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Relaxed Robust Stabilization of Nonlinear Systems with Parametric Uncertainties

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Abstract—**This paper addresses stability analysis and robust stabilization for nonlinear systems in the presence of parametric uncertainties. The Takagi-Sugeno (T-S) fuzzy model with parametric uncertainties is used as the model for the uncertain nonlinear system. Both continuous-time and discrete-time cases of the T-S fuzzy system are considered. In the two cases, sufficient conditions are proposed for robust stabilization in the sense of Lyapunov asymptotic stability, which are represented in the form of linear matrix inequalities. Finally, the T-S fuzzy model of the chaotic Lorenz system, which has complex nonlinearity, is developed as a simulation platform. The validity and applicability of the proposed approach are successfully demonstrated by means of the numerical simulation for the continuous-time nonlinear system.**

I. Introduction

It is well known that the most plants in the industry often have severe nonlinearity and uncertainties. So the stability analysis and synthesis of nonlinear systems is an important issue. Thus, the nonlinearity and uncertainties cause the additional difficulties to the control theory of general non-linear systems and the design of their controllers. In order to overcome these kinds of difficulties in the design of a controller for an uncertain nonlinear system, various methods have been developed in the last two decades. Among all the methods, a successful approach is fuzzy control. The fuzzy control technique represents a means of collecting human knowledge and expertise, and it has been applied to various industrial fields [1,2]. Recently, fuzzy control has attracted increasing attention, essentially because it can provide an effective solution to the controller design of the plants which are complex, uncertain, ill-defined, and have available qualitative knowledge from domain experts.

In spite of the usefulness of fuzzy control, there are still many basic issues that remain to be further addressed. Its main drawback comes from the lack of a systematic control design methodology. Particularly, the stability analysis of a fuzzy system is not easy, and the parameter tuning is generally a time-consuming procedure, due to the nonlinear and multi-parametric nature of the fuzzy control system. In order to resolve these problems, the idea that a linear system is adopted as the consequent part of a fuzzy rule has evolved into the innovative T-S fuzzy model. The T-S fuzzy system is proposed by Takagi and Sugeno in 1985 [3]. Recently, the T-S fuzzy model has become one of the useful control approaches for complex systems. It can provide an effective representation of complex nonlinear systems in terms of fuzzy sets [4]. Based on T-S fuzzy model, a great number of results appeared concerning stability analysis and design in the literature. The authors of [5-7] proved that the T-S fuzzy systems can approximate to any continuous functions in a compact set of R^n at any preciseness. A lot of nonlinear systems can be represented by T-S fuzzy systems and allows the designers to take advantage of conventional linear system to analyze and design fuzzy control systems. Originally, Tanaka and his colleagues provided a sufficient condition for the quadratic stability of the T-S fuzzy systems in the sense of Lyapunov by considering a common Lyapunov function of the sub fuzzy systems in a series of papers [8]. In reference [9], an interesting quadratic stabilization condition is reported to release the conservatism by collecting the interactions in a single matrix. Very recently, a more relaxed stabilization condition is proposed in [10]. It admits more freedom in guaranteeing the stability of T-S fuzzy control systems. In reference [11], a new LMI-based stabilization condition is obtained by relaxing the results in references before. A rigorous proof is given to show that the stabilization condition can include the interesting results published recently as special cases.

Although there are a lot references about the analysis of T-S model, most plants in the industry have uncertainties. So besides stability, another important requirement for a control system is its robustness, and this remains to be a central issue in the study of uncertain nonlinear control systems and the controller design. Kiriakidis [12] studied the issue of stability robustness against modeling errors in T-S fuzzy model-based control. Liu [13] provided the stability condition for T-S fuzzy systems with parametric uncertainties via a so-called fuzzy Lyapunov function which is a multiple Lyapunov function. Reference [13] deals with a robust design of fuzzy controllers for a class of uncertain nonlinear systems via a fuzzy Lyapunov function. In designing a robust fuzzy control system, the uncertain nonlinear systems are represented by T-S fuzzy model with parametric uncertainties. Lee [14] proposed some sufficient conditions in the linear matrix inequality (LMI) format and a systematic design procedure of

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the controller design for a general nonlinear system with parametric uncertainties, for both continuous-time and discrete-time T-S fuzzy systems. Specifically, some new solutions are proposed to the robust stabilization problem for a class of nonlinear systems with time-varying, but norm-bounded parametric uncertainties.

In this paper, we consider the problem of the robust fuzzy control for a class of uncertain nonlinear systems via a Lyapunov function. In the reference [14], the author divided the closed-loop system into two parts, and then considered each part as asymptotically stable. According to reference [11], this paper divided the closed-loop system into three parts. So the more interactions among the fuzzy subsystems should be considered. Then a more relaxed robust stabilization condition is obtained. The overall proposed design method presents a systematic and effective framework for continuous-time and discrete-time control of the complex systems such as chaotic systems.

