathbf{e}_u + z\mathbf{e}_z + z_i\mathbf{e}_{z_i}$. The transformation rule for connection 1-forms given in (3) after some lengthy computations yield the following as the only nonzero connection 1-forms; $\omega^v_w, \omega^v_{w_i}, \omega^v_u, \omega^v_z, \omega^{v_i}_w, \omega^{v_i}_{w_i}, \omega^{v_i}_u, \omega^{v_i}_{z_i}, \omega^u_w, \omega^u_{w_i}, \omega^u_u, \omega^z_v, \omega^z_{v_i}, \omega^z_{w_i}, \omega^z_u, \omega^{z_i}_w, \omega^{z_i}_{v_i}, \omega^{z_i}_{w_i}, \omega^{z_i}_u$. The actual expressions of these connection 1-forms are too lengthy to be displayed here.

The constrained dynamics in the frame $\mathbf{e}$ are as follows where $\gamma' = v\mathbf{e}_v + w\mathbf{e}_w + v_i\mathbf{e}_{v_i} + w_i\mathbf{e}_{w_i} + u\mathbf{e}_u + z\mathbf{e}_z + z_i\mathbf{e}_{z_i}$;

$$\begin{aligned}
\dot{v} + (w\omega^v_w + w_i\omega^v_{w_i} + u\omega^v_u + z\omega^v_z)(\gamma') &= \frac{h_i}{\sin(\theta_i - \theta)}u^1 \\
\dot{w} &= u^2 \\
\dot{v}_i + (w\omega^{v_i}_w + w_i\omega^{v_i}_{w_i} + u\omega^{v_i}_u + z_i\omega^{v_i}_{z_i})(\gamma') &= \frac{h}{\sin(\theta_i - \theta)}u^1 \\
\dot{w}_i &= u^3 \\
\dot{u} + (w\omega^u_w + w_i\omega^u_{w_i} + u\omega^u_u)(\gamma') &= u^1 \\
\dot{z} + (v\omega^z_v + w\omega^z_w + w_i\omega^z_{w_i} + u\omega^z_u)(\gamma') &= 0 \\
\dot{z}_i + (w\omega^{z_i}_w + v_i\omega^{z_i}_{v_i} + w_i\omega^{z_i}_{w_i} + u\omega^{z_i}_u)(\gamma') &= 0
\end{aligned}$$

Recall that for $\gamma'(0) \in \mathcal{D}$, $\overset{A}{\nabla}$ restricts γ' to $\mathcal{D}$. The choice of the frame $\mathbf{e}$ is such that $\mathbf{e}_z, \mathbf{e}_{z_i} \in \mathcal{D}^\perp$ and the functions z, z_i will remain identically zero leading to the identity: $h_i u = \sin(\theta_i - \theta)v$. The above constrained dynamics also give us the following;

$$\begin{aligned}
v_i &= \frac{h}{h_i}v \\
w_i &= a\frac{(\dot{u}h_i - \dot{v}\sin(\theta_i - \theta))}{a_{w_iu}u + a_{vw_i}v} \\
&\quad - \frac{(a_{vw}vw + a_{wu}wu + a_{vu}vu + a_{uu}u^2)}{a_{w_iu}u + a_{vw_i}v}
\end{aligned} \tag{9}$$

where $a_{vw} = \omega^v_{vw}\sin(\theta_i - \theta) - \omega^u_{vw}h_i + (\omega^v_{v_iw}\sin(\theta_i - \theta) - \omega^u_{v_iw}h_i)\frac{h}{h_i}$, $a_{wu} = \omega^v_{uw}\sin(\theta_i - \theta) - \omega^u_{uw}h_i$, $a_{w_iu} = \omega^v_{uw_i}\sin(\theta_i - \theta) - \omega^u_{uw_i}h_i$, $a_{vw_i} = \omega^v_{vw_i}\sin(\theta_i - \theta) - \omega^u_{vw_i}h_i + (\omega^v_{v_iw_i}\sin(\theta_i - \theta) - \omega^u_{v_iw_i}h_i)\frac{h}{h_i}$, $a_{vu} = \omega^v_{vu}\sin(\theta_i - \theta) - \omega^u_{vu}h_i + (\omega^v_{v_iu}\sin(\theta_i - \theta) - \omega^u_{v_iu}h_i)\frac{h}{h_i}$, $a_{uu} = \omega^v_{uu}\sin(\theta_i - \theta) - \omega^u_{uu}h_i$.

