2008 American Control Conference
Westin Seattle Hotel, Seattle, Washington, USA
June 11-13, 2008

Estimation of Model Uncertainties in Closed-loop Systems

Henrik Niemann
Dept. of Electrical Engineering, Automation
Technical University of Denmark
DK-2800 Kgs. Lyngby, Denmark
hhn@elektro.dtu.dk

Niels Kjølstad Poulsen
Dept. of Informatics and Math. Modeling
Technical University of Denmark
DK-2800 Kgs. Lyngby, Denmark
nkp@imm.dtu.dk

Abstract— **This paper describe a method for estimation of parameters or uncertainties in closed-loop systems. The method is based on an application of the dual YJBK (after Youla, Jabr, Bongiorno and Kucera) parameterization of all systems stabilized by a given controller. The dual YJBK transfer function is a measure for the variation in the system seen through the feedback controller. It is shown that it is possible to isolate a certain number of parameters or uncertain blocks in the system exactly. This is obtained by modifying the feedback controller through the YJBK transfer function together with pre- and post-filters. The estimation is then derived using standard methods.**

I. INTRODUCTION

One of the key issue in robust control is to get an estimate/measure of the model uncertainties. This is both relevant in connection with controller design as well as in connection with validation of the designed robust controller, [12].

There exist a number of different approaches for estimation of model uncertainties for closed-loop systems. One of the most well known approaches is based on the so-called Hansen scheme, see e.g. [5], [2]. The approach is based on using the dual YJBK parameterization to describe the variation of the system. Based on the dual YJBK transfer function, the variations of the system can then be estimated indirectly. In [13], the dual YJBK transfer function is estimated directly and used in a following controller design. In this method, an explicit estimation of the model uncertainties are not given.

Some other relevant methods has been considered in details in the thesis by Callafon, Hakvoort and van den Boom, [3], [4], [14] and in the papers [15], [16].

The main contribution in this paper is to use the controller design in connection with obtaining a simple way to estimation of the model uncertainties. A simple way to modify a controller is by using the YJBK parameterization, see e.g. [11]. By modifying the feedback controller, it is shown that it will be possible to get a transfer function based on available input and output vectors, that is the model uncertainties. This will simplify an estimation of either upper bounds on the uncertainties or a complete estimation of the model uncertainties. However, the method does not allow an unlimited number of parameters/uncertain blocks to be estimated directly. The number of parameters/dim. of the uncertain blocks is bounded by the number of measurement signals and control signals. Another drawback with the approach is that the feedback controller is modified. In some cases, the modification of the controller is equivalent to a decoupling of the feedback controller, which is not acceptable. Instead, the modification of the controller can be done for a certain limit frequency range. This is especially relevant in connection with uncertain parameters, where it is possible to estimate these based in a single periodic input signal. Further, the frequency for a test signal might be selected away from the operation frequency range.

II. SYSTEM SET-UP

Let a general system be given by:

$$\Sigma_P : \begin{cases} z = G_{zw}w + G_{zd}d + G_{zu}u \\ e = G_{ew}w + G_{ed}d + G_{eu}u \\ y = G_{yw}w + G_{yd}d + G_{yu}u \end{cases} \tag{1}$$

where $d \in \mathcal{R}^r$ is a disturbance signal vector, $u \in \mathcal{R}^m$ the control input signal vector, $e \in \mathcal{R}^q$ is the external output signal vector to be controlled, $y \in \mathcal{R}^p$ is the measurement vector, $w \in \mathcal{R}^k$ and $z \in \mathcal{R}^k$ are external input and output vectors. The connection between the external output and the external input is given by

$$w = \Delta z$$

where Δ represents the model uncertainties in the system. It is without loss of generality to assume that Δ is a square matrix. The derived results can easily be generalized to the non-square case. The model uncertainties can either be structures or unstructured. A special case for structured uncertainties is when Δ is given as a diagonal matrix including real parameter uncertainties/variations given by:

$$\Delta = diag(\delta_1, \cdots, \delta_i, \cdots, \delta_k)$$

where δ_i is the $i'th$ uncertain parameter.

