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Adaptive Synchronization for a Class of Complex Delayed Dynamical Networks

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Abstract— **This paper introduces a complex dynamical network with time-varying coupling delays and investigates its locally and globally synchronization. Based on the Lyapunov-Krasovskii functional method, some decentralized adaptive synchronization criteria are derived. The coupling terms are bounded by high-order polynomials, which are applicable to a large class of complex dynamical networks. The effectiveness of the proposed synchronization scheme are illustrated by a numerical example.**

I. INTRODUCTION

Complex networks are currently being studied across many fields of science and engineering, and the following reasons naturally stimulate the present researches. There are many inherent complexity issues that lead to tremendous difficulties in understanding various aspects of such complex networks, including the structural complexity, network evolution, connection diversity, dynamical complexity, node diversity, meta-complication, etc. The topology of a network often affects its function, thus we should consider not only the dynamics of each individual node in the network, but also the topological connectivity of a network in order to better investigate the dynamical behaviors of various complex networks.

Traditionally, a network of complex topology is described by a completely random graph, which was introduced by Paul Erdös and Alfréd Rényi [1]. Recently, Watts and Strogatz introduced the small-world networks [2], which demonstrates the transition from a regular network to a random network. Another significant recent discovery is the observation that a number of complex networks are essentially scale-free [3]-[4], which exhibit power-law distribution. Among all kinds of complex networks, Small-world networks and scale-free networks are most noticeable. Synchronization is a kind of typical collective behaviors and basic motions in nature. Synchronization of coupled oscillators can well explain many natural phenomena. Hereafter, synchronization of all dynamical nodes in complex dynamical networks have been the subject of considerable recent interest within science and technology communities. Most of the existing works have been focused on randomly coupled networks or completely regular networks, such as the continuous-time cellular neural network (CNN) and the discrete-time coupled map lattice (CML), and so on [5]-[7]. However, many real complex networks are neither completely regular or completely random. More recently, a general scale-free dynamical network model with constant, symmetric and irreducible coupling configuration was addressed in [8], synchronization criteria can be easily given in terms of checking simultaneous stability of several low-dimensional dynamic systems. Inspired by this model and proposed technique, a number of synchronization criteria have been put forward [9]-[16]. Furthermore, this technology has also been generalized to deal with networks with time delays [18]-[21].

Inspired by papers [12], [13], [22], [24], [25]. The main objective of the present paper is to derive synchronization criteria for a complex dynamical network which has time-varying delay coupling and the coupling terms are bounded by high-order polynomials. Based on the Lyapunov-Krasovskii functional method, we will derive some local and global adaptive synchronization criteria for the delayed dynamical network.

The rest of this paper is organized as follows. Section 2 gives the problem statement and preliminaries. The local and global adaptive synchronization criteria are presented in section 3. A numerical example is given in section 4, followed by conclusions in section 5.

II. MODEL DESCRIPTION AND PRELIMINARIES

In this section, we will introduce a complex dynamical network model with a time-varying coupling delay and give some preliminaries.

A. A Complex Delayed Dynamical Network Model

Consider an uncertain complex dynamical network with time-varying delay consisting of N identical nonlinear oscillators with uncertain nonlinear diffusive couplings, which is described by

$$\begin{aligned}\dot{x}_i(t) = f(x_i,t) + g_i(x_1,x_2,\cdots,x_N,x_1(t-d(t)),\\ x_2(t-d(t)),\cdots,x_N(t-d(t))) + u_i,\end{aligned} \tag{1}$$

where $x_i = (x_{i1},x_{i2},\cdots,x_{in})^T \in R^n$ is the state vector of the ith node, $f:\Omega\times R^+ \to R^n$ is a smooth nonlinear vector field, $\Omega \subseteq R^n$, $g_i:\Omega\times\ldots\times\Omega \to R^n$ are smooth but unknown nonlinear coupling functions, which indicate the nonlinear interconnections among the current states and the delayed

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states of the ith node and jth node, and $u_i \in R^m$ are the control inputs, the unknown scalar function $d(t)(1 \le i, j \le N)$ denotes any nonnegative, continuous, and bounded time-varying delay satisfying

$$0 \le d(t) \le d < \infty,\ \dot{d}(t) \le d^* < 1, 1 \le i, j \le N, \qquad (2)$$

where d and d^* are positive scalars.

It is noticed that the diffusive coupling configuration ensures that the coupling control terms will vanish on the synchronization manifold: $x_1(t,\phi_1) = x_2(t,\phi_2) = \ldots = x_N(t,\phi_N)$, namely, when the network achieves synchronization, $h_i(s(t), s(t), \cdots, s(t), s(t-d(t)), s(t-d(t)), \cdots, s(t-d(t))) + u_i = 0$, where $s(t)$ is a synchronous solution of the node system $\dot{x} = f(x,t)$, $\phi_i(i = 1,2,\ldots,N)$ are initial conditions and will be described later. Obviously, $S(t) = (s^T(t), s^T(t), \ldots, s^T(t))^T$ is a synchronous solution of the dynamical network (1) since it is a diffusive coupling network. Here, $s(t)$ can be either an equilibrium point, or a nontrivial periodic orbit, or an orbit of a chaotic attractor.

The objective here is to design controller u_i to guide the complex delayed dynamical network (1) to synchronize.

B. Mathematical Preliminaries

In this subsection, we will recall the concepts of network synchronization and network synchronization manifold.

