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Self-tuning for disturbance transmission decoupling in active vehicle suspensions

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***Abstract*— With active vehicle suspension, one can tailor a vehicles response to load and inertial disturbances without affecting the vehicle response to road disturbances. This decoupling is achieved in [1] and [2] using a filtered combination of measured signals. These filters require exact knowledge of certain vehicle parameters including vehicle mass to achieve the desired decoupling. Here we propose a parameter adaptive version of these filters which does not require knowledge of vehicle parameters.**

I. INTRODUCTION

Traditionally, passive suspension design for vehicles is concerned primarily with the trade off between good vehicle handling and passenger comfort. Good handling prevents the car from excessive rolling and pitching during manoeuvres while passenger comfort isolates vehicle passengers from road disturbances. Thus, there are two main sources of disturbance inputs to a vehicle suspension. One is a high frequency road disturbance due to road roughness that enters the suspension system through tire contact with the road. The other low frequency load disturbance is typically due to load and inertial forces experienced by, or generated, by the car.

With active suspension and appropriate actuators and sensors, one can eliminate the above trade off. Recently a number of vehicle applications have arisen that require fundamentally different performance specifications. For example, vehicle rollover prevention [3] systems and vehicle emulation applications [6] motivate the need for active suspension systems that not only isolate the cabin and its passengers from road disturbances, but also generate variations in the ride height to achieve some control task. Currently, suspension systems designed to mitigate rollover, for example, are designed without reference to the isolation task, raising the possibility of discomfort through noise transmission from the road to cabin.

In [1] and [2] an active suspension control strategy is presented. that uses combined filtered sensor measurements so that the ride height controller is not affected by road disturbance inputs. We refer to this arrangement as the Williams filter. In its most basic form, the Williams filter, for an active suspension, gives a method of specifying the transmission from road disturbance to chassis vertical acceleration, while at the same time responding to load disturbances from the car body. While the Williams filter provides a method of achieving these objectives, it suffers from the problem that the design assumes perfect knowledge of some suspension and vehicle parameters, some of which may be time varying. In this paper, we show how one may readily obtain an adaptive version of the Williams filter which does not require knowledge of the vehicle parameters and which can accommodate changes in vehicle parameters. We also consider a specific adaption scheme and demonstrate its stability using Lyapunov type arguments.

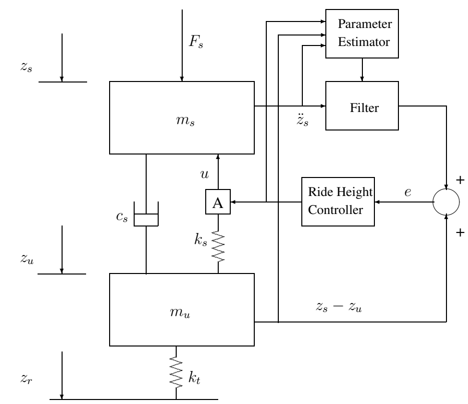

Fig. 1. The quarter-car model

II. DECOUPLING CONTROL STRUCTURE

The quarter car model illustrated in Figure 1 can be described by

$$m_s\ddot{z}_s = -c_s(\dot{z}_s-\dot{z}_u)-k_s(z_s-z_u-u)+F_s \quad (1)$$
$$m_u\ddot{z}_u = c_s(\dot{z}_s-\dot{z}_u)+k_s(z_s-z_u-u)-k_t(z_u-z_r) \quad (2)$$

The control input u is the deflection of the active suspension system which is in series with the suspension spring of stiffness k_s. We will assume, as in [1], that $\ddot{z}_s$ and z_s-z_u can be measured.

We suppose that the passive suspension coefficients k_s and c_s have been chosen so that, with u and F_s zero, the response of the vehicle to the road disturbance z_r is acceptable for passenger comfort. We now wish to design a controller for u to achieve some desirable response of the vehicle to the disturbance F_s without affecting the vehicle response to z_r. To achieve this, we first obtain a signal which is independent of z_r and use this as input to the "ride height controller" which

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TABLE I
NOTATION

z_s	Deflection of sprung mass	m
z_u	Deflection of unsprung mass	m
z_r	Road deflection	m
F_s	Inertial force	N
u	Control input	m
m_s	Sprung mass	kg
m_u	Unsprung mass	kg
c_s	Suspension damping coefficient	Ns/m
k_s	Suspension stiffness coefficient	N/m
k_t	Tyre stiffness coefficient	N/m

generates u. To this end, note that equation (1) can be written as

$$e_1 = k_s u + F_s \tag{3}$$

where

$$e_1 := m_s \ddot{z}_s + c_s(\dot{z}_s - \dot{z}_u) + k_s(z_s - z_u) \tag{4}$$

It follows from (3) that e_1 is completely unaffected by the disturbance z_r. Definition (4) tells us that e_1 can be obtained from the measurement signals $\ddot{z}_s$ and $z_s - z_u$. To avoid differentiation of the measurement $z_s - z_u$, we now consider for controller input any signal e of the form

$$e = F(s)e_1 = m_s F(s)\ddot{z}_s + (c_s s + k_s)F(s)(z_s - z_u)$$

where the scalar transfer function $F(s)$ has at least relative degree one. Recalling (3), we obtain that $e = F(s)(k_s u + F_s)$.

