2008 American Control Conference
Westin Seattle Hotel, Seattle, Washington, USA
June 11-13, 2008



# On further optimizing prediction dynamics for robust model predictive control

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***Abstract*— This paper presents a new method for further optimizing the prediction dynamic of constrained robust model predictive control. More slack variables are deployed in the proposed linear matrix inequality (LMI) formulation in order to provide extra degrees of freedom for the dynamic controller. As illustrated by a canonical example, the extra degrees of freedom allow for better performance and wider applicability. In addition, such design can be performed offline leaving only a simple optimization problem for online realization.**

***Keywords*: Constrained model predictive control; invariant set; robust control; prediction dynamics.**

## I. Introduction

Model predictive control (MPC) is one strategy that deals with controller design for systems with physical constraints. The basic idea of MPC is first found in several textbooks on the optimal control theory [1]–[3]. Over the last few decades MPC has attracted notable attention in many fields where slow dynamical systems are met (typically chemical processes). The idea of MPC can be summarized as follows: at each control interval, an explicit process model is utilized to predict the future response of a plant, and a constrained optimization problem is then solved to yield a sequence of future manipulated variable control adjustments in order to optimize future plant behavior. Only first move of the sequence of manipulated variable control adjustments will be implemented to the plant and then the entire calculation is repeated at subsequent control intervals (see e.g. García, Prett, & Morari [4]).

MPC strategy has received much consideration since it is possible to handle constrains on input, state and output signals during the design procedure of the controller. However, it has also been cited in many papers ( [5], etc) that the computational complexity is the key factor that limits MPC (especially for robust MPC) to slow dynamic or low dimensional systems. Recently a lot of work that aims at reducing the online computational complexity has been reported. For example, [6] attempts to use an open-loop parametric MPC where the objective function penalizes the nominal system while the constraints are guaranteed for all possible uncertainty. And in [7] the author tries to recast a min-max MPC problem with quadratic cost to a quadratically constrained quadratic program, which can be efficiently solved by specially tailored interior-point methods whose computational cost are linear in the number of variables. In [8], [9], an explicit solution of linear MPC in piecewise affine form has been derived by using multi-parametric linear/quadratic programming, leaving only a simple point location problem to online realization. However the prohibitive online storage and offline partition complexity limits its application to low dimensional systems.

On the other hand [10], [11] use ellipsoidal constraints approximation to formulate the min-max MPC in an LMI framework and solve it online. Such method provides an efficient tool for robust MPC but the computational load remains prohibitive when fast dynamic systems are used. In view of this, [5], [12] introduce an alternative approach which deploys a fixed state feedback law and extra degrees of freedom through the use of perturbations, thereby moves all demanding computations offline leaving only a small fraction of the computational load for the online realization. This approach is improved in [13], [14] by using varying parameters in the dynamic feedback law. However the formulation in [13], [14] is non-convex hence leads to no guarantee on convergency of the solution. In [15] the dynamic feedback law is further optimized and formulated into a convex problem.

Inspired by advances in output feedback controller design methods [16], [17], this paper aims at reducing the design conservativeness by introducing more slack variables such that the associated maximal stabilizable set is enlarged. By using a nonlinear parameter transformation technique, resulting optimization problem admits a convex formulation which can be solved efficiently. Despite this, since the demanding optimization can be performed offline, the proposed approach will not induce any extra online computational load.

The paper is organized as follows. In Section II, the problem is presented and some basic concepts concerning MPC are reviewed. In Section III, the new approach is present. In Section IV-A, a numerical example is presented followed by some conclusion remarks in Section V.

**Notation.** $\mathbb{R}$ is the set of real numbers. For a matrix $A$, $A^T$ denotes its transpose, and $A^{-1}$ its inverse (if exists). The matrix inequality $A > B$ ($A \geqslant B$) means that $A$ and $B$ are square and symmetric and $A - B$ is positive (positive semi-) definite. $I$ denotes the identity matrix. $x(k)$ or $x(k|k)$ denotes the state measured at real time $k$; and $x(k+i|k)$ $(i \geqslant 1)$ the state at prediction time $k+i$ predicted at real time $k$. The symbol $*$ within a matrix represents the symmetric entries. For positive definite matrix $Q$ and compatible column vector $x$, $\|x\|_Q \triangleq x^T Q x$.