Definition 1([11], [13]): Let $x_i(t,\varphi)(i = 1,2,\ldots,N)$ be a solution of the delayed dynamical network (1) and $C = C([-\tau,0],R^n)$ be the Banach space of continuous functions mapping the interval $[-\tau,0]$ into R^n with the norm $\|\phi\| = sup_{-\tau\le\theta\le0}|\phi(\theta)|$, where $\varphi = (\varphi_1^T, \varphi_2^T, \ldots, \varphi_N^T)^T$, $\varphi_i = \varphi_i(\Gamma) \in C([-d,0],R^n)$ are initial conditions. If there is a nonempty subset $E \subseteq \Omega$, with $\varphi_i \in E(i = 1,2,\ldots,N)$, such that $x_i(t,\varphi) \in \Omega$ for all $t \ge t_0, i = 1,2,\ldots,N$, and

$$\lim_{t\to\infty} \|x_i(t,\varphi) - s(t,s_0)\|_2 = 0, \quad i = 1,2,\ldots,N, \qquad (3)$$

where $\|.\|$ is the Euclidean norm, $s(t,s_0)$ is an asymptotically stable solution of the system $\dot{s} = f(s(t))$ with $s_0 \in \Omega$, then the dynamical networks (1) is said to realize synchronization and $E \times \ldots \times E$ is called the region of synchrony for the dynamical networks (1).

Definition 2([20]): The hyperplane $\mathbb{S} = \{(x_1^T(t), x_2^T(t), \ldots, x_N^T(t))^T \in R^{n\times N} : x_i(t) = x_j(t)\}$ for $i, j = 1,2,\ldots,N$ is said to be the synchronization manifold of the delayed dynamical network (1), where $x_i(t) = (x_{i1}(t), x_{i2}(t), \ldots, x_{in}(t))^T$ for $i = 1,2,\ldots,N$ is the state of node i.

III. Synchronization of Dynamical Networks with Time-Varying Delay

This section discusses the local synchronization and global synchronization of the uncertain delayed complex dynamical network (1). Several network synchronization criteria are given.

A. Local Synchronization

In order to investigate the stability of the synchronization manifold $\mathbb{S}$, let $e_i(t) = x_i(t) - s(t), i = 1,2,\ldots,N$, then we have

$$\begin{aligned}\dot{e}_i(t) = \tilde{f}(x_i,s,t) + \tilde{g}_i(x, x(t-d(t)), s, s(t-d(t))) + u_i,\\ i = 1,2,\ldots,N,\end{aligned} \qquad (4)$$

where $\tilde{f}(x_i,s,t) = f(s(t)+e_i(t)) - f(s(t))$ and $\tilde{g}_i(x, x(t-d(t)), s, s(t-d(t))) = g_i(x_1, x_2, \ldots, x_N, x_1(t-d(t)), x_2(t-d(t)), \ldots, x_N(t-d(t))) - g_i(s, s, \ldots, s, s(t-d(t)), s(t-d(t)), \ldots, s(t-d(t)))$.

It follows from the differentiability of the function f that

$$\begin{aligned}\dot{e}_i(t) = J(t)e_i(t) + \tilde{g}_i(x, x(t-d(t)), s, s(t-d(t))) + u_i,\\ i = 1,2,\ldots,N,\end{aligned} \qquad (5)$$

where $J(t) = Df(s(t),t) \in R^{n\times n}$ is the Jacobian matrix of f evaluated at $s(t)$.

Assumption 1: Suppose that there exist known positive scalars m_{ij}, n_{ij}, $r_{ijp}, s_{ijq}(1 \le i, j \le N, 1 \le p \le m_{ij}, 1 \le q \le n_{ij})$ satisfying $\|\tilde{g}_i(x, x(t-d(t)), s, s(t-d(t)))\|_2 \le \sum_{j=1}^N \sum_{p=1}^{m_{ij}} r_{ijp}\|e_j\|^p + \sum_{j=1}^N \sum_{q=1}^{n_{ij}} s_{ijq}\, \|e_j(t-d(t))\|^q$ for $1 \le i \le N$.

The inequalities in the Assumption 1 can be further rewritten as $\|\tilde{g}_i(x, x(t-d(t)), s, s(t-d(t)))\| \le \sum_{j=1}^N R_{ij}^T Y_{ij}(\|e_j\|) + \sum_{j=1}^N S_{ij}^T Z_{ij}(\|e_j(t-d(t))\|)$, where $R_{ij} = (r_{ij1}, r_{ij2}, \ldots, r_{ijp})^T$, $S_{ij} = (s_{ij1}, s_{ij2}, \ldots, s_{ijq})^T$, $Y_{ij}(.) = (\|e_j\|, \|e_j\|^2, \ldots, \|e_j\|^{m_{ij}})^T$, $Z_{ij}(.) = (\|e_j(t-d(t))\|, \|e_j(t-d(t))\|^2, \ldots, \|e_j(t-d(t))\|^{n_{ij}})^T$.

Obviously, $\{J(t), I\}$ is completely controllable, there must exist control gain matrices $K_i(t)$ and positive matrices $U_i(t)$ and $Q_i(t)$ satisfying $(J(t)+K_i(t))^T U_i(t) + U_i(t)(J(t)+K_i(t)) + \dot{U}_i(t) \le -Q_i(t)$ for $i = 1,2,\ldots,N$.

Remark 1: The coupling terms are bounded by high-order functions and the gains are assumed to be known in the Assumption 1. Therefore, the conditions in the Assumption 1 are weaker comparing with many existent results and applicable to a large class of complex dynamical networks.

Based on the above preliminaries, a locally network synchronization criterion is deduced as follows.