Closing the loop from w to z in Σ_P by using Δ, we get the following LFT (linear fractional transformation) description:

$$\Sigma_{P,\Delta} = \mathcal{F}_u(\Sigma_P, \Delta)$$

where $\Sigma_{P,\Delta}$ is given by:

$$\Sigma_{P,\Delta} : \begin{cases} e = G_{ed}(\Delta)d + G_{eu}(\Delta)u \\ y = G_{yd}(\Delta)d + G_{yu}(\Delta)u \end{cases} \tag{2}$$

Further, let the system be controlled by a stabilizing feedback controller given by:

$$\Sigma_C : \{ \ u = Ky \tag{3}$$

A. The YJBK Parameterization

Let a coprime factorization of the system G_{yu} from (1) and the stabilizing controller K from (3) be given by:

$$\begin{aligned} G_{yu} &= NM^{-1} = \tilde{M}^{-1}\tilde{N}, \quad N,M,\tilde{N},\tilde{M} \in \mathcal{RH}_\infty \\ K &= UV^{-1} = \tilde{V}^{-1}\tilde{U}, \quad\ \ U,V,\tilde{U},\tilde{V} \in \mathcal{RH}_\infty \end{aligned} \tag{4}$$

where the eight matrices in (4) must satisfy the double Bezout equation given by, see [13]:

$$\begin{aligned} \begin{pmatrix} I & 0 \\ 0 & I \end{pmatrix} &= \begin{pmatrix} \tilde{V} & -\tilde{U} \\ -\tilde{N} & \tilde{M} \end{pmatrix} \begin{pmatrix} M & U \\ N & V \end{pmatrix} \\ &= \begin{pmatrix} M & U \\ N & V \end{pmatrix} \begin{pmatrix} \tilde{V} & -\tilde{U} \\ -\tilde{N} & \tilde{M} \end{pmatrix} \end{aligned} \tag{5}$$

Based on the above coprime factorization of the system G_{yu} and the controller K, we can give a parameterization of all controllers that stabilize the system in terms of a stable transfer function Q, i.e. all stabilizing controllers are given by [13]:

$$K(Q) = (U+MQ)(V+NQ)^{-1},\ Q \in \mathcal{RH}_\infty \tag{6}$$

or by using a left factored form:

$$K(Q) = (\tilde{V}+Q\tilde{N})^{-1}(\tilde{U}+Q\tilde{M}),\ Q \in \mathcal{RH}_\infty \tag{7}$$

Using the Bezout equation, the controller given either by (6) or by (7) can be realized as an LFT in the parameter Q:

$$K(Q) = \mathcal{F}_l\left(\begin{pmatrix} UV^{-1} & \tilde{V}^{-1} \\ V^{-1} & -V^{-1}N \end{pmatrix}, Q\right) = \mathcal{F}_l(J_K, Q) \tag{8}$$

The YJBK parameterization is shown in Fig. 1.

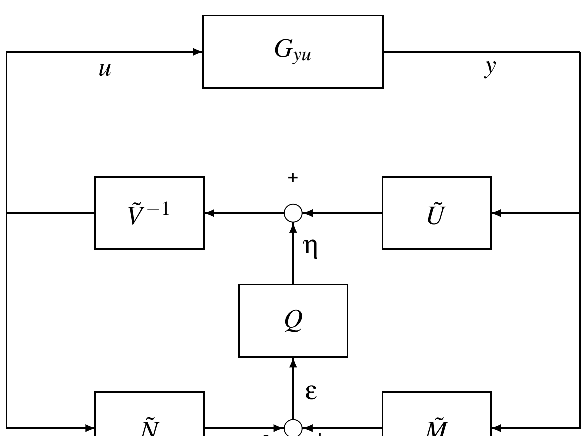

Fig. 1. The YJBK parameterization of all stabilizing controllers $K(Q)$ for a given system G_{yu}.

In the same way, it is possible to derive a parameterization in terms of a stable transfer function S of all systems that are stabilized by one controller, i.e. the dual YJBK parameterization. The parameterization is given by [13]:

$$G_{yu}(S) = (N+VS)(M+US)^{-1},\ S \in \mathcal{RH}_\infty \tag{9}$$

or by using a left factored form:

$$G_{yu}(S) = (\tilde{M}+S\tilde{U})^{-1}(\tilde{N}+S\tilde{V}),\ S \in \mathcal{RH}_\infty \tag{10}$$

An LFT representation of (9) or (10) is given by:

$$\begin{aligned} G_{yu}(S) &= \mathcal{F}_l\left(\begin{pmatrix} NM^{-1} & \tilde{M}^{-1} \\ M^{-1} & -M^{-1}U \end{pmatrix}, S\right) \\ &= \mathcal{F}_l(J_G, S) \end{aligned} \tag{11}$$

Further, S is given by, [13]:

$$S = \mathcal{F}_u(J_K, G_{yu}(S)) \tag{12}$$

B. Closed-loop Stability

Above, it was shown that it is required that both the YJBK transfer function Q and the dual YJBK transfer function S are required to be stable. Now, consider instead the closed-loop system consisting of $(G_{yu}(S), K(Q))$, shown in Fig. 2.

