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Networked control under synchronization errors

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Abstract— **This work concerns the observer-based control of a remote, Master-Slave system through the *Internet* network. This communication link introduces variable, asymmetric and unpredictable delays. The data-sampling effects are also taken into account, even in the aperiodic case. Previous strategies were requiring additional buffers, allowing the delay to become constant but greater and that the Slave and the Master share the same clock. In this article, these assumptions are not longer required. Thanks to an adequate Lyapunov-Krasovskii functional, the present result uses the information as soon as received. The proposed LMI ensure the asymptotic stability of the global closed loop system. The maximum admissive synchronization error is also computed. The last part of the paper provides an example where the Slave is a second-order system.**

I. Introduction and hypotheses

Internet technology appears as a natural and cheap way to ensure the communication link in remotely controlled systems [1]. Today, the available Quality of Service is often good enough for that kind of applications. However, such a communication link constitutes an additional dynamical system, which great influence on stability was already mentioned in the 60's [5]. Indeed, several dynamics and perturbations (communication delay, real-time sampling, packet dropout and synchronization errors) are unavoidably introduced and have to be taken into account during the design of the control/observation loop.

In the literature, many authors assume that the nodes of the NCS are synchronized [11]. However the synchronization is an fundamental issue of NCS since ensuring several nodes re synchronized is not easy and some error in it may reduce the performances of the controller [6]. The article focusses on the lake of time-synchronization and provides a robust controller for continuous networked control systems with synchronization error and to parameter uncertainties. A time-delay representation which takes into account the transmission delays, the sampling and the synchronization errors.

Previous works [8], [20] have shown that both sampling effects and communication time lags can be regrouped into a time-varying delay, homogenized representation. Recently this article was extended to solve the stabilization of the Master-Slave systems under packet loss [21]. The present study aims at including the effect of synchronization error on this unifying model. Once the global system is reduced to a system with time-varying delays, several control techniques can be involved (see for instance [14], [18]). Here, an observer-based instantaneous state feedback will be used. But, before presenting the details, a short overview of previous results will highlight the various assumptions on the communication delays $h_1(t)$ (from Master to Slave) and $h_2(t)$ (from Slave to Master).

Several works on tele-operation introduced the question of transmission delays in the *constant* case [2], [4], [16]. However, in networked control situations, the delays are basically variable (jitter phenomenon) and unpredictable. This is a source of problem when the classical predictor-based controllers are intended to be applied. These techniques generally need the constant delay, *i.e.* $h_i(t) = h_i$.

In the case of variable delays, some researches have used independent-of-delay conditions. Because such i.o.d. conditions may be conservative in general, particular cases such as constant or *symmetric* delays were considered [3]. These assumptions refers to the case where the transmission delays are equal, *i.e.* $h_1(t) = h_2(t) = R(t)/2$, where $R(t)$ denotes the round trip time (RTT). In [10] non-symmetric delays are considered, but only in the constant delay case, *i.e.* $h_1(t) = h_1 \neq h_2(t) = h_2$.

Another interesting approach was recently given in [22], which generalized the predictor techniques to the case of variable delays. In this case, a maximal bound of the delays is assumed to be known (h_M such that $0 \leq h(t) \leq h_M$), which is not that restrictive. The most constraining assumption is that a dynamical Ordinary Differential Equation (ODE) model of the delay is supposed to be available, which is possible in the case of a single-owner network.

When using *Internet*, the generated delays are not only time-varying and non-symmetric, but also unknown (no dynamical model of the delays is available, see [15]). To bypass this problem, it was proposed [12], [13], [17] to introduce two input buffers (Master and Slave) that make both the receivers wait until the maximum value h_M of the communication delay is reached. However, it is obvious that this situation maximizes the delays $h_1(t)$ and $h_2(t)$ up to their worst (largest) value (*i.e.*, $h_1(t) \hookrightarrow h_M$ and $h_2(t) \hookrightarrow h_M$) and, consequently, decreases the possible range of speed performance. To reduce this maximizing effect, [20] restricted the buffer to the only transmission from Master to Slave (thus, $h_1(t) \hookrightarrow h_M$), while the remote observer was computing the present Slave's state on the basis of the non-buffered Slave's information. The stabilizing gains were designed *via* some Lyapunov-Krasovskii functionals and, moreover, a guaranteed speed rate was computed. *A first original contribution of the present study is to get rid of any buffer.* We only assume the (non-symmetric) delays

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to have known minimal and maximal bounds h_m and h_M, so that the following holds:

$$\mathbf{A1}\ (maximal\ allowed\ delay):\quad h_m \leq h_i(t) \leq h_M. \qquad (1)$$

Since we aim at limiting the value of h_m, the use of UDP (User Datagram Protocol) is preferred to TCP (Transmission Control Protocol), the reliability mechanisms of which may needlessly slow down the feedback loop. In return, some data packets can be lost during the transmission, without being re-emitted. This phenomenon was treated in [21] and will not be treated in this article.