Theorem 1: For complex delayed dynamical network (1), suppose that Assumption 1 hold. Then the synchronization manifold $\mathbb{S}$ is locally asymptotically stable for any continuous time-varying function $d(t)$ satisfying (2) under the adaptive controllers

$$u_i = K_i(t)e_i - \theta_i(t)\frac{\partial V_i^T(e_i)}{\partial e_i} - \vartheta_i(t)\frac{\partial V_i^T(e_i)}{\partial e_i}, \qquad (6)$$

in which $\theta_i(t)$ and $\vartheta_i(t)$ are adaptive parameters with adaptive laws $\dot{\theta}_i = \frac{1}{2}\Gamma_i\|\frac{\partial V_i^T(e_i)}{\partial e_i}\|^2 - \frac{1}{2}\Gamma_i\rho_i\theta_i$, $\dot{\vartheta}_i = \frac{1}{2}\Pi_i\|\frac{\partial V_i^T(e_i)}{\partial e_i}\|^2 - \frac{1}{2}\Pi_i\rho_i\vartheta_i - \frac{1}{4}\Pi_i\rho_i\frac{\tilde{\theta}_i^2}{(\vartheta_i - \tilde{\theta}_i)}$, where Γ_i, Π_i and ρ_i are positive scalars, $\tilde{\theta}_i = \sum_{j=1}^N \frac{1}{2}(\frac{\|R_{ij}\|^2}{4\delta_{ij}} + \frac{\|S_{ij}\|^2}{4\lambda_{ij}(1-d^*)})$, and $h_i = max\{m_{ij}, n_{ij}\}(1 \le j \le N)$, $V_i(e_i) = \sum_{k=1}^{h_i} \frac{1}{k}(e_i^T U_i(t) e_i)^k$.

Proof. Construct a Lyapunov-Krasovskii functional candidate as

$$\tilde{V}(e,\theta,t) = \sum_{i=1}^{N}\{V_i(e_i) + \Gamma_i^{-1}(\theta_i - \tilde{\theta}_i)^2 + \Psi_i^{-1}(\vartheta_i - \tilde{\theta}_i)^2 + \sum_{j=1}^{N}\lambda_{ij}\int_{t-d(t)}^{t}\|e_j(s)\|^{2k}ds\}, \tag{7}$$

where λ_{ij} are sufficiently small positive scalars.

The time derivation of $\dot{\tilde{V}}$ along the solutions of the closed-loop systems is

$$\begin{aligned}\dot{\tilde{V}}(e,\theta,t) = &\sum_{i=1}^{N}\sum_{k=1}^{h_i}(e_i^T U_i(t)e_i)^{k-1}e_i^T((J(t)+K_i(t))^T U_i(t) \\ &+ U_i(t)(J(t)+K_i(t)) + \dot{U}_i(t))e_i \\ &+ \sum_{i=1}^{N}(\frac{\partial V_i(e_i)}{\partial e_i}\tilde{g}_i(x, x(t-d(t)), s, s(t-d(t)))) \\ &+ \sum_{i=1}^{N}\sum_{j=1}^{N}\lambda_{ij}(\|e_j\|^{2k} - (1-\dot{d}(t))\|e_j(t-d(t))\|^{2k}) \\ &+ \sum_{i=1}^{N}(2\Gamma_i^{-1}(\theta_i - \tilde{\theta}_i)\dot{\theta}_i + 2\Psi_i^{-1}(\vartheta_i - \tilde{\theta}_i)\dot{\vartheta}_i \\ &- \vartheta_i(t)\|\frac{\partial V_i^T(e_i)}{\partial e_i}\|^2 - \theta_i(t)\|\frac{\partial V_i^T(e_i)}{\partial e_i}\|^2).\end{aligned} \tag{8}$$

Based on Assumption 1, it is obvious that the following inequality is satisfied.

$$\begin{aligned}&\frac{\partial V_i(e_i)}{\partial e_i}\tilde{g}_i(x, x(t-d(t)), s, s(t-d(t))) \\ &\leq \sum_{j=1}^{N}R_{ij}^T Y_{ij}(\|e_j\|)\|\frac{\partial V_i(e_i)^T}{\partial e_i}\| \\ &\quad + \sum_{j=1}^{N}S_{ij}^T Z_{ij}(\|e_j(t-d(t))\|)\|\frac{\partial V_i(e_i)^T}{\partial e_i}\| \\ &\leq \sum_{j=1}^{N}(\frac{\|R_{ij}\|^2}{4\delta_{ij}}\|\frac{\partial V_i(e_i)^T}{\partial e_i}\|^2 + \delta_{ij}\|Y_{ij}(\|e_j\|)\|^2) \\ &\quad + \sum_{j=1}^{N}(\frac{\|S_{ij}\|^2}{4\lambda_{ij}(1-d^*)}\|\frac{\partial V_i(e_i)^T}{\partial e_i}\|^2 \\ &\quad + \lambda_{ij}(1-d^*)\|Z_{ij}(\|e_j(t-d(t))\|)\|^2) \\ &\leq 2\tilde{\theta}_i\|\frac{\partial V_i(e_i^T)}{\partial e_i}\|^2 + \sum_{j=1}^{N}(\sum_{k=1}^{m_{ij}}\delta_{ij}\|e_j\|^{2k} \\ &\quad + \sum_{k=1}^{n_{ij}}\lambda_{ij}(1-d^*)\|e_j(t-d(t))\|^{2k}).\end{aligned} \tag{9}$$

