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Connectivity Guaranteed Migration and Tracking of Multi-agent Flocks

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Abstract— **In this paper, we study the control strategy for connectivity guaranteed migration and trajectory tracking for multi-agent groups using artificial potential field (APF) based approach. Some agents, called active agents (AAs), that are attracted by the reference point (modeled as the virtual leader) lead the rest of the group to perform the tasks. An AA switching rule and the coupled flocking controller are proposed to realize the velocity consensus, inter-agent collision avoidance and joint connectivity of the group. Further, we show a geometric characterization and a stronger connectivity result of the group by the proposed controller under an additional assumption.**

I. Introduction

Connectivity is a key issue in the consensus and formation control of multi-agent systems [1], [2], [6], [5], [4], [3], [11], [12], [9], [17], [18]. A flock, although without universally accepted definition in control literature, is considered to be at least connected. But as a "loose" formation, a flock does not necessarily be in a unique geometric pattern. It is widely acknowledged that one of the pioneering works in constructing man-made flocking was done by Reynolds in computer graphics field. In [10], he proposes the well known Reynolds' rule by which a flocking simulation of a group of ideal agents, called boids, is successfully realized. In control society, many existing results on flocking control of multi-agent systems rely on the design of so called artificial potential fields or potential functions, from which the inter-agent virtual attraction/repulsion force is generated. It is hoped that by elaborately cooking the potential functions, all the agents in the group will eventually move in the same velocity, while simultaneously the configuration of the group converges to the local or global extremes of the collective potential function, which correspond to the desired geometric pattern or range.

The problem of flocking control for particle vehicles, with single or double integrator model, is worthy of study not only because it can provide high level control strategies for flocking control of multi-vehicle teams with more complex dynamics, but also due to its value in determining the effects of information flow in the distributed control of coupled systems. In the early paper [13], virtual leaders of the group are introduced and pair-wise potential not only exist between real agents in the group but also between a real agent and a virtual leader. But there, the aim of adding virtual leaders is to help shaping the potential function for the group so that it can be stabilized at the desired geometric pattern (not only a flock). The authors of [11] and [12] propose smooth or non-smooth pair-wise potential function whose gradient specify the inter-agent virtual attractive/repulsive force. It is proved in [11] that the control law combining the potential's gradient with velocity matching terms coincide with the Reynolds rules but will generically lead to regular fragmentation of the group. The paper further asserts (without proof) that by adding a destination feedback term in the control of each agent can successfully generate a flock with arbitrary initial conditions. Recently, the authors of [16] and [15] propose connectivity preserving controllers, by designing novel inter-agent potentials, to realize swarm aggregation and flocking of multi-agent systems under the initial connectivity assumption.

In this paper, we propose control strategies aimed at connectivity guaranteed migration and trajectory tracking of a group of agents with arbitrary initial positions and velocities. By "connectivity guaranteed", we mean that while the group performs migration or trajectory tracking, its graph is ensured to be jointly connected or eventually connected. A virtual leader is used to represent the stationary destination or the reference point on the trajectory being tracked by the group. Along the lines in [13], [11] and [12], we revisit APF based design approaches. It is assumed that some of the agents, called *active agents* (AAs), in the group are controlled by the virtual attractive force from the virtual leader as well as the attractive/repulsive force from its neighbors; while the others are only affected by the attractive/repulsive force from their neighbors. Unlike in Algorithm 2 in [11], where every agent can be seen active all the times, an AA switching rule and time-varying controller is presented to realize the velocity consensus, inter-agent collision avoidance, and some type of joint connectivity of the group. Further, under the additional assumption on the velocity consensus, we show that the proposed controller can drive the group to be eventually connected.

The rest of the paper is organized as follows: In Section II, we introduce some basics of graph theory and the properties of the potential functions used in this work. In Section III, we present our main results on the connectivity guaranteed migration and trajectory tracking control laws. Lastly, concluding remarks will be made in Section IV.

II. Preliminaries

A. Graph theory

To make this paper self-contained, we recall some basics of graph theory from the past literature, see, e.g. [7]. $\mathcal{G}(\mathcal{V},\mathcal{E})$

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is an undirected graph which consists of a vertex set $\mathcal{V}$ and an edge set $\mathcal{E} \subset \mathcal{V} \times \mathcal{V}$. For any $i, j \in \mathcal{V}$, the ordered pair $(i, j) \in \mathcal{E}$ if and only if i is a neighbor of j. A path from vertex i to j is defined as a sequence of directed edges $(v_1, v_2), (v_2, v_3), \cdots, (v_{n-2}, v_{n-1}), (v_{n-1}, v_n)$, where $n \geq 1, v_1 = i, v_n = j$, and $v_1, \cdots, v_n$ are distinct. An undirected graph is said to be connected if and only if there is a path between any pair of vertices.

Let $\mathcal{G}(\mathcal{V}, \mathcal{E}(t)), t \in \mathbb{R}$ be a graph with vertex set $\mathcal{V}$ and time-dependent edge set $\mathcal{E}(t)$, we use $\biguplus_t \mathcal{G}(\mathcal{V}, \mathcal{E}(t))$ to represent the graph composed of node set $\mathcal{V}$ and edge set $\bigcup_t \mathcal{E}(t)$. The graph $\mathcal{G}(\mathcal{V}, \mathcal{E}(t))$ is said to be jointly connected across $[t_1, t_2]$ if and only if, for any $i, j \in \mathcal{V}$, there is a path between i and j in $\biguplus_{t\in[t_1,t_2]} \mathcal{G}(\mathcal{V}, \mathcal{E}(t))$. In this case, it is also said that all the vertices in $\mathcal{V}$ are jointly connected across $[t_1, t_2]$. If for a given finite time t_1, there exists $t_2 \geq t_1$ such that $\mathcal{G}(\mathcal{V}, \mathcal{E}(t))$ is jointly connected over $[t_1, t_2]$, we say that $\mathcal{G}(\mathcal{V}, \mathcal{E}(t))$ (or all the vertices in $\mathcal{V}$) is (are) jointly after t_1.

