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Robust Adaptive Control of a Class of Switched Systems

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Abstract— **In this paper, an algorithm for robust adaptive control design for parametric-strict output feedback switched systems is developed. Under extensions of typical adaptive control assumptions, the control scheme guarantees system stability for bounded disturbances and parameters without requiring a priori knowledge on such parameters or disturbances. The problem reduces to an analysis of an exponentially stable and input-to-state stable system driven by piecewise continuous and impulsive inputs due to plant parameter switching and variation. This system is a modification of the closed loop error dynamics in standard adaptive control systems, through a generalized leakage-type adaptive controller. The results are illustrated via simulations and compared to other adaptive and non-adaptive control methods.**

Index Terms— **switched systems, robust adaptive control, hybrid systems.**

I. INTRODUCTION

Switched and hybrid systems have been gaining considerable interest in both research and industrial control communities. This is motivated by the need for systematic and formal methods to control such systems. These issues arise in systems with discrete changes in energy exchange elements due to intermittent interaction with other systems or with an environment or due to the nature of their constitutive relations. This is common in robotic and mechatronic systems with contact and impact effects, fluidic systems with valves or phase changes, and electrical circuits with switches.

Despite numerous publications on hybrid systems, there is a lack of constructive methods for control of a nontrivial class of switched systems with a priori stability and performance guarantees due to the challenging nature of these systems. In terms of stability and response of switched systems, several results have been obtained in recent years, e.g. [5], [11]. In this context, sufficient conditions for stability such as common Lyapunov functions and average dwell time [5] are the most commonly studied results. A corresponding control design requires switching controller gains such that all subsystems are made stable and such that a common Lyapunov function condition is satisfied, which for LTI systems requires system matrices to commute or be symmetric. In order to verify that such a condition is met, the system is partitioned into known subsystems and a set of linear matrix inequalities of the number of subsystems is solved, if a solution is feasible. The other class of results requires that all subsystems are stable (or with some known briefly visited unstable modes) and switching is slow enough on average, *average dwell time condition* [5]. The corresponding controller design requires gains to be adjusted to guarantee the stability of each frozen configuration and knowledge of worst case decay rate among subsystems and condition number of Lyapunov matrices in order to compute the maximum admissible switching speed. If plant switching exceeds this switching speed then stability can no longer be guaranteed.

On the other hand, an open problem in adaptive control is to extend the robustness of adaptive control to linearly-parameterized time varying systems. However, most of the results [6], [3], [7], [8], [10] are restricted to smoothly varying parameters with known bounds and typically require additional restrictive conditions such as slowly varying unknown parameters [10] or constant and known input vector parameters [7], in order ensure state boundedness. In this case, such a conclusion is of very little practical importance if the error can not be reduced to an acceptable level by increasing the adaptation or feedback gains or using a better nominal estimate of the plant parameters. Furthermore, performance with respect to rejection of disturbances as well as the transient response remain primarily unknown.

The developed control methodology, which is a generalization of fixed-sigma modification, yields strong robustness to time varying and switching parameters without requiring a priori known bounds on such parameters. This paper presents this adaptive controller methodology for time-varying switched systems, which was first introduced by the authors in [1] without a specific controller design for a particular class of systems, and develops a detailed design procedure based on the backstepping approach [4] for parametric-strict output feedback systems with time-varying switching parameters. The remainder of the paper is organized as follows. Section II presents the basic adaptive controller methodology. In section III, the controller design for parametric-strict output feedback switched systems is discussed. Section IV gives an example simulation demonstrating the key characteristics of the control system as well as comparing it with other non-adaptive and adaptive techniques. Conclusions are given in Section V.