Substituting (9) into (8), we have

$$\begin{aligned}\dot{\tilde{V}}(e,\theta,t) \leq &\sum_{i=1}^{N}(\sum_{k=1}^{h_i}(e_i^T U_i(t)e_i)^{k-1}(-e_i^T Q_i(t)e_i) \\ &+ \sum_{j=1}^{N}(\sum_{k=1}^{n_{ij}}\lambda_{ij}\|e_j\|^{2k} + \sum_{k=1}^{m_{ij}}\delta_{ij}\|e_j\|^{2k})) \\ &+ \sum_{i=1}^{N}(2\Gamma_i^{-1}(\theta_i - \tilde{\theta}_i)\dot{\theta}_i + 2\Psi_i^{-1}(\vartheta_i - \tilde{\theta}_i)\dot{\vartheta}_i) \\ &- \sum_{i=1}^{N}(\theta_i - \tilde{\theta}_i)\|\frac{\partial V_i^T(e_i)}{\partial e_i}\|^2 - \sum_{i=1}^{N}(\vartheta_i - \tilde{\theta}_i)\|\frac{\partial V_i^T(e_i)}{\partial e_i}\|^2 \\ \leq &\sum_{i=1}^{N}(-\sum_{k=1}^{h_i}(e_i^T U_i(t)e_i)^{k-1}(e_i^T Q_i(t)e_i) + (\theta_i - \tilde{\theta}_i)(\rho_i\theta) \\ &+ (\vartheta_i - \tilde{\theta}_i)(-\rho_i\theta - \frac{1}{2}\rho_i\frac{\tilde{\theta}_i^{\,2}}{(\vartheta_i - \tilde{\theta}_i)}) \\ &+ \sum_{j=1}^{N}(\sum_{k=1}^{n_{ij}}\lambda_{ij}\|e_j\|^{2k} + \sum_{k=1}^{m_{ij}}\delta_{ij}\|e_j\|^{2k})),\end{aligned} \tag{10}$$

if we choose parameters $\lambda_j = \max_i\{\lambda_{ij}\}$ and $\delta_j = \max_i\{\delta_{ij}\}$ for $1 \leq i \leq N$. The following inequalities hold:

$$\begin{aligned}&\sum_{i=1}^{N}(-\sum_{k=1}^{h_i}(e_i^T U_i(t)e_i)^{k-1}(e_i^T Q_i(t)e_i) \\ &\quad + \sum_{j=1}^{N}(\sum_{k=1}^{n_{ij}}\lambda_{ij}\|e_j\|^{2k} + \sum_{k=1}^{m_{ij}}\delta_{ij}\|e_j\|^{2k})) \\ &\leq \sum_{i=1}^{N}(-\sum_{k=1}^{h_i}(e_i^T U_i(t)e_i)^{k-1}(e_i^T Q_i(t)e_i) \\ &\quad + \sum_{j=1}^{N}(\sum_{k=1}^{h_j}\lambda_j\|e_j\|^{2k} + \sum_{k=1}^{h_j}\delta_j\|e_j\|^{2k})) \\ &= \sum_{i=1}^{N}\sum_{k=1}^{h_i}(-(e_i^T U_i(t)e_i)^{k-1}(e_i^T Q_i(t)e_i) \\ &\quad + N\lambda_i\|e_i\|^{2k} + N\delta_i\|e_i\|^{2k}),\end{aligned}$$

where $\lambda_j = 0(n_{ij} < j \leq h_j)$ and $\delta_j = 0(m_{ij} < j \leq h_j)$. Considering the following equality:

$$\begin{aligned}&(\theta_i - \tilde{\theta}_i)(-\rho_i\theta_i) + (\vartheta_i - \tilde{\theta}_i)(-\rho_i\vartheta_i - \frac{1}{2}\rho_i\frac{\tilde{\theta}_i^{\,2}}{(\vartheta_i - \tilde{\theta}_i)}) \\ &= -\rho_i(\theta_i - \frac{1}{2}\tilde{\theta}_i)^2 + \frac{1}{4}\rho_i\tilde{\theta}_i^2 - \rho_i(\vartheta_i - \frac{1}{2}\tilde{\theta}_i^2)^2 - \frac{1}{4}\rho_i\tilde{\theta}_i^2 \\ &= -\rho_i(\theta_i - \frac{1}{2}\tilde{\theta}_i)^2 - \rho_i(\vartheta_i - \frac{1}{2}\tilde{\theta}_i^2)^2,\end{aligned}$$

then we have

$$\begin{aligned}\dot{\tilde{V}}(e,\theta,t) \leq &\sum_{i=1}^{N}\sum_{k=1}^{h_i}(-\lambda_{min}(U_i(t))^{k-1}\lambda_{min}(Q_i(t))\|e_i\|^{2k} \\ &+ N\lambda_i\|e_i\|^{2k} + N\delta_i\|e_i\|^{2k}),\end{aligned}$$

if we choose parameters λ_{ij} and δ_{ij} small enough to render that the following inequality holds:

$$-\lambda_{min}^{k-1}(U_i(t))\lambda_{min}(Q_i(t)) + N\lambda_i + N\delta_i = -\Delta_i < 0,$$

where Δ_i are positive scalars. Furthermore, we have

$$\dot{\tilde{V}}(e,\theta,t) \leq \sum_{i=1}^{N}(-h_i\Delta_i\|e_i\|^{2k}).$$

Based on the Lyapunov-Krasovskii stability theorem and Barbalat Lemma, the proposed decentralized adaptive state feedback controllers can ensure that the synchronization manifold $\mathbb{S}$ is locally asymptotically stable for any continuous time-varying function $d(t)$ satisfying (2). This completes the proof of the Theorem 1.

Remark 2: If delay $d(t)=0$ and the coupling term bounded by first-order polynomials, namely, $\tilde{g}_i(x,x(t-d(t)),s,s(t-d(t))) \leq \sum_{j=1}^{N}\gamma_{ij}\|e_j\|$, where γ_{ij} are known but nonnegative constants for $i,j=1,2,\ldots,N$, then it is not difficult for us to obtain the Theorem 1 in [12].

In the following subsection, we discuss the global synchronization case.