In this work, we use $\mathcal{G}_p(\mathcal{V}, \mathcal{E}(t))$, sometimes simply $\mathcal{G}_p(t)$, to denote the group induced graph. The vertex set $\mathcal{V}$ and the edge set $\mathcal{E}(t), t \geq t_0$ are defined as:

$$\mathcal{V} = \{1, 2, \cdots, N\}, \tag{1}$$

$$\mathcal{E}(t) = \{(i, j) : \|x_i(t) - x_j(t)\| \leq r_{nb},\ i, j \in \mathcal{V}\}, \quad t \geq t_0 \tag{2}$$

where N is the number of agents in the group, r_{nb} is a positive real number less than r_s, the physical sensing and communication range of each agent. From the definitions above, we see that the graph $\mathcal{G}_p(t)$ is a undirected graph. A vertex (agent) set $\mathcal{V}_1(t) \subseteq \mathcal{V}$ is said to be a *subgroup* of the group at time t if $\forall\, i \in \mathcal{V}_1(t), \forall\, j \in \mathcal{V}\backslash\mathcal{V}_1(t),\ (i, j) \notin \mathcal{E}(t)$. A subgroup is said to be connected if all the vertices (agents) in it are connected.

The adjacency matrix $A_p(t) \in \mathbb{R}^{N\times N}$ and the Laplacian $L_p(t) \in \mathbb{R}^{N\times N}$ of the graph $\mathcal{G}_p(t)$ are, by convention, defined as [7]:

$$A_p(t) = [a_{ij}(t)],\ \text{with}\ a_{ij}(t) = \left\{ \begin{array}{ll} a^*_{ij} > 0, & \text{if}\ (i, j) \in \mathcal{E}(t) \\ 0, & \text{otherwise} \end{array} \right. \tag{3}$$

where $a^*_{ij} = a^*_{ji}, \forall\, i, j \in \mathcal{V}$; and

$$L_p(t) = [l_{ij}(t)],\ \text{with}\ l_{ij}(t) = \left\{ \begin{array}{lll} \sum_{k\neq i} a_{ik}(t) & , & \text{if}\ i = j \\ -a_{ij}(t) & , & \text{otherwise} \end{array} \right. \tag{4}$$

In this paper, we call an agent, which utilizes the motion information of the virtual leader as a reference in its controller, an *active agent* (AA) of the group. The set of the AAs at time $t, t \geq t_0$, is denoted as $\mathcal{W}(t)$. In addition, we define matrices

$$B(t) = diag\{b_1(t), \cdots, b_N(t)\}, \tag{5}$$

$$L_a(t) = L_p(t) + B(t). \tag{6}$$

with

$$b_i(t) = \left\{ \begin{array}{lll} b^*_i > 0 & , & \text{if}\ i \in \mathcal{W}(t) \\ 0 & , & \text{otherwise} \end{array} \right. \tag{7}$$

B. Potential functions

In this subsection, we introduce our potential functions that characterize, respectively, the inter-agent and leader-agent attraction and repulsion.

1) Inter-agent potential: Inter-agent potential function $\psi_a(\cdot) : (0, +\infty) \to [0, +\infty)$ is a C^2 function with the following properties: for some positive numbers d_a, r_a satisfying $0 < d_a < r_a < r_{nb}$,

a) $\frac{d\psi_a(x)}{dx} < 0, x \in (0, d_a)$; $\frac{d\psi_a(x)}{dx} > 0, x \in (d_a, r_a)$; $\frac{d\psi_a(x)}{dx} = 0, x \in [r_a, +\infty)$;
b) $\lim_{x\to 0} \psi_a(x) = +\infty$;
c) $\psi_a(x)$ has a unique minimum at $x = d_a$.

Following the idea in the work [11], we pick an example of inter-agent potential as:

$$\psi_a(x) = \int_{d_a}^{x} 10\cdot\left(-\frac{1}{\xi^2} + \frac{1}{d_a^2}\right) \varrho_h\left(\frac{\xi}{r_a}\right) d\xi, \quad x \in (0, +\infty) \tag{8}$$

where $\varrho_h(z)$ is a bump function which is defined in [11]:

$$\varrho_h(z) = \left\{ \begin{array}{ll} 1, & z \in [0, h) \\ \frac{1}{2}\left[1 + \cos\left(\pi\frac{z-h}{1-h}\right)\right], & z \in [h, 1] \\ 0, & z \in (1, +\infty) \end{array} \right. \tag{9}$$

2) Leader-agent potentials: The leader-agent potential $\psi_l(\cdot) : [0, +\infty) \to (0, +\infty)$ is a C^1 function with the properties:

a) $\frac{d\psi_l(x)}{dx} \cdot \frac{1}{x}$ is locally Lipschitz over $[0, +\infty)$, and therefore is uniformly continuous on $[0, x^*]$ for any $x^* < +\infty$;
b) $\frac{d\psi_l(x)}{dx} = 0, x = 0$; $\frac{d\psi_l(x)}{dx} > 0, x \in (0, +\infty)$;
c) $\lim_{x\to+\infty} \psi_l(x) = +\infty$;
d) For any given $x_* > 0$, $\exists\, \epsilon(x_*) > 0$ such that $\frac{d\psi_l(x)}{dx} > \epsilon$, $\forall\, x \geq x_*$.
e) $\exists\, C > 0$ such that $\left|\frac{d\psi_l(x)}{dx}\right| \Big/ |\psi_l(x)| < C, \forall\, x \in [0, +\infty)$,
f) $\exists\, C_l > 0$ such that $\psi_l(x) \geq C_l$, $\forall\, x \in [0, +\infty)$.

It is easy to see that $\psi_l(x)$ has a unique minimum, which is positive, at $x = 0$. An example of function ψ_l is $\frac{x^2}{2} + 1$.

