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Indirect Adaptive Control for Systems with Input Rate Saturation

Nazli E. Kahveci, Petros A. Ioannou

***Abstract*—An indirect adaptive control design with anti-windup augmentation for stable systems subject to input magnitude saturation is extended to address systems with significant input rate constraints. We use the method of interpreting the dynamics of the actuator, the plant and the unconstrained controller such that either the Linear Matrix Inequality (LMI) or the Riccati based anti-windup compensator design approaches for systems with input magnitude limitations can be exploited. We combine the control structure including the anti-windup compensator with an adaptive law and show that the stable performance of the nominal design is preserved in the presence of input rate saturation despite the unknown plant parameters. The proposed indirect adaptive control design methodology can be employed to recover the unconstrained tracking performance of plants with large parametric uncertainties by suppressing the adverse effects of dominant input rate saturation.**

I. Introduction

ALTHOUGH input rate saturation nonlinearity is considered to be relatively less common than input magnitude saturation, input rate constraints are also observed in many practical control systems.

The effects of the magnitude and rate limits of the actuators on the performance of a combustion chamber pressure control system are simulated in [1]. The dominating effect of rate saturation in the control of jet engine compressors is recognized in [2]. The magnitude and rate limits in practical implementation of bleed valves as actuators for active stall control in aircraft engines are discussed in [3].

The time delays in an aircraft control system can sharply degrade the handling qualities. [4] The destabilizing effects of the time delays associated with rate limiting phenomena are considered in [5].

The actuator rate saturation in aircraft control is known to lead Pilot Induced Oscillations (PIOs) which in the recent past have been recognized as the cause of the YF-22A prototype advanced tactical fighter crash in April, 1992 [6] and subsequent JAS 39 Gripen fighter crash at an air show in August, 1992 [7], displaying the significance of addressing actuator rate saturation in order to avoid relevant unstable oscillations and the risk of crash landing.

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Nazli E. Kahveci was with the Electrical Engineering Department, University of Southern California, Los Angeles, CA 90089 USA. She is now with Ford Research and Advanced Engineering, Ford Motor Company, Dearborn, MI 48121 USA. nkahveci@ford.com

Petros A. Ioannou is a Professor in the Electrical Engineering Department, University of Southern California, Los Angeles, CA 90089 USA. ioannou@usc.edu

In the quest for high performance, or in the event of a control surface failure, it is reasonable to expect additional complexities if the rate saturation is dominant. [8], [9] Several control methodologies to handle input magnitude and rate constraints can be found in [10]–[13].

Recently considerable attention has been focused on handling input magnitude saturation through anti-windup compensation. The rate saturation nonlinearity, which remains inadequately addressed in control design, has also been an active research topic for the anti-windup community in the past decade. [14]–[20] Input magnitude saturation of plants with uncertain models has been taken into account in developing anti-windup schemes in [21], [22], and the robustness of an LMI based linear anti-windup design addressing input magnitude constraints is examined in [23]. The robustness characteristics of anti-windup design methodologies which can handle input rate saturation and permit large parametric uncertainties are yet to be investigated.

The control of input magnitude constrained systems with unknown plant parameters is addressed using an LMI based adaptive anti-windup augmentation in [24]. The stability analysis of the respective design is provided in [25].

In this paper we propose an adaptive control structure with anti-windup compensation to remedy the rate saturation nonlinearities imposed on the control input, in particular if plants with unknown parameters are involved. The emphasis is on the mathematical formulation of the control problem under investigation and the stability properties of the proposed scheme. Due to the space constraints the corresponding simulation results are deferred to subsequent publications of the authors.

The rate-limiter is introduced as an input nonlinearity in Section II where the actuator dynamics are absorbed partly into the plant and partly into the unconstrained controller chosen as a Linear Quadratic (LQ) design for the desired state tracking performance. In the same section a Riccati based anti-windup compensator is constructed based on the augmented nominal system. The plant parameters are assumed to be unknown in Section III, and an indirect adaptive control design is developed using the Certainty Equivalence Principle and the estimates of the linear plant parameters. Section IV presents a detailed stability analysis. Our conclusions appear in Section V.