B. Global Synchronization

In the previous section, we have given a sufficient condition to guarantee that the trajectory of the dynamical network (1) will come to the locally stable synchronization manifold $\mathbb{S}$. However, it is more interesting to study the global synchronization of the dynamical network (1).

Rewrite node dynamics $\dot{x}_i(t)=f(x_i(t),t)$ as $\dot{x}_i(t)=Bx_i(t)+h(x_i(t),t)$, where $B\in R^{n\times n}$ is a constant matrix and $h:\Omega\to R^n$ is a continuous differentiable nonlinear function. Thus, the dynamical networks (1) is described by

$$\begin{aligned}\dot{x}_i(t) = {} & Bx_i(t)+h(x_i(t),t)+g_i(x_1,x_2,\cdots,x_N,x_1(t-d(t)),\\ & x_2(t-d(t)),\cdots,x_N(t-d(t)))+u_i,\ i=1,2,\ldots,N,\end{aligned} \tag{11}$$

let $e_i(t)=x_i(t)-s(t), i=1,2,\ldots,N$, then one can get the following error system

$$\begin{aligned}\dot{e}_i(t) = {} & Be_i(t)+\tilde{h}(x_i(t),s(t),t)\\ & +\tilde{g}_i(x,x(t-d(t)),s,s(t-d(t)))+u_i,\ i=1,2,\ldots,N,\end{aligned} \tag{12}$$

where $\tilde{h}(x_i(t),s(t),t)=h(x_i(t),t)-h(s(t),t)$, $\tilde{g}_i(x,x(t-d(t)),s,s(t-d(t)))=g_i(x_1,x_2,\ldots,x_N,x_1(t-d(t)),x_2(t-d(t)),\ldots,x_N(t-d(t)))-g_i(s,s,\ldots,s,s(t-d(t)),s(t-d(t)),\ldots,s(t-d(t)))$. Based on the fact that $s(t)$ is bounded, we use the differential mean value theorem, along with (12), we have the following matrix equation:

$$\begin{aligned}\dot{e}_i(t) = {} & (B+\Sigma(t))e_i(t)+\tilde{g}_i(x,x(t-d(t)),s,s(t-d(t)))+u_i,\\ & i=1,2,\ldots,N,\end{aligned} \tag{13}$$

where $\Sigma(t)=diag\{\Sigma_1(t),\Sigma_2(t),\ldots,\Sigma_n(t)\}\in R^{n\times n}$ is the derivative of $h(x_i(t),t)$ at $\theta(t)$, $s(t)\leq\theta(t)\leq x_i(t)$.

Since $\{(B+\Sigma(t)),I\}$ is completely controllable, there must exist control gain matrices $K_i(t)$ and positive matrices $U_i(t)$ and $Q_i(t)$ satisfying $((B+\Sigma(t))+K_i(t))^TU_i(t)+U_i(t)((B+\Sigma(t))+K_i(t))+\dot{U}_i(t)\leq -Q_i(t)$ for $i=1,2,\ldots,N$.

Then one can get the following global network synchronization criterion.

Theorem 2: For complex delayed dynamical network (1), suppose that Assumption 1 hold. Then the synchronization manifold $\mathbb{S}$ is globally asymptotically stable for any continuous time-varying function $d(t)$ satisfying (2) under the adaptive controllers

$$u_i = K_i(t)e_i-\theta_i(t)\frac{\partial V_i^T(e_i)}{\partial e_i}-\vartheta_i(t)\frac{\partial V_i^T(e_i)}{\partial e_i}, \tag{14}$$

in which $\theta_i(t)$ and $\vartheta_i(t)$ are adaptive parameters with adaptive laws $\dot{\theta}_i=\frac{1}{2}\Gamma_i\|\frac{\partial V_i^T(e_i)}{\partial e_i}\|^2-\frac{1}{2}\Gamma_i\rho_i\theta_i$, $\dot{\vartheta}_i=\frac{1}{2}\Pi_i\|\frac{\partial V_i^T(e_i)}{\partial e_i}\|^2-\frac{1}{2}\Pi_i\rho_i\vartheta_i-\frac{1}{4}\Pi_i\rho_i\frac{\tilde{\theta}_i^2}{(\vartheta_i-\tilde{\theta}_i)}$, $h_i=max\{m_{ij},n_{ij}\}(1\leq j\leq N)$, $V_i(e_i)=\sum_{k=1}^{h_i}\frac{1}{k}(e_i^TU_i(t)e_i)^k$, where Γ_i,Π_i and ρ_i are positive scalars, $\tilde{\theta}_i=\sum_{j=1}^{N}\frac{1}{2}(\frac{\|R_{ij}\|^2}{4\delta_{ij}}+\frac{\|S_{ij}\|^2}{4\lambda_{ij}(1-d^*)})$.

Proof: The proof is very similar to that of the Theorem 1, so is omitted here.

Assumption 2: Suppose that there exist known positive scalars m_{ij},n_{ij}, $r_{ijp},s_{ijq}(1\leq i,j\leq N, 1\leq p\leq m_{ij}, 1\leq q\leq n_{ij})$ satisfying $\|\tilde{g}_i(x,x(t-\tau),s,s(t-\tau))\|_2\leq \sum_{j=1}^{N}\sum_{p=1}^{m_{ij}}r_{ijp}\|e_j\|^p+\sum_{j=1}^{N}\sum_{q=1}^{n_{ij}}s_{ijq}\|e_j(t-\tau)\|^q$ for $1\leq i\leq N$, where time-varying delay $d(t)=\tau$, τ is a positive scale.