The remainders of this paper are organized as follows. Section II gives the models of both the continuous-time and discrete-time cases. Section III presents the controller design method for robust stabilization of T-S fuzzy systems in the presence of parametric uncertainties in both continuous-time and discrete-time. In Section IV, we show a controller design examples and simulation results. Finally, we summarize our work in Section V.

II. PRELIMINARIES

In this section, two fuzzy models, continuous-time and discrete-time models, proposed by Takagi and Sugeno, are described by If-Then rules which represent local linear input-output relations of nonlinear systems. In order to consider uncertain nonlinear system described by T-S fuzzy model with parametric uncertainties, consider the continuous-time T-S fuzzy system in which the i th rule is formulated as follows.

Plant Rule i :

If $z_1(t)$ M_{i1} and …and $z_s(t)$ is M_{is}

Then $\dot{x}(t) = (A_i + \Delta A_i)x(t) + (B_i + \Delta B_i)u(t).$ (1)

where $M_{ij}(i = 1,2,\ldots,r, j = 1,2,\ldots,s)$ is the fuzzy set and r is the number of If-Then rules. $z_i(t)\,(i = 1,2\ldots,s)$ are the premise variables. $x(t) \in R^n$ is the state vector, $u(t) \in R^m$ is the input vector. Assume $A_i \in R^{n\times n}$ and $B_i \in R^{n\times m}$ are system matrix and input matrix respectively, ΔA_i and ΔB_i represent the parametric uncertainties. Given a pair of $(x(t),u(t))$, and using the center of gravity method for defuzzification, the final output of the fuzzy system is inferred as follows.

$$\dot{x}(t) = \sum_{i=1}^{r} h_i(z(t))[(A_i + \Delta A_i)x(t) + (B_i + \Delta B_i)u(t)], \quad (2)$$

where $\omega_i(z(t)) = \prod_{j=1}^{s} M_{ij}((z_j(t)),\ h_i(z(t)) = (\omega_i(z(t))/\sum_{i=1}^{r}\omega_i(z(t))),$ and $M_{ij}(z_j(t))$ is the grade of membership of $z_j(t)$ in M_{ij} and $\omega_i(z(t))$ represents the weight of the ith rule. It is easy to check that $h_i(z(t)) \ge 0,\ i = 1,2\ldots,r$ and $\sum_{i=1}^{r} h_i(z(t)) = 1.$

Then, the state feedback controller for the continuous-time T-S fuzzy system is expressed as

Controller Rule i :

If $z_1(t)$ is M_{i1} and $z_2(t)$ is $M_{i2},\cdots, z_s(t)$ is M_{is}

Then $u(t) = K_i x(t),$ (3)

where, $K_i\ (i = 1,2\ldots,r)$, are the constant control gains to be determined. The designed fuzzy controller shares the same fuzzy sets in the premise parts with the plant and has local linear controllers in the consequent parts.

The output of fuzzy state feedback controller can be represented by

$$u(t) = \sum_{i=1}^{r} h_i(z(t))K_i x(t). \quad (4)$$

Similarly to the continuous-time case, the discrete-time T-S fuzzy model and the corresponding model-based state-feedback controller are constructed as follows.

Plant Rule i :

If $z_1(k)$ is M_{i1} and…and $z_s(k)$ is M_{is}

Then $x(t+1) = \sum_{i=1}^{r} h_i(z(k))(G_i + \Delta G_i)x(k) + (H_i + \Delta H_i)u(k).$ (5)

Controller Rule i :

If $z_1(k)$ is M_{i1} and…and $z_s(k)$ is M_{is}

Then $u(k) = K_i x(k),$ (6)

where k and $k+1$ denote the indexes of the time steps.

Because of the uncertain matrices, it is not easy to design the controller gain matrices. In order to find these gain matrices K_i , the uncertain matrices should be removed. Therefore, we assume, as usual, that the uncertain matrices ΔA_i and ΔB_i are admissibly norm-bounded and structured.

Assumption 1: The parameter uncertainties considered here are norm-bounded, i.e. they are in the form

$$\begin{bmatrix}\Delta A_i(t) & \Delta B_i(t)\end{bmatrix} = D_i F_i(t)[E_{1i} \quad E_{2i}],$$

where D_i , E_{1i} and E_{2i} are known real constant matrices of appropriate dimensions, $F_i(t)$ is an unknown matrix function with Lebesgue-measurable elements satisfying $F_i^T(t)F_i(t) \le I$, in which I is the identity matrix of appropriate dimension.