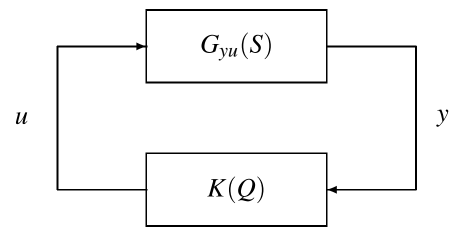

Fig. 2. The closed loop feedback system including the YJBK parameterization of all stabilizing controllers $K(Q)$ and the dual YJBK parameterization of all systems $G_{yu}(S)$.

The closed-loop system shown in Fig. 2 is not guaranteed to be stable by requiring that Q and S are stable transfer matrices. Instead, it has been shown in [13], that the closed loop system shown in Fig. 2 is closed loop stable if and only if the nominal feedback loop given by (G_{yu}, K) and the the feedback loop given by (Q, S) are both closed loop stable. This is shown in Fig. 3. This result can be derived by using the connection between S and Q given by (12). This gives directly that the transfer function between η and ε in Fig. 1 is S, resulting in the second feedback loop in Fig. 3.

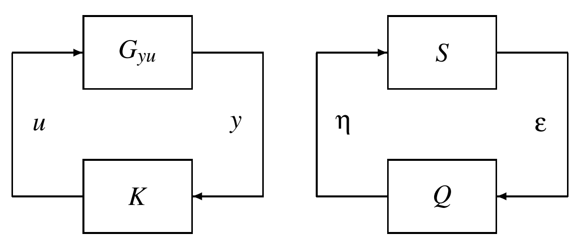

Fig. 3. The two closed loop feedback systems in appearing from the YJBK and the dual YJBK parameterization.

III. Estimation of Uncertainties

Uncertainties or parameter variations in a closed-loop system cannot in general be determined directly. An estimation can only be derived by using available/measurable inputs and outputs. The available vectors are the control vector u, the measurement vector y and the two internal vectors in the controller η, ε, see Fig. 4.

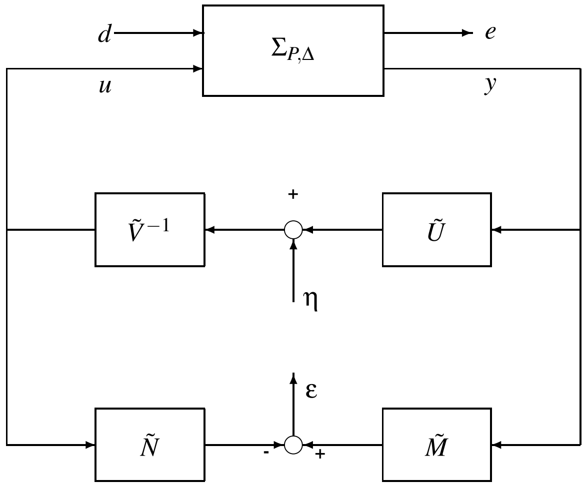

Fig. 4. A set-up for estimation of variations in the system.

The transfer function from inputs to outputs are given by:

$$\Sigma_{P,C} : \begin{cases} e = P_{ed}(\Delta)d + P_{e\eta}(\Delta)\eta \\ \varepsilon = P_{\varepsilon d}(\Delta)d + P_{\varepsilon\eta}(\Delta)\eta \end{cases} \tag{13}$$