The synchronization of the Slave and Master clocks is not assumed to be achieved. It means that the time t_S given by the Slave clock and the time t_M delivered by the Master clock do not have the same sense. For instance, if the reference time is assumed to be given by the Slave, it means that $t_M = t_S + \varepsilon(t)$ where ε corresponds to a time-varying error of synchronization. We take the following evaluation:

$$\mathbf{A2}:\quad The\ synchronization\ errors\ is\ time-varying\ but\ bounded: \quad |\varepsilon(t)| \leq \bar{\varepsilon} \qquad (2)$$

The proposed method will allow for computing some admissible (in the sense: non destabilizing) value of $\bar{\varepsilon}$.

An other feature of *Internet* is that the packets are not always arriving in their chronological emission order, while UDP does not automatically re-organize them. Then, the reception function of Master and Slave will be added a re-ordering mechanism, based on some "time-stamps" added in the control and measurements packets. This additional mechanism implies that the transmission delay variation satisfies:

$$\mathbf{A3}\ (packet\ reordering):\quad \dot{h}_i(t) < 1. \qquad (3)$$

The last disturbance implied by the network comes from samplers and zero-holders needed for a discrete-time implementation. Following the lines of [8], we consider they produce an additional variable delay $t - t_k$, where t_k is the k^{th} sampling instant. Moreover, because of the computer architecture and operating system, the sampling is generally not periodic, *i.e.* there is no exact period T such that $t_k = kT$. So, we only assume there exists a known maximum sampling interval T so that:

$$\mathbf{A4}\ (max.\ sampling\ interval):\ 0 \leq t_{k+1} - t_k \leq T. \qquad (4)$$

The global delays resulting from the communication-plus-sampling phenomena will be denoted by $\delta_i(t_k) = h_i(t) + t - t_k$, for which the condition $\dot{\delta}_i(t) \leq 1$ holds. Note that the limit case $\dot{\delta}_i = 1$ occurs.

The paper is organized as follows. Section II describes the remote system features. Section III considers the problem of robust stability with respect to synchronization errors. Section IV gives an illustrative example and Section V proposes some concluding remarks.

II. FEATURES OF THE REMOTE SYSTEM

For the sake of simplicity, the Slave is considered to fit a linearized model. The exchanged data correspond to the control (sent by the Master to the Slave) and to the output of the remote system (sent by the Slave to the Master). The Slave is not supposed to have a large computation power and its functions are limited to: receive control packets, apply control, send output measurement data. Thus the control and observation complexity is to be concentrated in the Master which has to: receive output measurements, estimate present state of Slave, compute and send the control value. Our purpose is to guarantee the asymptotic stability of the global Master-Slave system. In particular, the global system must ensure the closed-loop stability whatever the delay, the errors in the synchronization and the possible aperiodicity of the real-time sampling processes.The stability must be robust with respect to the resulting, global delay. This property will be proven by using adequate Lyapunov-Krasovskii functionals, leading to an LMI optimization of the controller and observer gains. The system has the features which are exposed in Fig.1 and explained in the sequel:

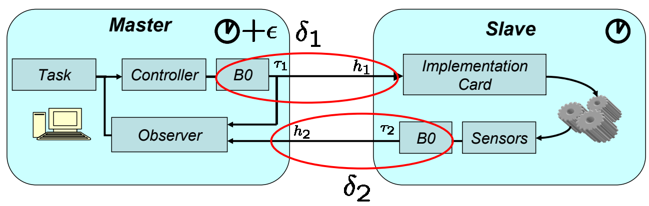

Fig. 1. Structure of the remote system under time-varying delays, samplings and synchronization errors.

- The Master computes and forwards the control to the Slave. The forwarding cannot be instantaneous. It induces a time-varying delay $h_1(t)$, assumed to satisfy **A1** and (thanks to packet re-ordering) **A3**, with synchronization error for which **A2** holds, as well as sampling effects which create the variable delay $\tau_1(t)$ satisfying **A4**.
- The Slave is driven by a controllable and observable, known model (A, B, C), influenced by an input delay $\delta_1(t)$ to be defined later on in subsection II-D:

$$\begin{cases} \dot{x}(t) = & Ax(t) + Bu(t - \delta_1(t)), \\ y(t) = & Cx(t). \end{cases} \qquad (5)$$

- The Slave measures its sampled output variables y, that the Master receives after a delay $h_2(t)$ which is assumed to belong to the same interval (see **A1**). This assumption is not restrictive since it is obtained by the union of the intervals of variation of h_1 and h_2. The delay $\tau_2(t)$ due to the sampling is added. It means that the Master only can access $y(t - \delta_2(t))$, where δ_2 corresponds to the resulting delay. The Master includes an observer for estimating $\hat{x}$ of the complete Slave state x at the present time. Based upon this estimation, the Master elaborates the control law.