Lemma 1: Given constant matrices D and E and a symmetric constant matrix S of appropriate dimensions, the following inequality holds $S + DFE + E^T F^T D^T < 0$, where F satisfies $F^T F \le R$, if and only if for some $\alpha > 0$,

$$S + \begin{bmatrix}\alpha^{-1}E^T & \alpha D\end{bmatrix}\begin{bmatrix}R & 0\\ 0 & I\end{bmatrix}\begin{bmatrix}\alpha^{-1}E\\ \alpha D^T\end{bmatrix} < 0 .$$

III. ROBUST STABILIZATION OF THE T-S FUZZY MODEL

In this section, two kinds of stabilization conditions are presented. The sufficient conditions have guaranteed the global asymptotic stability of the controlled T-S fuzzy system with parametric uncertainties. One is fuzzy state feedback stabilization problem for continuous-time system. The other is state feedback stabilization issue for discrete-time models.

A. The stabilization for continuous-time fuzzy model

Consider a continuous-time T-S fuzzy model, with parametric uncertainties, described by the state-space equation (2). The objective is to design a T-S fuzzy model based state feedback controller for robust stabilization of the system (2) in the form of (4).

By substituting (4) into (2), the corresponding closed-loop fuzzy system can be represented as

$$\dot{x}(t)=\sum_{i=1}^{r}\sum_{j=1}^{r}h_i(z(t))h_j(z(t))(A_i+\Delta A_i+(B_i+\Delta B_i)K_j)x(t). \quad (7)$$

The closed-loop system of (7) is found as follows.

$$\begin{aligned}\dot{x}(t)=&\sum_{i=1}^{r}h_i^3(A_i+\Delta A_i+(B_i+\Delta B_i)K_i)x(t)\\&+\sum_{i=1}^{r}\sum_{\substack{j=1\\j\neq i}}^{r}h_i^2h_j(2(A_i+\Delta A_i)+A_j+\Delta A_j+(B_i+\Delta B_i)(K_i+K_j)\\&+(B_i+\Delta B_i)K_i)x(t)\\&+\sum_{i=1}^{r-2}\sum_{j=i+1}^{r-1}\sum_{l=j+1}^{r}h_ih_jh_l(2(A_i+A_j+A_l+\Delta A_i+\Delta A_j+\Delta A_l)\\&+(B_i+\Delta B_i)(K_i+K_j)+(B_l+\Delta B_l)(K_i+K_j)\\&+(B_j+\Delta B_j)(K_i+K_l)+(B_i+\Delta B_i)(K_j+K_l))x(t) \qquad (8)\end{aligned}$$

The main result on the global asymptotic stability of the continuous-time T-S fuzzy model with parametric uncertainties is summarized in the following theorem. New stabilization condition is expressed in terms of LMIs.

Theorem 1: Consider system (7), if there exist a symmetric and positive definite matrix P, some matrices K_i, and some positive scalars α_{ijl} $(i,j,l=1,2\ldots,r)$ such that the following LMIs are satisfied, then the continuous-time T-S fuzzy system (7) is asymptotically stabilizable via the T-S fuzzy model-based state feedback controller (4).

$$\begin{bmatrix}\psi_{iii} & * & * \\ E_{1i}Q+E_{2i}N_i & -\alpha_{iii}I & * \\ D_i^T & 0 & -\alpha_{iii}^{-1}I\end{bmatrix}<0,\ i=1,2,\ldots,r, \qquad (9)$$

$$\begin{bmatrix}\gamma_{iij} & * & * & * & * & * & * \\ E_{1i}Q+E_{2i}N_i & -\alpha_{iii}I & * & * & * & * & * \\ E_{1i}Q+E_{2i}N_j & 0 & -\alpha_{iij}I & * & * & * & * \\ E_{1j}Q+E_{2j}N_i & 0 & 0 & -\alpha_{ijj}I & * & * & * \\ D_i^T & 0 & 0 & 0 & -\alpha_{iii}^{-1}I & * & * \\ D_i^T & 0 & 0 & 0 & 0 & -\alpha_{iij}^{-1}I & * \\ D_j^T & 0 & 0 & 0 & 0 & 0 & -\alpha_{ijj}^{-1}I\end{bmatrix}<0,$$

$$i=1,2,\ldots,r,\ j\neq i,\ j=1,2,\ldots,r, \qquad (10)$$

$$\begin{bmatrix}\Lambda & * \\ \Pi & \mathrm{N}\end{bmatrix}<0,\ i=1,2,\ldots,r-2,\ j=i+1,\ldots,r-1,\ l=j+1,\ldots,r, \quad (11)$$

where $\psi_{iii}=QA_i^T+A_iQ+N_i^TB_i^T+B_iN_i$,

$$\gamma_{iij}=2QA_i^T+2A_iQ+QA_j^T+A_jQ+N_i^TB_i^T+B_iN_i+N_j^TB_i^T+B_iN_j+N_i^TB_j^T+B_jN_i,$$

