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Game-theoretical Analysis for Choosing the Topology in Multi-loop Control Systems

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Abstract— **The focus of the paper is the review of the control structure selection for a given system. If a framework, derived from game theory is used for the control system design of multi-loop systems, the question arises, how unequally distributed or incomplete information affects the selection of the players strategies. In the field of game theory, information is an essential component. Different information sets lead to different strategy selections provided by the controllers and the belonging control laws. In the present paper, an asymmetric triangular multi-loop control structure was chosen as basis to study and discuss the effects of different information sets to the solution of each game. The solution of the game provides a Pareto-optimal set of controller parameters for the multi-loop control system.**

I. INTRODUCTION

According to [1], the disadvantages of using single-input/single-output (SISO) proportional-integral (PI) and proportional-integral-derivative (PID) controllers for multi-loop systems are the lack of loop interaction consideration and the existence of few powerful tools for its design. In contrast, standard techniques for controller tuning of multi-loop systems assume that the control loops can be adjusted individually by loop decoupling, thereby neglecting the interactions of the different control loops. A detailed literature research leads to the conclusion that there is no good method for simultaneous tuning of several controllers that significally improved performance over that of a single loop controller, see for example [2].

This was the most important motivation of developing a new method, considering several interacting controllers, see [3], [4], and [5]. In this method, a game-theoretic framework is designed, where the control system is viewed as a differential cooperative game. The differential game models a dynamic decision process evolving in time including more than one player. Each player has his own cost function and possibly access to different information, see [6]. In the present paper, the controllers are viewed as players i and the strategies of each player depend on the controller parameters and the control laws of the system. A cost function J_i is assigned to each player, such that the control system design consists in minimizing jointly all indices. This leads to a multi-objective optimization (MOO) problem, see [7], which is solved here using a genetic algorithm.

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In [3], the design method was satisfactory applied to solve controller interactive design in cascade structures and in [5] it was applied to the control of a two-input/two-output (TITO) multi-loop control structure of a reverse osmosis desalination plant. In [3], and [5], constraints for control signals were included during the control system design in a weighted sum manner. However, the selection of the weights is not simple and there is no guarantee, that the constraints will not be violated. Thus, in [8], constraints for the control signals are explicitly formulated and maintained during the control system design. The consideration of the system requirement on robust stability is proposed in [19].

The application of game theory in control theory is not new. According to [6], differential game theory can be viewed as a child of the parents game theory and optimal control theory. For example, a game-theoretic approach to design controllers for safety specifications is given in [9]. [10] uses ideas of game theory, to treat the design process as a two player zero sum game between the controller of a player and the disturbance generated by the actions of the other player. The new approach for control system design, optimizing the performance of multi-loop systems in a game-theoretic framework, proposed in [3], [4], and [8] by the authors, is now used to study the effects of different information distribution based on one basic structure, leading to different control structures for the same system.

II. GAME DESCRIPTION

The basic control structure for the multi-loop system is shown in Fig.1. The TITO system consists of two PI controllers $C_{11} = Q_{11}/P_{11}$, and $C_{22} = Q_{22}/P_{22}$, and three transfer functions $G_{11} = B_{11}/A_{11}$, $G_{21} = B_{21}/A_{21}$, and $G_{22} = B_{22}/A_{22}$ describing the process through polynomial equations. The polynomial description of the PI controllers is

$$C_{PI_i} = \frac{Q_{PI_i}}{P_{PI_i}} = \frac{q_{0i}z + q_{1i}}{z-1}. \quad (1)$$

The chosen control structure is triangular (asymmetric) in such a way that the upper control loop act as a disturbance on the lower control loop. Thus, the control loops of the multi-loop system interact only one-way. The control system design with optimal performance concerning the error convergence is now implemented using the authors approach in the game-theoretic framework.