II. METHODOLOGY

A. Parameterized Switched Systems

In this paper, we view a switching system as one parameterized by a time varying vector of parameters, which is piecewise differentiable, see Equation (1). This is a reasonable representation since it captures many physical systems

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that undergo switching dynamics, thus we will focus on such systems described by:

$$\begin{aligned} \dot{x} &= f(x,a,u,d) \\ y &= h(x,a) \\ a(t) &= a_i(t),\ t_{i-1} \le t < t_i,\ i = 1,2,\ldots \\ i(t)^{+} &= g(i(t),x,t) \end{aligned} \tag{1}$$

Therefore, we embed the switching behavior in the piecewise changes in $a(t)$, which again may be triggered by state or time driven events. $a_i(t) \in C^1$, i.e., at least one time continuously differentiable. This means $a(t)$ is piecewise continuous, with a well defined bounded derivative everywhere except at points t_i where $\dot{a} = d\,a/dt$ consists of dirac-delta functions. Also the points of discontinuity of a, which are distinct and form an infinitely countable set, are separated by a nonzero dwell time, i.e., there are no Zeno phenomena [5]. This is a reasonable assumption since this is how most physical systems behave.

B. Robust Adaptive Control

In this section, we discuss the basic methodology based on observation of the general structure of the adaptive control problem. In standard adaptive control for linearly-parameterized systems we usually have control and adaptation laws of the form:

$$\begin{aligned} u &= g(x_m,\hat{a},\dot{\hat{a}},y_r,t) \\ \dot{\hat{a}} &= f_a(x_m,\hat{a},y_r,t) \end{aligned} \tag{2}$$

where u is the control signal, $\hat{a}$ is an estimate of plant parameter vector $a \in S_a$, where S_a is an admissible set of parameters, x_m is measured state variables, and y_r is a desired reference trajectory to be followed. This yields the following closed loop error dynamics :

$$\begin{aligned} \dot{e}_c &= f_e(e_c,\tilde{a},t) + d(t) \\ \dot{\tilde{a}} &= f_a(e_c,\hat{a},t) - \dot{a} \end{aligned} \tag{3}$$

where e_c represents a generalized tracking error, includes state estimation error in general output feedback problems, $\tilde{a} = \hat{a} - a$ is parameter estimation error, and d is the disturbance.

In standard adaptive control we typically design the control and adaptation laws, Equation (2), such that $\forall\, a \in S_a$ we have:

$$e_c^T P f_e + \tilde{a}^T \Gamma^{-1} f_a \quad \le \quad -e_c^T C e_c \tag{4}$$

where matrices $P > 0$ and $C > 0$ are chosen depending on the particular algorithm, e.g. choice of reference model and the diagonal matrix $\Gamma > 0$ is the adaptation gain matrix. This is sufficient to stabilize the system with constant parameters and no disturbances. However, since the error dynamics is not ISS stable, stability is no longer guaranteed in the presence of bounded inputs such as d and $\dot{a}$. In order to deal with time varying and switching dynamics, a modification to the adaptation law will be pursued.

Now consider the following modified adaptation law:

$$\dot{\hat{a}} = f_a(e_c,\hat{a},t) - L(\hat{a} - a^*) \tag{5}$$

with the diagonal matrix $L > 0$ and $a^*(t)$ is an arbitrarily chosen piecewise continuous bounded vector, which is an additional estimate of the plant parameter vector. Then the same system in Equation (3) with the modified adaptation law becomes:

$$\begin{aligned} \dot{e}_c &= f_e(e_c,\tilde{a},t) + d(t) \\ \dot{\tilde{a}} &= f_a(e_c,\hat{a},t) - L\tilde{a} + L(a^* - a) - \dot{a} \end{aligned} \tag{6}$$

The modified adaptation law shown above is similar to leakage adaptive laws [2], which have been used to improve robustness with respect to unstructured uncertainties. The leakage adaptation law, also known as fixed-sigma, uses $L = \sigma\,\Gamma$, where $\sigma > 0$ is a scalar and the vector $a^*(t)$ above is usually not included or is a constant. In fact, the key contribution from the generalization presented here is not in the algebraic difference relative to leakage adaptive laws [2] but rather in how the algorithm is utilized and proven to achieve new properties for control of rapidly varying and switching systems. In particular, this leakage-type adaptive controller is shown to achieve internal exponential and ISS stability, for the class of systems under consideration, without need for persistence of excitation as required in [2], see Theorem 1 in [1]. As a result, the effect of plant variation and uncertainty is reduced to *inputs* acting on this ISS stable closed loop system. This, in turn, provides a separation between the robust stability and robust performance control problems.