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## II. Problem formulation

Consider the discrete-time uncertain linear system

$$\begin{aligned} x(k+1) &= A(k)x(k) + B(k)u(k) \\ y(k) &= Cx(k) \\ [A(k), B(k)] &\in \Omega := \mathrm{Co}\{[A_j, B_j], j = 0, 1, \ldots, l\} \end{aligned} \tag{1}$$

where $x \in \mathbb{R}^{n_x}$ is the state, $u \in \mathbb{R}^{n_u}$ is the input and $y \in \mathbb{R}^{n_y}$ is the output, $\mathrm{Co}\{\cdot\}$ denotes the convex hull while $[A_j, B_j]$ are vertices of the uncertain polytope $\Omega$. Constraints on the state $x$ and input $u$ are taken into account

$$Fx(k) + Eu(k) \leqslant f. \tag{2}$$

Suppose the infinite horizon linear quadratic performance index is given as

$$J(k) = \sum_{i=0}^{\infty} \|y(k+i|k)\|_{\mathbb{Q}} + \|u(k+i|k)\|_{\mathbb{R}} \tag{3}$$

where $\mathbb{Q}$ and $\mathbb{R}$ are given positive semi-definite symmetric matrices. Then the worst case performance index to be minimized is

$$\tilde{J}(k) = \max_{[A(k+i), B(k+i)] \in \Omega,\ i=0,1,\ldots} J(k) \tag{4}$$

subject to system dynamic for prediction

$$\begin{aligned} x(k+i+1|k) &= A(k+i|k)x(k+i|k) + B(k+i|k)u(k+i|k) \\ y(k+i|k) &= Cx(k+i|k) \end{aligned}. \tag{5}$$

In order to guarantee closed-loop stability, state/input constraint (2) must be satisfied along predicted trajectories for all possible future model uncertainty. Mathematically, this requirement can be written as and constraints along predicted trajectories

$$\begin{aligned} Fx(k+i|k) + Eu(k+i|k) \leqslant f, \\ \forall [A(k+j), B(k+j)] \in \Omega,\ j = 0, 1, \ldots, i-1. \end{aligned} \tag{6}$$

As was done in [13]–[15], in the following the system will be pre-stabilized with a state feedback controller $K$. Such controller can be optimized in varied senses, e.g., LQ optimal, feasible set maximal etc.

In the following we are ready to present the method proposed in this paper for optimizing the prediction dynamics.

## III. Main result

### A. Augmented system

Suppose the dynamic controller is modeled as

$$\begin{aligned} x_c(k+i+1|k) &= A_c x_c(k+i|k) + B_c y(k+i|k) \\ v(k+i|k) &= C_c x_c(k+i|k) + D_c y(k+i|k) \\ u(k+i|k) &= Kx(k+i|k) + v(i|k) \end{aligned} \tag{7}$$

where $x_c \in \mathbb{R}^{n_c}$ is the state of the dynamic controller, $A_c$, $B_c$, $C_c$ and $D_c$ are variables to be calculated. Please note that when $B_c = \mathbf{0}$ and $D_c = \mathbf{0}$, it recovers the formulation in [15]. Further more, if we define

$$A_c = \begin{bmatrix} \mathbf{0} & I_{n_u} & \cdots & \mathbf{0} \\ \vdots & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \cdots & I_{n_u} \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} \end{bmatrix},$$

$$C_c = [I_{n_u}, \mathbf{0}, \ldots, \mathbf{0}], \quad x_c = \begin{bmatrix} v(0|k) \\ v(1|k) \\ \vdots \\ v(N-1|k) \end{bmatrix},$$

where $N$ is the control horizon that $v(i|k) = 0$ for all $i \geqslant N$, (7) recovers the formulation in [12]. Therefore, with extra degrees of freedom provided by these parameters, we expect to find a less conservative design.

Given (7) and (5), the overall system can be written as

$$\xi(k+i+1|k) = \mathcal{A}(k+i|k)\xi(k+i|k) \tag{8}$$

where $\xi = [x^T, x_c^T]^T$ and $\mathcal{A}(k+i|k) \in \mathrm{Co}\{\mathcal{A}_j,\ j = 0, 1, \ldots, l\}$ with

$$\mathcal{A}_j = \begin{bmatrix} \Phi_j + B_j D_c C & B_j C_c \\ B_c C & A_c \end{bmatrix}, \quad \Phi_j = A_j + B_j K. \tag{9}$$

