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Fault recovery by nominal trajectory tracking

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Abstract— **Fault accommodation is meant to control the faulty system so as to preserve a given functionality. In model matching, the state dynamics is wished to be fault-invariant, leading to the family of pseudo-inverse and modified pseudo-inverse methods. In this paper, rather than the closeness between the faulty and the nominal system matrices, it is the closeness of their respective trajectories that is required. This problem setting allows to trade-off the quality of the accommodated trajectories and the energy of the accommodated control, and provides a clear definition of recoverable faults.**

Index Terms— **fault tolerant control, pseudo-inverse method, trajectory tracking.**

I. INTRODUCTION

Fault tolerant control (FTC) aims at preserving the functionality of a faulty system with acceptable performances when compared to normal operation [1]. Opposed to passive FTC, active FTC implements decisions that are specific to the diagnosed fault and to the functionality to be preserved: stability, disturbance attenuation [8], model matching [3], [4], predictive control [7], optimal control [10], [11]. Model matching and the pseudo-inverse method (PIM) have been first introduced in flight control systems, see e.g. [2], [3], [9], to deal with situations where pilots must keep faulty systems at hand. However, the PIM does not guarantee the stability of the obtained solution, a problem which has been later addressed by [3]. In [13] the model matching problem was revisited by searching for the solution within an admissible set of reference models, instead of finding the best approximation of an ideal one. This paper makes use of an idea first developed in [15], according to which, rather than requiring the faulty system to have the same model as the nominal system (in an approximate or admissible meaning), it is required that it follows the same trajectory (more precisely a best approximate of the nominal trajectory).

The paper is organized as follows : Section 2 describes and discusses the problem setting. Section 3 gives the problem solution, and discusses its stability and admissibility. Section 4 develops an illustrative example from the literature, and Section 5 provides some concluding remarks.

II. PROBLEM SETTING

Using standard notations, let

$$\dot{x}_n = A_n x_n + B_n u_n \tag{1}$$

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be the LTI model of the nominal system, where

$$u_n = K_n x_n \tag{2}$$

is the nominal state feedback, that results in the closed-loop behavior

$$\dot{x}_n = (A_n + B_n K_n) x_n \triangleq M_n x_n \tag{3}$$

where M_n is chosen so as to satisfy some nominal requirements (including stability). If the full state is not available, it is supposed that it can be reconstructed from the sensors, i.e. the system is completely observable. For brevity, sensor faults are not considered in the following development, indeed they can easily be taken into account (see [12]).

Assume a fault occurs at time t_f such that the faulty system can still be described by a LTI model, associated with the pair (A_f, B_f) . Let the control law be accommodated to $u_f = K_f x_f$, at time $t_0 > t_f$ then the post-fault trajectory obeys

$$\begin{array}{rcl} t & \in & [t_f, t_0[: \dot{x}_f = (A_f + B_f K_n) x_f \\ t & \geq & t_0 : \dot{x}_f = (A_f + B_f K_f) x_f \triangleq M_f x_f \end{array}$$

where the duration $t_0 - t_f$ is needed to detect, isolate, identify and accommodate the fault [14]. In the model matching setting, the state feedback K_f is chosen so as to obtain an unchanged input to state behavior, namely $M_f = M_n$. Since this is not always possible, the pseudo-inverse method defines an approximate solution by solving the problem

$$K_f = \arg\min \|M_f - M_n\|$$

where $\|\|$ is the Frobenius norm. As the solution may be unstable, further extensions solve this problem under stability [3], or general admissibility [13] constraints. Whatever the extension used, the trajectories of the nominal and accommodated systems are such that

$$\begin{array}{rcl} x_f(t) & = & \Phi_f(t - t_0) x_f(t_0) \\ x_n(t) & = & \Phi_n(t - t_0) x_n(t_0) \end{array}$$

where $\Phi_i(t - t_0) = e^{M_i(t-t_0)}$, $i = f, n$. Note that parametric faults are assumed here, hence the equilibrium state remains unchanged. Although both $x_f(t)$ and $x_n(t) \to 0$ as $t \to \infty$, there is no guarantee on the discrepancy between the two trajectories and on the discrepancy between the two control signals.