Throughout this paper, we use $\mathbb{N}, \mathbb{R}^+, \mathbb{Z}^+$ to denote, respectively, the set of natural numbers, nonnegative real numbers and nonnegative integers. In addition, we use $\mathbf{1}_N$ and $\mathbf{0}_N$ to represent the $N \times 1$ vectors with all the elements being 1 and 0. $\mathcal{L}^m_p[t_0, +\infty)$ is used to denote the set of all piecewise continuous functions $u : [t_0, +\infty) \to \mathbb{R}^m$ such that $\left(\int_{t_0}^{+\infty} \|u(t)\|^p dt\right)^{1/p} < +\infty$, [8].

III. Flocking Controller

In this section, we consider the model of each agent in the group as:

$$\dot{x}_i(t) = v_i(t), \quad \dot{v}_i(t) = u_i(t), \quad i \in \mathcal{V}, \tag{10}$$

where $x_i(t) \in \mathbb{R}^n$ and $v_i(t) \in \mathbb{R}^n$ $(n = 2, 3)$ are the position and velocity of the ith robot respectively; and $u_i(t)$ is the control input (acceleration) of the ith robot. The model for the virtual leader is in the same form as that of the agent, i.e.,

$$\dot{x}_l(t) = v_l(t), \quad \dot{v}_l(t) = u_l(t) \tag{11}$$

where "l" stands for the word "leader".

It is known that the mobility and limited sensing range of the agents in the group raise the issue that the neighboring relationship of the group may be time-varying. For this reason, to start with our discussion, we define the following time-dependent agent sets:

Definition 1: Agent sets $\mathcal{S}_i(t), \mathcal{N}_i(t), \mathcal{I}_i(t)$, $i \in \mathcal{V}, t \in [t_0, +\infty)$ are defined as

$$\begin{aligned}\mathcal{S}_i(t) &= \{j \in \mathcal{V} : \|x_i(t) - x_j(t)\| < r_s\}, &(12)\\ \mathcal{N}_i(t) &= \{j \in \mathcal{V} : \|x_i(t) - x_j(t)\| < r_{nb}\}, &(13)\\ \mathcal{I}_i(t) &= \{j \in \mathcal{V} : \|x_i(t) - x_j(t)\| < r_a\}, &(14)\end{aligned}$$

where r_s and r_{nb} have been mentioned in Subsection II-A; and r_a is as in Subsection II-B. Obviously, we have the relation: $r_a < r_{nb} < r_s$.

Note that in [12], the solutions of the switching closed-loop system is discussed using the tool of differential inclusion. But in this way, one cannot specify the single rate of change of the state when system switches since it is only can be said to lie in a set. In view of this, in the analysis of the closed-loop system, we introduce dwell time in the system dynamics. Our control strategy can be described as: Consider the time sequence

$$\mathcal{T} := \{t_0, t_1, \cdots\} \quad \text{with} \quad t_{k+1} - t_k = \tau_d > 0. \tag{15}$$

Each agent determines its neighbor set at every moment in $\mathcal{T}$. Simultaneously, the AAs of the group are determined by the rule:

(AASR) *For any $t \in [t_k, t_{k+1}), k \in \mathbb{Z}^+, t_k \in \mathcal{T}$, the set of AA is determined as*

$$\mathcal{W}(t) = \{i \in \mathcal{V} : i = \min SG_j(t_k), j = 1, 2, \cdots, m(t_k)\}, \tag{16}$$

where $SG_j(t_k), j = 1, 2, \cdots, m(t_k), 1 \le m(t_k) \le N$, are the subgroups of the group at time t_k such that $\forall\, j = 1, 2, \cdots, m(t_k), SG_j(t_k)$ is connected, and $\bigcup_{j=1}^{m(t_k)} SG_j(t_k) = \mathcal{V}$.

And for all $t \in [t_k, t_{k+1}), k \in \mathbb{Z}^+$, agent $i, i \in \mathcal{V}$ implements the control law

$$u_i^{as}(t) = \sum_{j \in \mathcal{N}_i(t_k)} f_a(d_{ij}) n_{ji} + \tilde{g}_i(t) f_l(d_{il}) n_{li} - \sum_{j \in \mathcal{N}_i(t_k) \bigcap \mathcal{S}_i(t)} a_{ij}^*(v_i - v_j) - b_i(t)(v_i - v_l) + u_l \tag{17}$$

where

$$f_a(d_{ij}) = \frac{d\psi_a(d_{ij})}{dd_{ij}},\ f_l(d_{il}) = \frac{d\psi_l(d_{il})}{dd_{il}}, d_{ij} = \|x_i - x_j\|,$$

$$d_{il} = \|x_i - x_l\|,\ n_{ji} = \frac{x_j - x_i}{d_{ij}},\ n_{li} = \frac{x_l - x_i}{d_{il}};$$

piecewise constant function $b_i(t)$ is as in (5); and time dependent function $\tilde{g}_i(\cdot) : [t_0, +\infty) \to \mathbb{R}^+$ is defined coupled with a new energy function $\varpi_i(t) : [t_0 - 0, +\infty) \to \mathbb{R}^+$ as

$$\tilde{g}_i(t) = \begin{cases} 0, & i \notin \mathcal{W}(t) \\ \frac{\varpi_i(T_1^i-)}{\psi_l(d_{il}(T_1^i))}, & i \in \mathcal{W}(t) \end{cases} \tag{18}$$

$$\begin{aligned}\varpi_i(t_0-) &= \psi_l(d_{il}(t_0)),\\ \varpi_i(t, d_{il}(t)) &= \begin{cases} \varpi_i(T_2^i-), & i \notin \mathcal{W}(t) \\ \tilde{g}_i(t)\psi_l(d_{il}(t)), & i \in \mathcal{W}(t) \end{cases}, t \ge t_0\end{aligned} \tag{19}$$

with

$$\begin{aligned}T_1^i(t) &\in \mathcal{T} \ \ s.t.\ \ i \in \mathcal{W}(\tau), \forall\, \tau \in [T_1^i, t] &(20)\\ T_2^i(t) &\in \mathcal{T} \ \ s.t.\ \ i \notin \mathcal{W}(\tau), \forall\, \tau \in [T_2^i, t] &(21)\end{aligned}$$

In the third term of (17), we must add $j \in \mathcal{S}_i(t)$ since, taking the sensing capability of the agents into consideration, it is possible that some agent in the set $\mathcal{N}_i(t_k)$ moves out of the sensing range of agent i at some $t \in [t_k, t_{k+1})$. However, $j \in \mathcal{S}_i(t)$ is not necessarily for the first term due to the property of the function $f_a(\cdot)$ that $f_a(d_{ij}) = 0$ for $d_{ij} \ge r_{nb}$.

