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Practical Robust Adaptive Control: Benchmark Example

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Abstract— **A novel disturbance rejection solution for a benchmark mass-spring-damper (MSD) system is presented. The proposed design, adaptive mixing control (AMC,) is a deterministic multiple model adaptive control (MMAC) approach that mixes candidate controllers by monitoring real-time estimates of an unknown parameter. Monte Carlo simulations are conducted to compare the performance of the AMC scheme with a stochastic robust MMAC (RMMAC) scheme, as well as a nonadaptive mixed-mu compensator.**

I. INTRODUCTION

All real systems are subjected to uncertainty due to unmodeled dynamics, unknown system parameters, disturbances, and process changes. A practical control design, therefore, must be able to maintain performance and stability robustly in the presence of these uncertainties. Two well-known design approaches have been developed for the control of uncertain systems: robust control [1], [2] and robust adaptive control [3]. Although robust control and robust adaptive control are effective for many systems, both techniques are somewhat limited in what can be achieved in terms of performance or robustness. For example, a robust controller, say a mixed-μ compensator, provides quantifiable robust performance guarantees but may fail to meet stringent control requirements in the presence of large parametric uncertainty [4]. Traditional robust adaptive control, on the other hand, avoids sacrificing achievable performance due to increased real uncertainty but performance guarantees have mainly focused on qualitative signal convergence and boundedness properties.

The multiple model adaptive control (MMAC) architecture provides an attractive framework for combining tools from adaptive control with robust non-adaptive schemes. The MMAC architecture comprises two levels of control: (1) a low-level system called the candidate controller set generates finely-tuned (to a subset of the uncertainty space) controls (2) a high-level system called the supervisor composes the control u from the candidate control signals by processing plant input/output data.

One promising MMAC approach is the so-called Robust MMAC (RMMAC) methodology that provides guidelines for designing both the candidate controller set and the supervisor [5]–[7]. The candidate controller set is designed by using state-of-the-art robust mixed-μ synthesis software [8], [9] in order to account simultaneously for robust-stability and

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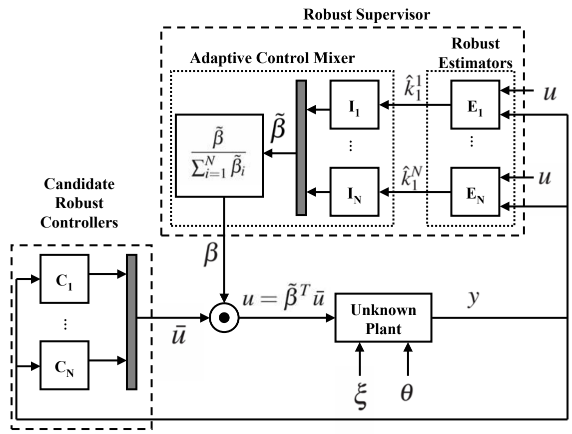


Fig. 1. The adaptive mixing control architecture

-performance. The supervisor is based on a dynamic hypothesis testing scheme that identifies the "nearest probabilistic neighbor." The results of [5], [6] demonstrate that, in the context of a benchmark example, the RMMAC scheme outperforms the best achievable non-adaptive scheme. However, since the supervisor utilizes Kalman filters to drive the hypothesis testing, the RMMAC scheme can suffer from poor performance due to either large initial state estimate error or inaccurate knowledge of the disturbance/noise statistics. Additionally, the complexity of the supervisor may hinder its application because every candidate controller requires a Kalman filter and a post posterior evaluation.