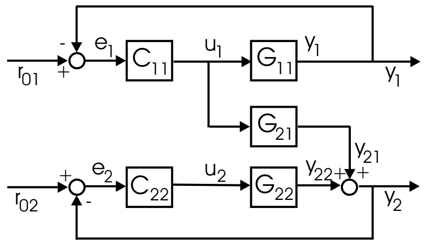

Fig. 1. Triangular control structure of a two-input/two-output system

A. The differential game description

The control system design of the two-input/two-output system in Fig. 1 is considered as a differential game between two players i with $i = 1, 2$ on the time period $[t_0, T]$. The strategies of the players are defined as

$$u_i(t) = \int_{t_0}^{T} c_{ij}(t) e_i(t - \tau) d\tau \tag{2}$$

with

$$\mathcal{L}\{c_{ij}(t)\} = C_{ij}(s) = Q_{ij}(s)/P_{ij}(s). \tag{3}$$

Q_{ij} and P_{ij}, with $j = 1, 2$, are the controller parameters of C_{ij} in Fig.1. The strategies of the players belong to the strategy sets $U_i = \{u_i | u_i \ is \ given \ by \ (2)\}$.
The differential game can now be described as the evolution of the errors e_i with

$$e_i^{(n)} = f_i(e_i^{(n-1)}, ..., \dot{e}_i, u_1, u_2) \tag{4}$$

and initial condition $e_i(t_0) = e_{i0}$, as well as costs J_i with

$$J_i = g_{i0}(e_{iT}). \tag{5}$$

The errors e_i belong to the set $E_i = \{e_i | e_i \ as \ solution \ of (4)\}$. Functions f_i are defined on $f_i : E_i \times U_1 \times U_2 \rightarrow \mathbb{R}^+$ and function g_{i0} on $g_{i0} : E_i \rightarrow \mathbb{R}^+$.

B. Cost function and constraint set up

A typical performance index applied to control problems is the integral square error (ISE) over the complete time interval with $t_0 = 0$ and $T = \infty$, which is used in the following for the costs J_i as

$$J_i = \int_0^{\infty} e_i^2(t) dt. \tag{6}$$

The terminal state e_{iT} as well as the cost functions J_i depend on the choice of u_1 and u_2. In contrast, the players strategies u_1 and u_2 depend on the controller parameters Q_{ij} and P_{ij} as well as the control system structure and the reference signals r_i. Equations of (6) are solved according to [20].
In a cooperative differential game involving two players, each player wants to minimize his cost criterion $J_i = g_{i0}(e_T)$ through the selection of his control strategies u_i.
A minimization of both costs J_i with given reference signals r_i and control structure leads to an optimization of the controller parameters Q_{ij} and P_{ij}.

C. Course of the game

To satisfy all cost functions J_i by tuning the controller parameters Q_{ij} and P_{ij}, a multi-objective optimization (MOO) problem has to be solved. The genetic algorithm of [11], which is already used in previous works, see [3], [4], and [5], is applied as solution method. The advantages of using a genetic algorithm to solve a MOO problem compared to other solution methods are: 1) GA's are Pareto methods, which are able to take care of all conflicting design objectives individually but compromising them concurrently, see [12] and 2) GA's have a multiple search property, see [13].
A range for each controller parameter Q_{ij} and P_{ij} must be specified at the beginning. The values for the starting population as well as all experimental parameter sets must be within this range. The final solution of the GA is a Pareto-optimal set of the controller parameters. A solution is Pareto-optimal, also called non-inferior, in the sense of Pareto, if there is no way of improving any cost of objective without leading to a degradation of at least one another. The chosen parameter sets have to ensure, that the final closed-loop system is stable, which is done during the evaluation of the cost functions J_i. In the present paper, only one system requirement for the different loops is considered during the optimization.
Using GA's to solve MOO problems is not new. If the reader is interested to know more about GA's, he is referred for example to [14] or [15].