$$\Lambda=\begin{bmatrix}\Xi_{ijl} & * & * & * & * & * & * \\ E_{1i}+E_{2i}N_j & -\alpha_{iij}I & * & * & * & * & * \\ E_{1i}+E_{2i}N_l & 0 & -\alpha_{iil}I & * & * & * & * \\ E_{1j}+E_{2j}N_i & 0 & 0 & -\alpha_{ijj}I & * & * & * \\ E_{1j}+E_{2j}N_l & 0 & 0 & 0 & -\alpha_{jjl}I & * & * \\ E_{1l}+E_{2l}N_i & 0 & 0 & 0 & 0 & -\alpha_{ill}I & * \\ E_{1l}+E_{2l}N_j & 0 & 0 & 0 & 0 & 0 & -\alpha_{jll}I\end{bmatrix},$$

$$\begin{aligned}\Xi_{ijl}=&2QA_i^T+2A_iQ+2QA_j^T+2A_jQ+2QA_l^T+2A_lQ+N_j^TB_i^T+B_iN_j\\&+N_l^TB_i^T+B_iN_l+N_i^TB_j^T+B_jN_i+N_l^TB_j^T+B_jN_l+N_i^TB_l^T\\&+B_lN_i+N_j^TB_l^T+B_jN_i,\end{aligned}$$

$$\mathrm{N}=\begin{bmatrix}\alpha_{iij}^{-1}I & * & * & * & * & * \\ 0 & \alpha_{iil}^{-1}I & * & * & * & * \\ 0 & 0 & \alpha_{ijj}^{-1}I & * & * & * \\ 0 & 0 & 0 & \alpha_{jjl}^{-1}I & * & * \\ 0 & 0 & 0 & 0 & \alpha_{ill}^{-1}I & * \\ 0 & 0 & 0 & 0 & 0 & \alpha_{jll}^{-1}I\end{bmatrix},$$

$$\Pi=\begin{bmatrix}D_i^T & 0 & 0 & 0 & 0 & 0 \\ D_i^T & 0 & 0 & 0 & 0 & 0 \\ D_j^T & 0 & 0 & 0 & 0 & 0 \\ D_j^T & 0 & 0 & 0 & 0 & 0 \\ D_l^T & 0 & 0 & 0 & 0 & 0 \\ D_l^T & 0 & 0 & 0 & 0 & 0\end{bmatrix}.$$

Moreover, in this case, the fuzzy local state feedback gains are $K_j=N_jQ^{-1}, j=1,2,\ldots,r$ and $Q=P^{-1}$, where $*$ denotes the transposed elements in the symmetric positions.

Proof: For system (7), choose the Lyapunov function

$$V(x(t))=x(t)^TPx(t), \qquad (12)$$

where $P>0$ is to be selected symmetric matrix. Then $V(x(t))$ is positive definite. The time derivative of this function along the trajectory of system in equation (12) is given by

$$\dot{V}(x(t))=\dot{x}^T(t)Px(t)+x^T(t)P\dot{x}(t). \qquad (13)$$

Substituting (8) into equation (13) we get

$$\begin{aligned}\dot{V}(x(t))=&\sum_{i=1}^{r}h_i^3x(t)^T((A_i+\Delta A_i+(B_i+\Delta B_i)K_i)^TP\\&+P(A_i+\Delta A_i+(B_i+\Delta B_i)K_i))x(t)\\&+\sum_{i=1}^{r}\sum_{\substack{j=1\\j\neq i}}^{r}h_i^2h_jx(t)^T(\omega^TP+P\omega)x(t)\\&+\sum_{i=1}^{r-2}\sum_{j=i+1}^{r-1}\sum_{l=j+1}^{r}h_ih_jh_lx(t)^T(\xi^TP+P\xi)x(t) \qquad (14)\end{aligned}$$

where

$$\omega=2(A_i+\Delta A_i)+A_j+\Delta A_j+(B_i+\Delta B_i)(K_i+K_j)+(B_i+\Delta B_i)K_i$$

$$\begin{aligned}\xi=&2(A_i+A_j+A_l+\Delta A_i+\Delta A_j+\Delta A_l)+(B_i+\Delta B_i)(K_i+K_j)+(B_l\\&+\Delta B_l)(K_i+K_j)+(B_j+\Delta B_j)(K_i+K_l)+(B_i+\Delta B_i)(K_j+K_l).\end{aligned}$$

If the time derivative of (13) is negative definite uniformly for all $x(t)$ and for all $t>0$, then the controlled fuzzy system (8) is asymptotically stable. Therefore, if assume each sum of the equation (14) to be negative definite, then controlled continuous-time T-S fuzzy system is asymptotically stable.

First, consider that the first sum of the last equation in (14) is negative definite

$$(A_i+\Delta A_i+(B_i+\Delta B_i)K_i)^T P+P(A_i+\Delta A_i+(B_i+\Delta B_i)K_i)<0,\quad i=1,2\ldots,r. \tag{15}$$

Using Assumption 1 to the (15) can be rewritten as

$$\Phi_{iii}+PD_iF_i(t)(E_{1i}+E_{2i}K_i)+\Delta A_i+(B_i+\Delta B_i)K_i)^T +P(A_i+\Delta A_i-B_iK_i)<0, \tag{16}$$

where $\Phi_{iii}=A_i^TP+PA_i+K_i^TB_i^TP+PB_iK_i$.