Since the RMMAC methodology separates control from identification, any suitably designed supervisor may be used. In this paper we present a novel supervisor scheme that is used in conjunction with the RMMAC candidate controller set with the aim of mitigating the above issues of RMMAC. The overall design is called adaptive mixing control (AMC) and is shown in Fig. 1. The AMC configuration chosen for this paper uses multiple robust parameter estimators to mix adaptively the candidate control laws. The RMMAC candidate controller set can be used in conjunction with other supervisory schemes, such as the ones from supervisory switching MMAC [10], [11], adaptive control with multiple models [12]–[14], and unfalsified control [15], [16]. A mixing approach was chosen to avoid poor transient performance after a controller switch. The performances of the AMC and RMMAC schemes (as well as a non-adaptive mixed-μ compensator) are compared on the two-cart benchmark problem. Due to space limitations, we assume the reader is familiar with the RMMAC methodology.

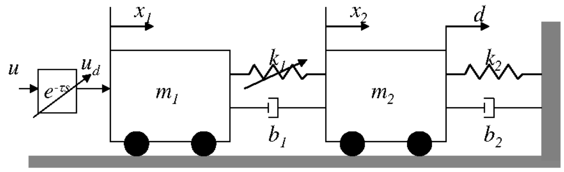


Fig. 2. The two-cart system possesses an unmodeled control delay, uncertain spring constant, process disturbance, and measurement noise.

II. The Two-Cart Model

The two-cart mass-spring-damper (MSD) system is used as a testbed for adaptive control design. The MSD system, shown in Fig. 2, is composed of two masses, m_1 and m_2, interconnected by springs and dashpots. The MSD system possesses important practical characteristics present in many real systems, such as parametric uncertainty, unmodeled dynamics, plant disturbance, and measurement noise.

The plant disturbance $d(t)$ is a low-frequency, stationary stochastic process that acts on m_2, which is generated by shaping a continuous-time white gaussian process as follows

$$d(s) = \frac{a}{s+a}\xi(s) \tag{1}$$

where $\xi(t)$ has zero mean and unit intensity ($\Xi = 1$), and $a > 0$ is the disturbance bandwidth.

The only measurement available is m_2's displacement plus additive white gaussian noise, i.e. $y(t) = x_2(t) + \theta(t)$, where the statistics of the sensor noise $\theta(t)$ are known to be

$$E\{\theta(t)\} = 0, \quad E\{\theta(t)\theta(\tau)\} = 10^{-6}\delta(t-\tau). \tag{2}$$

The state-space representation of the two-cart MSD model is given as

$$\dot{x}(t) = Ax(t) + Bu_d(t) + L\xi(t) \tag{3}$$
$$y(t) = Cx(t) + \theta(t) \tag{4}$$

where the state vector is

$$x = \begin{bmatrix} x_1(t) & x_2(t) & \dot{x}_1(t) & \dot{x}_2(t) & d(t) \end{bmatrix}^T \tag{5}$$

and

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ -\frac{k_1}{m_1} & \frac{k_1}{m_1} & -\frac{b_1}{m_1} & \frac{b_1}{m_1} & 0 \\ \frac{k_1}{m_2} & -\frac{k_1+k_2}{m_2} & \frac{b_1}{m_2} & -\frac{b}{m_2} & \frac{1}{m_2} \\ 0 & 0 & 0 & 0 & -a \end{bmatrix} \tag{6}$$

$$B = \begin{bmatrix} 0 & 0 & \frac{1}{m_1} & 0 & 0 \end{bmatrix}^T, \; L = \begin{bmatrix} 0 & 0 & 0 & 0 & a \end{bmatrix}^T \tag{7}$$
$$C = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \end{bmatrix}. \tag{8}$$
$$\tag{9}$$

The known parameters of the mechanical system are

$$m_1 = m_2 = 1, \quad k_2 = 0.15, \quad b_1 = b_2 = 0.1 \quad a = 0.1 \tag{10}$$

$$m \stackrel{\triangle}{=} m_1 + m_2, \quad b \stackrel{\triangle}{=} b_1 + b_2 \tag{11}$$

and the unknown constant parameter k_1 is restricted to the compact set

$$k_1 \in \Omega = \{k : 0.25 \leq k \leq 1.75\}. \tag{12}$$

The control $u(t)$ is applied to m_1 through a control channel with an unmodeled delay τ. The maximum delay through this channel is known to be 0.05 sec., i.e.

$$\tau \leq 0.05 \quad \text{sec.} \tag{13}$$

The performance variable $z(t) = x_2(t)$ is to be kept small. Therefore, the control objective is to design a control law $u(t)$ so that the effect of the disturbance and noise on $z(t)$ is attenuated.