D. Solution of the game

With the calculation of the optimal controller parameters Q_{ij} and P_{ij}, the costs J_i for the game are obtained, too. Some costs could be part of a solution concept for cooperative games, named the core, known to be the most attractive solution concept in cooperative game theory. In [16], the core is defined to be the subset of outcomes maximal with respect to the dominance relation; in other words, the subset of outcomes from which there is no tendency to move away - the equilibrium states. The idea of Pareto optimality can be obtained with the core.
Hence, the core collects cost pairs J_1, and J_2 (also called imputations) that are not dominated. All possible cost pairs are imputations where none of the players gets less than he would get if he plays alone.
In general the core is a selection from the set of imputations. For two player games the set of imputations coincides with the core.

E. Information

An essential component in the field of game theory is the information. According to [17], a player can make decisions (choose strategies) only dependent on his available information at that time. In this work, the available information is given through the control system structure.
To be able to classify the present information structure in a game, game theory distinguishes among others between

complete and incomplete information. In a game with complete information, all players know the strategy sets U_i and the costs J_i at any time. There are no private information like unknown strategies u_i for player $\neg i$ or even unknown costs J_i. In contrast, in a game with incomplete information, certain properties as for example the controller parameters of a player i are unknown to the teammates. A further distinction is done with symmetric and asymmetric incomplete information, where all players do not know a parameter, and asymmetric incomplete information, where only some of the teammates do not know this parameter.

Many games are characterized through unequally distributed or incomplete information. This is exactly the case given in the triangular control structure of Fig.1. In this structure, player 1 operates with his own information, that is the control law of the upper control loop and the parameters of controller C_{11}. In contrast, player 2 has information about his own control law and the parameters of controller C_{22} as well as information about player 1, indicated by the information flow from player 1's control signal u_1 over the transfer function of G_{21} to the output signal y_{22} of player 2.

In the following, different possible information structures and thus different control system design games with different control structures are implemented and their effects on the control system behaviour are studied.

III. Games with different information

Five reasonable and different games are described in this section, where the basic control structure of Fig.1 is used as first structure for comparison. The second, third and fourth controller structures are modified in the way that the control system contains different control laws leading to different information sets of the players.

A further differential game is considered here, which is not mentioned before in this paper, where the order of decision making and strategy selection is considered. The strategies of the players are, among others, dependent on the controller parameters Q_{ij} and P_{ij}. In this game, the controller parameters Q_{ij} and P_{ij} are not tuned according to a multiple parameter optimization simultaneously. In contrast, the controller parameters Q_{ij} and P_{ij} of the leader (player i) are optimized first. Dependent on the parameter set Q_{ij} and P_{ij}, the parameters $Q_{\neg ij}$ and $P_{\neg ij}$ of the other player $\neg i$ are optimized. In the previous work of the authors only the simultaneous parameter optimization was studied, but the basic control structure, analyzed in this paper allows the additional consideration of a leader-follower game, which is also known in the literature as stackelberg game.

The forthcoming descriptions of the varying games are restricted through those components which distinguish the games from each other. These components are the information sets of both players containing the error signals $e_i(s)$, with steps $(1/s)$ as references r_i, needed for the calculation of the cost functions J_i formulated in (6).

The applied constraints on the control signals u_i of [8] are neglected (wide ranges are set) in this study to clarify the pure influence of the information structure on the control system performance.

For shortage of space, the polynomials $A_{ij}(s)$, $B_{ij}(s)$, $P_{ij}(s)$, $Q_{ij}(s)$, $e_i(s)$ and $r_i(s)$ are abbreviated in the following as A_{ij}, B_{ij}, P_{ij}, Q_{ij}, e_i, and r_i.

A. Game I - the basic game

The first game (GI) is played based on the given control structure of Fig.1. The information set of player 1 consists of the error signal e_1, dependent on the controller parameter set Q_{11} and P_{11}, and the control law of the upper control loop. In contrast, the information set of player 2 includes his own control law, and his controller parameter set Q_{22} and P_{22} as well as the control law, and the controller parameters Q_{11} and P_{11} of player 1. According to the expressions of Section II-E the information structure of game I is incomplete and asymmetric.