II. Input Rate Saturation

A rate-limited actuator can be modeled as a first-order lag and a symmetric rate-limiting nonlinearity as shown in the block diagram in Figure 1. The time constant of the lag,

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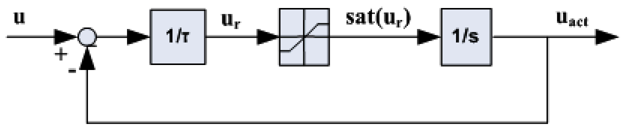

Fig. 1. A rate-limiter with first-order lag

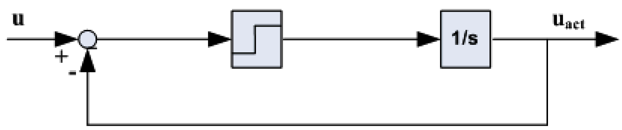

Fig. 2. An infinitely fast actuator model

τ determines the cut-off frequency of the equivalent linear actuator. [20] If the actuator dynamics are very fast, τ is very small, and the system can be modeled as an ideal relay instead of a saturation element. A pure rate limiter is shown in Figure 2. [5]

We consider a linear plant of order n with a scalar input $u_{act} \in \mathcal{R}$. The state vector $x \in \mathcal{R}^n$ is also defined as the output. The system matrices are thus $(A_p, B_p, C_p, 0)$, $A_p \in \mathcal{R}^{n\times n}, B_p \in \mathcal{R}^n, C_p \in \mathcal{R}^{n\times n}$. Since C_p is an identity matrix, the plant transfer function matrix can be written as

$$G(s)=(sI - A_p)^{-1}B_p \tag{1}$$

For any constant reference signal r the state tracking error

$$e = x - r \tag{2}$$

can be regulated to zero by solving the associated Algebraic Riccati Equation (ARE) and constructing the LQ controller

$$u = -K_1 e - K_2 \int_0^t e(\tau)d\tau \tag{3}$$

with parameters $K_1 \in \mathcal{R}^{1\times n}$ and $K_2 \in \mathcal{R}^{1\times n}$.

The control input rate is $sat(u_r)$ which can be defined using the saturation nonlinearity as

$$sat(u_r) = sign(u_r)\, min\,(\,|u_r|, \bar{u}_r\,) \tag{4}$$

where $\bar{u}_r$ is the rate saturation level posed at the actuator such that $\bar{u}_r > 0$, $\bar{u}_r \in \mathcal{R}$.

In the presence of input rate limitations the system can be summarized as depicted in Figure 3.

We follow the absorption method in [20] and subsume the time constant into the controller and the integrator block into the linear plant. The augmented plant, which is the original one modified by absorbing the integrator in the actuator dynamics, has the transfer function matrix:

$$\bar{G}(s) = \bar{C}_p(sI - \bar{A}_p)^{-1}\bar{B}_p \tag{5}$$

where

$$\bar{A}_p = \begin{bmatrix} A_p & B_p \\ 0 & 0 \end{bmatrix}, \quad \bar{B}_p = \begin{bmatrix} 0 \\ I \end{bmatrix}, \quad \bar{C}_p = \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix} \tag{6}$$

are the augmented system matrices, the dimensions of which can be specified as $\bar{A}_p \in \mathcal{R}^{(n+1)\times(n+1)}$, $\bar{B}_p \in \mathcal{R}^{n+1}$ and $\bar{C}_p \in \mathcal{R}^{(n+1)\times(n+1)}$. Notably, $\bar{C}_p$ is also an identity matrix.

The augmented linear plant has the input $sat(u_r) \in \mathcal{R}$ and the state vector $x_a \in \mathcal{R}^{n+1}$ which can be defined as

$$x_a = \begin{bmatrix} x^T & u_{act} \end{bmatrix}^T \tag{7}$$

Accordingly the augmented control law is formulated as

$$u_r = -(1/\tau)K_1 e - (1/\tau)K_2 \int_0^t e(\tau)d\tau - (1/\tau)u_{act} \tag{8}$$