III. Robust Adaptive Control Design

In this section, the candidate controller set will be described briefly and then the two subsystems of the high-level supervisor are described: (1) robust on-line parameter estimators and (2) adaptive control mixer.

A. Candidate Controller Set

The candidate controllers were systematically developed by the so-called % FNARC method [5], [6] to achieve 70% of the achievable performance if k_1 was known. The % FNARC method makes use of mixed-μ synthesis tools to partition the space Ω and to develop controllers that achieve the robust-performance and -stability requirements. When applied to the MSD system, the % FNARC method produced four candidate controllers

$$\mathscr{C} = \{C_1(s), \quad C_2(s), \quad C_3(s), \quad C_4(s)\} \tag{14}$$

for the partitioned uncertain parameter space

$$\begin{array}{ll} \Omega_1 = [1.02, 1.75], & \Omega_2 = [0.64, 1.02] \\ \Omega_3 = [0.40, 0.64], & \Omega_4 = [0.25, 0.40]. \end{array} \tag{15}$$

where both the candidate controller set and model set share the same index set $J = \{1, \quad 2, \quad 3, \quad 4\}$.

B. Robust Estimator Design

The first step in the robust on-line parameter estimator design is to derive a suitable estimation model. Since there is one unknown parameter we do not estimate the four unknown coefficients of the transfer function

$$G_0(s) = C(sI - A)^{-1}B. \tag{16}$$

A more practical design is to estimate the unknown parameter k_1 directly by exploiting the knowledge of how k_1 enters into the model. The plant output is given as

$$y(t) = G_0(s)e^{-s\tau}u(t) + G_\xi(s)\xi(t) + \theta \tag{17}$$
$$= G_0(s)(1 + \Delta(s))u(t) + G_\xi(s)\xi(t) + \theta \tag{18}$$

where $\Delta = e^{-\tau s} - 1$ is the multiplicative unmodeled dynamics due to the time delay and

$$G_\xi(s) = C(sI - A)^{-1}L \tag{19}$$
$$= \frac{N_\xi(s)}{D_0(s)(s+a)} \tag{20}$$

is the disturbance transfer function. The nominal plant model $G_0(s)$ can be written as

$$\begin{aligned} G_0(s) &= \frac{N_0(s)}{D_0(s)} \\ &= \frac{N_0^K(s)+k_1 N_0^U(s)}{D_0^K(s)+k_1 D_0^U(s)} \end{aligned}$$

to decompose the numerator and denominator into the known and unknown parts

$$N_0^K(s) = b_1 s, \quad N_0^U(s) = 1 \tag{21}$$
$$D_0^K(s) = m_1 m_2 s^4 + (bm_1 + b_1 m_2)s^3 \tag{22}$$
$$+ (m_1 k_2 + b_1 b_2)s^2 + b_1 k_2 s \tag{23}$$
$$D_0^U(s) = ms^2 + b_2 s + k_2. \tag{24}$$