The inequalities in the Assumption 2 can be further rewritten as $\|\tilde{g}_i(x,x(t-\tau),s,s(t-\tau))\|\leq\sum_{j=1}^{N}R_{ij}^T\ \ Y_{ij}(\|e_j\|)+\sum_{j=1}^{N}S_{ij}^TZ_{ij}(\|e_j(t-\tau)\|)$, where $R_{ij}=(r_{ij1},r_{ij2},\ldots,r_{ijp})^T$, $S_{ij}=(s_{ij1},s_{ij2},\ldots,s_{ijq})^T$, $Y_{ij}(.)=(\|e_j\|,\|e_j\|^2,\ldots,\|e_j\|^{m_{ij}})^T$, $Z_{ij}(.)=(\|e_j(t-\tau)\|,\|e_j(t-\tau)\|^2,\ldots,\|e_j(t-\tau)\|^{n_{ij}})^T$.

Obviously, $\{(B+\Sigma(t)),I\}$ is completely controllable, there must exist control gain matrices $K_i(t)$ and positive matrices $U_i(t)$ and $Q_i(t)$ satisfying $((B+\Sigma(t))+K_i(t))^TU_i(t)+U_i(t)((B+\Sigma(t))+K_i(t))+\dot{U}_i(t)\leq -Q_i(t)$ for $i=1,2,\ldots,N$.

Theorem 3: For complex delayed dynamical network (1), suppose that Assumption 2 hold. Then the synchronization manifold $\mathbb{S}$ is globally asymptotically stable for any constant coupling delay $d(t)=\tau$ under the adaptive controllers

$$u_i = K_i(t)e_i-\theta_i(t)\frac{\partial V_i^T(e_i)}{\partial e_i}-\vartheta_i(t)\frac{\partial V_i^T(e_i)}{\partial e_i}, \tag{15}$$

in which $\theta_i(t)$ and $\vartheta_i(t)$ are adaptive parameters with adaptive laws $\dot{\theta}_i=\frac{1}{2}\Gamma_i\|\frac{\partial V_i^T(e_i)}{\partial e_i}\|^2-\frac{1}{2}\Gamma_i\rho_i\theta_i$, $\dot{\vartheta}_i=\frac{1}{2}\Pi_i\|\frac{\partial V_i^T(e_i)}{\partial e_i}\|^2-\frac{1}{2}\Pi_i\rho_i\vartheta_i-\frac{1}{4}\Pi_i\rho_i\frac{\tilde{\theta}_i^2}{(\vartheta_i-\tilde{\theta}_i)}$, $h_i=max\{m_{ij},n_{ij}\}(1\leq j\leq N)$, $V_i(e_i)=\sum_{k=1}^{h_i}\frac{1}{k}(e_i^TU_i(t)e_i)^k$, where Γ_i,Π_i and ρ_i are positive scalars, $\tilde{\theta}_i=\sum_{j=1}^{N}\frac{1}{2}(\frac{\|R_{ij}\|^2}{4\delta_{ij}}+\frac{\|S_{ij}\|^2}{4\lambda_{ij}})$.

Proof. Construct a Lyapunov-Krasovskii functional candidate as

$$\begin{aligned}\tilde{V}(e,\theta,t) = {} & \sum_{i=1}^{N}\{V_i(e_i)+\Gamma_i^{-1}(\theta_i-\tilde{\theta}_i)^2+\Psi_i^{-1}(\vartheta_i-\tilde{\theta}_i)^2\\ & +\sum_{j=1}^{N}\lambda_{ij}\int_{t-\tau}^{t}\|e_j(s)\|^{2k}ds\},\end{aligned} \tag{16}$$

where λ_{ij} are sufficiently small positive scalars.

By taking the time derivative of $\dot{\tilde{V}}$ along the trajectories of the closed-loop system which gotten by substituting (15) into (13), we have

$$\begin{aligned}\dot{\hat{V}}(e,\theta,t) = &\sum_{i=1}^{N}\sum_{k=1}^{h_i}(e_i^T U_i(t)e_i)^{k-1}e_i^T(((B+\Sigma(t))\\&+K_i(t))^T U_i(t)+U_i(t)((B+\Sigma(t))+K_i(t))+\dot{U}_i(t))e_i\\&+\sum_{i=1}^{N}(\frac{\partial V_i(e_i)}{\partial e_i}\tilde{g}_i(x,x(t-\tau),s,s(t-\tau)))\\&+\sum_{i=1}^{N}\sum_{j=1}^{N}\lambda_{ij}(\|e_j\|^{2k}-\|e_j(t-\tau)\|^{2k})\\&+\sum_{i=1}^{N}(2\Gamma_i^{-1}(\theta_i-\tilde{\theta}_i)\dot{\theta}_i+2\Psi_i^{-1}(\vartheta_i-\tilde{\theta}_i)\dot{\vartheta}_i\\&-\vartheta_i(t)\|\frac{\partial V_i^T(e_i)}{\partial e_i}\|^2-\theta_i(t)\|\frac{\partial V_i^T(e_i)}{\partial e_i}\|^2).\end{aligned} \tag{17}$$

$$\begin{aligned}&\frac{\partial V_i(e_i)}{\partial e_i}\tilde{g}_i(x,x(t-\tau),s,s(t-\tau))\\&\leq\sum_{j=1}^{N}R_{ij}^T Y_{ij}(\|e_j\|)\|\frac{\partial V_i(e_i)^T}{\partial e_i}\|\\&\quad+\sum_{j=1}^{N}S_{ij}^T Z_{ij}(\|e_j(t-\tau)\|)\|\frac{\partial V_i(e_i)^T}{\partial e_i}\|\\&\leq\sum_{j=1}^{N}(\frac{\|R_{ij}^2\|^2}{4\delta_{ij}}\|\frac{\partial V_i(e_i)^T}{\partial e_i}\|^2+\delta_{ij}\|Y_{ij}(\|e_j\|)\|^2)\\&\quad+\sum_{j=1}^{N}(\frac{\|S_{ij}\|^2}{4\lambda_{ij}}\|\frac{\partial V_i(e_i)^T}{\partial e_i}\|^2+\lambda_{ij}\|Z_{ij}(\|e_j(t-\tau)\|)\|^2)\\&\leq 2\tilde{\theta}_i\|\frac{\partial V_i(e_i^T)}{\partial e_i}\|^2\\&\quad+\sum_{j=1}^{N}(\sum_{k=1}^{m_{ij}}\delta_{ij}\|e_j\|^{2k}+\sum_{k=1}^{n_{ij}}\lambda_{ij}\|e_j(t-\tau)\|^{2k}).\end{aligned} \tag{18}$$