where u_{act}, the input to the original plant, is given by

$$u_{act} = \frac{1}{s}\, sat(u_r) \tag{9}$$

The tracking error dynamics can be expressed as

$$\dot{e} = A_p e + A_p r + B_p u_{act} \tag{10}$$

and a possible realization of the augmented control is

$$\dot{x}_c = \bar{A}_c x_c + \bar{B}_c \begin{bmatrix} -e \\ -u_{act} \end{bmatrix} \tag{11}$$

$$u_r = \bar{C}_c x_c + \bar{D}_c \begin{bmatrix} -e \\ -u_{act} \end{bmatrix} \tag{12}$$

with $\bar{A}_c \in \mathcal{R}^{n\times n}$, $\bar{B}_c \in \mathcal{R}^{n\times(n+1)}$, $\bar{C}_c \in \mathcal{R}^{1\times n}$ and $\bar{D}_c \in \mathcal{R}^{1\times(n+1)}$ chosen as

$$\bar{A}_c = 0 \tag{13}$$

$$\bar{B}_c = \begin{bmatrix} I & 0 \end{bmatrix} \tag{14}$$

$$\bar{C}_c = (1/\tau)K_2 \tag{15}$$

$$\bar{D}_c = \begin{bmatrix} (1/\tau)K_1 & (1/\tau) \end{bmatrix} \tag{16}$$

Once the system subject to input rate saturation is interpreted as an augmented system with input magnitude saturation constraints as above, the relevant anti-windup compensator can be constructed.

If $\bar{G}(s) = \bar{N}(s)\,\bar{M}^{-1}(s)$ is a full-order right coprime factorization of the transfer function matrix of the augmented plant, the anti-windup compensator can be described by the following transfer function matrix:

$$\bar{K}_{aw}(s) = \begin{bmatrix} \bar{M}(s) - I \\ \bar{N}(s) \end{bmatrix} \tag{17}$$

The system matrices of the anti-windup compensator can hence be identified as $(\bar{A}_{aw}, \bar{B}_{aw}, \bar{C}_{aw}, \bar{D}_{aw})$ where

$$\bar{A}_{aw} = \bar{A}_p + \bar{B}_p F \tag{18}$$

$$\bar{B}_{aw} = \bar{B}_p \tag{19}$$

$$\bar{C}_{aw} = \begin{bmatrix} F \\ I \end{bmatrix} \tag{20}$$

$$\bar{D}_{aw} = 0 \tag{21}$$

with $\bar{A}_{aw} \in \mathcal{R}^{(n+1)\times(n+1)}$, $\bar{B}_{aw} \in \mathcal{R}^{n+1}$, $\bar{C}_{aw} \in \mathcal{R}^{(n+2)\times(n+1)}$, $\bar{D}_{aw} \in \mathcal{R}^{n+2}$, and F chosen such that $\bar{A}_p + \bar{B}_p F$ is a Hurwitz matrix.

Two equivalent methods to construct the anti-windup compensator guaranteeing similar $\mathcal{L}_2$ performance are LMI and

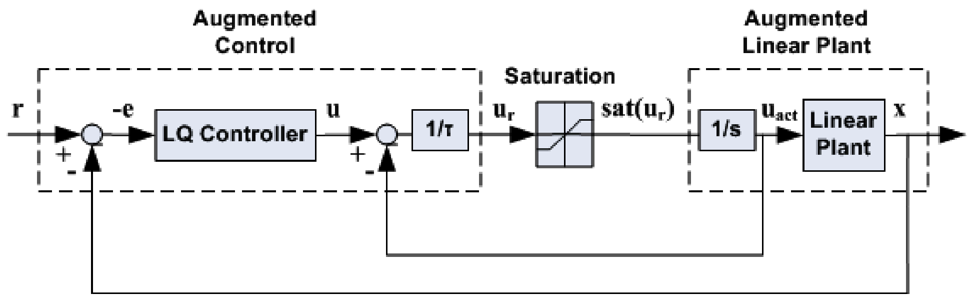

Fig. 3. The rate-limited system including the linear plant and the unconstrained controller

Riccati based synthesis techniques. Either one of them can be chosen to define the parameter F, and the implementation of the compensator into the system follows similarly afterwards regardless of the choice made.