The error functions e_1 and e_2 of game I are formulated as:

$$e_1 = \frac{A_{11}r_1}{A_{11}P_{11} + B_{11}Q_{11}} \tag{7}$$

for the first player, and

$$e_2 = \frac{A_{21}A_{22}(A_{11}P_{11} + B_{11}Q_{11})r_2 - B_{21}Q_{11}A_{11}A_{22}r_1}{A_{21}(A_{11}P_{11} + B_{11}Q_{11})(A_{22}P_{22} + B_{22}Q_{22})} \tag{8}$$

for the second player.

As a result, the cost function J_1 for player I contains only elements of the upper control loop while the cost function J_2 of the second player consists of elements of the lower control loop as well as elements of the upper control loop of player I.

B. Game II - a game with forward information flow

In this game no modifications on the information sets of both players are made. Only an additional controller is inserted to be able to add the error signal e_1 of the first player to the control signal u_2 of the second one as displayed in Fig.2. Hence, player 2 gets information about player 1 (his input) earlier. This makes it possible for him to react earlier on potential disturbances.

The additional controller C_{21}, together with controller C_{22} represent the second player, choosing the strategy u_2. The error functions e_1, and e_2 of game II (GII) are

$$e_1 = \frac{A_{11}r_1}{A_{11}P_{11} + B_{11}Q_{11}} \tag{9}$$

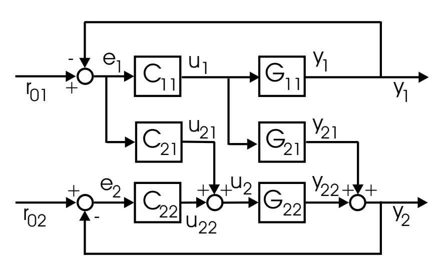

Fig. 2. Control structure of Game II and III

for the first player, and

$$e_2 = \frac{(A_{11}P_{11} + B_{11}Q_{11})A_{21}A_{22}r_2}{(A_{21}A_{22}P_{22} + B_{22}Q_{22}A_{21})(A_{11}P_{11} + B_{11}Q_{11})} - \frac{(B_{21}Q_{11}A_{22} + B_{22}Q_{21}A_{21})A_{11}r_1}{(A_{21}A_{22}P_{22} + B_{22}Q_{22}A_{21})(A_{11}P_{11} + B_{11}Q_{11})} \tag{10}$$

for the second player.
Like in the game before, the cost function J_1 for player 1 contains only elements of the upper control loop while the cost function J_2 of the second player consists of elements included in the lower control loop as well as elements of the upper control loop of player 1. There is no modification on the information sets neither for player 1 nor player 2.

C. Game III - a game with a decoupler

Game III (GIII) plays with the same control structure as game II, displayed in Fig.2 with the distinction that controller C_{21} now acts as a decoupler with

$$C_{21} = -\frac{G_{21}}{G_{11}} = -\frac{B_{21}A_{11}}{A_{21}B_{11}} \tag{11}$$

to reformulate

$$G = \begin{bmatrix} G_{11} & 0 \\ G_{21} & G_{22} \end{bmatrix} \tag{12}$$

as

$$G^* = \begin{bmatrix} 1 & 0 \\ C_{21} & 1 \end{bmatrix} \cdot \begin{bmatrix} G_{11} & 0 \\ G_{21} & G_{22} \end{bmatrix} = \begin{bmatrix} G_1 & 0 \\ 0 & G_2 \end{bmatrix} \tag{13}$$

The intention of a decoupler is to mathematically eliminate the effect of interactions, that means to transform the process matrix into a diagonal matrix. Hence, no parameters has to be tuned for this controller C_{21} and the error signals e_1 and e_2 are formulated like (9) and (10) except that $Q_{21} = -B_{21}A_{11}$ and $P_{21} = A_{21}B_{11}$.
However, this approach is limited through basics of control theory. The decoupling method could translate zeros to poles and unstable decoupling elements may result.