An alternative way of setting the accommodation problem is to request the trajectory of the accommodated system to mimic as much as possible the trajectory of the nominal

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one, trying to rub out the effect of the fault. Introducing two symmetric matrices $Q \geq 0$, and $R > 0$, and measuring the closeness by means of a quadratic cost

$$\begin{aligned} J &= \frac{1}{2}\int_{t_0}^{\infty} (x_f - x_n)^T Q (x_f - x_n)\, dt + ... \quad (4)\\ &... + \frac{1}{2}\int_{t_0}^{\infty} (u_f - u_n)^T R (u_f - u_n)\, dt \end{aligned}$$

provides a problem setting that allows to achieve a compromise between the discrepancies of the accommodated to nominal trajectory and the accommodated to nominal control signal.

III. PROBLEM SOLUTION

A. Optimality condition

From the classical theory of optimal control [6], one gets the set of necessary conditions

$$\begin{aligned} \dot{x}_f &= A_f x_f + B_f u_f \quad (5)\\ \dot{p}_f &= Q(x_f - x_n) - A_f^T p_f \quad (6)\\ 0 &= R(u_f - K_n x_n) - B_f^T p_f \quad (7) \end{aligned}$$

where ^{T} denotes transposition and p_f is the adjoint state vector. From (7), the accommodated control is

$$u_f = K_n x_n + R^{-1} B_f^T p_f \quad (8)$$

Following a classical approach, the adjoint state is taken under the form

$$p_f = H x_f + G x_n$$

where H and G are two matrices to be determined. Making use of (1), (5) and (8) one gets

$$\begin{aligned} \dot{p}_f = \; & H\left(A_f + B_f R^{-1} B_f^T H\right) x_f + \cdots \\ & \cdots + \left(H B_f K_n + B_f R^{-1} B_f^T G\right) x_n + \cdots \\ & \cdots + \left(G\left(A_n + B_n K n\right)\right) x_n \end{aligned}$$

From (6) it follows that

$$\dot{p}_f = Q x_f - Q x_n - A_f^T H x_f - A_f^T G x_n$$

and therefore

$$\left(Q - A_f^T H - H A_f + H B_f R^{-1} B_f^T H\right) x_f = ...$$
$$\left(\begin{array}{c} Q + H B_f K_n + H B_f R^{-1} B_f^T G - ... \\ ... - G A_n - G B_n K_n - A_f^T G \end{array} \right) x_n$$

so that H and G must satisfy

$$A_f^T H + H A_f + H B_f R^{-1} B_f^T H - Q = 0 \quad (9)$$
$$\begin{aligned} Q + H B_f K_n + G(A_n + B_n K_n) + .. \quad (10)\\ ... + \left(B_f R^{-1} B_f^T + A_f^T\right) G = 0 \end{aligned}$$

where (9) is a classical algebraic Riccati equation and (10) is a Ljapunov equation that is easily solved once H has been found.

B. Stability

From (7) one gets

$$u_f = u_n + R^{-1} B_f^T (H x_f + G x_n) \quad (11)$$

and therefore the accommodated control is obtained by adding the compensating term $R^{-1} B_f^T (H x_f + G x_n)$ to the nominal control, leading to the accommodated dynamics

$$\begin{aligned} \dot{x}_f = \; & \left(A_f + B_f R^{-1} B_f^T H\right) x_f ... \quad (12)\\ & + B_f \left(K_n + R^{-1} B_f^T G\right) x_n \end{aligned}$$

Let $z^T = \left(x_n^T \quad x_f^T \right)$ then from (3) and (12) one gets $\dot{z} = Mz$ with

$$M = \left(\begin{array}{cc} A_n + B_n K_n & 0 \\ B_f\left(K_n + R^{-1} B_f^T G\right) & A_f + B_f R^{-1} B_f^T H \end{array} \right).$$

Since K_n is such that the nominal closed-loop matrix $A_n + B_n K_n$ is stable, the stability of the accommodated system follows from the stability of $A_f + B_f R^{-1} B_f^T H$, which is well known to be achieved by a unique solution H provided the pair (A_f, B_f) is still stabilizable, and the pair (C, A_f) is detectable, where $Q = C^T C$.

C. Admissibility

Let (A_f, B_f) be a fault such that (A_f, B_f) is stabilizable and (C, A_f) is detectable, then there exists a unique pair (H, G) such that the accommodated control $u_f = u_n + R^{-1} B_f^T (H x_f + G x_n)$ stabilizes the faulty system and is optimal with respect to the cost (4). However, not any such fault is recoverable, because although minimal, the cost (4) might be too high for the accommodated behavior to be accepted as close enough to the nominal one.