### B. Offline optimization

The basic idea is to enlarge the feasible region associated with the unconstrained controller $K$ through use of augmented state vector $\xi$. That is, we want to find $A_c$, $B_c$, $C_c$, $D_c$ that gives the largest possible region of attraction. The optimization is done by looking for a a positive-definite matrix $Q_z$ which defines invariant ellipsoid $\mathcal{E}_z := \left\{\xi | \xi^T Q_z^{-1} \xi \leqslant 1\right\}$ that the volume of the projection from it onto the state space ( [12], [14]) is maximized, i.e., the volume of $\mathcal{E}_{xx} := \left\{x | x^T (T Q_z T^T)^{-1} x \leqslant 1\right\}$ (project function $x = Tz$) is maximized. Equivalently, this problem can be recast as follows

$$\max_{Q_z, Q_j, A_c, B_c, C_c, D_c} \ln\det(T Q_z T^T) \tag{10}$$

subject to stability constraint and state/input constraints in LMI formulation,

$$Q_j^{-1} - \mathcal{A}_j^T Q_j^{-1} \mathcal{A}_j \geqslant 0, \quad j = 0, 1, \ldots, l, \tag{11a}$$

$$Fx(k+i|k) + Eu(k+i|k) \leqslant f, \quad i = 0, 1, \ldots, \tag{11b}$$

and an extra constraint to ensure $\mathcal{E}_z := \left\{\xi | \xi^T Q_z^{-1} \xi \leqslant 1\right\}$ is invariant

$$Q_z \leqslant Q_j, \quad j = 0, 1, \ldots, l. \tag{11c}$$

*Theorem 3.1:* Consider discrete time uncertain system (1) with constraints (2).

(a) Existence: There exist matrices $Q_j$, $A_{c,j}$, $B_{c,j}$, $C_c$, $D_c$ of appropriate dimension satisfying (11a) and (11b) only if the following LMIs are feasible in positive definite matrices $X$, $Y$, and symmetric positive definite matrices

$W$, $\mathcal{Q}_{j,11}$, $\mathcal{Q}_{j,22}$, $Q_{xx}$ and matrices $N$, $R$, $S$, $T$, $D_c$, $\mathcal{Q}_{j,21}$ that for $j = 0, 1, \ldots, l$

$$\begin{bmatrix} X+X^T-\mathcal{Q}_{j,11} & * & * & * \\ I+N-\mathcal{Q}_{j,21} & Y+Y^T-\mathcal{Q}_{j,22} & * & * \\ \Phi_j X+B_j S & \Phi_j+B_j D_c C & \mathcal{Q}_{j,11} & * \\ R & Y^T\Phi_j+TC & \mathcal{Q}_{j,21} & \mathcal{Q}_{j,22} \end{bmatrix} \geqslant 0, \tag{12}$$

where

$$\begin{cases} N = Y^T X + V^T U, \\ R = Y^T(\Phi_j+B_j D_c C)X+V^T B_{c,j} CX+Y^T B_j C_c U \\ \quad +V^T A_{c,j} U, \\ S = D_c CX + C_c U, \\ T = Y^T B_j D_c + V^T B_{c,j}. \end{cases}$$

(b) Constraints: Constraints (11b) is guaranteed if following LMI holds

$$\begin{bmatrix} W & FX+EKX+ES & F+EK+ED_c C \\ * & X+X^T-\mathcal{Q}_{j,11} & I+N^T-\mathcal{Q}_{j,21}^T \\ * & * & Y+Y^T-\mathcal{Q}_{j,22} \end{bmatrix} \geqslant 0. \tag{13}$$

(c) Optimization: The offline maximization of the volume of $\mathcal{E}_{xx}$ is performed by solving the following optimization problem,

$$\max_{X,Y,W,N,R,S,T,D_c,\mathcal{Q}_{j,11},\mathcal{Q}_{j,21},\mathcal{Q}_{j,22},Q_{xx}} \ln\det Q_{xx} \tag{14}$$

subject to $\forall j = 0, 1, \ldots, l$

$$\mathcal{Q}_{j,11} \geqslant Q_{xx}, \tag{15}$$

and LMI (12), (13).