According to Lemma 1, the matrix inequality (16) satisfying $F_i^T(t)F_i(t)\le I$, if and only if there exists a constant $\alpha_{iii}^{1/2}$ such that

$$\Phi_{iii}+[\alpha_{iii}^{-1/2}(E_{1i}+E_{2i}K_i)^T\quad \alpha_{iii}^{1/2}PD_i]\times\begin{bmatrix}\alpha_{iii}^{-1/2}(E_{1i}+E_{2i}K_i)\\ \alpha_{iii}^{1/2}(PD_i)^T\end{bmatrix}$$

$$=\Phi_{iii}+[(E_{1i}+E_{2i}K_i)^T\quad PD_i]\times\begin{bmatrix}\alpha_{iii}^{-1}I & 0\\ 0 & \alpha_{iii}I\end{bmatrix}\times\begin{bmatrix}E_{1i}+E_{2i}K_i\\ PD_i\end{bmatrix}<0 \tag{17}$$

Then Applying the Schur complement to (17) we get

$$\begin{bmatrix}\Phi_{iii} & * & *\\ E_{1i}+E_{2i}K_i & -\alpha_{iii}I & *\\ D_i^TP & 0 & -\alpha_{iii}^{-1}I\end{bmatrix}<0. \tag{18}$$

The matrix inequality as above is not an LMI. In order to use the convex optimization technique, (18) must be converted to an LMI via some variable changes or transformations. Define the transformation matrix and take a congruence trans- formation. We get

$$\begin{bmatrix}P^{-1} & 0 & 0\\ 0 & I & 0\\ 0 & 0 & I\end{bmatrix}\begin{bmatrix}\Phi_{iii} & * & *\\ E_{1i}+E_{2i}K_i & -\alpha_{iii}I & *\\ D_i^TP & 0 & -\alpha_{iii}^{-1}I\end{bmatrix}\times\begin{bmatrix}P^{-1} & 0 & 0\\ 0 & I & 0\\ 0 & 0 & I\end{bmatrix}^T$$

$$=\begin{bmatrix}P^{-1}\Phi_{iii}P^{-1} & * & *\\ E_{1i}P^{-1}+E_{2i}K_iP^{-1} & -\alpha_{iii}I & *\\ D_i^T & 0 & -\alpha_{iii}^{-1}I\end{bmatrix}<0. \tag{19}$$

Letting $Q=P^{-1}$ and $K_i=N_iQ^{-1}, i=1,2,\ldots,r$ get the first LMI (9) in Theorem 1.

Similarly, consider the second sum of equation. Assume

$$\omega^TP+P\omega<0, \tag{20}$$

Then, using Assumption 1, (20) can be represented as

$$\Theta_{iij}+\begin{bmatrix}PD_i & PD_i & PD_j\end{bmatrix}\begin{bmatrix}F_i(t) & 0 & 0\\ 0 & F_i(t) & 0\\ 0 & 0 & F_j(t)\end{bmatrix}\begin{bmatrix}E_{1i}+E_{2i}K_i\\ E_{1i}+E_{2i}K_j\\ E_{1j}+E_{2j}K_i\end{bmatrix}$$

$$+\begin{bmatrix}E_{1i}+E_{2i}K_i\\ E_{1i}+E_{2i}K_j\\ E_{1j}+E_{2j}K_i\end{bmatrix}^T\begin{bmatrix}F_i(t) & 0 & 0\\ 0 & F_i(t) & 0\\ 0 & 0 & F_j(t)\end{bmatrix}^T\begin{bmatrix}PD_i & PD_i & PD_j\end{bmatrix}^T<0, \tag{21}$$

where

$$\Theta_{iij}=A_i^TP+PA_i+A_j^TP+PA_j+K_j^TB_i^TP+PB_iK_j+K_i^TB_j^TP+PB_jK_i,$$

Using Lemma 1 repeatedly, the matrix inequality (21) holds for all $F_i(t)$ satisfying

$$\begin{bmatrix}F_i(t) & 0 & 0\\ 0 & F_i(t) & 0\\ 0 & 0 & F_j(t)\end{bmatrix}^T\begin{bmatrix}F_i(t) & 0 & 0\\ 0 & F_i(t) & 0\\ 0 & 0 & F_j(t)\end{bmatrix}\le I,$$

if and only if there exists a constant such that

$$\Theta_{iij}+\begin{bmatrix}(E_{1i}+E_{2i}K_i)^T & (E_{1i}+E_{2i}K_j)^T & (E_{1j}+E_{2j}K_i)^T & PD_i & PD_i & PD_j\end{bmatrix}$$

$$\times\begin{bmatrix}\alpha_{iii}^{-1}I & 0 & 0 & 0 & 0 & 0\\ 0 & \alpha_{iij}^{-1}I & 0 & 0 & 0 & 0\\ 0 & 0 & \alpha_{iij}^{-1}I & 0 & 0 & 0\\ 0 & 0 & 0 & -\alpha_{iii}I & 0 & 0\\ 0 & 0 & 0 & 0 & -\alpha_{iij}I & 0\\ 0 & 0 & 0 & 0 & 0 & -\alpha_{iij}I\end{bmatrix}\begin{bmatrix}E_{1i}+E_{2i}K_i\\ E_{1i}+E_{2i}K_j\\ E_{1j}+E_{2j}K_i\\ (PD_i)^T\\ (PD_i)^T\\ (PD_j)^T\end{bmatrix}<0. \tag{22}$$