The transfer function between η and ε, $P_{\varepsilon\eta}(\Delta)$ is the dual YJBK transfer function as shown in (12). In the following, $S = P_{\varepsilon\eta}(\Delta)$ will be applied for this transfer function due to the relation with the dual YJBK parameterization. The dual YJBK transfer function is a function of the variations in the system. It is therefore relevant to analyze the dual YJBK transfer function in more details with respect to estimation of variations in the system. The dual YJBK transfer function has also been applied in connection with a closed-loop set-up for active fault diagnosis (AFD) considered in [8], [9], [10]. Following the line from AFD, [9], let's use the S for the estimation of the system variations. Using the general system set-up given by (1), an explicit equation for S can be derived. The equation for S is given by, [7]:

$$S(\Delta) = \tilde{M}G_{yw}\Delta(I - (G_{zw} + G_{zu}U\tilde{M}G_{yw})\Delta)^{-1}G_{zu}M \tag{14}$$

or in short

$$S(\Delta) = T_{12}\Delta(I - T_{22}\Delta)^{-1}T_{21}$$

Further, the transfer function from d to ε is given by, [7]:

$$P_{\varepsilon d}(\Delta) = (V - G_{yu}(\Delta)U)^{-1}G_{yd}(\Delta) \tag{15}$$

In the nominal case, $P_{\varepsilon d}(0) = \tilde{M}G_{yd}$.

(14) gives a direct description of the effect from variations in the system on the closed loop stability. If S gets unstable for some variations, the closed loop system will be unstable. An important observation is that

$$S(\Delta) = 0, \text{ for } \Delta = 0$$

This property has been used directly for fault detection in AFD. In this connection, S is named as the *fault signature matrix*, [9], [10].

$S(\Delta)$ given by (14) is also an indirect measure of the model uncertainties Δ in the system. It is possible to calculate an upper bound on S for bounded Δ. This can be done by using the skew-μ method, [12].

Instead of using the transfer function $S(\Delta)$ directly in connection with estimation of Δ, it is possible to modify the feedback controller K by introduction a Q as described in Section II-A. The auxiliary input vector is then given by:

$$\eta = Q\varepsilon + \bar{\eta} \tag{16}$$

The closed-loop transfer function from $\bar{\eta}$ to ε is then given by:

$$\begin{aligned} S(Q,\Delta) &= S(\Delta)(I - QS(\Delta))^{-1} \\ &= T_{12}\Delta(I - (T_{22} + T_{21}QT_{12})\Delta)^{-1}T_{21} \end{aligned} \tag{17}$$

Further, let's also include pre- and post-filters around $S(Q,\Delta)$ resulting in

$$\begin{aligned} S_W(Q,\Delta) &= W_O T_{12}\Delta(I - (T_{22} + T_{21}QT_{12})\Delta)^{-1}T_{21}W_I \\ &= W_O S(Q,\Delta)W_I \end{aligned} \tag{18}$$

This gives three transfer functions that can be designed with respect to get a simple and direct estimation of Δ. Based on (17) and (18), it is possible to apply standard estimation/identification methods. However, in some cases, it is possible to simplify the above closed-loop transfer function such that the estimation of Δ gets more simple. Assume that k satisfy:

$$k \leq min\{m, p\} \tag{19}$$

Let's consider $S_W(Q,\Delta)$ in a frequency range given by:

$$\bar{\omega} = [\omega_1,\ \omega_2] \tag{20}$$

The frequency range $\bar{\omega}$ can be the whole frequency range ($\omega_1 = 0$ and $\omega_2 = \infty$), a specified frequency range or a single frequency ($\omega_1 = \omega_2 = \omega_0$).

The feedback part of $S_W(Q,\Delta)$ given by:

$$T_{22} + T_{21}QT_{12}$$

is now considered. The condition in (19) guarantee that a left and right inverse of T_{12} and T_{21} exists and given by $T_{12}^{\dagger}$ and $T_{21}^{\dagger}$, respectively. Assume that T_{12} or T_{21} does not have zeros in the frequency range given by $\bar{\omega}$. It will always be possible to find a frequency range where this condition is satisfied. This assumption gives that there exist two transfer functions $\bar{T}_{12}, \bar{T}_{21} \in \mathcal{RH}_\infty$ that satisfy:

$$\begin{aligned} \bar{T}_{12}(j\omega) &= T_{12}^{\dagger}(j\omega), \\ \bar{T}_{21}(j\omega) &= T_{21}^{\dagger}(j\omega), \end{aligned} \qquad \text{for } \omega \in \bar{\omega}$$

i.e. there exist stable transfer functions that is exact equal to the inverses of T_{12} and T_{21} in the frequency range $\bar{\omega}$. Now, let $Q(j\omega)$ be given by

$$Q(j\omega) = -\bar{T}_{21}(j\omega)T_{22}(j\omega)\bar{T}_{12}(j\omega) \tag{21}$$

gives directly that

$$T_{22}(j\omega) + T_{21}(j\omega)Q(j\omega)T_{12}(j\omega) = 0, \quad \text{for } \omega \in \bar{\omega}$$