D. Game IV - a game with complete information

The most interesting game in the perspective of game theory is the game (GIV) where a modification of one information set is made. Thus, an extra controller C_{12} is added to the structure, see Fig.3, summating the error signal e_2 of the second player to the control signal u_1 of player 1. Player 1 extends its information set with the control law and the controller parameters Q_{22} and P_{22} of player 2 while the information set for the second player remains unchanged. The information structure is changed resulting in a game with complete information. All three controllers C_{11}, C_{12}, and C_{22} are tuned while minimizing (6) of both players. The corresponding error signals e_1 and e_2, needed to calculate the costs of (6), are

$$e_1 = \frac{TA_{11}r_1 - B_{11}Q_{12}A_{21}A_{22}r_2}{(A_{11}P_{11} + B_{11}Q_{11})T - B_{11}Q_{12}B_{21}Q_{11}A_{22}} \tag{14}$$

and

$$e_2 = \frac{(A_{11}P_{11} + B_{11}Q_{11})A_{21}A_{22}r_2 - B_{21}Q_{11}A_{22}A_{11}r_1}{(A_{11}P_{11} + B_{11}Q_{11})T - B_{11}Q_{12}B_{21}Q_{11}A_{22}} \tag{15}$$

with $T = A_{21}P_{12}A_{22} + B_{21}Q_{12}A_{22} + B_{22}Q_{22}A_{21}$.
Considering the elements of the error functions e_1 and e_2 in (14) and (15), one can conclude that there exists an analogy to the information sets of the players. Both, equations (14), and (15), contain elements (or information) of the other player.

E. Game V - a stackelberg game

According to [6], games, in which one player (called the leader) declares his strategy first and enforces it on the other player (called the follower) is called a stackelberg game.
As in the given basic triangular control structure of Fig.1 there is only one connection from player 1 to player 2. Player 2 has no effect (influence) on the control loop of player 1 at all. This leads to the idea that player 1 is treated as *leader* and player 2 as *follower* in a stackelberg game. The main advantage of this approach: no trade-off has to be met.
Player 1 minimizes his cost J_1 while choosing an optimal parameter set Q_{11} and P_{11}. Under consideration of the resulting strategy u_1, player 2 chooses his parameter set Q_{22} and P_{22} depending on the minimization of his cost and the use of the leader's strategy u_1, including the parameter set Q_{11} and P_{11}. Concerning game V (GV), the corresponding error functions e_1 and e_2 are evaluated as in (7) and (8) but with the attention of the order in decision making (strategy choosing).

IV. Application and Simulation Results

To be able to compare the five different games of Section III, the process transfer functions of a reverse osmosis desalination plant are used in this work with the appropriate basic control structure of Fig.1 and the following transfer functions from [18] with

$$G_{11} = \frac{0.002(0.056s + 1)}{(0.003s^2 + 0.1s + 1)}, \tag{16}$$

$$G_{12} = 0, \tag{17}$$

$$G_{21} = \frac{-0.51(0.35s + 1)}{(0.213s^2 + 0.7s + 1)}, \tag{18}$$

and

$$G_{22} = \frac{-57(0.32s + 1)}{(0.6s^2 + 1.8s + 1)}. \tag{19}$$

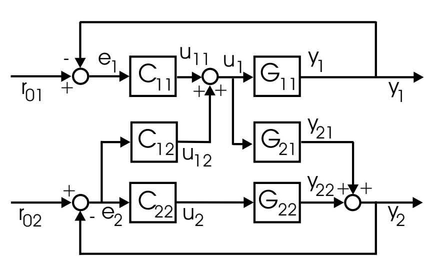

Fig. 3. Control structure of Game IV

The five different games are implemented using the genetic algorithm (GA) of [11]. For the discrete reverse osmosis system, the genetic algorithm operates with 1000 generations and 4 chromosomes, two for each controller. Four subpopulations with 50 individuals each are chosen and the number of cost functions is 2. For the stackelberg game the number of cost functions is 1, but the GA is run twice.