Applying Schur complement to (22) and taking the congruence transformation with $\begin{bmatrix}P & I & I & I & I & I\end{bmatrix}$ easily obtained

$$\begin{bmatrix}P^{-1}\Theta_{iij}P^{-1} & * & * & * & * & * & *\\ E_{1i}P^{-1}+E_{2i}K_iP^{-1} & -\alpha_{iii}I & * & * & * & * & *\\ E_{1i}P^{-1}+E_{2i}K_jP^{-1} & 0 & -\alpha_{iij}I & * & * & * & *\\ E_{1j}P^{-1}+E_{2j}K_iP^{-1} & 0 & 0 & -\alpha_{iij}I & * & * & *\\ D_i^T & 0 & 0 & 0 & -\alpha_{iii}^{-1}I & * & *\\ D_i^T & 0 & 0 & 0 & 0 & -\alpha_{iij}^{-1}I & *\\ D_j^T & 0 & 0 & 0 & 0 & 0 & -\alpha_{iij}^{-1}I\end{bmatrix}<0 \tag{23}$$

Denoting $Q=P^{-1}$ and $K_i=N_iQ^{-1}, i=1,2,\ldots,r$ have the second LMI (10).

Assume the third sum of equation

$$\xi^TP+P\xi<0, \tag{24}$$

Using Assumption 1, Lemma 1 and applying the Schur complement to (24) repeatedly, then (11) can be obtained. Thus $\dot V(x(t))<0$, if (15), (20) and (24) are satisfied. This completes the proof of the theorem.

B. The stabilization for discrete-time fuzzy model

In this section, we solves with the controller design problem for the discrete-time T-S fuzzy model with parametric uncertainties. The overall closed-loop fuzzy can be described as follows.

$$x(k+1)=\sum_{i=1}^{r}\sum_{j=1}^{r}h_i(z(k))h_j(z(k))(G_i+\Delta G_i+(H_i+\Delta H_i)K_j)x(k) \tag{25}$$

Theorem 2: Consider system (25). If there exist a symmetric and positive definite matrix P, some matrices α_{ijl}, $(i,j,l=1,2\ldots,r)$ such that the following LMIs are satisfied, then the discrete-time T-S fuzzy system (5) is asymptotically stabilizable via the T-S fuzzy model-based state-feedback controller (6).

$$\begin{bmatrix}-Q & * & * & *\\ G_iQ+H_iN_i & -Q & * & *\\ E_{1i}Q+E_{2i}N_i & 0 & -\alpha_{iii}I & *\\ 0 & D_i^T & 0 & -\alpha_{iii}^{-1}I\end{bmatrix}<0,\ i=1,2,\ldots,r, \tag{26}$$

$$\begin{bmatrix}\Sigma & *\\ \mathrm{O} & \mathrm{T}\end{bmatrix}<0,\ i=1,2,\ldots,r-2, j=i+1,\ldots,r-1, l=j+1,\ldots,r \tag{27}$$

$$\begin{bmatrix} -3Q & * & * & * & * & * & * & * \\ \mathrm{M}_{iij} & -3Q & * & * & * & * & * & * \\ G_iQ+H_iN_i & 0 & -\alpha_{iii}I & * & * & * & * & * \\ E_{1i}Q+E_{2i}N_j & 0 & 0 & -\alpha_{iij}I & * & * & * & * \\ E_{1j}Q+E_{2j}N_i & 0 & 0 & 0 & -\alpha_{ijj}I & * & * & * \\ 0 & D_i^T & 0 & 0 & 0 & -\alpha_{iii}^{-1}I & * & * \\ 0 & D_i^T & 0 & 0 & 0 & 0 & -\alpha_{iij}^{-1}I & * \\ 0 & D_j^T & 0 & 0 & 0 & 0 & 0 & -\alpha_{ijj}^{-1}I \end{bmatrix} < 0,$$

$$i = 1,2,\ldots,r,\ j \neq i,\ j = 1,2,\ldots,r \tag{28}$$

where

$$\Sigma = \begin{bmatrix} -6Q & * & * & * & * & * & * & * \\ \mathrm{E}_{ijl} & -6Q & * & * & * & * & * & * \\ E_{1i}+E_{2i}N_j & 0 & -\alpha_{iij}I & * & * & * & * & * \\ E_{1i}+E_{2i}N_l & 0 & 0 & -\alpha_{iil}I & * & * & * & * \\ E_{1j}+E_{2j}N_i & 0 & 0 & 0 & -\alpha_{ijj}I & * & * & * \\ E_{1j}+E_{2j}N_l & 0 & 0 & 0 & 0 & -\alpha_{jjl}I & * & * \\ E_{1l}+E_{2l}N_i & 0 & 0 & 0 & 0 & 0 & -\alpha_{ill}I & * \\ E_{1l}+E_{2l}N_j & 0 & 0 & 0 & 0 & 0 & 0 & -\alpha_{jll}I \end{bmatrix},$$