Instead of the LMI based anti-windup compensator design methodology followed in [24]–[26], here we use the alternative Riccati based method to construct the plant-order anti-windup compensator for stable plants.

Using the method given in [20] we define F to be

$$F = -\left(R + \frac{W^{-1}}{\epsilon}\right) \bar{B}_p{}^T P \tag{22}$$

where $P > 0$ satisfies the following ARE :

$$\bar{A}_p^T\, P + P\, \bar{A}_p - P\, \bar{B}_p\, R\, \bar{B}_p^T\, P + \bar{C}_p^T\, \bar{C}_p = 0 \tag{23}$$

The design parameters R, W and ϵ can be properly chosen for a similar $\mathcal{L}_2$ performance as promised by the respective LMI based design methodology.

The corresponding anti-windup design ensures the local stability of the nonlinear loop with the region of attraction defined by an ellipsoid as described in [20] where a simplified tuning approach is also provided for the choice of parameters to define the region of attraction and an upper bound on the local $\mathcal{L}_2$ gain. The reader not familiar with anti-windup synthesis techniques using Riccati equations might also want to consult [27].

When F is defined using (22) and (23), the matrices in (18,19,20,21) can be constructed, and the anti-windup compensator can be augmented into the system as

$$\dot{x}_{aw} = \bar{A}_{aw}x_{aw} + \bar{B}_{aw}(u_r - sat(u_r)) \tag{24}$$

$$y_{aw} = \bar{C}_{aw}x_{aw} + \bar{D}_{aw}(u_r - sat(u_r)) \tag{25}$$

with $y_{aw} = \begin{bmatrix} y_{aw_1} & y_{aw_2}^T \end{bmatrix}^T$, $y_{aw_1} \in \mathcal{R}$, $y_{aw_2} \in \mathcal{R}^{n+1}$.

The anti-windup modification term, y_{aw_2} is incorporated into the augmented control structure in (11,12) as

$$\dot{x}_{cm} = \bar{A}_c x_{cm} + \bar{B}_c \begin{bmatrix} -e \\ -u_{act} \end{bmatrix} - \bar{B}_c y_{aw_2} \tag{26}$$

$$u_m = \bar{C}_c x_{cm} + \bar{D}_c \begin{bmatrix} -e \\ -u_{act} \end{bmatrix} - \bar{D}_c y_{aw_2} \tag{27}$$

with $\bar{A}_c$, $\bar{B}_c$, $\bar{C}_c$ and $\bar{D}_c$ as defined previously.

On the other hand, the term y_{aw_1} modifies the control input u_m in (27), and is used to define u_r as

$$u_r = u_m - y_{aw_1} \tag{28}$$

The augmented linear plant subject to input magnitude saturation and the corresponding nominal control structure with anti-windup compensation are displayed in Figure 4.

III. An Indirect Adaptive Control Design

As the described control structure is based on the explicit plant model, in the unknown plant parameters case one can use the online estimations of these parameters and implement an adaptive control with anti-windup augmentation.

A first order filter $1/(s+\lambda)$, $\lambda > 0$ is used to avoid the high frequency sensor noise amplification by the derivative term. For any fixed set of plant parameters we thus obtain

$$\underbrace{\frac{s}{s+\lambda}\, x}_{\stackrel{def}{=} z} = A_p \frac{1}{s+\lambda}\, x + B_p \frac{1}{s+\lambda}\frac{1}{s}\, sat(u_r) \tag{29}$$

$$\Rightarrow z = \underbrace{\begin{bmatrix} A_p & B_p \end{bmatrix}}_{\stackrel{def}{=} \theta^{*T}} \underbrace{\begin{bmatrix} \frac{1}{s+\lambda}\, x \\ \frac{1}{s+\lambda}\frac{1}{s}\, sat(u_r) \end{bmatrix}}_{\stackrel{def}{=} \phi} \tag{30}$$

where $sat(u_r)$ is the time derivative of u_{act} which is the control input including the rate saturation nonlinearity and directly applied to the original plant. Note that as long as the rate of change of the control input does not hit the rate saturation limits, $\bar{u}_r$ or $-\bar{u}_r$, and remains in the region where $sat(u_r) = u_r$, the relation in (9) reduces to $u_{act} = (1/s)\, u_r$. Meanwhile, the anti-windup compensator remains inactive, and the output of the augmented controller, u_m is applied to the augmented plant as u_r.