- The sampling instants t_k may not be periodical *i.e.* $t_k \neq kT$, but it is supposed there exists a known T such that **A4** holds for any k.
- Instead of [20], both Master and Slave subsystems are not synchronized anymore, *i.e.* they do not share a common clock. However and lead each data packet includes an added time-stamp which corresponds to the time the packet was sent. By this way, the receiver can calculate an estimation of the transfer delays $h_i(t)$ as soon as it receives the packet but this estimation has an error $\varepsilon(t)$.

In the sequel, it is assumed that the reference clock is given by the Slave. The next subsections detail the features and notations.

A. The sampling delays

From a practical viewpoint, the system (including the controller, the observer, the network and the process) cannot be considered as a continuous-time one. Exchanging packets between Slave and Master in continuous time would mean that the network has a large bandwidth. Then, the packets only give discrete-time information. The corresponding sampling effect represents a possible disturbance to the stabilization and must be taken into account in the observer and contin [8], [19] such sampling effects is considered as continuous-time phenomena with variable time delays. Indeed, the sample $g(t_k)$ of a function $g(t)$ at time t_k can be written as $g(t_k) = g(t-[t-t_k]) = g(t-\tau(t))$; This notation replaces the sample-and-hold with an additional delay $\tau_k(t) = t-t_k,\ t \in [t_k, t_{k+1}[$. Thus, an aperiodic sampling is modeled as an unknown delay with the upper-bound T defined by (4). This change allows continuous-time techniques to be applied, e.g. Lyapunov-Krasovskii functionals for the stability study of sampled systems.

B. The control law

The controller computes a control law which considers some set-values to be reached by the Slave. The static state feedback control $u(t) = K\hat{x}(t)$ is defined considering the state estimate $\hat{x}$ given by the observer. The main difficulty is to design the linear gain K of controller in order to guarantee stability despite the value of the time-varying delay $\delta_1(t)$. In [20], the controller design was achieved using exponential stability criteria.

C. Transmission of the control u

The k^{th} packet sent by the Master to the Slave includes the designed control $u(t_{1,k})$ and a instant of time $t_{1,k}$ when the packet was sent. The Slave receives this information at time $t^r_{1,k}$. This time does not have the same meaning for both the Slave and the Master. Then, the term $t^r_{1,k} - t_{1,k}$, corresponding to the transmission delay, corrupted by ε is estimated by the Slave once the packet has reached it.

D. Receipt and processing of the control data

The control, sent at time $t_{1,k}$, is received by the Slave at time $t^r_{1,k} \geq t_{1,k} + h_m$. There is no raison that the Master also knows the time $t^r_{1,k}$ when the control $u(t_{1,k})$ will be injected into the Slave input. Finally, the slave process is governed by:

$$\dot{x}(t) = \quad Ax(t) + Bu(t_{1,k}) \tag{6}$$

where k is such that $h_m \leq t_{1,k} \leq h_M + T$.

E. Transmission of the measured output information

The Slave accesses its output y at discrete instants of time. A sent packet contains the output $y(t_{2,k'})$ and the measurement instant $t_{2,k'}$ which is the k'^{th} one. The Master receives the output data at time $t^r_{2,k'}$.

F. Observation of the process

For a given $\hat{k}$ and any $t \in [t_{1,\hat{k}} + (h_M - h_m)/2,\ t_{1,k+1} + (h_M - h_m)/2[$, there exists a k' such that the proposed observer is of the form:

$$\begin{cases} \dot{\hat{x}}(t) = & A\hat{x}(t) + Bu(t_{1,\hat{k}} + \varepsilon) - L(y(t_{2,k'}) - \hat{y}(t_{2,k'} - \varepsilon)), \\ \hat{y}(t) = & C\hat{x}(t). \end{cases} \tag{7}$$

The time stamp $t_{1,\hat{k}}$ correspond to the time where the control input is supposed to be implemented in the Slave. The index k' corresponds to the most recent output information the Master has received. Note that the Master is not supposed to know the time $t^r_{1,k}$ and the control $u(t_{1,k})$ (see Section II-D), which makes this observer realizable. The input delay approach to sampled-data signals allows considering a homogenized definition of the delays $\delta_1(t) \triangleq t - t_{1,k}$ where k corresponds to the real sampling implemented in the Slave process, $\hat{\delta}_1(t) \triangleq t - t_{1,\hat{k}}$ and $\delta_2(t) \triangleq t - t_{2,k'}$. The observer dynamics can be written as:

$$\begin{cases} \dot{\hat{x}}(t) = & A\hat{x}(t) + Bu(t - \hat{\delta}_1(t) + \varepsilon) \\ & -L(y(t-\delta_2(t)) - \hat{y}(t - \delta_2(t) - \varepsilon)), \\ \hat{y}(t) = & C\hat{x}(t), \end{cases} \tag{8}$$

where the features of the system lead to $h_m \leq \delta_i(t) \leq h_M + T$ for $i = 1,2$. Equivalently, if the average delay $\delta(h_m, h_M, T) = (h_M + T + h_m)/2$ and the maximum delay amplitude $\mu(h_m, h_M, T) = (h_M + T - h_m)/2$ is used, then:

$$\delta - \mu \leq \delta_i(t) \leq \delta + \mu, \quad \forall i = 1,2. \tag{9}$$

