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Simultaneous optimization for wind derivatives based on prediction errors

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Abstract— **Wind power energy has been paid much attention recently for various reasons, and the production of electricity with wind energy has been increasing rapidly for a few decades. In this work, we will propose a new type of weather derivatives based on the prediction errors for wind speeds, and estimate their hedge effect on wind power energy businesses. At first, we will investigate the correlation of prediction errors between the power output and the wind speed in a Japanese wind farm. Then we will develop a methodology that will optimally construct a wind derivative based on the prediction errors using nonparametric regressions. A simultaneous optimization technique of the loss and payoff functions for wind derivatives is demonstrated based on the empirical data.**

I. INTRODUCTION

Predicting the future weather conditions is considered important in real businesses for many industries including electricity producers and suppliers, because their profit or loss is largely affected by the weather conditions. Under these circumstances, we may have a new risk when the prediction error exists. In this work, we consider a hedging problem for the loss caused by prediction errors, and provide a new tipe of weather derivative (see, e.g., [5] for the introduction of weather derivatives) based on prediction errors on the wind condition. In contrast to the standard weather derivatives in which the underlying index is given by weather data only (such as temperature [1], [2], [3], [7], [8], [12], [13]), the proposed weather derivative uses prediction data and the payoff depends on the difference between the actual data and the prediction data.

Here we employ the power output from a wind farm (WF), where the power output is predicted using numerical weather prediction and the power generating properties for turbines. A possible sales contract of the power output using the prediction may be described as follows: The value of electricity generated by wind power is normally considered to be low due to the uncertainty of the tradable volume, e.g., 3 yen per 1 kWh. On the other hand, the value of the electricity would be estimated to be higher, if the tradable volume were quoted in advance by prediction, but the seller has to guarantee the quoted volume or has to pay the penalty in case of shortages. Suppose that the value of electricity with prediction is given as 7 yen per 1 kWh and that the penalty of the shortage is 10 yen per 1 kWh. These assumptions are not so far from the current situation discussed in the prediction business [9]. In this case, the loss function caused by prediction errors is depicted in Fig. 1, which shows the relation between the prediction error for the power output $P-\hat{P}$ (the actual power output minus its prediction) and the loss caused by the prediction error. Note that, even if the prediction error is positive, we can also think of this situation as an opportunity loss to sell the output with a suitable price.

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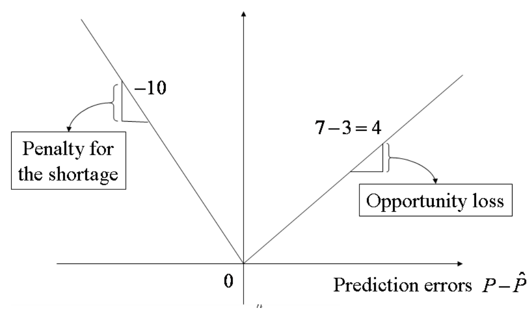

Fig. 1. An example of loss function

Based on the above discussions, we will first consider the following problems:

P1) Given the loss function and the payoff function of wind derivatives, find the optimal volume of wind derivative using a linear regression.
P2) Given the loss function, find the optimal payoff function of wind derivatives.

Then we will focus on a situation in which there already exists a standardized derivative contract with a certain payoff function, but there is some room for improvement on the loss function, e.g., for a WF owner. The problem can be thought of as a reverse problem of P2), which is given as follows:

P3) Given the payoff function of wind derivatives, find the optimal loss function against prediction errors of power output.

Finally, we will formulate a simultaneous optimization problem of payoff and loss functions as P4) below:

P4) Optimize the payoff function of wind derivatives and the loss function simultaneously.

We use the following notation: For a sequence of observations of a variable, $x_n,\ n=1,\dots,N$, the sample mean and the sample variance are denoted by $\mathrm{Mean}(x_n)$ and $\mathrm{Var}(x_n)$, respectively. $\mathrm{Cov}(x_n,\ y_n)$ and $\mathrm{Corr}(x_n,\ y_n)$ represent the sample covariance and the sample correlation, respectively, where $y_n,\ n=1,\dots,N$ is a sequence of observations for another variable. The set of real number is denoted by $\Re$, and an $n\times m$ matrix with real entries is denoted by $A\in\Re^{n\times m}$.