III. Application to Switched Systems in Parametric-strict Output Feedback Form

In this section we discuss the backstepping tuning functions design procedure for parametric output feedback systems [4]. The literature contains a few results extending the design procedure to systems with time varying parameters. However, the results are restricted to smoothly varying parameters as with most adaptive control results. Again only boundedness is concluded without clear tracking performance claims except when parameters become constant, at least asymptotically. In [7] the input vector parameters, vector $b(t)$ in Equation (7) below, is constant and known whereas other parameters are smooth time varying parameters of known bounds. A more recent result by the same authors in [8] applies to linear systems in parametric-strict output form and allows all parameters to be time varying without known bounds but are required to be smooth. Another recent result for linear-time-varying systems with smooth parameters [10] requires a priori information about rapidly varying parameters whereas completely unknown parameters are required to be slowly time varying. Again such results are yet to address switching systems where parameter smoothness is lost and time variation is a persistent intrinsic part of the system's behavior.

Consider systems in the parametric-strict output feedback form:

$$\begin{aligned}
\dot{x} &= Ax + \phi(y) + \Phi(y)a_p + b\beta(y)u + \bar{d} \\
y &= c^T x \qquad (7) \\
A &= \begin{bmatrix} 0 & \\ \vdots & I_{(n-1)\times(n-1)} \\ 0 \; \ldots & 0 \end{bmatrix} \\
c &= \begin{bmatrix} 1 \\ 0_{(n-1)\times 1} \end{bmatrix}, \; b = \begin{bmatrix} 0_{(r-1)\times 1} \\ b_m \\ \vdots \\ b_o \end{bmatrix} \\
\Phi &= \begin{bmatrix} \varphi_{1,1}(y) \ldots & \varphi_{q,1}(y) \\ \vdots & \vdots \\ \varphi_{1,n}(y) \ldots & \varphi_{q,n}(y) \end{bmatrix}, \phi = \begin{bmatrix} \varphi_{0,1}(y) \\ \vdots \\ \varphi_{0,n}(y) \end{bmatrix}
\end{aligned}$$

Where vectors a_p and b are unknown system parameters. The vector $\bar{d} = [0, d_2, \ldots, d_n]^T$ is the disturbance. Note the disturbance in the first state equation $d_1 = 0$ since it is enforced that if one exists that it will be augmented in the time varying vector of parameters a_p. This is needed since the effect of such a disturbance will appear in the closed loop dynamics as state dependent terms rather than just a disturbance due to the nature of the backstepping design procedure.

Assumption 3.1: The output y is measured.

Assumption 3.2: The sign of $b_m(t) \neq 0 \; \forall t$ is known and constant.

Assumption 3.3: The relative degree $r = n - m \geq 1$ of the system given by Equation (7) is a well defined and known constant.

Assumption 3.4: The zero dynamics of the system given by Equation (7) is uniformly exponentially stable.

Assumption 3.5: Parameter vectors $a_p(t)$ and $b(t)$ are piecewise differentiable bounded functions with finite discontinuities in finite time.

Assumption 3.6: $\varphi_{i,j}(y) \; \forall i = 0 \ldots q$ and $\forall j = 1 \ldots n$ and $\beta(y)$ are known smooth functions and $\beta(y) \neq 0 \; \forall y$.

Assumption 3.7: The reference trajectory y_r and its first r derivatives $y_r^{(1)}, \ldots, y_r^{(r)}$, are known, bounded and, piecewise continuous.

Assumption 3.8: $\bar{d} \in \mathbb{R}^n$ is uniformly bounded and piecewise continuous.