Remark 1: By AASR, each connected subgroup needs to identify the agent with minimum index, which is distributedly doable but not a scalable process as the group size increases. Here, as preliminary research, we assume that the AA switching process by AASR is completed instantly.

Note that by AASR, the following Assumption 1 naturally holds.

Assumption 1: For all $t \ge t_0$, there is a path connecting any agent in $\mathcal{V} \backslash \mathcal{W}(t)$ to some agent in $\mathcal{W}(t)$.

Also, it is not difficult to see that the functions $\tilde{g}_i(t)$ and $\varpi_i(t, d_{il}(t))$ are well defined. In addition,

- $\tilde{g}_i(t)$ can be obtained at every moment the agent i changes from non-AA to AA, or from AA to non-AA; and it is constant in each time interval $[t_k, t_{k+1}), k \in \mathbb{Z}^+$;
- $\varpi_i(t, d_{il}(t))$ can be seen as a shaped leader-agent potential, which is continuous w.r.t. t over $[t_0, +\infty)$, and is constant over $[t_k, t_{k+1})$ if $i \notin \mathcal{W}(t)$, $\forall\, t \in [t_k, t_{k+1})$.
- If $i \in \mathcal{W}(t), \forall\, t \in [t_0, +\infty)$, then $\tilde{g}_i(t) = 1$, $\varpi_i(t, d_{il}(t)) = \psi_l(d_{il}(t)), \forall\, t \in [t_0, +\infty)$; if $i \notin \mathcal{W}(t), \forall\, t \in [t_0, +\infty)$, then $\tilde{g}_i(t) = 0, \varpi_i(t, d_{il}(t)) = \psi_l(d_{il}(t_0)), \forall\, t \in [t_0, +\infty)$. Since agent 1 is always an AA in the group by AASR, we have $\tilde{g}_1(t) = 1$, and $\varpi_1(t, d_{il}(t)) = \psi_l(d_{1l}(t)), \forall\, t \ge t_0$.

Before moving on to analyze the controller (16) and (17), we need to point out that if the AAs in the group are fixed as the system evolves (AASR is not applied) as in Algorithm 2 in [11], then the the connectivity may not be guaranteed by (17) even Assumption 1 holds. This can be illustrated by the following example:

Example 1: Consider a group of $N = 40$ agents moving in $\mathbb{R}^2$ space. Assume that all agents are the AAs at any time $t \ge t_0$. In addition, suppose that at some time $t_1 \ge t_0$, the velocities of the group satisfy $v_i(t_1) = v_l(t_1), \forall\, i \in \mathcal{V}$; and the positions of the agents are:

For $i \in \mathcal{V}_1 := \{1, 2, \cdots, 10\}$,

$$\begin{aligned} x_{i_1}(t_1) &= x_{l_1}(t_1) + 1.4557 \cos(2\pi(i-1)/10), \\ x_{i_2}(t_1) &= x_{l_2}(t_1) + 1.4557 \sin(2\pi(i-1)/10), \end{aligned} \quad (22)$$

and for $i \in \mathcal{V}_2 := \{11, 12, \cdots, 40\}$,

$$\begin{aligned} x_{i_1}(t_1) &= x_{l_1}(t_1) + 3.0503 \cos(2\pi(i-11)/30), \\ x_{i_2}(t_1) &= x_{l_2}(t_1) + 3.0503 \sin(2\pi(i-11)/30). \end{aligned} \quad (23)$$

where $x_i = [x_{i_1}, x_{i_2}]^T$.

If $r_s = 1.5$, and the inter-agent and leader-agent potentials are chosen as in Section I with $d_a = 1, r_a = 1.2$ and $h = 0.8$. Then each agent in the subgroup $\mathcal{V}_1$ and $\mathcal{V}_2$ is only affected by the virtual forces applied by the virtual leader and the two closest agents in the same subgroup. And, according to (17), it is straightforward to check that

$$u_i(t_1) = u_l(t_1), \quad i \in \mathcal{V} \quad (24)$$

which implies that the inter-agent and leader-agent distances will keep constant for all $t \in [t_1, +\infty)$. Thus, the group can never be connected after time t_1 since no agent in $\mathcal{V}_1$ has a neighbor in $\mathcal{V}_2$.

Now define the energy functions:

$$\begin{aligned} V_a(x) &= \frac{1}{2} \sum_{i=1}^{N} \sum_{j \neq i} \psi_a(d_{ij}), & (25) \\ V_{sl}(t, x, x_l) &= \sum_{i=1}^{N} \varpi_i(t, d_{il}(t)), & (26) \\ H(v, v_l) &= \frac{1}{2} \sum_{i=1}^{N} \|v_i - v_l\|^2, & (27) \\ J_s(x, x_l, v, v_l) &= V_a(x) + V_{sl}(t, x, x_l) + H(v, v_l), & (28) \end{aligned}$$

where $x = [x_1^\top, \cdots, x_N^\top]^\top, v = [v_1^\top, \cdots, v_N^\top]^\top$. V_a, V_{sl} are called, respectively, the collective inter-agent and leader-agent potentials; H is the collective kinematic energy w.r.t. the virtual leader; and J_s is the total energy of the group.