According to (6) and (7) and for given k and any $t \in [t^r_{1,k} + h_m,\ t^r_{1,k+1} + h_m[$, there exist $\hat{k}$ and k' such that the system is governed by:

$$\begin{array}{ll} \dot{x}(t) = & Ax(t) + BK\hat{x}(t_{1,k}), \\ \dot{\hat{x}}(t) = & A\hat{x}(t) + BK\hat{x}(t_{1,\hat{k}} - \varepsilon) - LC(x(t_{2,k'}) - \hat{x}(t_{2,k'} + \varepsilon)). \end{array} \tag{10}$$

Rewriting the equations by using the error $e(t) = x(t) - \hat{x}(t)$, the dynamics become:

$$\begin{array}{ll} \dot{x}(t) = & Ax(t) + BKx(t_{1,k}) - BKe(t_{1,k}), \\ \dot{e}(t) = & Ae(t) + LCe(t_{2,k'}) - BK\int_{t_{1,k}}^{t_{1,\hat{k}}+\varepsilon} [\dot{x}(s) - \dot{e}(s)]ds \\ & +LC\int_{t_{2,k'}-\varepsilon}^{t_{2,k'}} [\dot{x}(s) - \dot{e}(s)]ds. \end{array}$$

Applying the input delay representation [8] leads to:

$$\begin{aligned}\dot{x}(t) = &\; Ax(t) + BKx(t-\delta_1(t)) - BKe(t-\delta_1(t)),\\ \dot{e}(t) = &\; Ae(t) + LCe(t-\delta_2(t)) - BK\int_{t_{1,k}}^{t_{1,\hat{k}}+\varepsilon}[\dot{x}(s)-\dot{e}(s)]ds\\ &+LC\int_{t_{2,k'}-\varepsilon}^{t_{2,k'}}[\dot{x}(s)-\dot{e}(s)]ds.\end{aligned} \quad (11)$$

with $\delta_1(t) = t - t_{1,k}$ and $\delta_2(t) = t - t_{2,k'}$. Knowing that the communication delays belong to the interval $[h_m,\ h_M]$ where h_m and h_M are given by the network properties, the condition (9) on the delays still holds.

In an ideal case $\varepsilon = 0$ (from **A2**, no synchronization error) and the Master to Slave delay is assumed to be well known, ie. $\delta_1(t) = \hat{\delta}_1(t)$ (see [20]), then the global system can then be rewritten using the error vector $e(t) = x(t) - \hat{x}(t)$ as:

$$\dot{x}(t) = Ax(t) + BKx(t-\delta_1(t)) - BKe(t-\delta_1(t)) \quad (12a)$$

$$\dot{e}(t) = Ae(t) + LCe(t-\delta_2(t)) \quad (12b)$$

For this ideal case, Theorem 2 and 3 in [20] deliver controller and observer gains.

III. Stabilization under synchronization error

This section focusses on the development of asymptotic stability criteria for the system detailed in Fig. 1. It is now accepted that $\delta_1(t) \neq \hat{\delta}_1(t)$ and that an error may appear in the synchronization process. As a synchronization error leads to a disturbance in the delay measurements, the stability of the controller and of the observer is not ensured anymore by Theorem 2 and 3 in [20].

As in equation (11), there are some interconnection terms between the two variables x and e. A separation principle thus is no longer applicable to prove the global stabilization. The proof of the stability requires to consider now both variables simultaneously.

Theorem 1: For given K and L, suppose that there exist positive definite matrices : P_{q1}, S_q, R_{qa}, $R_{q\varepsilon}$, S_{xe} and R_{xe} and matrices of size $n \times n$: P_{q2}, P_{q3}, Z_{ql} for $l = 1,2,3$, $Y_{ql'}$ for $l' = 1,2$ and for q representing the subscript x or e, such that the following LMI's hold :

$$\begin{bmatrix} \Theta_x & \Theta_{x12} & \mu P_x^T A_K & P_x^T A_K & \mu P_x^T A_K \\ * & -S_x & 0 & 0 & 0 \\ * & * & -\mu R_{xa} & 0 & 0 \\ * & * & * & -S_{xe} & 0 \\ * & * & * & * & -\mu R_{xe} \end{bmatrix} < 0, \quad (13)$$