The regressor vector is defined using the original plant state vector x and the saturated control input $sat(u_r)$ as

$$\phi = \frac{1}{s+\lambda} \begin{bmatrix} x^T & \frac{1}{s}\, sat(u_r) \end{bmatrix}^T \tag{31}$$

$$= \frac{1}{s+\lambda} \begin{bmatrix} x^T & u_{act} \end{bmatrix}^T \tag{32}$$

which is the filtered form of the state vector of the augmented plant as given in (7).

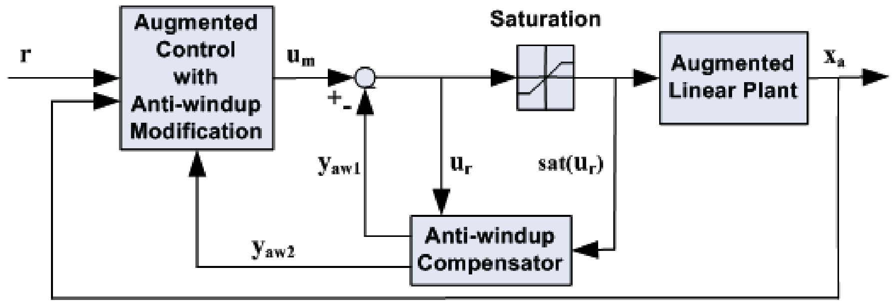

Fig. 4. The nominal control structure with anti-windup compensator for the rate-limited system in the augmented form

The estimation model can be expressed as

$$\hat{z}_i = \theta_i^T \phi\,, \quad i = 1, 2, ..., n \tag{33}$$

with θ_i as the estimate of θ_i^*, the i^{th} column of θ^* :

$$\theta_i(t) = \left[\begin{array}{ccccc} \hat{a}_{i1} & \hat{a}_{i2} & ... & \hat{a}_{in} & \hat{b}_i \end{array}\right]^T \tag{34}$$

where $\hat{a}_{ij}$ and $\hat{b}_i$ are the estimates of a_{ij} and b_i at time t, a_{ij} is the ij^{th} entry of $A_p \in \mathcal{R}^{n\times n}$, and b_i is the i^{th} entry of $B_p \in \mathcal{R}^n$.

The normalized estimation error ϵ_i can be constructed as

$$\epsilon_i = \frac{z_i - \theta_i^T \phi}{m^2}\,, \quad i = 1, 2, ..., n \tag{35}$$

by choosing the following proper normalization signal:

$$m^2 = 1 + \phi^T \phi \tag{36}$$

so that $\phi\,/\,m \in \mathcal{L}_\infty$ can be guaranteed.

The parameter errors are defined in vector form using

$$\tilde{\theta}_i = \theta_i - \theta_i^*\,, \quad i = 1, 2, ..., n \tag{37}$$

and the matrices of parameter errors are derived as

$$\tilde{A}_p = \hat{A}_p - A_p\,, \quad \tilde{B}_p = \hat{B}_p - B_p \tag{38}$$

For the adaptive law we employ the Gradient Algorithm:

$$\dot{\theta}_i = \Gamma_i \epsilon_i \phi\,, \quad \Gamma_i = \Gamma_i^T > 0 \tag{39}$$

For any constant θ_i^* we can thus show the following:

$$\|\dot{\theta}_i\| = \|\dot{\tilde{\theta}}_i\| \le \|\Gamma_i\|_{i,2}\, \|\epsilon_i m\|\, \frac{\|\phi\|}{m} \tag{40}$$

where $\|\Gamma_i\|_{i,2}$ is the induced $\mathcal{L}_2$ norm of Γ_i, and $\|\dot{\theta}_i\|, \|\dot{\tilde{\theta}}_i\|$, $\|\epsilon_i m\|, \|\phi\|$ denote the Euclidean norms of $\dot{\theta}_i$, $\dot{\tilde{\theta}}_i$, $\epsilon_i m$, ϕ.