It can be proved that if $\tau_d < \min\{r_s - r_{nb}, r_{nb} - r_a\}/(2\sqrt{2J_s(t_0)})$, then for all $t \in [t_k, t_{k+1})$, we have $j \in \mathcal{S}_i(t)$ for any $j \in \mathcal{N}_i(t_k)$, and $j \notin \mathcal{I}_i(t)$ for any $j \notin \mathcal{N}_i(t_k)$. This gives that the control law (17) can be put into the form: $\forall\, i \in \mathcal{V}$, $\forall\, t \in [t_k, t_{k+1})$,

$$\begin{aligned} u_i^{ds}(t) = &-\sum_{j \neq i} \nabla_{x_i} \psi_a(d_{ij}) - \tilde{g}_i(t) \nabla_{x_i} \psi_l(d_{il}) - \\ &\sum_{j \in \mathcal{N}_i(t_k)} a_{ij}^*(v_i - v_j) - b_i(t)(v_i - v_l) + u_l \quad (29) \end{aligned}$$

or compactly,

$$\begin{aligned} u^{ds}(t) = -\nabla_x V_a - \nabla_x V_{sl} - (L_a(t_k) \otimes \mathbf{1}_N)(v - v_l) \\ + \mathbf{1}_N \otimes u_l, \;\; \forall\, t \in [t_k, t_{k+1}). \quad (30) \end{aligned}$$

In the rest of this section, we always assume that τ_d is small enough and the initial condition (x_0, v_0) satisfies $x_0^i \neq x_0^j, \forall\, i, j \in \mathcal{V}, i \neq j$.

Before presenting the main results in this paper, we introduce an extension of the celebrated Barbalat Lemma, which will play an important role in the proofs of some results followed.

Definition 2: The function $f(\cdot) : \mathbb{R} \to \mathbb{R}$ is said to be piecewise uniformly continuous over $[t_0, +\infty)$ w.r.t. an infinite sequence $\{\hat{t}_i\}_{i=0}^{\infty}$, with $\hat{t}_0 = t_0$ and $\inf \hat{t}_i - \hat{t}_{i-1} \geq \hat{\tau} > 0$, if $\forall\, t \in [\hat{t}_{i-1}, \hat{t}_i), i \in \mathbb{N}$, $\forall\, \epsilon > 0$, $\exists\, \hat{\delta}_\epsilon > 0$, $|f(\tilde{t}) - f(t)| < \epsilon$, $\forall\, \tilde{t} \in B_{\hat{\delta}_\epsilon}(t) \bigcap [\hat{t}_{i-1}, \hat{t}_i)$, where $B_{\hat{\delta}_\epsilon}(t)$ is the open ball centered at t with the radius $\hat{\delta}_\epsilon$.

Lemma 1: Let $f(\cdot) : \mathbb{R} \to \mathbb{R}$ be piecewise uniformly continuous over $[t_0, +\infty)$ w.r.t. $\{\hat{t}_i\}_{i=0}^{\infty}$, and $h(\cdot) : \mathbb{R} \to \mathbb{R}$ satisfy $\lim_{t \to +\infty} h(t) = 0$. Suppose that $\lim_{t \to +\infty} \int_{t_0}^{t} (f(s) + h(s)) ds$ exists and is finite. Then $\lim_{t \to +\infty} f(t) = 0$.

Remark 2: If the function f is uniformly continuous over $[t_0, +\infty)$, then the conclusion in Lemma 1 naturally follows.

Now, we are in a position to propose the main theorem of this section.

Theorem 1: By the control strategy (16)-(17), $\lim_{t \to +\infty} \|v_i(t) - v_l(t)\| = 0$, $\forall\, i \in \mathcal{V}$; $\lim_{t \to +\infty} d_{1l}(t) = 0$; inter-agent collision is avoided; and for any $T \in [t_0, +\infty)$, the group is jointly connected across $[T, +\infty)$.

Proof: Let $\tilde{x}_i = x_i - x_l$, $\tilde{v}_i = v_i - v_l$, and $\tilde{x} = [\tilde{x}_1^\top, \cdots, \tilde{x}_N^\top]^\top$, $\tilde{v} = [\tilde{v}_1^\top, \cdots, \tilde{v}_N^\top]^\top$. And define new functions

$$\begin{aligned} \tilde{V}_a(\tilde{x}) &:= \frac{1}{2} \sum_{i=1}^{N} \sum_{j \neq i} \psi_a(\tilde{d}_{ij}) = V_a(x), \\ \tilde{V}_{sl}(t, \tilde{x}) &:= \sum_{i=1}^{N} \varpi_i(t, \tilde{d}_i(t)) = V_{sl}(t, x, x_l), \quad (31) \end{aligned}$$

where $\tilde{d}_{ij} = d_{ij}, \tilde{d}_i = d_{il}$. It can be easily seen that $\nabla_x V_a(x) = \nabla_{\tilde{x}} \tilde{V}_a(\tilde{x}),$, $\nabla_x V_{sl}(t, x, x_l) = \nabla_{\tilde{x}} \tilde{V}_{sl}(t, \tilde{x})$.

Note that since $\tilde{g}_i(t)$ is constant over $[t_k, t_{k+1}), \forall\, k \in \mathbb{Z}^+$, it follows that $\forall\, t \in (t_k, t_{k+1}), \forall\, k \in \mathbb{Z}^+$,

$$\frac{d\varpi_i(t, \tilde{d}_i(t))}{dt} = \left(\nabla_{\tilde{x}_i} \varpi_i(t, \tilde{d}_i(t))\right)^\top \tilde{v}_i, \quad (32)$$

Taking derivative of J_s w.r.t. t along the system (10), (16), (17), we have $\forall\, t \in (t_k, t_{k+1}), k \in \mathbb{Z}^+$,

$$\begin{aligned} \dot{J}_s &= \dot{\tilde{V}}_a(\tilde{x}) + \dot{\tilde{V}}_{sl}(t, \tilde{x}) + \tilde{v}^\top (u^{ds} - \mathbf{1}_N \otimes u_l) \\ &= (\nabla_{\tilde{x}} \tilde{V}_a)^\top \tilde{v} + (\nabla_{\tilde{x}} \tilde{V}_{sl})^\top \tilde{v} + \\ &\qquad \tilde{v}^\top (-\nabla_{\tilde{x}} \tilde{V}_a - \nabla_{\tilde{x}} \tilde{V}_{sl} - (L_a(t_k) \otimes \mathbf{1}_N) \tilde{v}) \\ &= -\tilde{v}^\top (L_a(t_k) \otimes \mathbf{1}_N) \tilde{v} \quad (33) \end{aligned}$$