$\mathrm{M}_{iij} = 2G_iQ + H_iN_i + H_iN_j + G_jQ + H_jN_i$,

$\mathrm{E}_{ijl} = 2G_iQ + 2G_jQ + 2G_lQ + H_iN_j + H_iN_l + H_jN_i + H_jN_l + H_lN_i + H_jN_i$,

$$\mathrm{O} = \begin{bmatrix} 0 & D_i^T & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & D_i^T & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & D_j^T & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & D_j^T & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & D_l^T & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & D_l^T & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\mathrm{T} = \begin{bmatrix} \alpha_{iij}^{-1}I & * & * & * & * & * \\ 0 & \alpha_{iil}^{-1}I & * & * & * & * \\ 0 & 0 & \alpha_{ijj}^{-1}I & * & * & * \\ 0 & 0 & 0 & \alpha_{jjl}^{-1}I & * & * \\ 0 & 0 & 0 & 0 & \alpha_{ill}^{-1}I & * \\ 0 & 0 & 0 & 0 & 0 & \alpha_{jll}^{-1}I \end{bmatrix}.$$

Moreover, in this case, the fuzzy local state feedback gains are $K_j = N_jQ^{-1}, j = 1,2,\ldots,r$ and $Q = P^{-1}$, where $*$ denotes the transposed elements in the symmetric positions.

Proof: For system (25), we choose the following discrete Lyapunov function

$$V(x(k)) = x(k)^T Px(k), \tag{29}$$

which is positive definite. By evaluating the difference of the $V(x(k))$.

$$\begin{aligned} \Delta V(x(k)) &= V(x(k+1)) - V(x(k)) \\ &= \sum_{i=1}^{r} h_i^3 x^T(k) \begin{bmatrix} -P & * \\ P((G_i+\Delta G_i)+(H_i+\Delta H_i)K_j) & -P \end{bmatrix} x(k) \\ &+ \sum_{i=1}^{r}\sum_{\substack{j=1 \\ j\neq i}}^{r} h_i^2 h_j x^T(k) \begin{bmatrix} -3P & * \\ \Omega & -3P \end{bmatrix} x(k) \\ &+ \sum_{i=1}^{r-2}\sum_{j=i+1}^{r-1}\sum_{l=j+1}^{r} h_i h_j h_l x^T(k) \begin{bmatrix} -6P & * \\ \mathrm{Z} & -6P \end{bmatrix} x(k), \end{aligned} \tag{30}$$

where

$\Omega = 2P(G_i + \Delta G_i) + P(G_j + \Delta G_j) + (H_i + \Delta H_i)K_i + (H_i + \Delta H_i)K_j + (H_j + \Delta H_j)K_i$,

$\mathrm{Z} = 2P(G_i + \Delta G_i + G_j + \Delta G_j + G_l + \Delta G_l) + (H_i + \Delta H_i)(K_j + K_l) + (H_j + \Delta H_j)(K_i + K_l) + (H_l + \Delta H_l)(K_i + K_j)$.

Therefore, if the three sums in (30) are all uniformly negative definite, $\Delta V(x(k))$ is negative definite so the system is asymptotically stable. The proof applied the same method as in theorem 1, so the process of derivation is omitted.

IV. Simulation Results

In this section, we simulate the control of the chaotic Lorenz system with parametric uncertainties. The control objective is to show the effectiveness of the proposed robust stabilization technique.

The Lorenz equations are as follows

$$\frac{d}{dt}\begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} = \begin{bmatrix} -\sigma x_1(t) + \sigma x_2(t) \\ rx_1(t) - x_2(t) - x_1(t)x_3(t) \\ x_1(t)x_2(t) - bx_3(t) \end{bmatrix}. \tag{31}$$

The nonlinear system (30) is exactly represented by the following T-S fuzzy model.

Plant Rule 1: If $x_1(t)$ is about M_1

Then $\dot{x}(t) = (A_1 + \Delta A_1)x(t) + (B_1 + \Delta B_1)u(t)$,

Plant Rule 2: If $x_1(t)$ is about M_2

Then $\dot{x}(t) = (A_2 + \Delta A_2)x(t) + (B_2 + \Delta B_2)u(t)$.