A linear parametric model (LPM) for k_1 is obtained by multiplying both sides of (18) by $D_0(s)$ and then collecting terms of k_1

$$z(t) = k_1\phi(t) + \eta(t) \tag{25}$$

where

$$z(t) = D_0^K(s)F(s)y(t) - N_0^K(s)F(s)u(t) \tag{26}$$
$$\phi(t) = N_0^U(s)F(s)u(t) - D_0^U(s)F(s)y(t) \tag{27}$$
$$\eta(t) = \Delta(s)N_0(s)F(s)u(t) + \frac{N_\xi(s)}{(s+a)}F(s)\xi + D_0(s)F(s)\theta \tag{28}$$
$$F(s) = \frac{6^5}{(s+6)^5}F_\eta(s) \tag{29}$$

where $F_\eta(s)$ is an elliptical bandpass filter whose state-space realization (A_E, B_E, C_E, D_E) was generated by the Matlab® commands

```
Wp = [.3 5]; Ws = [.1 10];
Rp =0.25 ; Rs = 80;
[N, Wn] = ellipord(Wp,Ws,Rp,Rs,'s');
[AE,BE,CE,DE] = ellip(N,Rp,Rs,Wn,'s');
```

The function of F_η is to attenuate the effects of η on estimation. The LPM (25) can be used to design a wide-class of robust adaptive laws [3]. In this study, an adaptive law using the gradient method based on integral cost will be used. The robust adaptive laws are implemented as

$$\dot{\hat{k}}_1^i = \Pr_{\tilde{\Omega}_i}\left(-5\left(r\hat{k}_1^i + q\right)\right) \quad i = 1,2,3,4 \tag{30}$$
$$\dot{r} = -0.1r + \frac{\phi^2}{m^2}, \qquad r(0) = 0 \tag{31}$$
$$\dot{q} = -0.1q - \frac{z\phi}{m^2}, \qquad q(0) = 0 \tag{32}$$

where $\Pr_{\tilde{\Omega}_i}$ is the projection operator (cf. [17, section 4.4]) which restricts $\hat{k}_1^i(t)$ to the projection set $\tilde{\Omega}_i$ given by

$$\begin{aligned} &\tilde{\Omega}_1 \triangleq [0.92, 1.75], \quad \tilde{\Omega}_2 \triangleq [0.58, 1.12], \\ &\tilde{\Omega}_3 \triangleq [0.36, 0.70], \quad \tilde{\Omega}_4 \triangleq [0.25, 0.44]. \end{aligned} \tag{33}$$

and $m^2(t)$ is the dynamic normalization signal

$$m^2 = 1 + n_d \tag{34}$$
$$\dot{n}_d = -0.04 n_d + u_f^2 + y_f^2, \quad n_d(0) = 0 \tag{35}$$
$$u_f = F_\eta(s)u, \quad y_f = F_\eta(s)y. \tag{36}$$

The role of $m(t)$ is to guarantee that $\phi/m, \eta/m \in \mathscr{L}_\infty$ and to slow down adaptation when η becomes large.

C. Adaptive Control Mixer Design

Fig. 1 shows the internal structure of the adaptive control mixer block driven by the four parallel on-line parameter estimates. For simplicity, each estimator differs only by its initialization $\hat{k}_1^i(0)$ and the projected set $\tilde{\Omega}_i$. In this configuration, the estimate $\hat{k}_1^i$ is evaluated by the membership function $I_i : \Omega \to [0,1]$, to generate the pre-normalized weight $\tilde{\beta}_i \triangleq I_i(\hat{k}_1^i)$. The control weighting signal is then generated as

$$\beta = \frac{\tilde{\beta}}{\sum_{i=1}^4 \tilde{\beta}_i}. \tag{37}$$

There are many choices for the shapes of the membership function I_i. For this study, trapezoidal functions are chosen because of their low computation cost. The trapezoidal function is defined as

$$T(x;a,b,c,d) = \begin{cases} 0, & x < a, \quad \text{or} \quad x > d \\ \frac{x-a}{b-a}, & a \le x \le b \\ 1, & b < x < c \\ \frac{d-x}{d-c}, & c \le x \le d \end{cases} \tag{38}$$

and the membership functions are, in turn, defined as

$$I_1(x) \triangleq T(x; 0.92, 1.12, 1.75, 2), \tag{39}$$
$$I_2(x) \triangleq T(x; 0.58, 0.70, 0.92, 1.12), \tag{40}$$
$$I_3(x) \triangleq T(x; 0.36, 0.44, 0.58, 0.70), \tag{41}$$
$$I_4(x) \triangleq T(x; 0, 0.25, 0.36, 0.44). \tag{42}$$