Since the group can only has finite interconnection topologies, it follows from Lemma 3 in [14] that there exists a positive real constant λ_2^* such that, for all $k \in \mathbb{Z}^+$,

$$\dot{J}_s \leq -\lambda_2^* \|\tilde{v}\|^2. \quad (34)$$

We know that $J_s(t)$ is continuous over $[t_0, +\infty)$ by the continuity of $\varpi_i(t), \forall\, i \in \mathcal{V}$. This, together with (33) and the positiveness of J_s, give that $\forall\, t \geq t_0$,

$$2\lambda_2^* \int_{t_0}^{t} H(s) ds = \lambda_2^* \int_{t_0}^{t} \|\tilde{v}(s)\|^2 ds \leq J_s(t_0) \quad (35)$$

On the other hand, from the boundedness of J_s, we have V_a and H are bounded over $[t_0, +\infty)$, which leads to the boundedness of $\nabla_x V_a$ and $\tilde{v}(t)$. Also the boundedness of $J_s(t)$ gives that $\varpi_i(t), i \in \mathcal{V}$ are bounded over $[t_0, +\infty)$. By property e) of the function ψ_l, it follows that,

$$\begin{aligned}\|\nabla_{x_i} V_{sl}\| &= \|\nabla_{x_i}\varpi_i(t, d_{il})\| = \|\tilde{g}_i(t)\nabla_{x_i}\psi_l(d_{il})\| \\ &= |\tilde{g}_i(t)\psi_l(d_{il})| \cdot \frac{\|\nabla_{x_i}\psi_l(d_{il})\|}{|\psi_l(d_{il})|} \\ &= \begin{cases} 0, & i \notin \mathcal{W}(t) \\ |\varpi_i(t)| \cdot \frac{\left|\frac{d\psi_l(d_{il})}{dd_{il}}\right|}{|\psi_l(d_{il})|}, & i \in \mathcal{W}(t) \end{cases}\end{aligned}$$

is bounded over $[t_0, +\infty)$. Therefore,

$$\begin{aligned}\frac{dH}{dt} &= \tilde{v}^\top(-\nabla_x V_a - \nabla_x V_{sl} - (L_a(t) \otimes I_N)\tilde{v}) \\ &\in \mathcal{L}_\infty[t_0, +\infty), \end{aligned} \tag{36}$$

which implies that H is uniformly continuous with respect to t for all $t \geq t_0$. Thus, by Barbalat Lemma, $\lim_{t\to+\infty} H(v(t), v_l(t)) = 0$, which means that $\forall\, i \in \mathcal{V}, \|v_i(t) - v_l(t)\| \to 0$ as $t \to +\infty$.

The collision avoidance argument is easily justified by the boundedness of $V_a(t)$ over $[t_0, +\infty)$; and the fact that inter-agent potential approaches infinity when d_{ij} goes to zero.

Now, we prove that the group is jointly connected across $[T, +\infty)$ for any finite $T \geq t_0$. First, we show that $\lim_{t\to+\infty} d_{1l}(t) = \lim_{t\to+\infty}\|x_1(t) - x_l(t)\| = 0$. If this is not true, there exist $\delta_d > 0$ and a time sequence $\{\tilde{T}_i\}_{i=1}^\infty$ such that $d_{1l}(\tilde{T}_i) > \delta_d$ for all $i \in \mathbb{N}$. Suppose $\|\tilde{v}(t)\| \leq \gamma_v$, $\forall\, t \in [t_0, +\infty)$, where $\gamma_v \in (0, +\infty)$. Then we have for all $t \in [\tilde{T}_i, \tilde{T}_i + \Delta T)$, $\forall\, i \in \mathbb{N}$

$$d_{1l}(t) \geq \delta_d/2, \quad \text{and} \quad \angle(n_{l1}(t), n_{l1}(\tilde{T}_i)) \leq \frac{\delta_d/2}{\delta_d/2} = 1\, rad. \tag{37}$$

where $\Delta T = \frac{\delta_d}{2\gamma_v}$.

Let $\Delta\tilde{T} = \min\{\Delta T, \tau_d\}$ and $m_f = \inf_{d_{1l}\geq\delta_d/2} f_l(d_{1l})$. By property d) of ψ_l, we have $m_f > 0$. Since $\lim_{t\to+\infty}\|\tilde{v}(t)\| = 0$, there exists $K \in \mathbb{N}$ such that

$$\|\tilde{v}(t)\| < \epsilon := \frac{m_f \Delta\tilde{T}\cos(1)}{4N + b_1^*\Delta\tilde{T}}, \quad \forall\, t \geq \tilde{T}_K. \tag{38}$$

From the definition of $\Delta\tilde{T}$, it is easy to see that there is at most one element in $\mathcal{T}$ which lies in the interval $[\tilde{T}_K, \tilde{T}_K + \Delta\tilde{T}]$. Assume this element exists and denote it by t_I. (If it is not the case, the proof is similar and thus omitted.) Now, according to the AASR, agent 1 is always an AA in the group; and what is more, the only AA in the connected subgroup it resides. Denote $SG_1(t), t \in [t_0, +\infty)$ as the connected subgroup at time t with agent 1 as the AA, then,

$$\begin{aligned}4N\epsilon &\geq \left\|\sum_{j\in SG_1(t_I)} \tilde{v}_j(\tilde{T}_K + \Delta\tilde{T}) - \tilde{v}_j(t_I)\right\| + \\ &\qquad \left\|\sum_{j\in SG_1(\tilde{T}_K)} \tilde{v}_j(t_I) - \tilde{v}_j(\tilde{T}_K)\right\| \\ &\geq \left|\int_{\tilde{T}_K}^{\tilde{T}_K+\Delta\tilde{T}} \left(\sum_{j\in SG_1(t)} (u_j(t) - u_l(t))\right)^\top n_{l1}(\tilde{T}_K)dt\right| \\ &= \left|\int_{\tilde{T}_K}^{\tilde{T}_K+\Delta\tilde{T}} (f_l(d_{1l}(t))n_{l1}(t) - b_1^*\tilde{v}_1(t))^\top n_{l1}(\tilde{T}_K)dt\right| \\ &\geq m_f \cdot \cos(1)\cdot\Delta\tilde{T} - b_1^*\Delta\tilde{T}\epsilon,\end{aligned}$$

which contradicts with (38).