Further, let the two filters $W_I(j\omega)$ and a $W_O(j\omega)$ be given by:

$$W_O = \bar{T}_{12}, \quad W_I = \bar{T}_{21} \tag{22}$$

Using the controller given by (21) and the pre- and post-filter satisfying the conditions in (22) gives the following $S_W(Q,\Delta)$ for $\omega \in \bar{\omega}$:

$$S_W(Q,\Delta) = \Delta, \text{ for } \omega \in \bar{\omega} \tag{23}$$

A stabilizing controller $Q(s)$ satisfying the interpolation constraint in (21) will give an exact decoupling as given by (23) for $\bar{\omega}$. Using (12) and (16), Δ is then the transfer function between $\bar{\eta}_q$ and ε_q, i.e.

$$\varepsilon_q = S_W(Q,\Delta)\bar{\eta}_q = \Delta\bar{\eta}_q, \quad \text{for } \omega \in \bar{\omega} \tag{24}$$

The output vector ε_q given by (24) is derived for the disturbance free case. However, the effect from disturbance need to be included in the equation for ε_q when it should be applied in connection with identification/estimation of Δ. Using the YJBK parameterization of the feedback controller given by (6) in (15) gives directly the following transfer function from disturbance d to ε_q:

$$P_{\varepsilon_q d}(Q,\Delta) = W_O((V+NQ) - G_{yu}(\Delta)(U+MQ))^{-1}G_{yd}(\Delta) \tag{25}$$

In the nominal case, $P_{\varepsilon_q d}(Q,0)$ is independent of Q, i.e. the transfer function is given by:

$$P_{\varepsilon_q d}(Q,0) = W_O\tilde{M}G_{yd}$$

All together, the complete equation for ε_q is then given by:

$$\varepsilon_q = \Delta\bar{\eta}_q + W_O((V+NQ) - G_{yu}(\Delta)(U+MQ))^{-1}G_{yd}(\Delta)d \tag{26}$$

for $\omega \in \bar{\omega}$.

A. Special Cases

The above results can be used for a direct estimation of the uncertain block Δ in a specified frequency range $\bar{\omega}$. Standard identification/estimation methods as e.g. recursive least squares (RLS) or recursive maximum likelihood can be applied to this estimation.

The uncertainty set-up as shown in (2) is a very general set-up. In many cases, it is possible to apply a more simple model for the uncertainties, which will also simplify the above derivation of the two weight matrices W_I, W_O and the feedback controller Q.

Parametric uncertainties: First, let's consider the case where the Δ block represent parameter variations. The uncertain block is then given by:

$$\Delta = diag(\delta_1, \cdots, \delta_i, \cdots, \delta_k)$$

where $\delta_i \in R$ is the $i'th$ uncertain parameter. The estimation of the parameters can then be done by using a periodic signal with a fixed frequency ω_0, because the parameters δ_i does not depend of the frequencies.

Additive model uncertainties: The next case is when the model uncertainties is described as additive uncertainties, i.e. the system is described by:

$$G_{yu}(\Delta_A) = G_0 + \Delta_A \tag{27}$$

The associated dual YJBK transfer function S is given by, [7]:

$$S_A = \tilde{M}\Delta_A(I - U\tilde{M}\Delta_A)^{-1}M \tag{28}$$

If the nominal system G_0 is stable, then the inverse of M and $\tilde{M}$ are stable. The feedback controller Q given by (21)

$$Q_A(j\omega) = -M^{-1}U(j\omega) \tag{29}$$

Further, the two filters $W_I(j\omega)$ and $W_O(j\omega)$ are given by:

$$W_O = \tilde{M}^{-1}, \quad W_I = M^{-1} \tag{30}$$

If the nominal system is unstable, it will only be possible to calculate the uncertain block Δ_A in a certain frequency range.

The transfer function from disturbance d to the output ε_q given by (25) can also be calculated and is given by:

$$P_{\varepsilon_q d}(Q,\Delta_A) = \tilde{M}^{-1}G_{yd}(\Delta_A) \tag{31}$$

The above case shows clearly the effect of changing the feedback controller. Using the feedback controller Q given by (29) in the general controller given by (6) or (7) will decouple the controller exact, i.e. $K(Q) = 0$. However, when the nominal system is stable, it is quite simple to derive an equation for the uncertain block in (27). Instead of using using the above method, it is more easy to derive the model uncertainties directly by using

$$\Delta_A = G - G_0$$

when the system is stable.