A. Observer Design

In this section, an observer for estimation of unmeasured states are developed. We choose a filter gain vector k such that the matrix $A_o = A - kc^T$ is Hurwitz, which is possible due to observability of the pair (A, c^T) by construction. Therefore, we have the following filter equations [4]:

$$\begin{aligned}
\dot{\xi} &= A_o\xi + ky + \phi(y) \\
\dot{\Xi} &= A_o\Xi + \Phi(y) \\
\dot{\lambda} &= A_o\lambda + e_n\beta(y)u \\
v_j &= A_o^j\lambda, \quad j = 0, \ldots, m \\
\Omega^T &= [v_m, \ldots, v_1, v_0, \Xi] \qquad (8)
\end{aligned}$$

where e_i is the i^{th} unit vector. Now we need to define the error variable of the state estimator ε, which is given by:

$$\varepsilon = x - \left(\xi + \int_0^t \dot{\Omega}^T(\tau)\theta(\tau)d\tau\right)$$

Where $\theta = [b, a_p]^T$. Note that the definition of ε given above differs from that in standard backstepping designs [4] in order to allow time varying parameters. From this we have the following estimation error equation, which is an exponentially stable linear system with eigenvalues dictated by choice of filter gain vector k:

$$\dot{\varepsilon} = A_0\varepsilon + \bar{d}$$

B. Control Design

Based on the backstepping tuning functions design procedure [4], we define the following terms to construct our closed loop system :

$$\begin{aligned}
z_1 &= y - y_r \\
z_i &= v_{m,i} - \alpha_{i-1} - \hat{\rho}\, y_r^{(i-1)} \quad , i = 2, \ldots, r \\
\omega &= [v_{m,2}, v_{m-1,2}, \ldots, v_{0,2}, \Phi_{(1)} + \Xi_{(2)}]^T \\
\bar{\omega} &= [0, v_{m-1,2}, \ldots, v_{0,2}, \Phi_{(1)} + \Xi_{(2)}]^T \\
\omega_o &= \varphi_{0,1} + \xi_2 \\
\tau_1 &= (w - \hat{\rho}(\bar{\alpha}_1 + \dot{y}_r)e_1)\, z_1 - \Gamma_o^{-1}L_o(\hat{\theta} - \theta^*) \\
\tau_i &= \tau_{i-1} - \frac{\partial\alpha_{i-1}}{\partial y}\omega\, z_i \quad , i = 2, \ldots, r \\
\alpha_1 &= \hat{\rho}\bar{\alpha}_1, \; \bar{\alpha}_1 = -(c_1 + d_1)z_1 - \omega_o - \bar{\omega}^T\hat{\theta} \\
\alpha_2 &= -\hat{b}_m\, z_1 - \left[c_2 + d_2\left(\frac{\partial\alpha_1}{\partial y}\right)^2\right] z_2 + \frac{\partial\alpha_1}{\partial\hat{\theta}}\Gamma\tau_2 \\
&\quad + \left(\dot{y}_r + \frac{\partial\alpha_1}{\partial\hat{\rho}}\right)\dot{\hat{\rho}} + \beta_2 \\
\alpha_i &= -z_{i-1} - \left[c_i + d_i\left(\frac{\partial\alpha_{i-1}}{\partial y}\right)^2\right] z_i + \frac{\partial\alpha_{i-1}}{\partial\hat{\theta}}\Gamma\tau_i \\
&\quad + \left(y_r^{(i-1)} + \frac{\partial\alpha_{i-1}}{\partial\hat{\rho}}\right)\dot{\hat{\rho}} + \beta_i \\
&\quad - \sum_{j=2}^{i-1}\frac{\partial\alpha_{j-1}}{\partial\hat{\theta}}\Gamma\frac{\partial\alpha_{i-1}}{\partial y}\omega\, z_j \quad , i = 3, \ldots, r \\
\beta_i &= \frac{\partial\alpha_{i-1}}{\partial y}\left(\omega_o + \omega^T\hat{\theta}\right) + k_i\, v_{m,1} \\
&\quad + \sum_{j=1}^{m+i-1}\frac{\partial\alpha_{i-1}}{\partial\lambda_j}(-k_j\lambda_1 + \lambda_{j+1}) \\
&\quad + \sum_{j=1}^{i-1}\left[\frac{\partial\alpha_{i-1}}{\partial a*^{(j-1)}}a *^{(j)} + \frac{\partial\alpha_{i-1}}{\partial y_r^{(j-1)}}y_r^{(j)}\right] \\
&\quad + \frac{\partial\alpha_{i-1}}{\partial\Xi}(A_o\Xi + \Phi) + \frac{\partial\alpha_{i-1}}{\partial\xi}(A_o\xi + ky + \phi)
\end{aligned}$$