Comment : Note that e is completely unaffected by the disturbance z_r and can be used to achieve multiple control objectives.

If, as in [2], we consider $F(s) = 1/(c_s s + k_s)$ then,

$$e = W(s)\ddot{z}_s + z_s - z_u \tag{5}$$

where

$$W(s) = \frac{\beta}{s+\alpha} \tag{6}$$

with

$$\alpha = k_s/c_s\,, \qquad \beta = m_s/c_s\,. \tag{7}$$

(In [1], Williams considers $F(s) = 1/(m_s s^2 + c_s s + k_s)$). Note that e can be realized from the following state space description:

$$\begin{array}{rcl} \dot{x}_f & = & -\alpha x_f + \beta \ddot{z}_s \\ e & = & x_f + z_s - z_u \end{array} \tag{8}$$

III. THE ADAPTIVE WILLIAMS FILTER

The filters proposed in [1] and [2] can never be implemented due to uncertainty in the suspension and vehicle parameters. Therefore road noise will not be cancelled exactly at the controller. Since the filter (6) or (8) only contains two parameters, α and β, one approach is to replace α and β in (8) with some adaptively tuned estimates $\hat{\alpha}$ and $\hat{\beta}$. This results in the adaptive Williams filter:

$$\boxed{\begin{array}{rcl} \dot{x}_f & = & -\hat{\alpha} x_f + \hat{\beta} \ddot{z}_s \\ e & = & x_f + z_s - z_u \end{array}} \tag{9}$$

To obtain adaptation laws for the estimated parameters, we note that equation (1) can be written as

$$\dot{\eta} + \alpha(\eta - u) + \beta \ddot{z}_s + \gamma = w \tag{10}$$

where $\eta := z_s - z_u$. If we view F_s as consisting of a constant part F_{s_0} and a slowly time-varying part $F_s - F_{s_0}$, then $\gamma = -F_{s_0}/c_s$, $w = (F_s - F_{s_0})/c_s$, $F_s = c_s(w - \gamma)$ and γ is a constant. If w is a low frequency signal, we can mitigate its effect by applying a high pass filter T to the above differential equation to obtain

$$\boxed{\dot{\tilde{\eta}} + \alpha(\tilde{\eta} - \tilde{u}) + \beta \tilde{u}_2 + \tilde{\gamma} = \tilde{w}} \tag{11}$$

where

$$\tilde{\eta} := T(\eta)\,, \quad \tilde{u} := T(u)\,, \quad \tilde{u}_2 := T(\ddot{z}_s)\,, \quad \tilde{\gamma} := T(\gamma)$$

Comment : We now note that (11) describes a simple linear first order system which depends linearly on the constant parameters α, β and $\tilde{\gamma}$; also all the signals $\tilde{\eta}$, $\tilde{u}$ and $\tilde{u}_2$ are available. Thus, we can use some standard parameter estimation schemes to obtain estimates $\hat{\alpha}$, $\hat{\beta}$, and $\hat{\gamma}$ of α, β and $\tilde{\gamma}$.

A. A specific parameter estimator

Here we will use the following simple parameter estimation scheme which is based on those used in the adaptive control literature; see, for example [4].

$$\begin{array}{l} \dot{\hat{\eta}} + \hat{\alpha}(\hat{\eta} - \tilde{u}) + \hat{\beta}\tilde{u}_2 + \hat{\gamma} + l_0 \nu + l_4(\hat{\eta} - \tilde{u})^2 \nu = 0 \\ \nu = \hat{\eta} - \tilde{\eta} \\ \dot{\hat{\alpha}} = l_1 \nu (\hat{\eta} - \tilde{u}) \\ \dot{\hat{\beta}} = l_2 \nu \tilde{u}_2 \\ \dot{\hat{\gamma}} = l_3 \nu \end{array} \tag{12}$$

where

$$l_0 \geq 0\,, \qquad l_1, l_2, l_3, l_4 > 0\,.$$

To demonstrate stability of the above scheme, introduce

$$\delta\alpha := \hat{\alpha} - \alpha\,, \quad \delta\beta := \hat{\beta} - \beta\,, \quad \delta\gamma := \hat{\gamma} - \tilde{\gamma}\,, \qquad \tilde{u}_1 := \hat{\eta} - \tilde{u}\,.$$

Equations (11) and (12) yield

$$\begin{array}{l} \dot{\nu} + (\alpha + l_0)\nu + \delta\alpha \tilde{u}_1 + \delta\beta \tilde{u}_2 + \delta\gamma + l_4 \tilde{u}_1^2 \nu = -\tilde{w} \\ \dot{\delta\alpha} = l_1 \nu \tilde{u}_1 \\ \dot{\delta\beta} = l_2 \nu \tilde{u}_2 \\ \dot{\delta\gamma} = l_3 \nu \end{array}$$