Multiplicative input uncertainties: A very used uncertain model description is the multiplicative input and output uncertain models. Using these two uncertain descriptions, the model uncertainties is transformed either into the input or the output of the system and represented in general by a full complex uncertain block, [12]. First, let's consider the case where the uncertainties is represented by a multiplicative input uncertainties. The system is then described by:

$$G_{yu}(\Delta_I) = G_0(I + \Delta_I) \tag{32}$$

The associated dual YJBK transfer function S is given by, [7]:

$$S_I = \tilde{M}\Delta_I(I - N\tilde{U}\Delta_A)^{-1}N \tag{33}$$

It will not in general be possible to obtain an exact calculation of Δ_I for all frequencies, because the coprime matrix N will not in general be invertible. The YJBK controller Q_I must be designed such that:

$$N\tilde{U} + NQ_I\tilde{M} = 0 \tag{34}$$

This gives

$$Q_I(j\omega) = -\tilde{U}\tilde{M}^{-1}(j\omega) \tag{35}$$

when the inverse of $\tilde{M}$ is stable. Else, an approximative inverse of $\tilde{M}$ for a certain frequency range need to be applied. The pre- and post-filter W_I and W_O must be selected as the inverse of N and $\tilde{M}$, respectively. These will not in general exist, so approximative inverses need to be applied instead.

The transfer function from disturbance d to the output ε_q given by (25) can also be calculated and is given by:

$$P_{\varepsilon_q d}(Q,\Delta_A) = W_O G_{yd}(\Delta_I) \tag{36}$$

Multiplicative output uncertainties: Equivalent for the multiplicative output uncertainties. Let the model be given by:

$$G_{yu}(\Delta_O) = (I+\Delta_O)G_0 \tag{37}$$

The associated dual YJBK transfer function S is given by, [7]:

$$S_O = \tilde{N}\Delta_I(I-U\tilde{N}\Delta_A)^{-1}M \tag{38}$$

The associated YJBK controller Q_O is then given by:

$$Q_O(j\omega) = -M^{-1}U(j\omega) \tag{39}$$

when the inverse of M is stable. Else, an approximative inverse need to be used. The pre- and post-filter W_I and W_O must be selected as the inverse of M and $\tilde{N}$, respectively. These will not in general exist, so approximative inverses need to be applied instead.

The transfer function from disturbance d to the output ε_q given by (25) is the same as in the above case.

B. Estimation Methods

Based on the isolation of the uncertainties/parameter variations described above, the next step is an estimation of Δ. This can be done in a number of different ways.

The most simple way is to use $\bar{\eta}_q$ as a periodic input signal given by

$$\bar{\eta}_q = a_q \sin(\omega_0 t)$$

where ω_0 is the frequency to be designed. This will give information about the gain and phase at a single frequency. If Δ is constant or almost constant, it is a simple way to estimate the uncertainties.

When Δ is not constant, estimation based on a single periodic signal will not give a reasonably estimate. Instead, an input signal including a number of periodic signals can be applied. This will give information about Δ at a number of specific frequencies.

The estimate of Δ can also be obtained by means of standard methods from system identification. One such method is spectral analysis in which the extended probe signal $\bar{\eta}_q$ with spectral density Φ_η is applied to the system. If the cross spectral density, $\Phi_{\varepsilon\bar{\eta}_q}$ is estimated (e.g. with FFT techniques) then

$$\hat{\Delta} = \Phi_{\varepsilon\bar{\eta}_q}\Phi_\eta^{-1}$$

is an estimate of Δ.

Another way of estimating Δ could be to apply a parametric method. If the structure of Δ is known then it is straight forward to apply e.g. an output error method. If the structure is unknown then it can be search for or a high order model can be applied. In the latter case the uncertainties on the estimates can be used for assessing an upper bound on the uncertainty on Δ rather than Δ itself.

IV. EXAMPLE

The well known four tank system is described in e.g.[6]. The main issue in this example is to give a description how the necessary calculation are derived to use the estimation methods described in this paper. The calculation is derived in state space.