$$\begin{bmatrix} \Theta_e & \Theta_{e12} & \mu P_e^T A_L & \bar{\varepsilon} P_e^T A_L & \bar{\varepsilon} P_e^T A_L & \beta P_e^T A_K & \beta P_e^T A_K \\ * & -S_e + S_{xe} & 0 & 0 & 0 & 0 & 0 \\ * & * & -\mu R_{ea} & 0 & 0 & 0 & 0 \\ * & * & * & -\bar{\varepsilon} R_{e\varepsilon} & 0 & 0 & 0 \\ * & * & * & * & -\bar{\varepsilon} R_{x\varepsilon} & 0 & 0 \\ * & * & * & * & * & -\beta R_{e\varepsilon} & 0 \\ * & * & * & * & * & * & -\beta R_{x\varepsilon} \end{bmatrix} < 0, \quad (14)$$

$$\begin{bmatrix} R_q & Y_{q1} & Y_{q2} \\ * & Z_{q1} & Z_{q2} \\ * & * & Z_{q3} \end{bmatrix} \geq 0, \quad q \in \{x, e\}, \quad (15)$$

where $\beta = 2(\mu + \bar{\varepsilon})$, $P_q = \begin{bmatrix} P_{q1} & 0 \\ P_{q2} & P_{q3} \end{bmatrix}$ and

$$\begin{aligned}\Theta_x = &\; \Theta_x^n + \begin{bmatrix} 0 & 0 \\ 0 & 4(\mu+\bar{\varepsilon})R_{x\varepsilon} \end{bmatrix},\\ \Theta_e = &\; \Theta_e^n + \begin{bmatrix} 0 & 0 \\ 0 & 4(\mu+\bar{\varepsilon})R_{e\varepsilon} + 2\mu R_{xe} \end{bmatrix},\\ \Theta_q^n = &\; P_q^T \begin{bmatrix} 0 & I \\ A & -I \end{bmatrix} + \begin{bmatrix} 0 & I \\ A & -I \end{bmatrix}^T P_q\\ &+ \begin{bmatrix} S_q + Y_{q1} + Y_{q1}^T + \delta Z_{q1} & Y_{q2} + \delta Z_{q2} \\ * & \delta R_q + 2\mu R_{qa} + \delta Z_{q3} \end{bmatrix},\\ \Theta_{x12} = &\; P_x^T A_K - \begin{bmatrix} Y_{x1}^T \\ Y_{x2}^T \end{bmatrix}, \Theta_{e12} = P_e^T A_L - \begin{bmatrix} Y_{e1}^T \\ Y_{e2}^T \end{bmatrix}.\end{aligned}$$

and where $A_K = \begin{bmatrix} 0 \\ BK \end{bmatrix}$ and $A_L = \begin{bmatrix} 0 \\ LC \end{bmatrix}$.

Then, the system (10) is asymptotic stable.

Proof: To analyze asymptotic stability of such a system, equations (11) are rewritten by using the descriptor representation introduced in [7], [9] with $\bar{x}(t) = col\{x(t), \dot{x}(t)\}$, $\bar{e}(t) = col\{e(t), \dot{e}(t)\}$. Consider the following Lyapunov-Krasovskii functional:

$$V = V_{xn} + V_{xa} + V_{x\varepsilon} + V_{en} + V_{ea} + V_{e\varepsilon} + V_{xe} \quad (16)$$

where the sub-Lyapunov-Krasovskii functionals are, for q representing the subscript of the variables 'x' and 'e':

$$\begin{aligned}V_{qn}(t) = &\; \bar{q}^T(t)EP_q\bar{q}(t) + \int_{-\delta}^{0}\int_{t+\theta}^{t}\dot{q}^T(s)R_q\dot{q}(s)dsd\theta\\ &+\int_{t-\delta}^{t}q^T(s)S_qq(s)ds,\\ V_{qa}(t) = &\; \int_{-\mu}^{\mu}\int_{t+\theta-\delta}^{t}\dot{q}^T(s)R_{qa}\dot{q}(s)dsd\theta,\\ V_{q\varepsilon}(t) = &\; 2\int_{-\mu-\bar{\varepsilon}}^{\mu+\bar{\varepsilon}}\int_{t+\theta-\delta}^{t}\dot{q}^T(s)R_{x\varepsilon}\dot{q}(s)dsd\theta\\ V_{xe}(t) = &\; \int_{-\mu}^{\mu}\int_{t+\theta-\delta}^{t}\dot{e}^T(s)R_{xe}\dot{e}(s)dsd\theta\end{aligned}$$

with $E = diag\{I_n, 0\}$ and P_x, P_e defined in Theorem 1.