Remark 1: Achievable RMS performance levels can be predicted by using H_2 tools. Fig. 3 shows the expected RMS performances for AMC and RMMAC when $\tau = 0$. Note that near model boundaries, AMCs control mixing reduces the peaks of the achievable RMS performance.

Remark 2: A multiple estimator design was chosen for this benchmark example to reduce transients due to rapid, large parameter changes. Since each estimate remains in a neighborhood of its model, AMC can respond rapidly to sudden changes in the unknown parameter.

Remark 3: The estimators can share the states r, q, and n_d; therefore, if an additional candidate controller C_{N+1} is added, the AMC supervisor requires only one extra state ($\hat{k}_1^{N+1}$.) Contrast this with dynamic hypothesis testing, which requires n (the dimension of the plant) additional states for the Kalman filter and one for the posterior probability evaluator.

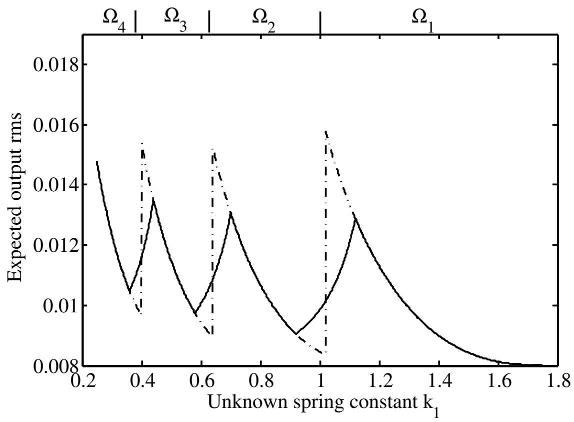


Fig. 3. Expected rms values for AMC (solid) and RMMAC (dash-dot) assuming perfect identification

Remark 4: Extending the above design to systems with multiple unknown parameters can be accomplished by extending the robust estimators to the vector case [17]. Limitations in conventional adaptive control – linear-in-the-parameters model and convexity of Ω requirements – apply to AMC as well. These limitation do not hold for the RMMAC methodology and supervisory control.

IV. Simulation Results

The effectiveness of AMC is demonstrated by Monte Carlo simulations. In [5], [6] the RMMAC scheme demonstrated superior performance over a fixed mixed-μ controller designed for the entire parameter space Ω, called the global non-adaptive robust controller (GNARC,) which represents the best non-adaptive control design. Those results demonstrated the power of combining adaptive and robust control techniques. The focus of this paper is the proposal of an alternative supervisor scheme, one based on robust parameter estimation opposed to dynamic hypothesis testing. The motivation for this approach is to provide an alternative approach that may achieve improved performance in certain applications, specifically applications with uncertain or non-stationary disturbance models.

To compare approaches, the AMC, RMMAC, and GNARC schemes are simulated side-by-side. Both AMC and RMMAC schemes use the same candidate controller set $\{C_i(s)\}$. Refer to [5], [6] for details on the RMMAC and GNARC designs and the construction of the candidate controller set. In all simulations, the plant's initial conditions are given as

$$\begin{aligned} x_0 &= \begin{bmatrix} x_1(0) & x_2(0) & \dot{x}_1(0) & \dot{x}_2(0) & d(0) \end{bmatrix}^T \\ &\sim \big[\mathscr{U}[-0.1,0.1] \quad \mathscr{U}[-0.1,0.1] \\ &\qquad \mathscr{U}[-1,1] \quad \mathscr{U}[-1,1] \quad \mathscr{U}[-0.1,0.1] \big]^T \end{aligned}$$

where $\mathscr{U}[a,b]$ is a uniform distribution on the interval $[a,b]$. The choice of the intervals were motivated by observations of the approximate range of the plant state when the GNARC is used to control the plant. The control channel time-delay for all simulations was chosen as $\tau = 0.01$.