The four tank system is described by the following linear state space system:

$$\begin{aligned} G_{yu} &= \left[\begin{array}{c|c} A & B_u \\ \hline C_y & 0 \end{array}\right] \\ &= \left[\begin{array}{cccc|cc} -\frac{1}{T_1} & 0 & \frac{A_3}{A_1T_3} & 0 & \frac{\gamma_1 k_1}{A_1} & 0 \\ 0 & -\frac{1}{T_2} & 0 & \frac{A_4}{A_2T_4} & 0 & \frac{(1-\gamma_2)k_1}{A_3} \\ 0 & 0 & -\frac{1}{T_3} & 0 & 0 & \frac{(1-\gamma_2)k_1}{A_3} \\ 0 & 0 & 0 & -\frac{1}{T_4} & \frac{(1-\gamma_1)k_1}{A_4} & 0 \\ \hline k_c & 0 & 0 & 0 & 0 & 0 \\ 0 & k_c & 0 & 0 & 0 & 0 \end{array}\right] \end{aligned}$$

T_i is given by

$$T_i = \frac{A_i}{a_i}\sqrt{\frac{2h_i^0}{g}}$$

where the data for the parameters can be found in [6].

An analysis of the system shown that it is minimum phase for $1 < \gamma_1+\gamma_2 < 2$ and non-minimum phase for $0 < \gamma_1+\gamma_2 < 1$. γ_1 and γ_2 that describe the opening of the two valves in the system, are important parameters for the system. It is therefore relevant to estimate these two parameters, if they cannot be measured directly. For doing this, let the two parameters be described by:

$$\gamma_1 = \gamma_{10}+\delta_1, \quad \gamma_2 = \gamma_{20}+\delta_2$$

where γ_{i0} is the nominal value of γ_i and δ_i is the variation around the nominal value.

The system need to be described in the standard set-up given by (1) or in state space

$$\Sigma_P : \left[\begin{array}{c|cc} A & B_w & B_u \\ \hline C_z & D_{zw} & D_{zu} \\ C_y & D_{yw} & D_{yu} \end{array}\right]$$

(the disturbance input d and the external output e has been removed in the above state space description).

Now, let Δ be given by

$$\Delta = diag(\delta_1,\ \delta_2)$$

the input matrix B can then be written as

$$\begin{aligned} B &= B_u(0) + \begin{bmatrix} \frac{k_1}{A_1} & 0 \\ 0 & \frac{k_2}{A_2} \\ 0 & \frac{-k_1}{A_3} \\ \frac{-k_1}{A_4} & 0 \end{bmatrix} \begin{bmatrix} \delta_1 & 0 \\ 0 & \delta_2 \end{bmatrix} \\ &= B_u(0) + B_w \Delta D_{zu} \end{aligned}$$

where $B_u(0)$ is the nominal input matrix and $D_{zu} = I$. Further, C_z, D_{zw} and D_{yw} are zero matrices of suitable dimensions.

Let the four tank system be controlled be a full-order observer based controller given by

$$\Sigma_C : \left[\begin{array}{c|c} A + B_u F + L C_y & -L \\ \hline F & 0 \end{array}\right]$$

where F is the state feedback gain and L is the full order observer gain. If the applied feedback controller is not a full order based controller, the controller can in a number of cases be transfered into an observer based controller by using the transformation described in [1]. An alternative is to use the state space description directly in the equations for the coprime factors. The equations for the coprime factors based on a general controller can be found in [13].

The system set-up for the four tank system gives a multiplicative input uncertain description. The YJBK controller is given by (35). The two coprime factors $\tilde{U}$ and $\tilde{M}$ are given by:

$$\begin{bmatrix} \tilde{U} \\ \tilde{M} \end{bmatrix} = \left[\begin{array}{c|c} A + L C_y & L \\ \hline -F & 0 \\ C_y & I \end{array}\right]$$

When the system is stable, the decoupling controller is given by

$$Q_I = \left[\begin{array}{c|c} A & L \\ \hline F & 0 \end{array}\right]$$

When the open loop system is unstable, the inverse of $\tilde{M}$ is not stable. $\tilde{M}$ is a non-minimum phase system with the unstable poles as right half plane zeros. This problem can be handled by a factorization of $\tilde{M}$ into a minimum phase part and an all-pass factor given by, [12]:

$$\tilde{M} = C(z)\tilde{M}_{\min}$$

where $C(z)$ is an all-pass factor. The result is that it is not possible to design a controller Q_I that will satisfy the condition in (34) for all frequencies. An approximative solution can be derived below or above the right half plane zeros (unstable poles in the system) in $\tilde{M}$.