(d) Controller: Suppose optimization problem in (c) admits a solution. The resulting dynamic controller is given in the following equations. Given $U$, $V$ that satisfy $N = Y^T X + V^T U$,

$$\begin{cases} A_{c,j} = V^{-T}(R - Y^T(\Phi_j+B_j D_c C)X - V^T B_{c,j} CX \\ \qquad -Y^T B_j C_c U)U^{-1}, \\ B_{c,j} = V^{-T}(T - Y^T B_j D_c), \\ C_c = (S - D_c CX)U^{-1}, \\ D_c = D_c. \end{cases}$$

*Proof:*

(a) By using Shur complement, (11a) can be recast as

$$\begin{bmatrix} Q_j & * \\ \mathcal{A}_j Q_j & Q_j \end{bmatrix} \geqslant 0. \tag{16}$$

Pre and post multiplying the above inequality by $\mathrm{diag}[G^T Q_j^{-1}, I]$ and $\mathrm{diag}[Q_j^{-1} G, I]$ respectively yields

$$\begin{bmatrix} G^T Q_j^{-1} G & * \\ \mathcal{A}_j G & Q_j \end{bmatrix} \geqslant 0. \tag{17}$$

Since $(G^T - Q_j)Q_j^{-1}(G - Q_j) \geqslant 0$, it follows that $G + G^T - Q_j \geqslant G^T Q_j^{-1} G$. Therefore (17) is satisfied if

$$\begin{bmatrix} G + G^T - Q_j & * \\ \mathcal{A}_j G & Q_j \end{bmatrix} \geqslant 0. \tag{18}$$

Define

$$G = \begin{bmatrix} X & U_1 \\ U & \bullet \end{bmatrix}, \quad G^{-1} = \begin{bmatrix} Y & V_1 \\ V & \bullet \end{bmatrix} \tag{19}$$

where $\bullet$ indicates blocks of $G$ and $G^{-1}$ that are determined uniquely by $X$, $U$, $U_1$ and $Y$, $V$, $V_1$.
Pre and post multiplying (18) by

$$\begin{bmatrix} Z^T & 0 \\ 0 & Z^T \end{bmatrix}, \quad \begin{bmatrix} Z & 0 \\ 0 & Z \end{bmatrix} \quad \text{with} \quad Z = \begin{bmatrix} I & Y \\ 0 & V \end{bmatrix}$$

respectively yields (12).

(b) Following [15] it's easy to show that constraints (11b) is equivalent to the following LMI,

$$\begin{bmatrix} W & [F+EK+ED_c C, \ EC_c] \\ * & Q_j^{-1} \end{bmatrix} \geqslant 0 \quad W_{ii} \leqslant f_i^2. \tag{20}$$

Pre and post multiplying (20) by $\mathrm{diag}[I, G^T]$ and $\mathrm{diag}[I, G]$ respectively and using similar technique yields

$$\begin{bmatrix} W & [F+EK+ED_c C, \ EC_c]G \\ * & G+G^T-Q_j \end{bmatrix} \geqslant 0 \quad W_{ii} \leqslant f_i^2. \tag{21}$$

Pre and post multiplying (21) by

$$\begin{bmatrix} I & 0 \\ 0 & Z^T \end{bmatrix}, \quad \begin{bmatrix} I & 0 \\ 0 & Z \end{bmatrix}$$

respectively yields (13).

(c, d) It's straight forward from above derivation hence is omitted here for brevity.

■

*Remark 3.2:* The above theorem considers the largest possible region of attraction. With the extra parameters $N$, $R$, $S$, $T$, $D_c$, $\mathcal{Q}_{j,11}$, $\mathcal{Q}_{j,21}$, $\mathcal{Q}_{j,22}$, it's safe to say that we can reduce the design conservativeness.

*Remark 3.3:* In Theorem 3.1, $A_c$ and $B_c$ are allowed to take vertex dependent value, i.e., $A_{c,j}$ and $B_{c,j}$. Thus for any $A(k)$, $B(k)$ in polytope $\Omega$, there exist $A_c(k) \in \mathrm{Co}\{A_{c,j}, j = 0, 1, \ldots, l\}$ and $B_c(k) \in \mathrm{Co}\{B_{c,j}, j = 0, 1, \ldots, l\}$ so that $\mathcal{E}_z := \left\{\xi | \xi^T Q_z^{-1} \xi \leqslant 1\right\}$ is invariant. This also apply to the following result.

We can take into account prediction cost in the optimization problem [15].

*Theorem 3.4:* Consider discrete time uncertain system (1) with constraints (2).