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II. STANDARD MINIMUM VARIANCE HEDGING PROBLEM

A. Loss and payoff functions

For simplicity, consider a wind power energy trade between two basic positions, a seller and a buyer, for the electricity output. Assume that the seller and the buyer carry out sales contracts based on the prediction of the power output. Let $n=1,\ldots,N$ be the time index (say, hourly index) and define the following variables:

- P_n: Total power output at time n
- $\hat{P}_n$: Prediction of P_n (which is computed, e.g., 1 day in advance)

The buyer is willing to buy the power output by using the reference $\hat{P}_n$, and may require a penalty if the prediction error exceeds a certain level.

Let $\varepsilon_{p,n}$ $(n=1,\ldots,N)$ be the prediction error of the power output at time n, which causes a loss for the seller due to the penalty or opportunity loss to sell the output. Suppose that the loss associated with the prediction error of the power output is defined using a loss function as $\phi(\varepsilon_{p,n})$. For instance, the loss function may be given as the one shown in Fig. 1 if the seller is a WF owner. Also, there is a case in which the prediction is sufficiently accurate or the prediction error is less than a certain (small) level. In this case, the seller can be thought of getting a bonus because of a higher price of power output with prediction, which results in a profit for the seller and makes the loss negative, i.e., $\phi(\varepsilon_{p,n})<0$. We assume that

$$\mathrm{Mean}(\phi(\varepsilon_{p,n}))=0 \tag{1}$$

so that the sum of profit/loss is zero on average.

We will consider a situation in which the seller with ϕ would like to compensate their loss on $\varepsilon_{p,n}$ using a weather derivative on the prediction error of the wind speed. To this end, define the following variables:

- W_n: Wind speed at time n
- $\hat{W}_n$: Prediction of W_n (which is computed, e.g., 1 day in advance)

Let $\varepsilon_{w,n}$ be the prediction error of the wind speed, and assume that the payoff of the wind derivative is defined by using a suitable payoff function of $\varepsilon_{w,n}$ as $\psi(\varepsilon_{w,n})$. Also, suppose that the weather derivative contract with a payoff function ψ is carried out in advance without any cost and that $\psi(\varepsilon_{w,n})$ satisfies the following condition:

$$\mathrm{Mean}(\psi(\varepsilon_{w,n}))=0. \tag{2}$$

Note that condition (2) indicates that the physical probability measure provides a risk neutral probability measure.

B. Minimum variance hedge

With the notation and definitions introduced in the previous subsection, the first optimization problem, P1), is formulated as follows:

Contract volume optimization problem:

$$\min_{\Delta\in\Re}\ \mathrm{Var}(\phi(\varepsilon_{p,n})+\Delta\psi(\varepsilon_{w,n})). \tag{3}$$

The contract volume optimization problem may be considered as the standard "minimum variance hedge," and the optimal volume Δ^* may be computed analytically as

$$\Delta^*=-\frac{\mathrm{Cov}(\phi(\varepsilon_{p,n}),\ \psi(\varepsilon_{w,n}))}{\mathrm{Var}(\psi(\varepsilon_{w,n}))}. \tag{4}$$

To estimate the hedge effect, we define the variance reduction rate (VRR) as follows:

$$\mathrm{VRR}:=\frac{\mathrm{Var}(\phi(\varepsilon_{p,n})+\Delta^*\psi(\varepsilon_{w,n}))}{\mathrm{Var}(\phi(\varepsilon_{p,n}))}. \tag{5}$$

Because the minimum variance can be computed as

$$\begin{aligned}&\mathrm{Var}(\phi(\varepsilon_{p,n})+\Delta^*\psi(\varepsilon_{w,n}))\\ =\ &\mathrm{Var}(\phi(\varepsilon_{p,n}))\Big(1-[\mathrm{Corr}(\phi(\varepsilon_{p,n}),\ \psi(\varepsilon_{w,n}))]^2\Big),\end{aligned} \tag{6}$$

we obtain

$$\mathrm{VRR}=1-[\mathrm{Corr}(\phi(\varepsilon_{p,n}),\ \psi(\varepsilon_{w,n}))]^2. \tag{7}$$

Note that VRR satisfies

$$0\le\mathrm{VRR}\le 1 \tag{8}$$

and that a smaller VRR provides a better hedge effect in terms of minimum variance.