Where constants $c_i, d_i > 0$ and diagonal matrices $\Gamma_o, L_o > 0$ and $a^* = [\theta^*, \rho^*]$ is an estimate of plant parameters θ and ρ to be used in the update laws, to be presented next.The following set of control and update laws are used, which are modifications of those used in the standard backstepping tuning functions design procedure [4]:

$$
\begin{aligned}
u &= \begin{cases} \frac{1}{\beta(y)}[\alpha_r + \hat{\rho} y_r^{(r)} - v_{m,r+1}] & \text{if } r > 1 \\ \frac{1}{\beta(y)}[\alpha_1 + \hat{\rho} \dot{y}_r - v_{m,2}] & \text{if } r = 1 \end{cases} \\
\dot{\hat{\theta}} &= \begin{cases} \Gamma_o \tau_r & \text{if } r > 1 \\ \Gamma_o \bar{\omega} z_1 - L_o(\hat{\theta} - \theta^*) & \text{if } r = 1 \end{cases} \\
\dot{\hat{\rho}} &= -\gamma_\rho \, sign(b_m)[\bar{\alpha}_1 + \dot{y}_r] z_1 - L_\rho(\hat{\rho} - \rho^*) \qquad (9)
\end{aligned}
$$

Where the diagonal matrix $\Gamma_o > 0$ and scalar $\gamma_\rho > 0$ are adaptation gains and adaptation filter gains are diagonal matrix $L_o > 0$ and scalar $L_\rho > 0$. Denote the vector $\hat{a} = [\hat{\theta}^T, \hat{\rho}]$, which is an estimate of the total parameter vector $a = [\theta^T, \rho]^T$ with the chosen parameter estimate vector $a^* = [\theta *^T, \rho^*]^T \in C^{r-1}$, i.e., $r-1$ times continuously differentiable. Theorem 1 below states the main result of this section for the backsepping based design procedure for systems in parametric-strict output feedback form.

Theorem 1: If the system given by Equation (7) satisfies the assumptions (3.1-3.8) then the adaptive feedback control given by Equations (9) and filter Equations (8) yields:
(i) Uniformly internally exponentially stable and ISS system with state $x_c = [e_c, \tilde{a}]^T$.
(ii) states $x_c = [e_c, \tilde{a}]^T$, x, and filter states Λ, ξ are bounded with tracking error satisfying:
$\|y - y_r\| \le c_1 \|x_c(t_o)\| e^{-\alpha(t-t_o)} + c_2 \int_{t_o}^{t} e^{\alpha(\tau - t)} \|v(\tau)\| \, d\tau$.
where c_1, c_2 are constants, $\alpha = \underline{\lambda}(diag(P^{-1}C, L))$, and $v = [P^{1/2} d, \Gamma^{-1/2}(L(a^* - a) - \dot{a})]^T$.

The proof is given in the Appendix.

IV. Example Simulation

Consider the following unstable 2^{nd} order plant of relative degree 1 with a 2-mode periodic switching:

$$
\begin{aligned}
\dot{x}_1 &= a_1 x_1^3 + x_2 + (1 + x_1^2) b_1 u + d \\
\dot{x}_2 &= a_2 x_1 + (1 + x_1^2) b_2 u \\
y &= x_1
\end{aligned}
$$

Where u, d, and y are control signal, disturbance, and measured output, respectively. Whereas, the plant parameters are given by:

$$
\begin{aligned}
a_1 &= 3 + 30 \, square(2\pi\omega \, t) \, , a_2 = -2 - 20 \, square(2\pi\omega \, t) \\
b_2 &= 20 + 10 \, square(2\pi\omega \, t), \; b_1 = 5 + \; square(2\pi\omega \, t)
\end{aligned}
$$

Where $square$ denotes a square wave with unity amplitude and ω is the plant switching frequency in Hz. The control design is based on the backstepping design procedure of Section III. Let us choose the nominal gains $C = 100$ (feedback gain), adaptation filter gain $L = I$, where I is the identity matrix, then we have from Theorem 1 that the decay rate $\alpha = 1$ rad/sec. This should yield a settling time of at most 4 seconds for the closed loop system. Also the nominal value of the adaptation gain $\Gamma = 100I$ will be used. Whereas, a^* is chosen to be a constant vector a_{ave} taking the average values of the parameters a_1, a_2, b_1, b_2, i.e., when square functions are set to zero.

