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Model Reduction for Switched Linear Parameter Varying Systems With Average Dwell Time

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Abstract— **In this paper, the model reduction problem is studied for a class of discrete-time switched linear parameter varying systems under average dwell time switching. A parameterized reduced-order model is constructed and the corresponding existence conditions of such models are derived via LMI formulation. The minimal average dwell time among all the subsystems and the desired reduced system are obtained such that the resulting model error system is exponentially stable and has a guaranteed $l_2 - l_\infty$ error performance. A numerical example is given to demonstrate the potential and effectiveness of the developed theoretical results.**

I. INTRODUCTION

Switched systems have been extensively studied during past decades and many useful results have been reported in the literature, see, for instance, [1], [2], [6], [15], [17] and references therein. Despite of a diversity of forms, the switching patterns can be classified as autonomous or controlled ones, which, respectively, developed by systems themselves (objective behavior) [3] or designers' intervention (subjective behavior) [10]. The switching signals in both autonomous and controlled switched systems can be represented as functions of time. As a result, considerable research on the switched systems under arbitrary switching or average dwell time (ADT) one have been conducted in recent years, see, for example, [2], [5], [14], [15].

Arbitrary switching means that the switching time sequence (when to switch) is completely random and the switching order of subsystems (which one is switched) is real time accessible. Note that the so-called jump systems governed by some kind of stochastic process can be included in this category as a special case. The ADT switching means that the time interval between consecutive switches is required to be no less than a given positive number on the average. As expected, such a switching rule not only displays more flexible controlled switching mechanism dependent on time, but also reflects the time property of most autonomous and controlled switching logics, such as hysteresis switching [4]. Therefore, many advanced results for the switched systems with ADT switching have been reported in the continuous-time context including both linear and nonlinear case, see for example, [5], [11], [12]. Very recently, motivated by practical applications such as active magnetic bearing system [8], missile autopilot system [7], F-16 aircraft system [9], a special class of switched linear parameter-varying systems (LPV) under ADT switching has been modeled and studied. However, it must be noted that the corresponding modeling process often results in a high-order system, which needs to be simplified or reduced. Due to the switching and parameters varying features, the model reduction results for the underlying systems can not be straitforwardly achieved using the existing model simplification techniques for general dynamic systems.

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This paper presents the model reduction for a class of switched linear discrete-time systems with time-varying parameters under ADT switching. A reduced-order model is constructed to approximate the original system in the sense of classical $l_2 - l_\infty$ performance criterion, namely, energy (of input signal) to peak (of output error) bounded. The stability criterion for general discrete-time switched systems is introduced and the μ-dependent technique, proposed in [14], is adopted to obtain LMI-based existence conditions for a parameterized reduced model. The basic functions and gridding technique are utilized to solve the corresponding parameterized convex problem. An illustrative example is given to demonstrate the feasibility and efficiency of the constructed reduced model.

Notation: The notation used in this paper is fairly standard. The superscript "T" stands for matrix transposition, $\mathbb{R}^n$ denotes the n dimensional Euclidean space and $\mathbb{N}$ represents the set of nonnegative integers, the notation $\| \quad \|$ refers to the Euclidean vector norm. $l_2[0,\infty)$ is the space of square summable infinite sequences and, for $u = \{u(k)\} \in l_2[0,\infty)$, its norm is given by $\|u\|_2 = \sqrt{\sum_{k=0}^{\infty} |w(k)|^2}$, $l_\infty[0,\infty)$ is the space of all essentially bounded functions and, for $e = \{e(k)\} \in l_\infty[0,\infty)$, its norm is given by $\|e\|_\infty = \sqrt{\sup_k \{e^T(k)e(k)\}}$. $\mathcal{C}^1$ denotes the space of continuously differentiable functions, and a scalar function $\beta : [0,\infty) \to [0,\infty)$ is said to be of class $\mathcal{K}_\infty$ if it is continuous, strictly increasing, unbounded, and $\beta(0) = 0$. In addition, in symmetric block matrices or long matrix expressions, we use * as an ellipsis for the terms introduced by symmetry, and $diag\{\cdots\}$ stands for a block-diagonal matrix. Matrices, if their dimensions are not explicitly stated, are assumed to be compatible for algebraic operations. The notation $P > 0$ (≥ 0) means that P is symmetric and positive (semi-positive) definite. I and 0 represent, respectively, identity matrix and zero matrix.