Let $\epsilon_s = x_f - x_n$ and $\epsilon_u = u_f - u_n$. Using (11) one gets

$$\epsilon_s^T Q \epsilon_s + \epsilon_u^T R \epsilon_u = z^T S z$$

where

$$S = \left(\begin{array}{cc} Q + G^T B_f R^{-1} B_f^T G & -Q + G^T B_f R^{-1} B_f^T H \\ -Q + H^T B_f R^{-1} B_f^T G & Q + H^T B_f R^{-1} B_f^T H \end{array} \right)$$

The cost can now be easily computed. Since M is stable, there is a symmetric negative definite matrix P such that

$$MP + PM = S$$

It follows that

$$\frac{d}{dt} z^T P z = z^T S z$$

and

$$J = \frac{1}{2}\int_{t_0}^{\infty} \frac{d}{dt} z^T P z dt = -\frac{1}{2} z^T(t_0) P z(t_0)$$

Let η be the admissibility limit, then recoverable faults are such that

$$-\frac{1}{2} z^T(t_0) P z(t_0) \leq \eta \quad (13)$$

1) [*Remarks*]
2) The state discrepancy on the time window $[t_f, t_0[$ is not taken into account in the cost (4) since it depends on the fault only. Indeed, the control has not yet been accommodated, this is why there is no control discrepancy.
3) Note that the bigger the fault the bigger the initial state difference $\epsilon_s(t_0) = x_f(t_0) - x_n(t_0)$. If the nominal control destabilizes the faulty system, the same applies as the diagnosis and accommodation delay increases.
4) The Model Matching approach can in no case provide any optimal solution, since it defines a matrix M_f such that $\dot{x}_f = M_f x_f$: whatever the way M_f is computed, the input x_n is never taken into account, as (11) shows it should be.

D. Real time acceleration

After some fault occurs at time t_f, the duration $t_0 - t_f$ is needed before the accommodated control can be applied. Let $t_1 \in]t_f, t_0[$ be the time at which fault detection, isolation and estimation have been performed, therefore the updated matrices (A_f, B_f) are available at t_1, and $t_0 - t_1$ is the time that is necessary to solve the algebraic Riccati equation (9) and the Ljapunov equation (10). The shorter the delay $t_0 - t_f$, the better the fault tolerance performances, as illustrated by the example in the next section. As far as the computation of the accommodated solution is concerned, real time constraints can be handled by using so-called *anytime algorithms*, whose solution improves as the iteration number grows. Therefore, the result can be applied as soon as the first iteration, and improved in further iterations, leading to the so-called *progressive accommodation* of the fault that has been shown to drastically improve the post-fault behavior during the transient period $[t_1, t_f]$ [14]. For algebraic Riccati equations, the Newton-Raphson iteration, first proposed in [5] results in the following algorithm :

$$\begin{gathered}(A_f B_f F_{k-1})^T H_k + H_k (A_f B_f F_{k-1}) + \cdots \\ \cdots + F_{k-1} B_f (-R) F_{k-1} - Q = 0\end{gathered}$$

with

$$F_{k-1} = -R^{-1} B_f H_{k-1}$$

where $k = 1, 2, ...$ and the initial H_0 is given such that $A_f + B_f R^{-1} B_f^T H_0$ is Hurwitz. It follows that [5] :

$$\begin{array}{ll}(a) & A_f + B_f R^{-1} B_f^T H_k \text{ is Hurwitz for all } k = 1, 2, .. \\ (b) & H \leq \cdots \leq H_{k+1} \leq H_k \leq \cdots \leq H_0, \quad k = 1, 2, \cdots \\ (c) & \lim_{k\to\infty} H_k = H\end{array}$$

Finally, let T be the iteration period of the above computation scheme, applying the control law

$$u_f(t) = u_n(t) + R^{-1} B_f^T [H_k x_f(t) + G_k x_n(t)]$$

on the time interval $t \in [kT, (k+1)T[$ where

$$\begin{gathered}Q + H_k B_f K_n + G_k (A_n + B_n K_n) + \cdots \\ \cdots + \left(B_f R^{-1} B_f^T + A_f^T\right) G_k = 0\end{gathered}$$

allows to start accommodating the fault as soon as $t_1 + T$ with a much better result than the one obtained if the solution was waited to converge before it is applied.