The signification of each sub-Lyapunov-Krasovskii functional has to be explain. The first functionals V_{xn} and V_{en} deal with the stability of the Slave and the observer systems subject to the constant delay δ while V_{xa} and V_{ea} refer to the disturbances due to the delay variations. Even if the functionals do not explicitly depend on each time-varying delay, it will be considered both different delays δ_1 and δ_2. The functionals $V_{x\varepsilon}$ and $V_{e\varepsilon}$ are concerned with synchronization error. The last functional V_{xe} deals with the interconnection between the variables x and e. This will appear more clearly later on. According to Theorem 2 in [19], if LMI (15) holds for $'q = x'$, the following inequality holds:

$$\dot{V}_{xn}(t) + \dot{V}_{xa}(t) \leq \xi_x^T(t) \begin{bmatrix} \Psi_{x1} & P_x^T \begin{bmatrix} 0 \\ BK \end{bmatrix} \\ * & -S_x \end{bmatrix} \xi_x(t) + \eta_x(t), \quad (17)$$

where $\xi_x(t) = col\{x(t), \dot{x}(t), x(t-\delta)\}$ and:

$$\begin{aligned}\eta_x(t) = &\; -2\bar{x}^T(t)P_x^T \begin{bmatrix} 0 & (BK)^T \end{bmatrix} e(t-\delta_1(t))\\ \Psi_{x1} = &\; \Theta_x^n + \mu P_x^T \begin{bmatrix} 0 & (BK)^T \end{bmatrix}^T R_{xa}^{-1} \begin{bmatrix} 0 & (BK)^T \end{bmatrix} P_x,\end{aligned}$$

Noting that $e(t-\delta_1(t)) = e(t-\delta) - \int_{t-\delta_1(t)}^{t-\delta}\dot{e}(s)ds$ and using a classical LMI bounding, the following inequality holds for $i = 1,2$:

$$\begin{aligned}\eta_x(t) \leq &\; \bar{x}^T(t)P_x^T \begin{bmatrix} 0 \\ BK \end{bmatrix} (S_{xe}^{-1} + \mu R_{xe}^{-1}) \begin{bmatrix} 0 \\ BK \end{bmatrix}^T P_x\bar{x}(t)\\ &+e^T(t-\delta)S_{xe}e(t-\delta) + \left|\int_{t-\delta_1(t)}^{t-\delta}\dot{e}^T(s)R_{xe}\dot{e}(s)ds\right|\end{aligned} \quad (18)$$

where S_{xe} and R_{xe} are positive definite matrices which represent the presence of the error vector in the state equation. Then, the following inequality holds:

$$\begin{array}{l} \dot{V}_{xn}(t)+\dot{V}_{xa}(t) \leq \xi_x^T(t) \begin{bmatrix} \Psi_{x2}^n & P_x^T \begin{bmatrix} 0 \\ BK \end{bmatrix} - \begin{bmatrix} Y_{x1}^T \\ Y_{x2}^T \end{bmatrix} \\ * & -S_x \end{bmatrix} \xi_x(t) \\ \quad +e^T(t-\delta)S_{xe}e(t-\delta) + |\int_{t-\delta_1(t)}^{t-\delta} \dot{e}^T(s)R_{xe}\dot{e}(s)ds|, \end{array} \quad (19)$$

where

$$\Psi_{x2}^n = \Theta_x^n + P_x^T \begin{bmatrix} 0 \\ BK \end{bmatrix} (S_{xe}^{-1} + \mu R_{xa}^{-1} + \mu R_{xe}^{-1}) \begin{bmatrix} 0 \\ BK \end{bmatrix}^T P_x$$

Concerning the errors dynamics, LMI (15) with $q = e$ yields:

$$\begin{array}{rl} \dot{V}_{en}(t)+\dot{V}_{ea}(t) \leq & \xi_e^T(t) \begin{bmatrix} \Psi_{e1} & P_e^T \begin{bmatrix} 0 \\ LC \end{bmatrix} - Y_e^T \\ * & -S_e \end{bmatrix} \xi_e(t) \\ & -\eta_{e1}^x(t) + \eta_{e1}^e(t) - \eta_{e2}^x(t) + \eta_{e2}^e(t), \end{array} \quad (20)$$

where $\xi_e(t) = col\{e(t), \dot{e}(t), e(t-\delta)\}$ and

$$\begin{array}{rl} \Psi_{e1} = & \Theta_e^n + \mu P_e^T \begin{bmatrix} 0 & (LC)^T \end{bmatrix}^T R_{ea}^{-1} \begin{bmatrix} 0 & (LC) \end{bmatrix} P_e, \\ \eta_{e1}^q(t) = & 2\bar{e}^T(t) \times P_e^T \begin{bmatrix} 0 & (BK)^T \end{bmatrix}^T \int_{t_{1,k}}^{t_{1,\hat{k}}+\varepsilon} \dot{q}(s)ds \\ \eta_{e2}^q(t) = & -2\bar{e}^T(t) \times P_e^T \begin{bmatrix} 0 & (LC)^T \end{bmatrix}^T \int_{t_{2,k'}-\varepsilon}^{t_{2,k'}} \dot{q}(s)ds \end{array}$$

where q represents either x or e. Note that the functions $\eta_{ei}^q(t)$, for $q =$‘x’,‘e’ and $i = 1,2$ correspond to the disturbances due to the synchronization error. Consider now $q =$‘x’ and $i = 1$:

$$-\eta_e^x(t) = -2\bar{e}^T(t)P_e^T \begin{bmatrix} 0 \\ BK \end{bmatrix} \int_{t_{1,k}}^{t_{1,\hat{k}}+\varepsilon} \dot{x}(s)ds$$