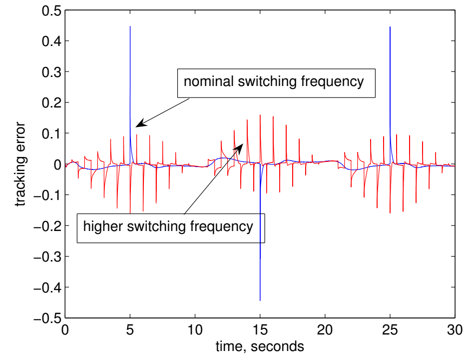

Fig. 1. Tracking error for different plant switching frequencies for modified adaptive controller.

Figure 1 shows the response of the modified adaptive controller for the output of the plant tracking a sinusoidal reference of amplitude 2 and frequency 0.3 rad/sec; the disturbance is set to zero for this case. The response follows the predicted theoretical behavior with the system responding to the corresponding impulse change in $\dot{a}$ and step change in $a(t)$ due to switching in plant parameter vector $a(t)$ with the error settling after exponentially decaying transients according to the system decay rate α. Whereas, by increasing the plant switching frequency, the same trend follows with no concern of instability with high frequency attenuation observed.

Figure 2 shows the effect of different choices of the additional parameter estimate a^* for the nominal case of Figure 1. The figure shows that the average tracking error is larger when a larger size of the input $a^* - a$ due to larger uncertainty and performance is improved with switching to a better choice for a^*.

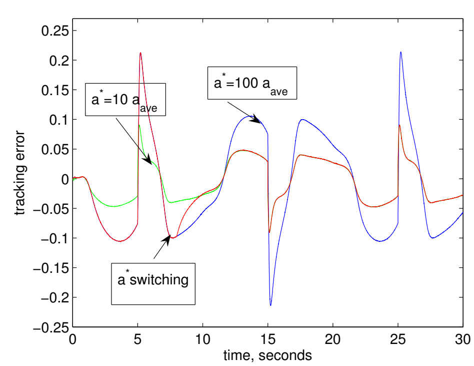

Fig. 2. Effect of parameter estimate a^* on tracking error for modified adaptive controller.

Figure 3 displays the response of the nominal case of Figure 1 for switching frequency $\omega = 0.1 \; Hz$ with the addition of a sinusoidal disturbance $d = 50 \sin(\pi \, t)$, which introduces a clear sinusoidal content to the tracking error. Whereas, increasing feedback gain, significantly reduces the tracking error due to both plant switching (jumps and other

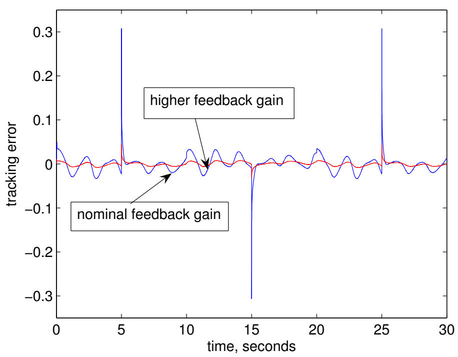

Fig. 3. Effect of feedback gain on tracking error for modified adaptive controller.

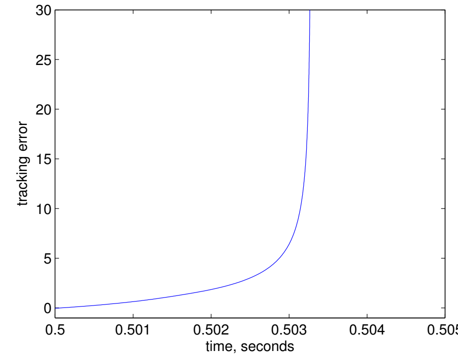

Fig. 5. Tracking error for projection adaptive controller with small parameter bound $M = 1$.

steady errors) as well as the disturbance-induced error.