In the case of standard minimum variance hedge, the optimal volume is also found by solving a linear regression problem, where $\phi(\varepsilon_{p,n})$ is regressed with respect to $\psi(\varepsilon_{w,n})$, and the regression coefficient gives the optimal volume for fixed loss and payoff functions. On the other hand, we can expect to obtain a better hedge effect if we could optimize the payoff function of the weather derivative directly. This can be done by applying non-parametric regression technique introduced in the next section, and we will find that using a non-parametric regression corresponds to optimizing the derivative contract directly by choosing a suitable payoff function.

III. MINIMUM VARIANCE HEDGING USING NON-PARAMETRIC REGRESSION

In this section, we first introduce a non-parametric regression technique, and then formulate the second optimization problem, P2).

A. Generalized additive models

The non-parametric regression technique introduced here is to find a (cubic) smoothing spline that minimizes the so-called penalized residual sum of squares (PRSS) among all regression spline functions with two continuous derivatives. Let y_n and x_n be dependent and independent variables, respectively, and express y_n as

$$y_n=h(x_n)+\varepsilon_n,\quad \mathrm{Mean}(\varepsilon_n)=0 \tag{9}$$

using a smooth function h and residuals ε_n. Here the function h is a (cubic) smoothing spline that minimizes the following PRSS,

$$\mathrm{PRSS}=\sum_{n=1}^{N}(y_n-h(x_n))^2+\lambda\int_{-\infty}^{\infty}\left(h''(x)\right)^2\mathrm{d}x \tag{10}$$

among all functions h with two continuous derivatives, where λ is a given parameter. In (10), the first term measures closeness to the data while the second term penalizes curvature in the function. Note that, if $\lambda = 0$ and h is given by a polynomial function, the problem is reduced to the standard regression polynomial and is solved by the least squares method. It is shown that (10) has an explicit and unique minimizer and that a candidate of optimal λ may be found by using the so-called generalized cross validation criteria (See [10]). Note that regression splines can be extended to the multivariable case with additive sums of smoothing splines, known as generalized additive models (GAMs; see e.g., [6]). Also note that GAMs can be computed using free software "R (`http://cran.r-project.org/`)," and we will refer to the class of smoothing splines for non-parametric regression as GAMs in this paper. We will apply GAMs to solve P2)–P4) and estimate the hedge effect of wind derivatives.

Note that, instead of writing the problem as an unconstrained optimization problem, we can reformulate it as an optimization problem constrained on h as follows:

$$\begin{aligned} \min_{h} \quad & \sum_{n=1}^{N} (y_n - h(x_n))^2 \\ \text{s.t.} \quad & \int_{-\infty}^{\infty} \left(h''(x)\right)^2 \mathrm{d}x \leq \alpha \end{aligned} \tag{11}$$

where α is a given parameter. Based on the similar argument to that in [10], we can verify that the objective function of problem (11) is quadratic subject to a convex constraint and that the minimization problem (11) is equivalent to its Lagrange dual problem. Therefore, we see that fixing λ in (10) corresponds to fixing α in (11) and that the non-parametric regression problem using GAM may be recast as a minimization problem of the sample variance with a smooth constraint.

B. Optimization of derivative contracts

It is in a position to formulate the the second optimization problem, i.e., the payoff function optimization problem, in the context of minimum variance hedge using non-parametric regression as follows, where $\phi = \overline{\phi}$ is a given loss function:

Payoff function optimization problem:

$$\begin{aligned} \min_{\psi} \quad & \mathrm{Var}\left(\overline{\phi}\left(\varepsilon_{p,n}\right) + \psi\left(\varepsilon_{w,n}\right)\right) \\ \text{s.t.} \quad & \int_{-\infty}^{\infty} \left(\psi''(x)\right)^2 \mathrm{d}x \leq \alpha. \end{aligned} \tag{12}$$