In order to guarantee the boundedness of parameter estimates a projection algorithm needs to be included. Together with $\|\phi\|\,/\,m \in \mathcal{L}_\infty$ and $\epsilon_i m \in \mathcal{L}_2 \bigcap \mathcal{L}_\infty$, it is implied by (40) that $\dot{\theta}_i \in \mathcal{L}_2 \bigcap \mathcal{L}_\infty$. Hence, we have $\theta_i \in \mathcal{L}_\infty$ and ϵ_i, $\epsilon_i m$, $\dot{\theta}_i \in \mathcal{L}_2 \bigcap \mathcal{L}_\infty$ for $i = 1, 2, ..., n$.

The reader is referred to [28] for the relevant discussion on the stabilizability problem in the indirect Adaptive Pole Placement control.

As the plant parameters are unknown, the controller gains K_1 and K_2 cannot be evaluated by solving the associated ARE. The modified controller in (26,27) thus cannot be constructed. Similarly, F cannot be obtained using (22), and the anti-windup controller in (24,25) cannot be implemented.

Instead we implement an adaptive control design based on the Certainty Equivalence Principle. The adaptive control gain $\hat{K}$ is evaluated through the solution of the associated

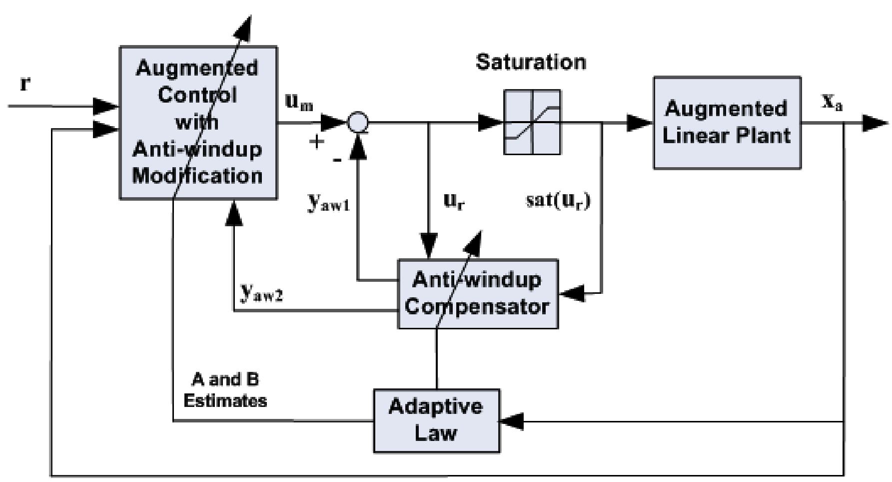

Fig. 5. The adaptive control design for rate-limited systems with unknown plant parameters

ARE using the parameter estimates from the adaptive law. Accordingly, a new parameter $\hat{F}$ can be defined through (22) using the solution of (23) with the parameter estimates.

As we work on an artificial plant augmentation, we do not need to estimate all elements of the matrices $\bar{A}_p$, $\bar{B}_p$ and $\bar{C}_p$ but only the original plant parameters, A_p and B_p which are required to construct the estimate of $\bar{A}_p$. We do not estimate $\bar{B}_p$ and $\bar{C}_p$ since they are known.

The block diagram of the adaptive control structure including the anti-windup scheme is illustrated in Figure 5 where both the augmented control and the anti-windup compensator parameters are shown to be evaluated online.

IV. Stability Analysis

We simply add and subtract the terms $\hat{A}_p e$ and $\hat{B}_p u_{act}$ to the tracking error dynamics in (10) and obtain

$$\dot{e} = \hat{A}_p e - \hat{B}_p\, u_{act} + A_p r - \tilde{A}_p e - \tilde{B}_p\, u_{act} \tag{41}$$

where $-\tilde{A}_p\, e$ and $-\tilde{B}_p\, u_{act}$ are the terms due to parameter estimation errors.