Now, define the set of agents that are jointly connected to agent 1 after any finite time by $\mathcal{V}_1$. If $\mathcal{V}_1 \neq \mathcal{V}$, let $p = \min\{i : i \in \mathcal{V} - \mathcal{V}_1\}$. Clearly, there exists $\tilde{t} \geq t_0$ such that there is no connection between the sets $\mathcal{V}$ and $\mathcal{V} - \mathcal{V}_1$. Thus, by the AASR, we have agent p is an AA in the group and the only AA in the subgroup it resides after $\tilde{t}$, which, by the similar arguments above, gives that $\lim_{t\to+\infty}\|x_p(t) - x_l(t)\| = 0$. This implies that there exists $t_c \geq t_0$ such that $\|x_1(t_c) - x_p(t_c)\| < r_{nb}$ for all $t \geq t_c$, which is a contradiction. Therefore, it can be concluded that $\mathcal{V}_1 = \mathcal{V}$. ■

In the following, by putting some additional assumption on the velocity consensus of the group, we render a stronger result than Theorem 1.

Theorem 2: In Theorem 1, if $\tilde{v}(t) \in \mathcal{L}_1[t_0, +\infty)$, then there exists $T_f \geq t_0$ such that the group is connected at $t_k \in \mathcal{T}\bigcap[T_f, +\infty)$. Moreover, the configuration of the group a.e. converges to some local minimum of the collective potential $V_a = \frac{1}{2}\sum_{i=1}^N\sum_{j\neq i}\psi_a(d_{ij})$.

Proof: From assumption that $\int_{t_0}^{+\infty}\|\tilde{v}(t)\|dt < +\infty$, it is not difficult to show that $\forall\, i \in \mathcal{V}$, $\lim_{t\to+\infty} x_i(t) - x_l(t) = d_{il}^* n_{il}^*$; and $\forall\, i, j \in \mathcal{V}$, $\lim_{t\to+\infty} x_i(t) - x_j(t) = d_{ij}^* n_{ij}^*$, where $d_{il}^* \geq 0$, $d_{ij}^* > 0$, $n_{il}^*, n_{ij}^* \in \mathbb{R}^n$, $\|n_{il}^*\| = \|n_{ij}^*\| = 1$. This implies $\exists\, d_M > 0$ such that $d_{il}(t) < d_M, \forall\, i \in \mathcal{V}$, $\forall\, t \in [t_0, +\infty)$; and $\forall\, i, j \in \mathcal{V}$,

$$\lim_{t\to+\infty}\nabla_{x_i}\psi_a(d_{ij}) = f_a(d_{ij}^*)n_{ij}^*. \tag{39}$$

By Theorem 1, we know that $\forall\, i \in \mathcal{V}$, $\int_{t_0}^{+\infty} u_i(t) - u_l(t)dt = \lim_{t\to+\infty}\tilde{v}_i(t) - \tilde{v}_i(t_0) = -\tilde{v}_i(t_0)$; and, in (29), the term $-\sum_{j\in\mathcal{N}_i(t_k)} a_{ij}(v_i - v_j) - b_i(t)(v_i - v_l)$ approaches zero as $t \to +\infty$. On the other hand, since $\tilde{g}_i(t), \forall\, i \in \mathcal{V}$ is constant over $[t_k, t_{k+1}), \forall\, k \in \mathbb{Z}^+$, and $d_{il}(t)$ is bounded over $[t_0, +\infty)$, it can be concluded that $-\sum_{j\neq i}\nabla_{x_i}\psi_a(d_{ij}) - \tilde{g}_i(t)\nabla_{x_i}\psi_l(d_{il})$ is piecewise uniformly continuous over $[t_0, +\infty)$ w.r.t. $\{t_k\}_{k=0}^\infty$. Hence, by Lemma 1, it follows that $\forall\, i \in \mathcal{V}$,

$$-\sum_{j\neq i}\nabla_{x_i}\psi_a(d_{ij}) - \tilde{g}_i(t)\nabla_{x_i}\psi_l(d_{il}) \to 0 \;\text{ as }\; t \to +\infty. \tag{40}$$

(40) and (39) give that $\lim_{t\to+\infty}\tilde{g}_i(t)\nabla_{x_i}\psi_l(d_{il})$ exists for any $i \in \mathcal{V}$. But we know from Theorem 1 that $d_{1l}^* = 0$, which implies $d_{il}^* \neq 0$ for all $i \neq 1$. So, $\lim_{t\to+\infty}\nabla_{x_i}\psi_l(d_{il}) = f_l(d_{il}^*)n_{il}^* \neq \mathbf{0}_n$ for all $i \neq 1$. As a result, $\lim_{t\to+\infty}\tilde{g}_i(t)$ exists for any $i \in \mathcal{V}$ (recall that $\tilde{g}_1(t) = 1$ for all $t \geq t_0$).