The minimization problem (12) may be recast as (11) by taking $y_n = \overline{\phi}(\varepsilon_{p,n})$, $x_n = \varepsilon_{w,n}$, and $h = -\psi$, and therefore, can be solved by applying GAM. Let ψ^* be the optimal payoff function. Then VRR may be defined as

$$\mathrm{VRR} := \frac{\mathrm{Var}\left(\overline{\phi}\left(\varepsilon_{p,n}\right) + \psi^*\left(\varepsilon_{w,n}\right)\right)}{\mathrm{Var}\left(\overline{\phi}\left(\varepsilon_{p,n}\right)\right)}. \tag{13}$$

Although it is possible to find the optimal payoff function by solving GAM once, it may be worthwhile to mention that we have a slight improvement by applying a linear regression after finding the optimal payoff function ψ^* as

$$\min_{a \in \Re} \mathrm{Var}\left(\overline{\phi}\left(\varepsilon_{p,n}\right) + a\psi^*\left(\varepsilon_{w,n}\right)\right). \tag{14}$$

In this case, VRR may be given as

$$\mathrm{VRR} = \frac{\mathrm{Var}\left(\overline{\phi}\left(\varepsilon_{p,n}\right) + a^*\psi^*\left(\varepsilon_{w,n}\right)\right)}{\mathrm{Var}\left(\phi\left(\varepsilon_{p,n}\right)\right)}. \tag{15}$$

or equivalently,

$$\mathrm{VRR} = 1 - \left[\mathrm{Corr}(\overline{\phi}\left(\varepsilon_{p,n}\right),\ \psi^*\left(\varepsilon_{w,n}\right))\right]^2. \tag{16}$$

where $a^* \in \Re$ is the regression coefficient to solve (14). Note that (16) is independent of a^*, or any scaling parameter to $\psi^*(\varepsilon_{w,n})$, and that it can be computed if ψ^* is specified. Therefore, we use the right hand side of (16) as a proxy of VRR. It is readily confirmed that VRR in (13) is actually an upper bound of (16). However, as indicated in the end of Subsection V-B, the gap between (13) and (16) is very small from our numerical experience.

IV. Optimization with loss functions and simultaneous optimization

A. Optimal loss function

Next, we will consider a case in which a payoff function of wind derivative is given but we would like to find a loss function that is desirable for using the wind derivative, i.e., in a case where there already exists a standardized derivative contract with a certain payoff function, but there is some room for improvement on the loss function, e.g., for a WF owner. We assume that possible losses on $\varepsilon_{p,n}$, $\phi(\varepsilon_{p,n})$, has the same mean and variance, i.e., $\mathrm{Mean}(\phi(\varepsilon_{p,n})) = 0$ and

$$\mathrm{Var}\left(\phi\left(\varepsilon_{p,n}\right)\right) = c. \tag{17}$$

We will compute an optimal loss function satisfying (17), for given payoff function $\psi = \overline{\psi}$.

The problem is formulated as follows:

Loss function optimization problem:

$$\begin{aligned} \min_{\phi} \quad & \mathrm{Var}\left(\phi\left(\varepsilon_{p,n}\right) + \overline{\psi}\left(\varepsilon_{w,n}\right)\right) \\ \text{s.t.} \quad & \int_{-\infty}^{\infty} \left(\phi''(x)\right)^2 \mathrm{d}x \leq \alpha \quad \text{and (17).} \end{aligned} \tag{18}$$

Note that the constraint (17) is also quadratic if ϕ is given by a cubic natural spline function, and hence, the problem might be reformulated as an unconstrained optimization problem by introducing another Lagrangian term for the variance constraint. On the other hand, we can still apply GAM directly to solve the problem without the variance constraint (17), similar to the payoff function optimization problem (12). Then we can scale the minimizing function so that it satisfies the variance constraint (17).