Using the proposed approach of [3], and [8], Pareto-optimal sets as solutions of the different games were obtained, where the parameter vector χ of the controllers is of the form

$$\chi = [q_{01}\ q_{11}\ q_{02}\ q_{12}]\,, \tag{20}$$

depending on the game and providing the chromosomes for the GA.

The range of the parameters are set equally for all games in the range around the controller parameters of [18]. The selection of a parameter set from the Pareto-optimal set is done with no predefined choice in this paper. For the solution of the game, it is primary necessary to satisfy all constraints and belonging to the Pareto-optimal set. The required decision maker, choosing a single parameter set from control theoretic view is still an open question.

Obtained controller parameters are listed in Table I. As mentioned in the description of the games, the presented solutions are part of the core, since this two player game is fully cooperative.

To be able to display the dependency of the systems' outputs and the control structure derived from different information access from the game-theoretic view, the step responses of the outputs are shown and discussed in the following.

In Fig.4, the step responses for the outputs y_1, and y_2 to a change in the setpoint of the permeate flux, from $232\ m^3/h$ to $341\ m^3/h$ are shown. Considering the step responses of the flux in the upper part of Fig.4, three groups of amplitudes are distinguished. Those step responses for games II and III, showing fast rise time but overshoots, and those for games I and V with no overshoot and more slowly rise times. The step response for game IV lies somewhere in between with middle rise time and small overshoot. The step responses of the flux for games II, III, and IV achieve a setpoint convergence after 24 seconds while the step response for game V requires 42 seconds and the one for game I requires 51 seconds for setpoint convergence. The display range of Fig.4 is cutted at 24 seconds to be able to identify the single step responses of the conductivity.

TABLE I

CONTROLLER AND OPTIMIZATION PARAMETERS

	GI	GII	$GIII$	GIV	GV
K_{P11}	424.33	800	696.34	550.44	485
K_{I11}	8376.4	80000	80000	18480	10560
K_{P12}	$-$	$-$	$-$	13.058	$-$
K_{I12}	$-$	$-$	$-$	-5000	$-$
K_{P21}	$-$	-23.174	$-$	$-$	$-$
K_{I21}	$-$	-2298.6	$-$	$-$	$-$
K_{P22}	-89.79	-500	-500	-500	-99.525
K_{I22}	-42154	-43307	-10000	-48750	-10000

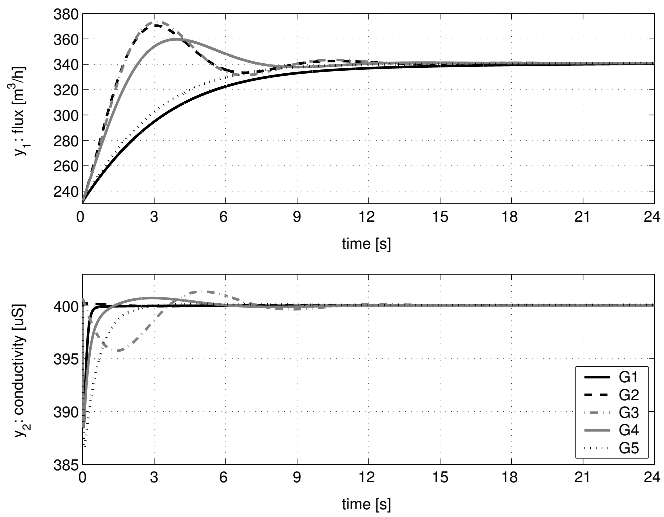

Fig. 4. Step responses of the systems' outputs y_1, and y_2 for all games to a step change in the setpoint of the permeate flux