TABLE I

Assumptions Satisfied

	k_1	Transient RMS (%D)	Long-term RMS (%D)
AMC		0.066	0.010
RMMAC	1.385	0.100 (44%)	0.010 (0%)
GNARC		0.040 (−39%)	0.032 (233%)
AMC		0.029	0.010
RMMAC	1.020	0.088 (204%)	0.018 (70%)
GNARC		0.038 (32%)	0.032 (213%)
AMC		0.025	0.011
RMMAC	0.830	0.133 (444%)	0.011 (0%)
GNARC		0.034 (38%)	0.030 (180%)
AMC		0.027	0.014
RMMAC	0.640	0.077 (182%)	0.016 (17%)
GNARC		0.041 (50%)	0.027 (99%)
AMC		0.025	0.012
RMMAC	0.520	0.079 (215%)	0.011 (−4%)
GNARC		0.038 (50%)	0.026 (119%)
AMC		0.023	0.0145
RMMAC	0.400	0.046 (98%)	0.0155 (7%)
GNARC		0.041 (76%)	0.033 (128%)
AMC		0.039	0.014
RMMAC	0.325	0.064 (65%)	0.012 (−13%)
GNARC		0.059 (53%)	0.044 (226%)

TABLE II

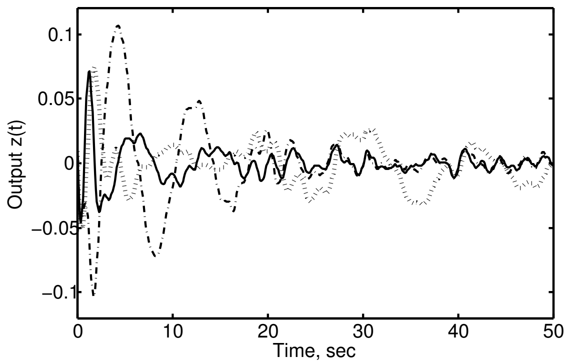
Assumptions Violated

	Violation	Transient RMS (%D)	Long-term RMS (%D)
AMC	Dist. Bandwidth	0.175	0.180
RMMAC	$a = 3$ rad/s	0.384 (120%)	0.376 (109%)
GNARC		0.214 (23%)	0.233 (30%)
AMC	Dist. Power	0.102	0.095
RMMAC	$\Xi = 100$	0.247 (142%)	0.215 (125%)
GNARC		0.280 (173%)	0.288 (202%)
AMC	Time Varying k_1	0.016	0.012
RMMAC	Slow sin	0.081 (394%)	0.014 (12%)
GNARC		0.038 (134%)	0.039 (226%)
AMC	Time Varying k_1	0.269	0.043
RMMAC	Fast sin	0.221 (−18%)	0.149 (248%)
GNARC		0.066 (−76%)	0.053 (23%)
AMC	Time Varying k_1	0.020	0.014
RMMAC	Step	0.031 (55%)	0.013 (−3%)
GNARC		0.037 (82%)	0.036 (165%)