IV. EXAMPLE

The following LTI system was used in [3] to illustrate the pseudo-inverse (PIM) and modified pseudo-inverse (MPIM) methods,

$$A_n = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \quad B_n = \begin{pmatrix} 1 \\ 5 \end{pmatrix}$$

along with the reference model

$$M_n = \begin{pmatrix} -2 & 0 \\ -5 & -1 \end{pmatrix}$$

and the fault

$$A_f = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \quad B_f = \begin{pmatrix} -1 \\ 1 \end{pmatrix}.$$

This example is here extended to a family of parameterized faults :

$$A_f = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \quad B_f(\theta) = \begin{pmatrix} 1 - 2\theta \\ 5 - 4\theta \end{pmatrix}$$

where $\theta \in [0, 1]$ is a parameter, such that $\theta = 0$ means no fault while $\theta = 1$ gives the fault in [3]. Note that neither the healthy nor the faulty system are controllable, but both are stabilizable. The state feedback $u = k_1 x_1 + k_2 x_2$ gives the closed loop matrix

$$M(k_1, k_2, \theta) = \begin{pmatrix} k_1(1-2\theta) - 1 & k_2(1-2\theta) \\ k_1(5-4\theta) & k_2(5-4\theta) - 1 \end{pmatrix}$$

and its optimal value with respect to the minimization of the Frobenius norm is

$$\begin{aligned} k_1(\theta) &= \frac{22\theta - 26}{(1-2\theta)^2 + (5-4\theta)^2} \\ k_2(\theta) &= 0 \end{aligned}$$

It is easily seen that the norm can be zeroed (Exact Model Matching) only in the nominal case $\theta = 0$ and is associated with the nominal control $u_n(t) = \begin{pmatrix} -1 & 0 \end{pmatrix} x_n(t)$. For $\theta \neq 0$, the PIM results in the closed loop matrix

$$M_f^{PIM}(\theta) = \begin{pmatrix} \frac{-64\theta^2+118\theta-52}{20\theta^2-44\theta+26} & 0 \\ \frac{-88\theta^2+214\theta-130}{20\theta^2-44\theta+26} & -1 \end{pmatrix}$$

whose eigenvalues are

$$\lambda_1(\theta) = -1 \quad \lambda_2(\theta) = \frac{-64\theta^2+118\theta-52}{20\theta^2-44\theta+26}$$

It can be checked that it provides an unstable solution for all faults such that $\theta > 0.728$ (remember that $\theta = 1$ in the example of [3]), so that in that case, MPIM has to be used.

In the proposed approach, using $Q = I$ and $R = 1$ one gets the optimal control:

$$u_f(\theta) = H(\theta)x_f + G(\theta)x_n$$

where $H(\theta)$ is given by

$$H(\theta)B_fR^{-1}B_f^TH(\theta) - 2H(\theta) - Q = 0$$

with

$$B_fR^{-1}B_f^T = \begin{pmatrix} (1-2\theta)^2 & (1-2\theta)(5-4\theta) \\ (5-4\theta)(1-2\theta) & (5-4\theta)^2 \end{pmatrix}$$

while $G(\theta)$ is the solution of :

$$Q + H(\theta)\begin{pmatrix} (2\theta-1) & 0 \\ (4\theta-5) & 0 \end{pmatrix} + G(\theta)\begin{pmatrix} -2 & 0 \\ -5 & -1 \end{pmatrix} + \cdots$$
$$+\begin{pmatrix} (1-2\theta)^2-1 & (1-2\theta)(5-4\theta) \\ (5-4\theta)(1-2\theta) & (5-4\theta)^2-1 \end{pmatrix}G(\theta) = 0.$$

A. Stable case.

Let us first illustrate the case where PIM provides a stable closed loop by choosing $\theta = 0.6$. The control

$$u_f^{PIM}(0.6) = -1.88235x_1^{PIM}(t)$$

gives :

$$M_f^{PIM}(0.6) = \begin{pmatrix} -0.6235 & 0 \\ -4,8941 & -1 \end{pmatrix}$$

The LQ optimal control u_f is defined by the pair

$$H(0.6) = \begin{pmatrix} -0.4986 & -0.0181 \\ -0.0181 & -0.2650 \end{pmatrix}$$
$$G(0.6) = \begin{pmatrix} 0.2789 & 0.0181 \\ -0.1258 & 0.2650 \end{pmatrix}$$

Fig. 1 shows the state trajectories $x_n(t), x_f^{PIM}(t)$ and $x_f(t)$, for a 2 s. fault diagnosis and accommodation delay, i.e. during the first 2 seconds, the faulty system is still controlled by the nominal control. As a result, trajectories x_f^{PIM} and x_f are identical for $t \in [10, 12[$, and x_f shows a behavior closer to x_n only after $t = 12$.