For consensus of the N subsystems, we want control over the functions v, w, u. The identity $h_i u = \sin(\theta_i - \theta)v$ along with (9) implies v_i approaches v and w_i approaches w as $u, \dot{u}$ approaches zero (assuming $v > 0$). Hence controllers that drive $v \in [v^{min}, v^{max}]$, $w, u = 0$ trivially satisfy the actuator/operating constraints of (7). Two sets of exponentially stabilizing nonlinear control laws are designed for v, w, u; one to drive the formation towards a desired waypoint, and the other to drive $v = \frac{v^{min}+v^{max}}{2}$, $w, u = 0$ [6]. Figure (1) shows simulation results for six agents moving through a given set of waypoints while maintaining formation for the proposed motion planning algorithm. The time history of the functions v_i, w_i corresponding to "speed" and "steer" for each of the six agents for the above results are shown in Fig.(2). The lower and upper bounds of v_i, w_i are also shown.

For details of the motion planning algorithm including the distributed communication architecture and the exponentially stabilizing nonlinear control laws, we refer the reader to [6].

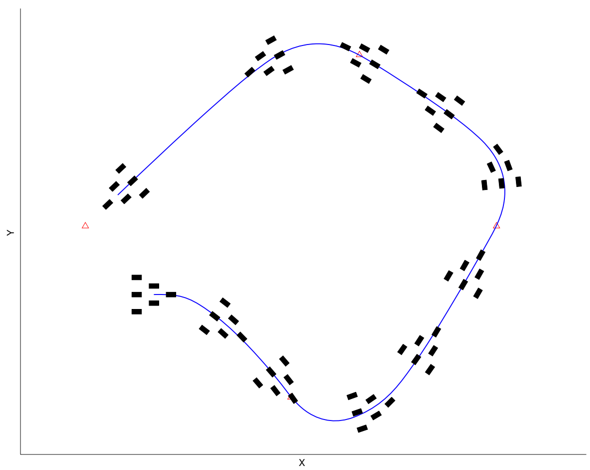

Fig. 1. Formation keeping motion for six mobile agents

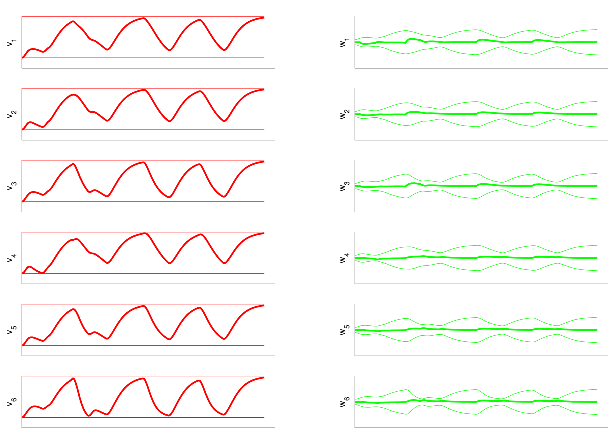

Fig. 2. "Speed" and "Steer" controls for each of the six agents for the coordinated motion shown in Fig.1

VII. CONCLUSION

A class of problems in formation control is considered. An intrinsic geometric formulation of the associated constraints unifies this class of problems. The constraints include nonholonomic, holonomic and actuator constraints. The constrained dynamics as well as the real-time trajectory generating algorithm is validated through simulations for the rigid formation keeping example.

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