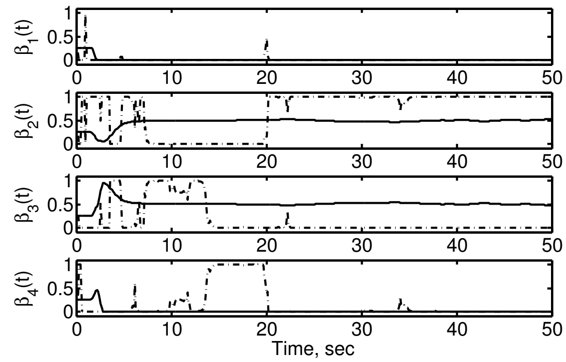
For each experiment, we run five Monte-Carlo simulations, and the averaged results are given in Tables I and II. Two RMS values are given: (1) transient and (2) long-term. Transient RMS is calculated for the first half of the simulation time and is used to judge learning performance. Long-term RMS is calculated over the final half of the simulation time and is used to predict asymptotic performance. Also, the metric "% decrease from AMC" is given and is defined as

$$\%D \triangleq \frac{RMS - RMS_{AMC}}{RMS_{AMC}} 100\% \tag{43}$$

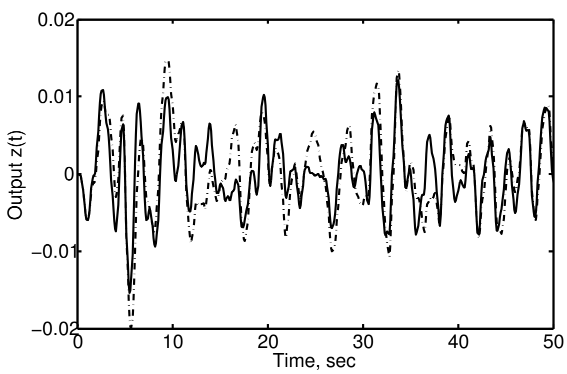
which quantifies AMC's improvement over the other designs. Above, "RMS" is either RMMAC's or GNARC's RMS, depending on the context.



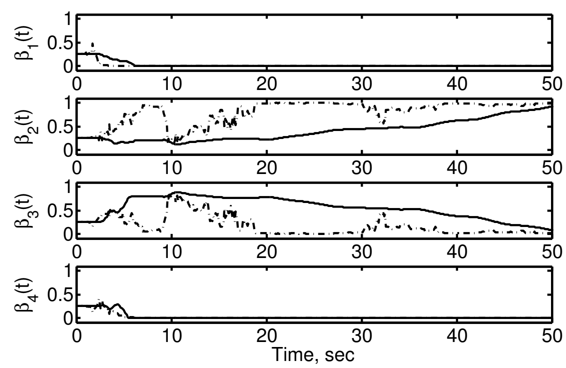
(a) Output, $k_1 = 0.64$, nonzero plant intial conditions



(b) Control weights, $k_1 = 0.64$, nonzero plant intial conditions



(c) Output, $k_1 = 0.64$, zero plant intial conditions



(d) Control weights, $k_1 = 0.64$, zero plant intial conditions

Fig. 4. Experiment 3: Results for difficult identification, $k_1 = 0.64$. PEMMAC (*solid*), RMMAC (*dash-dot*), and GNARC (*dot-dot*)

A. Experiment Set 1: Satisfied Nominal Assumptions

These experiments test the AMC, RMMAC, and GNARC schemes when the plant matches the model (1)-(13). Note that the model still contains the control delay, uncertainty in k_1, process disturbance $d(t)$, and measurement noise $\theta(t)$. Table I shows the results. Both adaptive designs possess superior disturbance rejection performance relative to the GNARC design. The AMC and RMMAC schemes have comparable long-term performance. In general, we see that Fig. 3 does an acceptable job predicting long-term RMS output. Observe the improved long-term performance for the cases when persistent control mixing occurs (i.e., $k_1 \in \{1.020,\ 0.640\ 0.400\}$.)