Note that the optimal volume of wind derivative with the given payoff $\overline{\psi}$ and the optimal loss function ϕ^* will be found by solving the standard minimum variance hedging problem in Subsection II-B, and VRR may be computed as

$$\mathrm{VRR} = 1 - \left[\mathrm{Corr}\left(\phi^*\left(\varepsilon_{p,n}\right),\ \overline{\psi}\left(\varepsilon_{w,n}\right)\right)\right]^2. \tag{19}$$

B. Simultaneous optimization

It may be interesting to consider a simultaneous optimization of the payoff and loss functions, $\psi(\varepsilon_{w,n})$ and $\phi(\varepsilon_{p,n})$. Recall that VRR can be computed using the correlation between the payoff and the loss functions. Since the larger correlation the smaller VRR, the minimization of VRR boils down to the maximization of correlation between $\phi(\varepsilon_{p,n})$ and $\psi(\varepsilon_{w,n})$. Therefore, the simultaneous optimization of the payoff and the loss functions may be formulated as follows:

Simultaneous optimization problem:

$$\begin{aligned} \max_{\phi,\ \psi} \quad & \mathrm{Corr}\left(\phi\left(\varepsilon_{p,n}\right), \psi\left(\varepsilon_{w,n}\right)\right) \\ \text{s.t.} \quad & \int_{-\infty}^{\infty} \left(\psi''(x)\right)^2 \mathrm{d}x \le \alpha_\psi \\ & \int_{-\infty}^{\infty} \left(\phi''(x)\right)^2 \mathrm{d}x \le \alpha_\phi \quad \text{and (17)} \end{aligned} \tag{20}$$

The simultaneous optimization problem may be solved using an iterative algorithm by solving the payoff function optimization problem with $\phi = \overline{\phi}$ fixed, or the loss function optimization problem with $\psi = \overline{\psi}$ fixed, at each step. The following is the iterative algorithm:

Iterative algorithm:

1) Given $\phi = \overline{\phi}$, find ψ to solve the payoff function optimization problem. Let ψ^* be the optimal function, and let $\overline{\psi} = \psi^*$.
2) Given $\psi = \overline{\psi}$, find ϕ to solve the loss function optimization problem. Let ϕ^* be the optimal loss function and let $\overline{\phi} = \phi^*$.
3) Repeat Steps 2 and 3 until the objective function in (20) does not change.

Note that the optimal loss function obtained from the above iterative algorithm satisfies (17) and that we can consider additional constraints to take more realistic situations into account for the loss and payoff functions.

V. EMPIRICAL ANALYSIS AND NUMERICAL EXPERIMENT

In this section, we demonstrate the solutions P1)–P4) and estimate their hedge effect using empirical data for the power output, wind speed, and their predictions. Here we consider the power output from a wind farm (WF) located in Japan, where the power output from the WF is predicted based on the numerical weather prediction and the power generating properties for turbines. The numerical weather prediction consists of the following two steps:

- Japan Meteorological Agency announces the hourly data of regional spectral models for the next 51 hours twice a day (9am and 9pm).
- Using them as initial and boundary values, a public weather forecasting company computes more sophisticated values for the next day's hourly data by 12pm.

A. Preliminary

1) Data description: In this paper, we use the prediction data obtained from the Local Circulation Assessment and Prediction System (LOCALS [4]) developed by the ITOCHU Techno-Solutions Corporation for the wind speed and the power output of a wind farm in Japan. The data set is given as follows:[1]

Data specifications:
Realized and predicted values of total power output for the WF, and those of wind speed for the observation tower in the WF.

Data period:
2002–2003 (1 year), hourly data, everyday

Total number of data points:
8,000 for each variable excluding missing values

Let $n = 1, \ldots, N$ be the time index (where $N \simeq 8,000$), and assume that the actual power output and the wind speed at time n are, respectively, denoted by P_n and W_n. Also, let $\hat{P}_n$ and $\hat{W}_n$ be the predictions of the corresponding power output and the wind speed obtained from LOCALs, which are computed by noon one day before the actual data is observed. Note that, in the data set used in this paper, the power output P_n and its prediction $\hat{P}_n$ are normalized so that P_n's maximum equals 100.