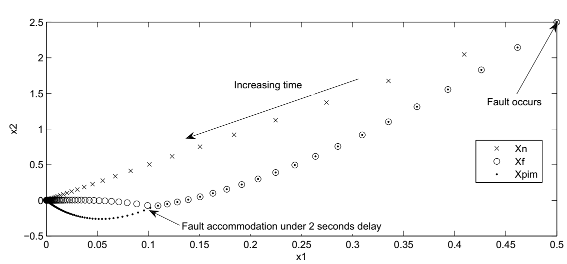

Fig. 1. State trajectories for a 2s delay

Fig. 2 shows the significative improvement in the quadratic costs associated with the discrepancies $\left(x_n - x_f^{PIM}, u_n - u_f^{PIM}\right)$ and $(x_n - x_f, u_n - u_f)$ still for a 2s. delay.

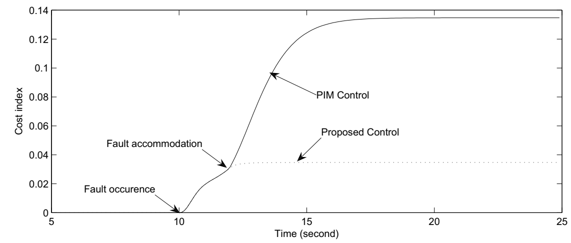

Fig. 2. Comparison of the PIM and the LQ optimal costs

B. Instable case.

Let now $\theta = 1$ as in [3]. The PIM control is:

$$u_f^{PIM}(1) = -2x_1(t)$$

and gives the following closed loop which is instable.

$$M_f^{PIM}(1) = \begin{pmatrix} 1 & 0 \\ -2 & -1 \end{pmatrix}$$

The MPIM solution provided in [3] is:

$$u_f^{MPIM}(t) = -0.8x_1(t)$$

which results in the stable closed loop:

$$M_f^{MPIM}(1) = \begin{pmatrix} -0.2 & 0 \\ -0.8 & -1 \end{pmatrix}$$

The LQ optimal control u_f is defined by the pair

$$H(1) = \begin{pmatrix} -0.4330 & -0.0670 \\ -0.0670 & -0.4330 \end{pmatrix}$$
$$G(1) = \begin{pmatrix} 0.0311 & 0.0670 \\ -0.5311 & 0.4330 \end{pmatrix}$$

Fig. 3 shows the state trajectories $x_n(t), x_f^{MPIM}(t)$ and $x_f(t)$ for three different delays, namely 0 s. (ideal case), 1 s. and 2 s. while Fig. 4 displays the quadratic costs associated with the discrepancies $\left(x_n - x_f^{MPIM}, u_n - u_f^{MPIM}\right)$ and $(x_n - x_f, u_n - u_f)$ for the 2s. case.

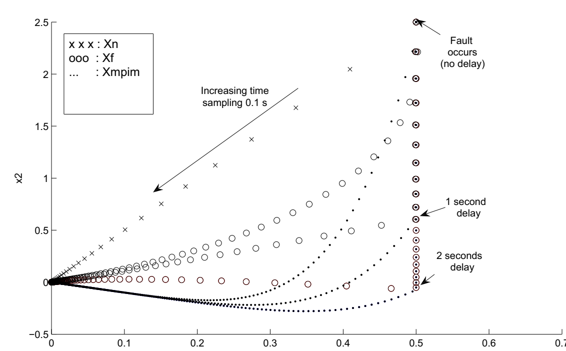

Fig. 3. Comparison of state trajectories for respective delays 0, 1 and 2 seconds

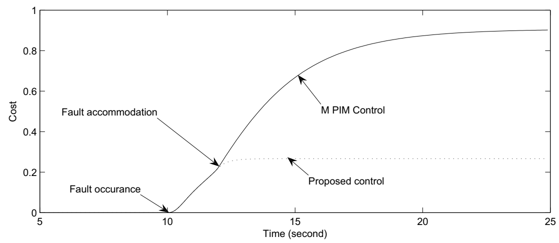

Fig. 4. Comparison of the MPIM and the LQ optimal costs

Fig. 5 shows how LQ optimal costs increase with the diagnosis and accommodation delay. It follows that for small delays the fault may be recoverable, while it becomes unrecoverable for bigger ones, since the cost may be unaffordable (i.e. associated with inadmissible dynamic behavior).