Let us compare the system's response with the developed adaptive controller to other control techniques. We consider the same system with switching frequency $\omega = 1$ Hz case. The system is required to follow a constant reference of amplitude 2. First consider a non-adaptive backstepping controller, where the parameter estimate $\hat{a}$, in the developed control scheme of is replaced with a fixed value $\hat{a} = a_{ave}$. Figure 4 shows that the non-adaptive controller yields an unstable closed loop despite using the same assumed value of plant parameter vector, which has been used by the modified adaptive controller with $a^* = a_{ave}$.

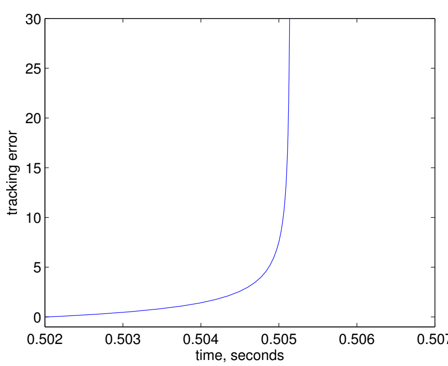

Fig. 4. Tracking error for non-adaptive backstepping controller with $\hat{a} = a_{ave}$.

Whereas, using a standard equivalent adaptive controller, by setting $L = 0$ in the modified adaptive controller of Equation (5) some of the parameter estimates $\hat{a}$ grew unbounded due to lack of ISS stability. This is a known issue with standard adaptive control in the presence of parameter variations or even disturbances, which is usually referred to as parameter drift [**?**], [2]. Next, we consider a parameter projection modification to the standard adaptive controller of Equation (2). The projection modification [2] used here is given by:

$$\dot{\hat{a}} = \begin{cases} f_a & \text{if } \|\hat{a}\| \le M \text{ or } \hat{a}^T f_a \le 0 \\ f_a - \frac{\hat{a}\,\hat{a}^T}{\|\hat{a}\|^2}\left(\frac{\|\hat{a}\|^2 - M^2}{M^2}\right) f_a & \text{otherwise} \end{cases}$$

Which uses an assumed bound on parameters $\|a\| \le M$. This assumption is critical to projection algorithms. Figure 5 shows the tracking error growing unbounded when a projection algorithm was implemented with a tight bound $M = 1$ due large switching in plant parameters. This is in contrast to the developed adaptive controller, which does not require such information to achieve stability.

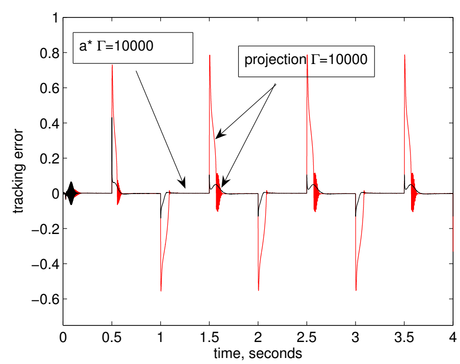

Fig. 6. Tracking error comparison for modified adaptive controller and a projection adaptive controller.

Nevertheless, it was possible to obtain a choice for the projection bound, $M = 10$, where the system remained stable. Figure 6 compares the tracking error for this projection adaptive controller and the developed adaptive controller with $a^* = a_{ave}$ for the same adaptation gain. The developed adaptive controller achieved smaller tracking error. More importantly, the projection controller does not display the systematic dependence on the adaptation gain Γ, Figure 7, unlike the proposed adaptive controller where a clear reduction in tracking error is observed with increasing Γ, in accordance with the scaling relationship in Theorem 1.

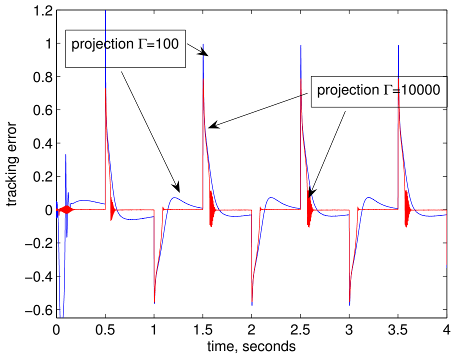

Fig. 7. Effect of adaptation gain Γ on tracking error for projection adaptive controller.