We first attempt to express the control input u_{act}, the original state vector x, and thus the tracking error e in terms of the normalized estimation error given by

$$\epsilon = \begin{bmatrix} \epsilon_1 & \epsilon_2 & \ldots & \epsilon_n \end{bmatrix}^T \tag{42}$$

One can derive ϵm^2 using (33) and (35) as follows:

$$\epsilon m^2 = -\tilde{A}_p \frac{1}{s+\lambda} x - \tilde{B}_p \frac{1}{s+\lambda}\frac{1}{s} sat(u_r) \tag{43}$$

$$= -\tilde{A}_p \frac{1}{s+\lambda}(e+r) - \tilde{B}_p \frac{1}{s+\lambda}\frac{1}{s} sat(u_r) \tag{44}$$

We operate on both sides of (44) with $(s+\lambda)$ and combine the corresponding relation with (41) to further derive

$$\begin{aligned}\dot{e} = {} & \hat{A}_p e + (s+\lambda)\epsilon m^2 + \dot{\hat{A}}_p \frac{1}{s+\lambda} e \\ & + \dot{\hat{B}}_p \frac{1}{s+\lambda}\frac{1}{s} sat(u_r) + \tilde{A}_p r + \dot{\hat{A}}_p \frac{1}{s+\lambda} r \\ & - \hat{B}_p \frac{1}{s} sat(u_r) + A_p r\end{aligned} \tag{45}$$

A new vector variable can then be defined as

$$\bar{e} = e - \epsilon m^2 \tag{46}$$

and substituted into (45) in order to obtain

$$\begin{aligned}\dot{\bar{e}} = {} & \hat{A}_p \bar{e} + \underbrace{(\lambda I + \hat{A}_p)}_{\in \mathcal{L}_\infty} \underbrace{\epsilon m}_{\in \mathcal{L}_2} m + \underbrace{\dot{\hat{A}}_p}_{\in \mathcal{L}_2} \frac{1}{s+\lambda} e \\ & + \underbrace{\dot{\hat{B}}_p}_{\in \mathcal{L}_2} \underbrace{\frac{1}{s+\lambda}\frac{1}{s} sat(u_r)}_{\in \mathcal{L}_\infty} + \underbrace{\hat{A}_p r}_{\in \mathcal{L}_\infty} \\ & + \underbrace{\dot{\hat{A}}_p}_{\in \mathcal{L}_2} \underbrace{\frac{1}{s+\lambda} r}_{\in \mathcal{L}_\infty} - \underbrace{\hat{B}_p \frac{1}{s} sat(u_r)}_{\in \mathcal{L}_\infty}\end{aligned} \tag{47}$$

The implementation of a similar projection function as discussed in [25] is essential in order to have all the roots of $det(sI - \hat{A}_p(t))$ in the negative half-plane and to establish exponential stability of the homogeneous part of (47).

The homogeneous part of (47) is not affected by neither the modified LQ controller output u_m nor the implemented u_{act} when the system is in the saturation phase since the rate of change of the actual effective control input to the plant is $sat(u_r)$ which is a constant equal to either $\bar{u}_r$ or $-\bar{u}_r$. The integrator action would result in an effective control input of $(1/s)\, \bar{u}_r$ or $-(1/s)\, \bar{u}_r$ which can again be shown to be bounded for any time instant assuming that the initial effective control input is bounded. The parameter estimates $\hat{A}_p$ and $\hat{B}_p$ are bounded by projection, the reference signal r is bounded by definition, and the adaptive law guarantees that ϵm, $\dot{\hat{A}}_p$, $\dot{\hat{B}}_p \in \mathcal{L}_\infty$.

A fictitious normalizing signal m_f is defined as

$$m_f{}^2 = c + \|e_t\|_{2\delta}^2 \tag{48}$$

where $c > 0$ is some constant, and $\|e_t\|_{2\delta}^2$ is the exponentially weighted $\mathcal{L}_2$ norm of e. Using (46) we can write

$$\|e_t\|_{2\delta} \leq \|\bar{e}_t\|_{2\delta} + \|(\epsilon m^2)_t\|_{2\delta} \tag{49}$$

and obtain the following bound for m_f^2 :

$$m_f^2 \leq c + c\,\|\bar{e}_t\|_{2\delta}^2 + c\,\|(\epsilon m^2)_t\|_{2\delta}^2 \tag{50}$$