(a) Existence: For a given positive scalar $\gamma$, it can be guaranteed that $\tilde{J} \leqslant \gamma$ with LMIs (11a) and (11b) hold only if the following LMIs admit a solution in positive definite matrices $X$, $Y$, and symmetric positive definite matrices $W$, $\mathcal{Q}_{j,11}$, $\mathcal{Q}_{j,22}$ and matrices $N$, $R$, $S$, $T$, $D_c$, $\mathcal{Q}_{j,21}$, for $j = 0, 1, \ldots, l$.

$$\begin{bmatrix} X+X^T-\mathcal{Q}_{j,11} & * & * & * & * & * \\ I+N-\mathcal{Q}_{j,21} & Y+Y^T-\mathcal{Q}_{j,22} & * & * & * & * \\ \Phi_j X + B_j S & \Phi_j + B_j D_c C & \mathcal{Q}_{j,11} & * & * & * \\ R & Y^T\Phi_j + TC & \mathcal{Q}_{j,21} & \mathcal{Q}_{j,22} & * & * \\ \mathbb{Q}^{\frac{1}{2}} CX & \mathbb{Q}^{\frac{1}{2}} C & 0 & 0 & \gamma & * \\ \mathbb{R}^{\frac{1}{2}}(KX+S) & \mathbb{R}^{\frac{1}{2}}(K+D_cC) & 0 & 0 & 0 & \gamma \end{bmatrix} \geqslant 0. \tag{22}$$

(b, c, d) The same as those of Theorem 3.1.

*Proof:* In the following we will only prove (a) since (b, c, d) is the same as those in proof to Theorem 3.1.

Suppose that at time $k$ Lyapunov function is given as $V(i,j) = \xi(k+i|k)^T Q_j^{-1}(\xi k+i|k)$. Replace (11a) with

$$Q_j^{-1} - \mathcal{A}_j^T Q_j^{-1}\mathcal{A}_j \geqslant \frac{1}{\gamma}\begin{bmatrix} C & 0 \\ K+D_cC & C_c\end{bmatrix}^T \begin{bmatrix} \mathbb{Q} & 0 \\ 0 & \mathbb{R}\end{bmatrix} \begin{bmatrix} C & 0 \\ K+D_cC & C_c\end{bmatrix} \quad j = 0, 1, \ldots, l. \tag{23}$$

Then it follows that

$$V(i,j) - V(i+1,j) \geqslant \frac{1}{\gamma}\left(\|y(k+i|k)\|_{\mathbb{Q}} + \|u(k+i|k)\|_{\mathbb{R}}\right). \tag{24}$$

Sum (24) from $i = 0$ to $\infty$ and with $V(\infty, j) = 0$

$$1 \geqslant V(0,j) \geqslant \tfrac{1}{\gamma}\textstyle\sum_{i=0}^{\infty} \|y(k+i|k)\|_{\mathbb{Q}} + \|u(k+i|k)\|_{\mathbb{R}}, \quad \forall j = 0, 1, \ldots, l. \tag{25}$$

hence $\tilde{J}(k) \leqslant \gamma$.

By using Shur complement, (23) can be rewritten as

$$\begin{bmatrix} G+G^T-Q_j & * & * \\ \mathcal{A}_j G & Q_j & * \\ \begin{bmatrix} \mathbb{Q} & 0 \\ 0 & \mathbb{R}\end{bmatrix}^{\frac{1}{2}} \begin{bmatrix} C & 0 \\ K+D_cC & C_c\end{bmatrix} & 0 & \gamma \end{bmatrix} \geqslant 0. \tag{26}$$

Pre and post multiplying (26) by

$$\begin{bmatrix} Z^T & 0 & 0 \\ 0 & Z^T & 0 \\ 0 & 0 & Z^T \end{bmatrix}, \quad \begin{bmatrix} Z & 0 & 0 \\ 0 & Z & 0 \\ 0 & 0 & Z \end{bmatrix}$$

respectively yields (22). ∎

### C. Online realization

By augmenting the original state $x$ with predictive controller dynamics, we are able to ensure the ellipsoidal stability constraint at current time instant rather than at the end of the control horizon. This in turn reduces the online computational load although the optimality is sacrificed to some extent, which can be reduced to a negligible level by allowing a line search outside the ellipsoid ( [5], [13]).