Noting that from assumption **A4**, inequality $t_{1,\hat{k}} + \varepsilon - t_{1,k} \leq \bar{\varepsilon} + 2\mu$ holds, then a classical bounding leads to:

$$\begin{array}{rl} \eta_{e1}^x(t) \leq & (\bar{\varepsilon}+2\mu)\bar{e}^T(t)P_e^T \begin{bmatrix} 0 \\ BK \end{bmatrix} R_{x\varepsilon}^{-1} \begin{bmatrix} 0 \\ BK \end{bmatrix}^T P_e\bar{e}(t) \\ & + \int_{t_{1,k}}^{t_{1,\hat{k}}+\varepsilon} \dot{x}^T(s)R_{x\varepsilon}\dot{x}(s)ds. \end{array} \quad (21)$$

By the same method, the following inequalities hold:

$$\begin{array}{rl} \eta_{e1}^e(t) \leq & (\bar{\varepsilon}+2\mu)\bar{e}^T(t)P_e^T \begin{bmatrix} 0 \\ BK \end{bmatrix} R_{e\varepsilon}^{-1} \begin{bmatrix} 0 \\ BK \end{bmatrix}^T P_e\bar{e}(t) \\ & + \int_{t_{1,k}}^{t_{1,\hat{k}}+\varepsilon} \dot{e}^T(s)R_{e\varepsilon}\dot{e}(s)ds. \\ \eta_{e2}^x(t) \leq & \bar{\varepsilon}\bar{e}^T(t)P_e^T \begin{bmatrix} 0 \\ LC \end{bmatrix} R_{x\varepsilon}^{-1} \begin{bmatrix} 0 \\ LC \end{bmatrix}^T P_e\bar{e}(t) \\ & + \int_{t_{2,k'}-\varepsilon}^{t_{2,k'}} \dot{x}^T(s)R_{x\varepsilon}\dot{e}(s)ds. \\ \eta_{e2}^e(t) \leq & \bar{\varepsilon}\bar{e}^T(t)P_e^T \begin{bmatrix} 0 \\ LC \end{bmatrix} R_{e\varepsilon}^{-1} \begin{bmatrix} 0 \\ LC \end{bmatrix}^T P_e\bar{e}(t) \\ & + \int_{t_{2,k'}-\varepsilon}^{t_{2,k'}} \dot{e}^T(s)R_{e\varepsilon}\dot{e}(s)ds. \end{array} \quad (22)$$

Finally, the following inequality holds:

$$\begin{array}{rl} \dot{V}_{en}(t)+\dot{V}_{ea}(t) \leq & \xi_e^T(t) \begin{bmatrix} \Psi_{e2}^n & P_e^T \begin{bmatrix} 0 \\ LC \end{bmatrix} - Y_e^T \\ * & -S_e \end{bmatrix} \xi_e(t) \\ & + \sum_{q=x,e} \int_{t_{1,k}}^{t_{1,\hat{k}}+\varepsilon} \dot{q}^T(s)R_{qp}\dot{q}(s)ds \\ & + \sum_{q=x,e} \int_{t_{2,k'}-\varepsilon}^{t_{2,k'}} \dot{q}^T(s)R_{qp}\dot{q}(s)ds, \end{array} \quad (23)$$

where

$$\begin{array}{l} \Psi_{e2}^n = \Theta_e^n + \mu P_e^T \begin{bmatrix} 0 \\ LC \end{bmatrix} R_{ea}^{-1} \begin{bmatrix} 0 \\ LC \end{bmatrix}^T P_e \\ +2(\bar{\varepsilon}+\mu)P_e^T \begin{bmatrix} 0 \\ BK \end{bmatrix} (R_{x\varepsilon}^{-1} + R_{e\varepsilon}^{-1}) \begin{bmatrix} 0 \\ BK \end{bmatrix}^T P_e \\ +\bar{\varepsilon}P_e^T \begin{bmatrix} 0 \\ LC \end{bmatrix} (R_{x\varepsilon}^{-1} + R_{e\varepsilon}^{-1}) \begin{bmatrix} 0 \\ LC \end{bmatrix}^T P_e. \end{array}$$