Because Δ only consists of two parameters, it is possible to estimate these parameters by using a single periodic input signal. This mean that Q_I need only satisfy:

$$Q_I(j\omega) = -\tilde{U}\tilde{M}^{-1}(j\omega), \text{ for } \omega = \omega_0$$

else

$$Q_I(j\omega) \approx 0, \text{ for } \omega \neq \omega_0$$

where ω_0 is the frequency of the periodic input signal. One way to obtain this is be using a band-pass filter together with Q_I.

The last part is the design of the two filters W_I and W_O are given as the inverse of N and $\tilde{M}$, respectively. N given by:

$$N = \left[\begin{array}{c|c} A + B_u F & B_u \\ \hline C_y & 0 \end{array}\right]$$

is not invertible for all frequencies because D_{yu} is zero. Further, when the system includes non minimum phase zeros, the inverse will include an unstable pole. Again, the two filters need only to be equal to the inverses for $\omega = \omega_0$.

V. CONCLUSION

Estimation of model uncertainties in closed-loop systems has been considered in this paper. Using the YJBK set-up, it has been shown that it is possible to estimate a limit number of parameters/uncertain blocks directly, when the system is applied in closed-loop. It will in general be impossible to obtain a direct estimation of the model uncertainties for all frequencies. Instead, it is shown how it is possible to get a direct estimate in a limit frequency range instead.

REFERENCES

[1] D. Alazard and P. Apkarian. Exact observer-based structures for arbitrary compensators. *International Journal of Robust and Nonlinear Control*, 9:101–118, 1999.

[2] F. De Bruyne, B.D.O. Anderson, and N. Linard. The Hansen scheme revisited. In *Proceedings of the 37th Conference on Dicision and Control*, pages 706–711, Tampa, FL, USA, 1998.

[3] R. de Callafon. *Feedback oriented identification for enhanced robust control*. PhD thesis, Delft University of Technology, Delft, The Netherlands, 1998.

[4] R.G. Hakvoort. *System identification for robust process control*. PhD thesis, Delft University of Technology, Delft, The Netherlands, 1994.

[5] F.R. Hansen and G.F. Franklin. On a representation approach to closed-loop experiment design. In *Proceedings of the American Control Conference*, pages 1319–1320, Atlanta, GA, USA, 1988.

[6] K.H. Johansson. The quadruple-tank process: A multivariable laboratory process with an adjustable zero. *IEEE Trans. on Control Technology*, 8(3):456–465, 2000.

[7] H.H. Niemann. Dual Youla parameterization. *IEE Proceedings - Control Theory and Applications*, 150(5):493–497, 2003.

[8] H.H. Niemann. Active fault diagnosis in closed-loop uncertain systems. In *Preprints of 6th IFAC Symposium on Fault Detection Supervision ans Safety for Technical Processes, SAFEPROCESS'2006*, pages 631–636, Beijing, China, 2006.

[9] H.H. Niemann. A setup for active fault diagnosis. *IEEE Transactions on Automatic Control*, 51(9):1572–1578, 2006.

[10] H.H. Niemann and N.K. Poulsen. Active fault diagnosis in closed-loop systems. In *Proceedings of the 16th IFAC World Congress*, page 6p, Prague, Czech Republic, 2005.

[11] H.H. Niemann, J. Stoustrup, and R.B. Abrahamsen. Switching between multivariable controllers. *Optimal Control - Application and Methods*, pages 51–66, 2004.

[12] S. Skogestad and I. Postlethwaite. *Multivariable feedback control: Analysis and Design*. Wiley, 2005.

[13] T.T. Tay, I.M.Y. Mareels, and J.B. Moore. *High performance control*. Birkhäuser, 1997.

[14] T. van den Boom. *MIMO system identification for $\mathcal{H}_\infty$ robust control*. PhD thesis, Eindhoven University of Technology, Eindhoven, The Netherlands, 1993.

[15] P. Van den Hof and R.A. de Callafon. Multivariable closed-loop identification: From indirect identification to dual Youla parameterization. In *Proceedings of the 33rd Conference on Dicision and Control*, pages 1397–1402, Kobe, Japan, 1996.

[16] P. Van den Hof and R.J.P. Schrama. Identification and control - Closed-loop issues. *Automatica*, 31(12):1751–1770, 1995.