Let $\mathcal{T}_i := \{t_{i_k}\}_{k=0}^\infty$, $i_k \in \mathbb{Z}^+$ with $t_{i_0} = t_0$ be the subset of $\mathcal{T}$ such that agent i switches between being active and inactive at each $t_{i_k}, k \in \mathbb{Z}^+$ ($\mathcal{T}_i$ might be finite, but the proof would be similar). Define $\mathcal{T}_l^i, i \in \mathcal{V}$ as the set of time at which agent i is an AA, i.e., $\mathcal{T}_l^i = \{t : t \in [t_0, +\infty), i \in \mathcal{W}(t)\}$. Now, we prove that $\forall\, i \in \mathcal{V}$, if $\mathcal{T}_l^i$ is not empty, then $\tilde{g}_i(t)$ is lower bounded above zero over T_l^i. Without loss of generality, assume $i \notin \mathcal{W}(t_0)$. Then we have that $\mathcal{T}_l^i = \bigcup_{k=0}^{\infty}[t_{i_{2k+1}}, t_{i_{2k+2}})$. By (18) and (19), it follows that for all $t \in [t_{i_{2K+1}}, t_{i_{2K+2}}), K \in \mathbb{Z}^+$,

$$\tilde{g}_i(t) = \prod_{k=0}^{K} \frac{\psi_l(d_{il}(t_{i_{2k}}))}{\psi_l(d_{il}(t_{i_{2k+1}}))}. \tag{41}$$

Since the function $\frac{d\psi_l(y)}{dy}$ is continuous over $[0, +\infty)$, there exists a Lipschitz constant β_L such that $\forall\, y_1, y_2 \in [0, d_M)$, $|\psi_l(y_1) - \psi_l(y_2)| \le \beta_L |y_1 - y_2|$. Therefore, combining property f) of ψ_l, we have $\forall\, K \in \mathbb{Z}^+$ and $\forall\, t \in [t_{i_{2K+1}}, t_{i_{2K+2}})$,

$$\begin{aligned}
\frac{1}{\tilde{g}_i(t)} &= \prod_{k=0}^{K}\left[1 + \frac{\psi_l(d_{il}(t_{i_{2k+1}})) - \psi_l(d_{il}(t_{i_{2k}}))}{\psi_l(d_{il}(t_{i_{2k}}))}\right] \\
&\le \prod_{k=0}^{K}\left[1 + \frac{\beta_L}{C_l}\left\|\int_{t_{i_{2k}}}^{t_{i_{2k+1}}} v_i(t) - v_l(t)dt\right\|\right] \\
&\le \exp\left\{\frac{\beta_L}{C_l}\int_{t_0}^{+\infty}\|\tilde{v}(t)\|dt\right\} \qquad (42)
\end{aligned}$$

By assumption $\int_{t_0}^{+\infty}\|\tilde{v}(t)\|dt < +\infty$, (42) implies that $\tilde{g}_i(t) \ge m_g > 0$ for all $t \in \mathcal{T}_l^i$.

Now, the two facts that for any $i \in \mathcal{V}$, $\lim_{t\to+\infty}\tilde{g}_i(t)$ exists; and $\tilde{g}_i(t) \ge m_g > 0, \forall\, t \in \mathcal{T}_l^i$; $\tilde{g}_i(t) = 0, \forall\, t \in [t_0, +\infty)\backslash\mathcal{T}_l^i$ show that there must be $T_f \in [t_0, +\infty)$ such that for all $t \ge T_f$, either $i \in \mathcal{W}(t)$ or $i \notin \mathcal{W}(t)$. Suppose that $\exists\, i \neq 1$, the former case holds. Then, by similar arguments as in Theorem 1, we have $\lim_{t\to+\infty}\|x_i(t) - x_l(t)\| = d_{il}^* = 0$, which is a contradiction. Therefore, for all $t \ge T_f$, agent 1 is the only AA in the group, which by the AASR, implies that the group is connected at $t_k \in \mathcal{T}\bigcap[T_f, +\infty)$.

The second part of the Theorem comes from (40) and the facts that $\tilde{g}_1(t) = 1, \forall\, t \in [t_0, +\infty)$, $\lim_{t\to+\infty}\nabla_{x_1}\psi_l(d_{1l}) = \mathbf{0}_n$, and $\tilde{g}_i(t) = 0, \forall\, t \in [t_f, +\infty), i \neq 1$. ∎

At the end of this section, we show a result on the robustness of the control law (16) and (17). Consider the controller, $\forall\, i \in \mathcal{V}$, $\forall\, t \in [t_k, t_{k+1}), k \in \mathbb{Z}^+$,

$$\begin{aligned}
\tilde{u}_i^{as}(t) = \sum_{j\in\mathcal{N}_i(t_k)} f_a(d_{ij})n_{ji} + \tilde{g}_i(t)f_l(d_{il})n_{li} - \\
\sum_{j\in\mathcal{N}_i(t_k)\bigcap\mathcal{S}_i(t)} a_{ij}^*(v_i - v_j) - b_i(t)(v_i - v_l) + u_l + \delta_u^i(t), \ (43)
\end{aligned}$$

where $\tilde{g}_i(t)$ is as in (18), and $\delta_u^i(t) : \mathbb{R} \to \mathbb{R}^n$ is the disturbance signal.

Theorem 3: If $\forall i \in \mathcal{V}$, $\delta_u^i(t) \in \mathcal{L}_1^n[t_0, +\infty)\bigcap\mathcal{L}_2^n[t_0, +\infty)\bigcap\mathcal{L}_\infty^n[t_0, +\infty)$, then by (16) and (43), $\lim_{t\to+\infty}\|v_i(t) - v_l(t)\| = 0, \forall\, i \in \mathcal{V}$; $\lim_{t\to+\infty} d_{1l}(t) = 0$; and inter-agent collision is avoided. Furthermore, if $\forall\, i \in \mathcal{V}$, $\delta_u^i(t) \to 0$ as $t \to +\infty$, then the group is jointly connected after any finite time. Lastly, if the assumption $\tilde{v}(t) \in \mathcal{L}_1[t_0, +\infty)$ also holds, then, there exists $T_f \ge t_0$ such that the group is connected at $t_k \in \mathcal{T}\bigcap[T_f, +\infty)$. Moreover, the configuration of the group a.e. converges to some local minimum of the collective potential V_a.

IV. Conclusions

It has been shown in this paper that when the group performs migration or trajectory tracking along with inter-agent collision avoidance, the connectivity of the group may not be guaranteed if all the members in the group are attracted by the destination or the moving reference point. For this problem, in this work, we put forward a switching strategy that can ensure some kind of joint connectivity, and by adding another assumption, the ultimate connectivity of the group. Our future work will focus on the design of decentralized and scalable control strategies for the connectivity guaranteed formation control of multi-agent systems.

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