2) Prediction error of the wind speed: At first, we applied a linear regression of the actual wind speed W_n with respect to its prediction $\hat{W}$ as

$$W_n = a_w \hat{W}_n + b_w + \varepsilon_{w,n}, \quad n = 0, \ldots, N, \ \mathrm{Mean}\left(\varepsilon_{w,n}\right) = 0 \tag{21}$$

where a_w and b_w are a regression coefficient and intercept, respectively, and $\varepsilon_{w,n}$ is a residual satisfying $\mathrm{Mean}\left(\varepsilon_{p,n}\right) = 0$. In this case, the sample variance of residuals (i.e., $\mathrm{Var}\left(\varepsilon_{w,n}\right)$) was found to be 5.12.

On the other hand, we computed the regression spline f satisfying

$$W_n = f\left(\hat{W}_n\right) + \varepsilon_{w,n}. \tag{22}$$

using GAM. In this case, the sample variance of the residuals (i.e., $\mathrm{Var}\left(\varepsilon_{w,n}\right)$) was found to be 4.95. Noting that the sample variance of the measured values is 11.0, we can say that the variance of the wind speed was reduced by 50% (from 11.0 to 5.12) using the predicted value and the linear regression and that it was improved a little using GAM, i.e., from 5.12 to 4.95. In this section, we define the prediction error of the wind speed as the one given by GAMs, i.e., $\varepsilon_{w,n}$ in (22).

3) Prediction error of the power output: Similarly, we applied a linear regression of the actual power output P_n with respect to the predicted value $\hat{P}_n$, and in this case, the sample variance of the residuals was found to be 249. On the other hand, we computed the regression spline function g satisfying

$$P_n = g\left(\hat{P}_n\right) + \varepsilon_{p,n}, \quad n = 0, \ldots, N \tag{23}$$

using GAM, and found that the sample variance of residuals in this case is 239. Noting that the sample variance of the measured value of the power output is 504, we can say that the variance of the wind speed was reduced to less than half

[1] All the data used in this paper were provided by ITOCHU Techno-Solutions Corporation.

(from 504 to 249) using the predicted value and the linear regression and that it was improved a little using GAM, i.e., 249 to 239, similar to the wind speed case.

Although we should be able to define the prediction error of the power output using the residual in (22), it might be worthwhile to mention that there is another way to define the prediction error of the power output. As stated in the beginning of this section, the power output is predicted using numerical weather prediction, and therefore, we can define a regression model such that the power output P_n is a dependent variable and the wind speed prediction $\hat{W}_n$ is an independent variable, i.e.,

$$P_n = h(\hat{W}_n) + \varepsilon_{p,n}, \tag{24}$$

where h is a regression spline that minimizes PRSS. For the same data sat, we applied GAM to find the regression spline h in (24), and in this case, the sample variance of the residuals (i.e., $\mathrm{Var}(\varepsilon_{p,n})$) was computed to be 254, which is, in fact, higher than the one given by (23). However, it will turn out that using the prediction error in (24) provides not only a better hedge effect but also a smaller variance of the hedged loss when combining with the optimal wind derivative. Therefore, we will use the residual $\varepsilon_{p,n}$ in (24) to define the prediction error of the power output. An empirical analysis using the prediction error defined by the residual in (23) may be found in [11].

B. Construction of wind derivatives and their hedge effect

1) Linear function's case: We first solve the minimum variance hedging problem for the simplest case where the loss and the payoff functions are both linear. Let

$$\phi(\varepsilon_{p,n}) = \varepsilon_{p,n}, \quad \psi(\varepsilon_{w,n}) = \varepsilon_{w,n} \tag{25}$$

without loss of generality. In this case, the problem is reduced to solving a linear regression for the following regression function:

$$\varepsilon_{p,n} = a_w \varepsilon_{w,n} + \eta_n, \tag{26}$$

where η_n is a residual. Since the linear regression computes a_w that minimizes variance of $\eta_n = \varepsilon_{p,n} - a_w \varepsilon_{w,n}$, the regression coefficient provides the optimal volume as $\Delta^* = -a_w$ in (3) under condition (25), where

$$a_w = \frac{\mathrm{Cov}(\varepsilon_{p,n},\ \varepsilon_{w,n})}{\mathrm{Var}(\varepsilon_{w,n})}. \tag{27}$$