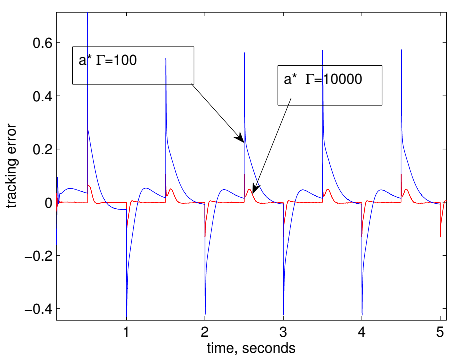

Fig. 8. Effect of adaptation gain Γ on tracking error for modified adaptive controller.

V. Conclusions

An algorithm for robust adaptive control design for switched systems in parametric-strict output feedback form is presented. Under typical adaptive control assumptions, the control scheme guarantees system stability for piecewise differentiable bounded parameters and piecewise continuous bounded disturbances without requiring a priori knowledge on such parameters. A leakage-type adaptive control modification is shown to achieve internal exponential and ISS stability, without need for persistence of excitation. The effect of plant variation and switching is reduced to piecewise continuous and impulsive inputs acting on this ISS stable closed loop system. The results are illustrated through example simulations demonstrating superior robustness of stability and performance relative to non-adaptive and other adaptive methods.

Appendix

Proof of Theorem 1

Proof:

The result of part (i) follows by direct application of Theorem 1 in [1]. The overall system follows the form discussed in Section II with $e_c = [z^T, \varepsilon^T]^T$ and $\tilde{a}$ as states. Also denote by overall adaptation and filters gains $\Gamma^{-1} = diag(\Gamma_o^{-1}, \gamma_\rho^{-1}|b_m|)$ and $L = diag(L_o, L_\rho)$. Using the Lyapunov function:

$$\begin{aligned} V(e_c, \tilde{a}) &= z^T z + 2\sum_{i=1}^{r} \varepsilon^T \frac{P_o}{d_i}\varepsilon + \tilde{\theta}^T \Gamma_o^{-1}\tilde{\theta} + |b_m(t)|\frac{\tilde{\rho}^2}{\gamma} \\ &\quad + \hat{d}^T \Gamma_d^{-1} \hat{d} \\ &= e_c^T P e_c + \tilde{a}^T \Gamma^{-1} \tilde{a} \end{aligned}$$

where, $P = diag(I, 2\sum_{i=1}^{r} \frac{P_o}{d_i})$, $C = diag(C_o, D)$, $C_o = diag(c_1, \ldots, c_r)$, and the matrix $D = diag(d_o, 3/4d_o, d_o, \ldots, d_o)$, where $d_o = \sum_{i=1}^{r} \frac{1}{d_i}$ and P_o is such that $A_0^T P_o + P_o A_0 < 0$ for the Hurwitz matrix A_0.

For part (ii) the boundedness of x_c follows from part (i). The boundedness of other signals x, Λ and ξ is proven next, which differs slightly from that usually done in [4] due to the modified definition of ε. From boundedness of $e_c = [z, \varepsilon]^T$ and reference y_r, this proves boundedness of y since $z_1 = y - y_r$, which shows boundedness of ξ and Ξ by stability of the filter dynamics and smoothness of ϕ and Φ. By the boundedness of y and uniform exponential stability of the zero dynamics, then u is bounded, see [4] for more analogous details. Therefore, v_i are also bounded by filter construction and thus Λ and $\dot{\Lambda}$ are bounded. By boundedness of y, u then from Equation (7), we have that $\dot{x} - Ax$ is bounded. Therefore, expanding Equation (9) we get:

$$\dot{x} - Ax + kc^T\varepsilon - \dot{\varepsilon} - \dot{\Omega}^T\theta = -A\left(\xi + \int_0^t \dot{\Omega}^T(\tau)\theta(\tau)d\tau\right)$$

From above the right hand side of this equation is bounded since $\dot{x} - Ax$, ε, $\dot{\varepsilon}$, $\dot{\Omega}$, and θ are all bounded. Therefore, by boundedness of ε and the right hand side of the equation above as well as the definition of ε, then x is also bounded. ■

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