A step in the setpoint of the permeate flux causes a disturbance in the step responses for the conductivity due to the information distribution during the games: player 2 of all games has information about the controller parameters Q_{11} and P_{11} and the control law of player 1. The disturbance of game V is the largest while it needs 6 seconds to get back to the setpoint. Game I shows a relative large disturbance of about $11\ \mu S$, too, but it is the first of all games in reaching the setpoint after 3 seconds. Those games with a forward of information flow show the smallest disturbances due to the earlier information flow and the possibility to compensate the disturbance earlier. The amplitude response of game IV show both, a relative large disturbance and a comparatively long time of about 14 seconds to reach the setpoint again. The step responses for the outputs y_1 and y_2 to a change in the setpoint of the permeate conductivity from $400\ \mu S$ to $430\ \mu S$ are shown in Fig. 5. Concerning the permeate flux, the amplitude responses for all games except game IV show no disturbance since only player 1 of game IV possesses additional information about player 2. The disturbance is about $0.82\ \mu S$ and needs about 7 seconds to get back to the setpoint. Again, the display range is cutted to identify the step responses of the conductivity.

The step responses of game II and game III concerning the permeate conductivity reach the setpoint already after 0.03 seconds with no overshoot. game IV and game V show amplitude responses with small overshoot and a setpoint convergence of 1, respectively 2 seconds. Game I is the slowest game in setpoint reaching with the largest overshoot.

A. Discussion

Game I and game V are working with the same cost functions only the order of the parameter optimization is different. The amplitude responses of Fig.4 and Fig.5 verify this difference. Game V first satisfies the cost function of the *leader*. Dependent on the result of this optimization the

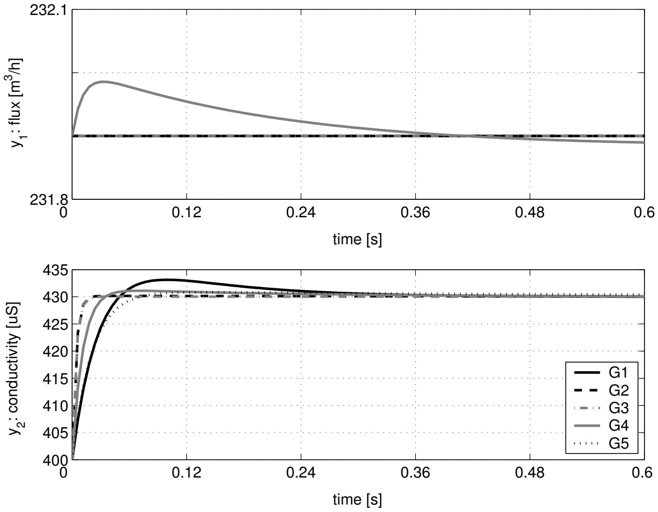

Fig. 5. Step responses of the systems' outputs y_1, and y_2 for all games to a step change in the setpoint of the permeate conductivity

cost function of the *follower* is optimized. Corresponding to this order of optimization, the step response of the flux for game V shows a faster setpoint convergence after a step than the one for game I. In contrast, the step response for the conductivity of game V converges more slowly to the setpoint compared to game I. Game I tries to find a trade-off between both cost functions while game V first satisfies the *leader's* cost function (concerning the flux) and than the one for the *follower* (concerning the conductivity).

Game II and game III consist of the same control structure with an additional controller from e_1 to u_{22}, leading to an earlier information flow. Comparing their step responses, both show similar behaviour. The redundant but earlier information flow from e_1 to u_{22} affects the rise time of the conductivity positively, but the step responses of the flux show a considerable overshoot of about 10% of the operating point.

Game IV, the game with the additional information of the first player, needs only a few time steps concerning the setpoint convergence with marginal overshoot in the step responses of the flux as well as in the step responses of the conductivity.

V. Conclusion

Games with a forward of information flow from player 1 to player 2 show a faster error convergence in the flux (y_1) but larger amplitudes in the conductivity (y_2) compared to game I and game V. An extension of the information flow as in game IV leads to a better trade-off compared to those games with incomplete information.

So far, the first study leads to the preliminary conclusion that game theory is a qualified tool to analyse, optimize and categorize different topologies, derived from different information distribution, in multi-loop systems.

Next step in this work will be the study of the effects if explicit constraints on the control signals are considered during the game. It is necessary, to clarify if this step improves the performance of game I and game V or if there arise other effects on the control system behaviour.

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