Substituting (9) into (8), we have

$$\begin{aligned}\dot{\hat{V}}(e,\theta,t)\leq&\sum_{i=1}^{N}(\sum_{k=1}^{h_i}(e_i^T U_i(t)e_i)^{k-1}(-e_i^T Q_i(t)e_i)\\&+\sum_{j=1}^{N}(\sum_{k=1}^{n_{ij}}\lambda_{ij}\|e_j\|^{2k}+\sum_{k=1}^{m_{ij}}\delta_{ij}\|e_j\|^{2k}))\\&+\sum_{i=1}^{N}(2\Gamma_i^{-1}(\theta_i-\tilde{\theta}_i)\dot{\theta}_i+2\Psi_i^{-1}(\vartheta_i-\tilde{\theta}_i)\dot{\vartheta}_i)\\&-\sum_{i=1}^{N}(\theta_i-\tilde{\theta}_i)\|\frac{\partial V_i^T(e_i)}{\partial e_i}\|^2-\sum_{i=1}^{N}(\vartheta_i-\tilde{\theta}_i)\|\frac{\partial V_i^T(e_i)}{\partial e_i}\|^2\\\leq&\sum_{i=1}^{N}(-\sum_{k=1}^{h_i}(e_i^T U_i(t)e_i)^{k-1}(e_i^T Q_i(t)e_i)\\&+(\theta_i-\tilde{\theta}_i)(\rho_i\theta)+(\vartheta_i-\tilde{\theta}_i)(-\rho_i\theta-\frac{1}{2}\rho_i\frac{\tilde{\theta}_i^2}{(\vartheta_i-\tilde{\theta}_i)})\\&+\sum_{j=1}^{N}(\sum_{k=1}^{n_{ij}}\lambda_{ij}\|e_j\|^{2k}+\sum_{k=1}^{m_{ij}}\delta_{ij}\|e_j\|^{2k})),\end{aligned} \tag{19}$$

if we choose parameters $\lambda_j=\max_i\{\lambda_{ij}\}$ and $\delta_j=\max_i\{\delta_{ij}\}$ for $1\leq i\leq N$. The following inequalities hold:

$$\begin{aligned}&\sum_{i=1}^{N}(-\sum_{k=1}^{h_i}(e_i^T U_i(t)e_i)^{k-1}(e_i^T Q_i(t)e_i)\\&\quad+\sum_{j=1}^{N}(\sum_{k=1}^{n_{ij}}\lambda_{ij}\|e_j\|^{2k}+\sum_{k=1}^{m_{ij}}\delta_{ij}\|e_j\|^{2k}))\\&\leq\sum_{i=1}^{N}(-\sum_{k=1}^{h_i}(e_i^T U_i(t)e_i)^{k-1}(e_i^T Q_i(t)e_i)\\&\quad+\sum_{j=1}^{N}(\sum_{k=1}^{h_j}\lambda_j\|e_j\|^{2k}+\sum_{k=1}^{h_j}\delta_j\|e_j\|^{2k}))\\&=\sum_{i=1}^{N}\sum_{k=1}^{h_i}(-(e_i^T U_i(t)e_i)^{k-1}(e_i^T Q_i(t)e_i)\\&\quad+N\lambda_i\|e_i\|^{2k}+N\delta_i\|e_i\|^{2k}),\end{aligned}$$

where $\lambda_j=0(n_{ij}<j\leq h_j)$ and $\delta_j=0(m_{ij}<j\leq h_j)$. Considering the following equality:

$$\begin{aligned}&(\theta_i-\tilde{\theta}_i)(-\rho_i\theta_i)+(\vartheta_i-\tilde{\theta}_i)(-\rho_i\vartheta_i-\frac{1}{2}\rho_i\frac{\tilde{\theta}_i^2}{(\vartheta_i-\tilde{\theta}_i)})\\&=-\rho_i(\theta_i-\frac{1}{2}\tilde{\theta}_i)^2+\frac{1}{4}\rho_i\tilde{\theta}_i^2-\rho_i(\vartheta_i-\frac{1}{2}\tilde{\theta}_i^2)^2-\frac{1}{4}\rho_i\tilde{\theta}_i^2\\&=-\rho_i(\theta_i-\frac{1}{2}\tilde{\theta}_i)^2-\rho_i(\vartheta_i-\frac{1}{2}\tilde{\theta}_i^2)^2,\end{aligned}$$

then we have

$$\begin{aligned}\dot{\hat{V}}(e,\theta,t)\leq&\sum_{i=1}^{N}\sum_{k=1}^{h_i}(-\lambda_{min}(U_i(t))^{k-1}\lambda_{min}(Q(t))\|e_i\|^{2k}\\&+N\lambda_i\|e_i\|^{2k}+N\delta_i\|e_i\|^{2k}),\end{aligned}$$

if we choose parameters λ_{ij} and δ_{ij} small enough to render that the following inequality holds:

$$-\lambda_{min}^{k-1}(U_i(t))\lambda_{min}(Q(t))+N\lambda_i+N\delta_i=-\Delta_i<0,$$

where Δ_i are positive scalars. Furthermore, we have

$$\dot{\hat{V}}(e,\theta,t)\leq\sum_{i=1}^{N}(-h_i\Delta_i\|e_i\|^{2k}).$$

Based on the Lyapunov-Krasovskii stability theorem, the proposed decentralized adaptive state feedback controllers can ensure that the synchronization manifold $\mathbb{S}$ is locally asymptotically stable for any time-delay τ. This completes the proof of the Theorem 3.