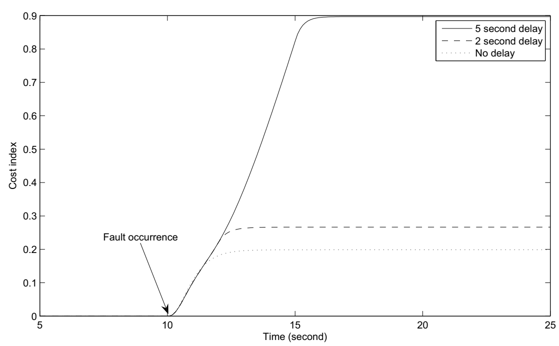

Fig. 5. The LQ optimal costs for the three diagnosis and accommodation delays.

Let us consider now, that the 2 seconds accommodation delay is the sum of 1 second for fault detection and 1 second to solve the algebraic Riccati [1]. The efficiency of fault accommodation can be improved by using Newton-Raphson iterations to solve the Riccati equation (9). In the considered example, the time-ratio between solving a Lyapunov equation and a Riccati equation is about 4, and the higher the dimension of the state space, the higher the ratio. For such an instable system, using the first iteration value H_1 (which is obtained after 0,2 second) instead of waiting the Riccati equation solution H during 1 second, allows to stabilize the system much sooner, and hence gives improved results. In this framework, the Newton-Raphson algorithm converges to H within 5 iterations, with the following sequences for H_k:

[1]In practice, strong data-handling limitations are indeed associated with multi-rate embedded computers, where the time dedicated to control is very short and/or with high dimensional state space problems.

$$
\begin{aligned}
H_1 &= \begin{pmatrix} -1 & 1 \\ 1 & -1 \end{pmatrix} \\
H_2 &= \begin{pmatrix} -0.7 & 0.2 \\ 0.2 & -0.7 \end{pmatrix} \\
H_3 &= \begin{pmatrix} -0.4839 & -0.0161 \\ -0.0161 & -0.4839 \end{pmatrix} \\
H_4 &= \begin{pmatrix} -0.4357 & -0.0643 \\ -0.0643 & -0.4357 \end{pmatrix} \\
H_5 &= \begin{pmatrix} -0.4330 & -0.0670 \\ -0.0670 & -0.4330 \end{pmatrix} = H
\end{aligned}
$$

Fig. 6 compares the direct accommodation control and the progressive accommodation one. It is seen that progressive accommodation practically ”rubs out” the effect of the accommodation delay, since the resulting trajectories are quite similar to the ones associated with a 1 second FDI + FTC delay.

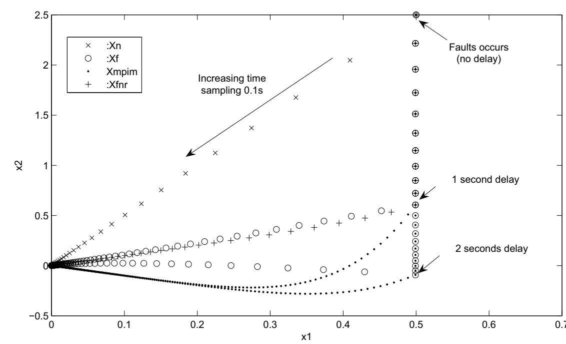

Fig. 6. Comparison os state trajectories for direct and iterative Ricatti solution.

V. CONCLUSION

Setting an active fault tolerant control problem rests on (1) a clear definition of the control objective that is wished to be invariant under the fault along with (2) the definition of acceptable degradations of this objective. In the PIM and the MPIM approaches, the second point is not specified, the result is that any fault is recoverable (there is always a solution to the fault accommodation problem). In the proposed approach, the control objective is to mimic as much as possible the trajectory of the healthy system, while achieving a trade-off with the control effort. As far as the approach rests on the optimization of a cost function, admissibility is easily defined, and the set of recoverable faults directly follows. This paper shows that recoverability is not an intrinsic property of faults : the faulty system must indeed satisfy some controllability (stabilizability) and observability (detectability) conditions for a solution to exist, but the same fault could be recoverable or not depending on the delay introduced by the diagnosis and accommodation processes.

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