For illustrative purposes Fig. 4 shows the averaged output and control weight time histories for $k_1 = 0.64$. Fig. 4(a) and 4(b) show the results when the initial conditions are chosen to be nonzero as described. Fig. 4(c) and 4(d) show the same simulations when the plant initial conditions are chosen to be zero. These plots show that initial conditions can have a large effect on RMMACs transient performance. This is not a surprising result because the RMMAC scheme uses Kalman filters to drive the dynamic hypothesis testing scheme, and one would expect that if the Kalman filters initial state estimation error is large then model identification may not be reliable until the state estimation error converges to zero. If the Kalman filters were initialized with true plant states then model identification will converge rapidly to the correct model. In this case, RMMACs transient performance is comparable to (and sometimes better than) AMCs transient performance. These observations motivate techniques to precondition the RMMACs Kalman filters if the initial state estimation error cannot be guaranteed to be small.

B. Experiment Set 2: Violated Model Assumptions

These simulations concern unknown violations of the model assumptions, namely perturbations in the disturbance model and a time-varying parameter. Uncertainty in the disturbance model is a real engineering issue because in many applications the disturbance model is poorly known or time-varying. In [5], [6], the RMMAC/XI design was proposed to mitigate the ill effects of unexpected increased disturbance power, and, as reported in [18] but not implemented here, the modification regained the performance of the known case. The cost of XI modification is increased complexity because twice the number of Kalman filters are required. Since the XI modification adds significant complexity that may limit its application, Experiment 2 is intended to investigate if AMC may be a suitable design approach when the disturbance model is uncertain.

There are two test cases concerning the disturbance model: first a 300% increase in disturbance bandwidth; and then a 100% increase in disturbance power. In both cases, k_1 is chosen to be 0.83, which is the center of model #2. The first two rows of Table II show the results. Comparatively, AMC demonstrates excellent transient and long-term RMS performance.

Next, the constant parameter assumption is violated as k_1 is allowed to vary. The issue of parameter variations is important to consider because it is one of the motivations for adaptive control.

The first five Monte Carlo simulations use the *slow sinusoid waveform* $k_1 = 1 - 0.75sin(0.01t)$. These experiments are simulated for 1,000 seconds and the results are shown in Table II. Both AMC and RMMAC schemes maintain long-term performance levels similar to the constant k_1 case.

Then the adaptive schemes are stressed by rapidly varying k_1. The next set of Monte Carlo simulations uses the *fast sinusoid waveform* $k_1 = 1 - 0.75sin(0.5t)$. These simulations run for 200 seconds. The final five Monte Carlo simulations use the *step waveform*

$$k_1(t) = \begin{cases} 0.325, & t \in [0,100) \bigcup [600,700] \\ 0.520, & t \in [100,200) \bigcup [500,600) \\ 0.830, & t \in [200,300) \bigcup [400,500) \\ 1.385, & t \in [300,400) \end{cases} \quad (44)$$

These simulations run for 700 seconds.

Table II shows the results for the two fast parameter waveforms. Both adaptive schemes perform poorly with the fast sinusoid parameter changes. Although AMC demonstrates poor transient performance, its long-term RMS is lower than GNARCs because, for the most part, AMC keeps C_1 and C_2 in the loop, and k_1 spends most of its time in Ω_1 and Ω_2. The RMMAC scheme, on the other hand, rapidly "switches" among all controllers. For the step waveform, both AMC and RMMAC schemes perform well by achieving long-term RMS values near nominal model levels.

V. Conclusions

A solution to a benchmark example has been developed by using the AMC approach with multiple estimators. The AMC approach mixes candidate controllers by monitoring real-time estimates of the unknown parameter. The motivation for this design was to develop a MMAC scheme that is capable of maintaining satisfactory performance despite uncertainty in the disturbance model while avoiding discontinuous switching among the candidate controller set. Monte Carlo simulations were conducted to compare the AMC scheme with the stochastic MMAC approach RMMAC and a non-adaptive mixed-μ compensator. The simulation results demonstrated that the AMC scheme achieved satisfactory performance despite perturbations in the disturbance model, non-zero plant initial conditions, and certain classes of parameter time variations. The first step in formalizing and analyzing the AMC approach with one estimator is the topic of [19].

VI. Acknowledgments

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