Fig. 2 shows a scatter plot of $\varepsilon_{w,n}$ vs. $\varepsilon_{p,n}$ with a linear regression line. The sample correlation (i.e., $\mathrm{Corr}(\varepsilon_{p,n},\ \varepsilon_{w,n})$) and VRR are computed to be 0.76, and

$$1 - \mathrm{Corr}(\varepsilon_{p,n},\ \varepsilon_{w,n})^2 \simeq 0.43. \tag{28}$$

We see that the prediction errors of the wind speed and the power output, $\varepsilon_{w,n}$ and $\varepsilon_{p,n}$, are highly correlated and that the sample variance is reduced to 43% from the original one using the wind derivative in the case where the loss and the payoff functions are both linear.

Now, we apply GAMs to compute an optimal payoff function. The solid line in Fig. 3 shows the optimal payoff curve obtained by solving the optimization problem (12) when ϕ is linear. In this case, the VRR is computed to be

$$\frac{\mathrm{Var}(\varepsilon_{p,n} + \psi^*(\varepsilon_{w,n}))}{\mathrm{Var}(\varepsilon_{p,n})} \simeq 0.407. \tag{29}$$

where ψ^* is the optimal payoff function found by solving the payoff function optimization problem (12). Moreover, the sample variance of the hedged loss $\varepsilon_{p,n} + \psi^*(\varepsilon_{w,n})$ is

$$\mathrm{Var}(\varepsilon_{p,n} + \psi^*(\varepsilon_{w,n})) \simeq 103. \tag{30}$$

The above variance is actually lower than that of the hedged loss using (23) with the optimal wind derivative, which is computed as 119. Therefore, we see that, even though the variance of the original loss might be larger, it can be reduced more effectively by combining it with the wind derivative if we define the prediction error by (24) instead of (23).

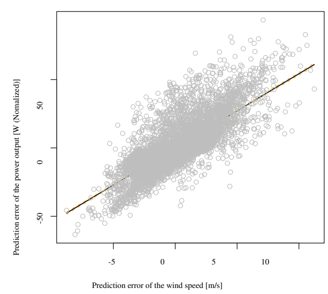

Fig. 2. $\varepsilon_{w,n}$ vs. $\varepsilon_{p,n}$

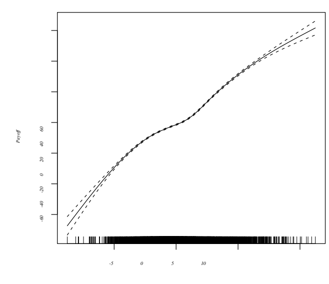

Fig. 3. Optimal payoff function

2) Piecewise linear function's case: Next, we will consider the case in which the loss function $\phi = \overline{\phi}$ is given as shown in Fig. 1 with zero mean constraint (1), i.e.,

$$\overline{\phi}(\varepsilon_{p,n}) := 4\left|\varepsilon_{p,n}\right|^+ + 10\left|\varepsilon_{p,n}\right|^- - \mu \tag{31}$$

where $\mu := \mathrm{Mean}\left(4\left|\varepsilon_{p,n}\right|^+ + 10\left|\varepsilon_{p,n}\right|^-\right)$, $|x|^+ := \max(x,\ 0)$ and $|x|^- := \min(x,\ 0)$. The solid line in Fig. 4 shows the optimal payoff function found by solving problem (12). In this case, VRR in (13) is computed to be

$$\mathrm{VRR} = 0.5461946\cdots \tag{32}$$

whereas the right hand side of (16) is found to be

$$1 - [\mathrm{Corr}(\phi(\varepsilon_{p,n}),\ \psi^*(\varepsilon_{w,n}))]^2 = 0.5461927\cdots. \tag{33}$$

From this example, we see that VRR can be approximated as in (16) with high accuracy.

C. Optimal loss function and simultaneous optimization

In this subsection, we first provide an illustrative example of solving P3) to compute an optimal loss function, and then solve the simultaneous optimization problem of P4).