Remark 3: If delay $\tau=0$ and the coupling term is bounded by first-order polynomials, namely, $\tilde{g}_i(x,x(t-\tau),s,s(t-\tau))\leq\sum_{j=1}^{N}\gamma_{ij}\|e_j\|$, where γ_{ij} are known but non-negative constants for $i,j=1,2,\ldots,N$, then we can get the Theorem 3 in [12].

IV. EXAMPLE

In this section, we will use an example to illustrate the effectiveness of the criteria derived in this paper. we consider

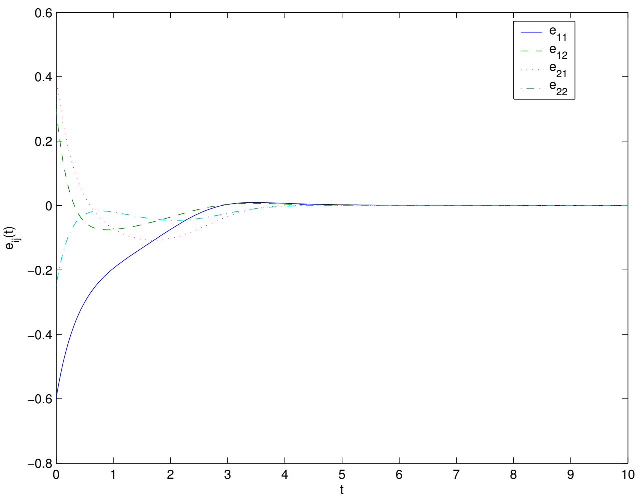

Fig.1. The synchronization errors $e_{ij}(t)$ for delayed network.

a lower-dimensional dynamical network with 2 nodes,

$$\begin{pmatrix} \dot{x}_{11} \\ \dot{x}_{12} \end{pmatrix} = \begin{pmatrix} -2 & 0 \\ 0 & -3 \end{pmatrix} \begin{pmatrix} x_{11} \\ x_{12} \end{pmatrix} + u_1$$
$$+ \begin{pmatrix} 0 \\ x_{11}+x_{22}+x_{11}(t-0.5sin(t))+x_{12}x_{21}(t-0.5sin(t)) \end{pmatrix},$$
$$\begin{pmatrix} \dot{x}_{21} \\ \dot{x}_{22} \end{pmatrix} = \begin{pmatrix} -2 & 0 \\ 0 & -3 \end{pmatrix} \begin{pmatrix} x_{21} \\ x_{22} \end{pmatrix} + u_2$$
$$+ \begin{pmatrix} 0 \\ x_{21}+x_{12}+x_{21}(t-0.5sin(t))+x_{22}x_{11}(t-0.5sin(t)) \end{pmatrix}.$$

We choose $\Gamma_i = 1$, $\rho_i = 1$, $\Pi_i = 1$, $\delta_{ij} = 0.1$, $\tau_{ij} = 0.1$, $\lambda_{ij} = 0.2$ and $K_i(t) = diag\{-1,-2\}$ and $U_i(t) = diag\{1,1\}$. Further calculation, we have $\|R_{ij}\|^2 = 1$, $\|S_{ij}\|^2 = 1$ and $U_i(t) = diag\{1,1\}$, therefore, the conditions of the Theorem 1 are satisfied, then we can obtain the following decentralized adaptive controllers:

$$u_1 = \begin{pmatrix} -1 & 0 \\ 0 & -2 \end{pmatrix} \begin{pmatrix} e_{11} \\ e_{12} \end{pmatrix} - 2(\theta_1(t)+\vartheta_1(t)) \begin{pmatrix} e_{11} \\ e_{12} \end{pmatrix},$$
$$u_2 = \begin{pmatrix} -1 & 0 \\ 0 & -2 \end{pmatrix} \begin{pmatrix} e_{21} \\ e_{22} \end{pmatrix} - 2(\theta_1(t)+\vartheta_1(t)) \begin{pmatrix} e_{21} \\ e_{22} \end{pmatrix},$$
$$\dot{\theta}_1(t) = 2e_{11}^2 + 2e_{12}^2 - \theta_1,$$
$$\dot{\theta}_2(t) = 2e_{21}^2 + 2e_{22}2 - \theta_2,$$
$$\dot{\vartheta}_1(t) = 2e_{11}^2 + 2e_{12}^2 - \vartheta_1 - \frac{2.5}{\vartheta_1 - 5},$$
$$\dot{\vartheta}_1(t) = 2e_{21}^2 + 2e_{22}^2 - \vartheta_2 - \frac{2.5}{\vartheta_2 - 5}$$

In Fig. 1, we plot the curves of the synchronization errors between the states of the two nodes.

V. CONCLUSIONS

We have investigated the locally and globally adaptive synchronization of the complex delayed dynamical network. Several network synchronization criteria are deduced based on the Lyapunov-Krasovskii functional approach. A time-varying delay in the coupling term is allowed, in addition, the assumptions in this paper are very common compared with some similar results and the proposed controllers are very simple in form.

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