The membership functions for the plant rules are shown in Fig. 1.

$$\Gamma_1^1(x_1(t)) = \frac{-x_1(t) + M_2}{M_2 - M_1}, \Gamma_1^2(x_2(t)) = \frac{-x_1(t) - M_1}{M_2 - M_1},$$

$$A_1 = \begin{bmatrix} -\sigma & \sigma & 0 \\ r & -1 & -M_1 \\ 0 & M_1 & -b \end{bmatrix}, A_2 = \begin{bmatrix} -\sigma & \sigma & 0 \\ r & -1 & -M_2 \\ 0 & M_2 & -b \end{bmatrix},$$

The input matrices B_1 and B_2 are as arbitrarily chosen as bellow which guarantees the system controllability. ΔB_1 and ΔB_2 are chosen as follows.

$$B_1 = B_2 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \Delta B_1 = \Delta B_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

In the simulation, the nominal values (σ, r, b) of are (10, 28, 8/3) for chaos to emerge. Assume all parameters uncertain bounded within 30% of their nominal values. Based on Assumption 1, we define

$$D_1 = D_2 = \begin{bmatrix} -0.3 & 0 & 0 \\ 0 & 0.3 & 0 \\ 0 & 0 & 0.3 \end{bmatrix}, E_{11} = E_{12} = \begin{bmatrix} -\sigma & \sigma & 0 \\ r & 0 & 0 \\ 0 & 0 & b \end{bmatrix},$$

$$E_{21} = E_{22} = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^T.$$

The control objective is to make the chaotic system be stabilized. By applying Theorem 1 and using LMI toolbox in the matlab, we obtain

$$K_1 = [114.70429 \quad 68.07884 \quad 10.21541],$$
$$K_2 = [62.32490 \quad 38.61411 \quad -10.48324].$$

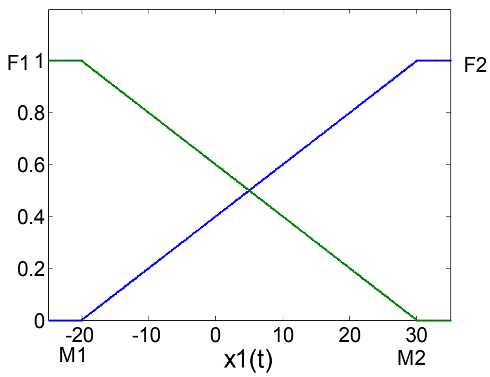

Fig. 1. The membership functions for the T-S fuzzy model of the Lorenz system.

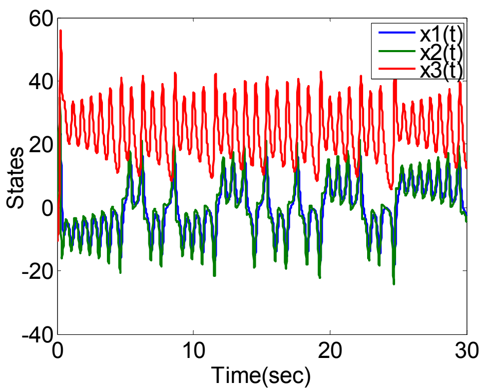

Fig. 2. The time response of states.

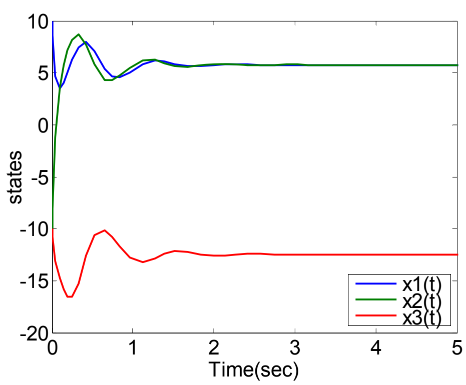

Fig. 3. The time response of state of the controlled system.

The response of uncontrolled states x_1, x_2, x_3 for the chaotic Lorenz system is show in Fig. 2. The control result, for the nonlinear continuous-time system with parametric uncertainties, is shown in Fig. 3. The simulation results show that the T-S fuzzy model-based state-feedback controller not only can stabilize the nonlinear systems, but also has strong robustness against admissible norm-bounded parametric uncertainties.

V. CONCLUSION

In this paper, we have developed and analyzed the controller for both continuous-time and discrete-time T-S fuzzy models, and we have proposed a new robust fuzzy controller for T-S fuzzy models with parametric uncertainties. Based on the Lyapunov function approach, we obtained some robust stability conditions in the linear matrix inequality format and a systematic design procedure for the controller design of a general nonlinear system with parametric uncertainties. The designed controller can globally asymptotically stabilize the closed-loop T-S fuzzy system subject to all admissible parametric uncertainties. The design scheme was applied to the stabilizing control of the Lorenz system. Simulation results show the effectiveness of the new approach in controlling nonlinear systems with parametric uncertainties.

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