Since the linear correlation between $\varepsilon_{p,n}$ and $\varepsilon_{w,n}$ is high in this example, it would be more interesting to consider the case where a payoff function is non-linear with respect to $\varepsilon_{w,n}$. Therefore, we assume that there already exists a

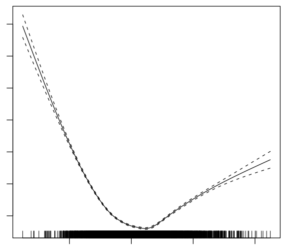

Fig. 4. Optimal payoff function on the wind speed prediction error $\varepsilon_{w,n}$

derivative contract with the payoff being proportional to the size of the wind speed prediction error, $|\varepsilon_{w,n}|$. Noting that $\psi(\varepsilon_{w,n})$ satisfies (2), such a payoff function may be given as

$$\psi(\varepsilon_{w,n}) = \overline{\psi}(\varepsilon_{w,n}) := |\varepsilon_{w,n}| - \text{Mean}(|\varepsilon_{w,n}|), \qquad (34)$$

Now we will solve P3) with the given payoff function in (34). Assume that the sample variance of the loss, $\phi(\varepsilon_{p,n})$, satisfies

$$\text{Var}(\phi(\varepsilon_{p,n})) = \text{Var}(\varepsilon_{p,n}) \qquad (35)$$

and we solve problem (18) with the assumption that the optimal loss function satisfies the above variance constraint. The solid line in Fig. 5 shows the optimal loss function, which is obtained by applying GAM and scaling the minimizing function. In this case, VRR is found to be 0.56.

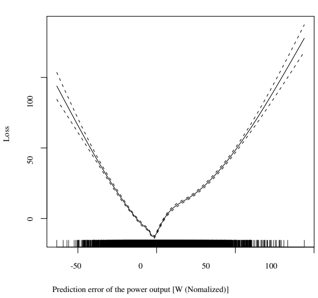

Fig. 5. Optimal loss function

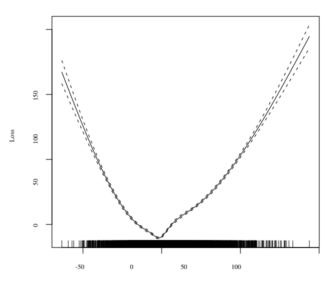

Fig. 6. Iterative algorithm

Next, we demonstrate the simultaneous optimization of P4). Here we also introduce a nonlinearity using the absolute value of $\varepsilon_{w,n}$. Assume that the payoff of the wind derivative is a function of $|\varepsilon_{w,n}|$, and consider the maximization of

$$\text{Corr}(\phi(\varepsilon_{p,n}), \psi(|\varepsilon_{w,n}|)). \qquad (36)$$

We apply the iterative algorithm for a fixed loss function ϕ or a fixed payoff function ψ at each step to maximize (36). Assume that the payoff function is initially set to the one given in (34) and we solve the loss function optimization problem. The initial loss function in this case is given by the one shown in Fig. 5. We repeatedly apply Steps 1 and 2 in the iterative algorithm until the objective function does not change or the relative change of the values of the objective function is less than a sufficiently small number. In this example, we obtained $\text{VRR} = 0.53$ after the 8th iteration, by applying the Iterative algirhtm. Fig. 6 shows the optimal loss function after the 8th iteration. We see that the loss function became smoother compared to the one given in Fig. 5.

VI. Concluding remarks

In this work, we have proposed a new type of weather derivatives based on the prediction errors for wind speeds and estimated their hedge effect on wind power energy businesses. At first, we explained some properties of the loss for a WF caused by prediction errors of the power output, and characterized it using a loss function on the error. We then formulated four types of optimization problems: 1) Contract volume optimization problem, 2) Payoff function optimization problem, 3) Loss function optimization problem, and 4) Simultaneous optimization problem. It was shown that the contract volume optimization problem may be reduced to the standard minimum variance hedge and is solved by applying linear regression. The idea of standard minimum variance hedging was generalized to the payoff function optimization problem by introducing a non-parametric regression technique using smooth splines (or GAMs). We also showed that the loss function optimization problem may be solved by applying GAMs, and a simultaneous optimization technique of the loss and payoff functions for wind derivatives was demonstrated by applying GAMs iteratively. An empirical analysis and numerical experiments were performed to illustrate the hedge effect of the proposed wind derivatives.

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