The online computation is as follows

$$\min_{x_c} \quad x_c^T W_c x_c \tag{27}$$

subject to

$$\begin{bmatrix} 1 & * & * \\ x & \mathcal{Q}_{j,11} & * \\ x_c & \mathcal{Q}_{j,21} & \mathcal{Q}_{j,22} \end{bmatrix} \geqslant 0 \tag{28}$$

where $W_c$ is the predefined weight matrix of appropriate dimension and $\mathcal{Q}_{j,11}$, $\mathcal{Q}_{j,21}$ and $\mathcal{Q}_{j,22}$ are obtained from the offline optimization.

In either case of a unique pair of $A_c$, $B_c$ (LTI systems) or they are from a polytopic set, the MPC law takes form of

$$u(k) = (K + D_cC)x(k) + C_c x_c(k). \tag{29}$$

Overall, the feasible invariant set and the stabilizing $K$ render the closed loop system robust and asymptotically stable.

## IV. Numerical example

### A. Example 1

First let's consider an LTI model. A constrained double integrator used in [14], [15] is given as follows

$$A = \begin{bmatrix} 1 & T_s \\ 0 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} T_s^2 \\ T_s \end{bmatrix}, \quad C = [1 \quad 0],$$

where $T_s = 0.5$, and with input constraint

$$|u| \leqslant 1,$$

and state constraint

$$|x_2| \leqslant 1.$$

Other parameters are set as follows

$$\mathbb{Q} = 1, \quad \mathbb{R} = 1, \quad K = [-0.9653 \quad -1.3655].$$

Therefore for $l = 0$, solving (14) subject to (22), (13) and (15) gives the largest possible invariant set. The sizes of the invariant sets with corresponding $\gamma$ are given in Table I. For comparison purpose, sizes of the invariant sets obtained in [15] are also provided.

| $\gamma$ | $10^2$ | $10^3$ | $10^4$ | $10^5$ | $10^6$ |
|---|---|---|---|---|---|
| $\det(Y)^{\frac{1}{2}}$ in [15] | 1.4626 | 3.1863 | 6.8689 | 14.8059 | 31.9032 |
| $\det(Q_{xx})^{\frac{1}{2}}$ here | 1.4675 | 3.1990 | 6.9235 | 14.8788 | 32.0051 |

TABLE I
COMPARISON OF SIZE OF INVARIANT SETS

From the results given in Table I we can see that the proposed method can reduce the design conservativeness and improve the optimization efficiency. We should also note that the improvement here is slight (around 4‰). This is because that for LTI systems, only parameters $B_c$ and $D_c$ contribute to the improvement compared to formulation in [15]. When LTV systems are adopted, other slack variables such as $Q_j$ and $G$ will reduce the design conservativeness to a further extent.

### B. Example 2

Consider uncertain LTV system using in [15], where

$$\begin{aligned} A_0 &= \begin{bmatrix} 0.3 & 0 & -0.5 & -0.5 \\ 0 & 0 & 0.6 & -0.4 \\ -0.5 & 0.6 & 0.2 & 0.2 \\ -0.5 & -0.4 & 0.2 & -0.7 \end{bmatrix}, B_0 = \begin{bmatrix} 0.6 \\ 1.0 \\ 0 \\ -1.1 \end{bmatrix}, \\ A_1 &= \begin{bmatrix} 0.4 & 0.3 & 0 & 0 \\ 0.3 & 0.3 & 0.1 & 0.2 \\ 0 & 0.1 & 0.7 & -0.1 \\ 0 & 0.2 & -0.1 & 0.6 \end{bmatrix}, B_1 = \begin{bmatrix} 0.1 \\ 0.2 \\ -0.8 \\ 0 \end{bmatrix}, \\ C &= [0.8 \;\; 0 \;\; 0 \;\; 0]. \end{aligned}$$

With input constraint $|u| \leqslant 1$, the maximal invariant set has $(\det Q_{xx})^{\frac{1}{2}} = 899489$. Compared to $(\det Y)^{\frac{1}{2}} = 539376$ from [15], there is a $66.7\%$ improvement.

From the optimization result, we can see that the proposed approach significantly reduces the design conservativeness, resulting in a much larger invariant feasible set.

## V. Conclusion

This paper presents a new method which further optimizes prediction dynamics for robust model predictive control. Slack variables are used in order to reduce the design conservativeness. Nonlinear parameter transformation technique is employed, leading the optimization to a convex problem which can be solved efficiently by LMI solvers. Through a canonical example the effectiveness of the proposed approach is evaluated.

## VI. Acknowledgement

This work is supported by project “Design of Advanced Controllers for Economic, Robust and Safe Manufacturing Performance”.

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