Based on (47) we can also write

$$\begin{aligned}\|\bar{e}_t\|_{2\delta}^2 \leq {} & c\,\|(\epsilon m^2)_t\|_{2\delta}^2 + c\,\|(\dot{\hat{A}}_p e)_t\|_{2\delta}^2 \\ & + c\,\|(\dot{\hat{B}}_p \frac{1}{s} sat(u_r))_t\|_{2\delta}^2 + c\,\|(\hat{A}_p r)_t\|_{2\delta}^2 \\ & + c\,\|(\dot{\hat{A}}_p r)_t\|_{2\delta}^2 + c\,\|(\hat{B}_p)_t\|_{2\delta}^2\end{aligned} \tag{51}$$

where $\|(\dot{\hat{B}}_p \frac{1}{s} sat(u_r))_t\|_{2\delta}^2$ and $\|(\dot{\hat{A}}_p r)_t\|_{2\delta}^2$ are known to be bounded by the properties of the adaptive law. The boundedness of $\|(\hat{A}_p r)_t\|_{2\delta}^2$ and $\|(\hat{B}_p)_t\|_{2\delta}^2$ can be established by projection. $\|(\dot{\hat{A}}_p e)_t\|_{2\delta}^2$ can be further shown to be bounded by $\|(\dot{\hat{A}}_p m_f)_t\|_{2\delta}^2$.

We partition ϕ in (32) as $\phi = \begin{bmatrix} \phi_1^T & \phi_2 \end{bmatrix}^T$ and define

$$\phi_1 = \frac{1}{s+\lambda} x\,, \quad \phi_2 = \frac{1}{s+\lambda} u_{act} \tag{52}$$

One can observe that the state x and the error e bound ϕ_1, and ϕ_2 is bounded by definition. Thus,

$$m = \sqrt{1 + \phi^T \phi} \tag{53}$$

can be shown to be bounded by m_f.

We then substitute (51) in (50) and obtain

$$m_f^2 \leq c + c\,\|(\epsilon m m_f)_t\|_{2\delta}^2 + c\,\|(\dot{\hat{A}}_p m_f)_t\|_{2\delta}^2 \tag{54}$$

$$\leq c + c \int_0^t e^{-\delta(t-\tau)} g^2(\tau) m_f^2(\tau) d\tau \tag{55}$$

where

$$g^2 = \epsilon^2 m^2 + \|\dot{\hat{A}}_p\|_{i,2}^2 \tag{56}$$

Since ϵm, $\dot{\hat{A}}_p \in \mathcal{L}_2$, it follows that $g \in \mathcal{L}_2$. Bellman-

Gronwall Lemma (see App.) can then be applied to show that $m_f \in \mathcal{L}_\infty$ which further implies that m , $\bar{e}$, x , $\phi \in \mathcal{L}_\infty$.

As a result we establish boundedness of all signals in the closed-loop when the system experiences saturation.

V. Conclusion

The control of linear systems with bounded inputs is a challenging task. The recently developed anti-windup compensator design techniques promise systematic formulation and stability during saturation. The control of uncertain systems prone to input rate saturation, however, is still recognized as an open problem.

In this paper an adaptive control structure including an adaptive anti-windup scheme is developed for systems with large parametric uncertainties and input rate saturation constraints. The signals in the closed-loop are shown to remain bounded in the saturation phase while the adaptive anti-windup augmentation is activated. The oscillations in the system response caused by the input rate saturation and the corresponding performance degradation can thus be suppressed by the proposed indirect adaptive control design.

Appendix

Bellman-Gronwall Lemma: If $\lambda(t)$, $g(t)$ and $k(t)$ are nonnegative piecewise continuous functions of time t , and

$$y(t) \leq \lambda(t) + g(t) \int_{t_0}^{t} k(s) y(s) ds \, , \ \forall \, t \geq t_0 \geq 0$$

then it can be shown [29] for any $t \geq t_0 \geq 0$ that

$$y(t) \leq \lambda(t) + g(t) \int_{t_0}^{t} \lambda(s) k(s) exp \left(\int_{s}^{t} k(\tau) g(\tau) d\tau \right) ds$$

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