As a candidate Lyapunov function, consider the positive definite function

$$V(\nu, \delta\alpha, \delta\beta, \delta\gamma) = \nu^2 + \delta\alpha^2/l_1 + \delta\beta^2/l_2 + \delta\gamma^2/l_3\,.$$

With $\tilde{w} = 0$,

$$\dot{V} = -2(\alpha + l_0)\nu^2 - 2l_4 \tilde{u}_1^2 \nu^2\,.$$

From this one can guarantee that ν, $\hat{\alpha}$, $\hat{\beta}$, $\hat{\gamma}$ are bounded. In addition ν^2 is an $\mathscr{L}_1$ function. Considering now $V_1(\nu) = \nu^2$, we obtain that

$$\begin{aligned} \dot{V}_1 &= -2(\alpha + l_0)V_1 - 2\nu(\delta\beta\tilde{u}_2 + \delta\gamma) - 2\delta\alpha(\tilde{u}_1 \nu) - 2l_4(\tilde{u}_1 \nu)^2 \\ &\leq -2\nu(\delta\beta\tilde{u}_2 + \delta\gamma) - 2l_4(\tilde{u}_1\nu + \delta\alpha/2l_4)^2 + (\delta\alpha)^2/2l_4 \\ &\leq -2\nu(\delta\beta\tilde{u}_2 + \delta\gamma) + (\delta\alpha)^2/2l_4 \end{aligned}$$

Since ν, $\tilde{u}_2$, $\delta\alpha$, $\delta\beta$, $\delta\gamma$ are bounded, it now follows that $\dot{V}_1$ is bounded above. Since V_1 is also nonnegative and $\mathscr{L}_1$, one can show that V_1 converges to zero. Hence ν converges to zero. To obtain convergence of $(\hat{\alpha},\hat{\beta})$ to (α,β) one needs the signals $\tilde{\eta}-\tilde{u}$ and $\tilde{u}_2$ to be "sufficiently rich".

IV. Simulation Results

For numerical simulation results, we used $m_s = 250$kg, $c_s = 4$kNs/m and $k_s = 12$kN/m from [2]; this results in $\alpha = 0.0625$ and $\beta = 3$. The ride height controller is chosen to be the robust loop shaping controller also obtained from [2]. The initial simulation results are shown in Figures 2 and 3. The parameters converge to their correct values after approximately 5 seconds and the effect of road noise is eliminated from the signal e. At 5 seconds a 100N step change in the down force is applied and at 10 seconds the sprung mass m_s is increased by 25%. The parameter estimates then converge to the new correct values and road noise is eliminated from e. In this experiment the road noise is generated using filtered white noise to approximately replicate actual road noise PSD [5]. If the road noise does not sufficiently excite the parameter estimator, noise can be added to the control signal to ensure parameter convergence. The adaptive control law was found to be relatively insensitive to acceleration sensor noise.

V. CONCLUSIONS

We have presented a decoupling control scheme which adaptively cancels the effect of road noise seen by the ride height controller. Stability of a specific adaptation scheme is proved using Lyapunov arguments. Simulation results demonstrate the effectiveness of the adaptive decoupling controller. Future work will involve developing adaptive decoupling controllers for half- and full-car models, for other suspension arrangements, and their integration into emulation and rollover prevention systems.

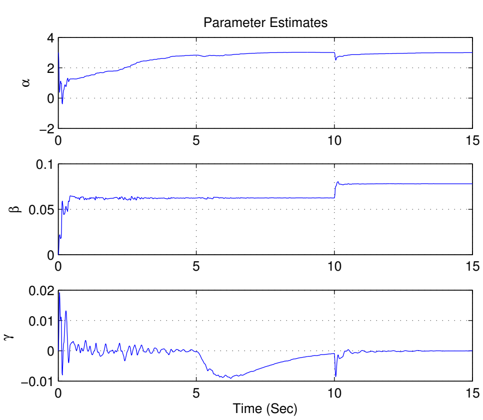

Fig. 2. Parameter Estimates

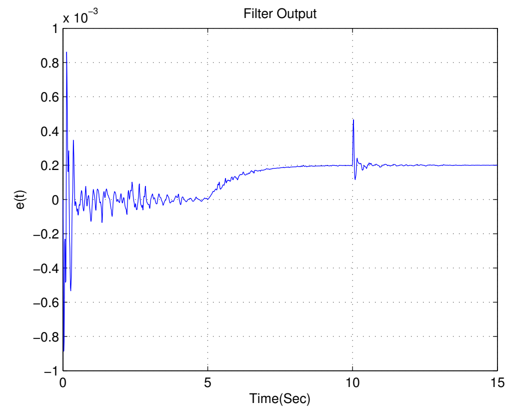

Fig. 3. Filter output e

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