Differentiating $V_{x\varepsilon}$, $V_{e\varepsilon}$ and V_{xe} leads to:

$$\begin{array}{rl} \dot{V}_{x\varepsilon}(t) = & 4(\mu+\bar{\varepsilon})\dot{x}^T(t)R_{x\varepsilon}\dot{x}(t) \\ & -2\int_{t-\delta-\mu-\bar{\varepsilon}}^{t-\delta+\mu+\bar{\varepsilon}} \dot{x}^T(s)R_{x\varepsilon}\dot{x}(s)ds \\ \dot{V}_{e\varepsilon}(t) = & 4(\mu+\bar{\varepsilon})\dot{e}^T(t)R_{e\varepsilon}\dot{e}(t) \\ & -2\int_{t-\delta-\mu-\bar{\varepsilon}}^{t-\delta+\mu+\bar{\varepsilon}} \dot{e}^T(s)R_{e\varepsilon}\dot{e}(s)ds, \\ \dot{V}_{xe}(t) = & 2\mu\dot{e}^T(t)R_{xe}\dot{e}(t) \\ & -2\int_{t-\delta-\mu}^{t-\delta+\mu} \dot{e}^T(s)R_{xe}\dot{e}(s)ds. \end{array} \quad (24)$$

Combining (19), (23) and (24) and noting that the sum of the negative integrals in (24) with the integrals in (22) is negative, the following inequality holds:

$$\begin{array}{rl} \dot{V}(t) \leq & \xi_x^T(t) \begin{bmatrix} \Psi_{x2} & P_x^T \begin{bmatrix} 0 \\ BK \end{bmatrix} \\ * & -S_x \end{bmatrix} \xi_x(t) \\ & +\xi_e^T(t) \begin{bmatrix} \Psi_{e2} & P_e^T \begin{bmatrix} 0 \\ LC \end{bmatrix} \\ * & -S_e + S_{xe} \end{bmatrix} \xi_e(t) \end{array} \quad (25)$$

where

$$\begin{array}{rl} \Psi_x = & \Psi_x^n + \begin{bmatrix} 0 & 0 \\ 0 & 4(\mu+\bar{\varepsilon})R_{x\varepsilon} \end{bmatrix}, \\ \Psi_e = & \Psi_e^n + \begin{bmatrix} 0 & 0 \\ 0 & 4(\mu+\bar{\varepsilon})R_{e\varepsilon} + 2\mu R_{xe} \end{bmatrix}, \end{array}$$

Then the Schur complement leads to the LMI’s given in (13) and (14). Then LMI’s from Theorem 1 are satisfied, the system (11) is asymptotically stable.

■

Remark 1: Theorem 1 allows the robust stability of the global remote to be guaranteed system with respect to the synchronization error and for observer and controller gains given in [20]. Since the problems of designing observer and controller gains are dual, the development of constructive LMI’s is not straightforward. Another solution would be to develop conditions in order to design the controller gain for a given observer gain and other conditions to solve the opposite problem.

IV. APPLICATION TO A MOBILE ROBOT

This study is illustrated on the model of a mobile robot (Slave) which can move in one direction. The identification phase gives the following dynamics:

$$\left\{ \begin{array}{rl} \dot{x}(t) = & \begin{bmatrix} 0 & 1 \\ 0 & -11,32 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 11,32 \end{bmatrix} u(t-\delta_1(t)), \\ y(t) = & \begin{bmatrix} 1 & 0 \end{bmatrix} x(t). \end{array} \right. \quad (26)$$

The characteristics of transmission delays in a classical network (between Lens and Lille in France (50km)) allows $h_m = 0,1s$ and $h_M = 0.4s$. Consider now that the bandwidth of the network allows the sampling period as $T = 0.1s$ to be defined. For these values, Theorems 2 and 3 in [20] produce the following gains $L = \begin{bmatrix} -0.9119 & -0.0726 \end{bmatrix}^T$ and $K = \begin{bmatrix} -0.9125 & -0.0801 \end{bmatrix}$. These gains ensure that the remote system is α-stable for $\alpha_x = \alpha_e = 1.05$ in an ideal case ($\varepsilon = 0$ and $\delta_1 = \hat{\delta}_1$. Theorem 1 ensures that the global system (12) is asymptotically stable and robust with respect to any time-varying synchronization error less than $\bar{\varepsilon} = 0.214s$ in (3). Moreover it guarantees asymptotic stability of the global system without the introduction of a buffer in the controller.

V. Concluding remarks

A characteristic feature of this control strategy is to consider that the Master runs in continuous time (*i.e.*, with small computation step) whereas the Slave provides discrete-time measurements. Thus, the observer keeps on providing a continuous estimation of the Slave state, even if the Slave information is not sent continuously.

This paper has proposed a strategy for an observer-based controller for a remote process. No buffering technique was involved, which allows both the Master and the Slave to use the available information as soon as received. Various perturbations were dealt with: (1) jittery, non-symmetric and unpredictable delays (*Internet*); (2) synchronization error and (3) aperiodic sampling (real-time). A remaining assumption in [20] which is that Master and Slave's clocks have to be synchronized is not required anymore.

Even if the proposed asymptotic stability conditions give satisfying results, it would be interesting to investigate in developing exponential stability criteria to establish the effect on synchronization error to the exponential decay rate of the solutions. Moreover new and less conservative results on the stability of systems with sampled-data control recently appear. It would be interesting to apply these technics on the present system.

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