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II. Problem Formulation and Preliminaries

Consider a class of discrete-time switched linear systems given by

$$\begin{aligned} x(k+1) &= A_{\sigma(k)}(\rho(k))x(k) + B_{\sigma(k)}(\rho(k))u(k) \\ y(k) &= C_{\sigma(k)}(\rho(k))x(k), \end{aligned} \tag{1}$$

where $x(k) \in \mathbb{R}^n$ is the state vector, $u(k) \in \mathbb{R}^l$ is the input vector which belongs to $l_2[0,\infty)$, $y(k) \in \mathbb{R}^m$ is the output vector. $\sigma(k)$ is a piecewise constant function of time, called a switching signal, which takes its values in the finite set $\mathcal{I} = \{1,\ldots,N\}$, $N > 1$ is the number of subsystems. At an arbitrary discrete time k, $\sigma(k)$, denoted by σ for simplicity, is dependent on k or $x(k)$, or both, or other switching rules. As in [2], we assume that the sequence of subsystems in switching signal σ is unknown *a priori*, but its instantaneous value is available in real time. Meanwhile, for the switching time sequence $k_0 < k_1 < k_2 < \ldots$ of switching signal σ, the holding time between $[k_l, k_{l+1}]$ is called the dwell time of the currently engaged subsystem, where $l \in \mathbb{N}$. In addition, when $\sigma(k) = i \in \mathcal{I}$, the matrices $(A_i(\rho(k)),\ B_i(\rho(k)),\ C_i(\rho(k)))$, denoting the ith subsystem, are known functions of measurable $\rho(k)$, where $\rho(k) = [\rho_1(k),\ldots,\rho_s(k)]^T$, $|\rho_v(k)| \le \bar{\rho}_v, \forall 1 \le v \le s$ is a vector of time-varying parameters, which belongs to a compact set $\Re_i \in \mathbb{R}^s$.

For switching signal $\sigma(k)$, we revisit the ADT property from the following definition.

Definition 1: [5] For switching signal and any $k_v > k_s > k_0$, let $N_{\sigma(k)}(k_s,k_v)$ be the switching numbers of $\sigma(k)$ over the interval $[k_s,k_v]$. If for any given $N_0 > 0$, $\tau_a > 0$, we have $N_{\sigma(k)}(k_s,k_v) \le N_0 + (k_v - k_s)/\tau_a$, then τ_a and N_0 are called average dwell time and the chatter bound, respectively.

Remark 1: Intuitively, the ADT property means the time interval between consecutive switching is at least τ_a on the average. Then, a basic problem in the analysis and synthesis for such systems is to specify the minimal τ_a and the corresponding admissible switching signals.

In this paper, we are interested in finding a reduced-order model to approximate the original system under ADT switching. Since the time-varying parameters are real-time measurable, the desired reduced-order model can be constructed by:

$$\begin{aligned} \hat{x}(k+1) &= \hat{A}_i(\rho_k)\hat{x}(k) + \hat{B}_i(\rho_k)u(k) \\ \hat{y}(k) &= \hat{C}_i(\rho_k)\hat{x}(k), \end{aligned} \tag{2}$$

where $\hat{x}(k) \in \mathbb{R}^q$ is the state vector of the reduced-order system with $q < n$ and $\hat{A}_i(\rho_k), \hat{B}_i(\rho_k)$ and $\hat{C}_i(\rho_k), i \in \mathcal{I}$ (we write $\rho(k)$ as ρ_k for notation simplicity) are matrices with compatible dimensions to be determined, with the same parameter dependence as in system (1). Also, the desired reduced model (2) is assumed to be switched synchronously by the switching signal σ in system (1).

Denoting $\xi(k) \triangleq [\ x^T(k) \quad \hat{x}^T(k)\ ]^T$, $e(k) \triangleq y(k) - \hat{y}(k)$ and augmenting the model of (1) to include the states of system (2), we obtain the following model error system

$$\begin{aligned} \xi(k+1) &= \bar{A}_i(\rho_k)\xi(k) + \bar{B}_i(\rho_k)u(k) \\ e(k) &= \bar{C}_i(\rho_k)\xi(k), \end{aligned} \tag{3}$$

where

$$\begin{aligned} \bar{A}_i(\rho_k) &= \begin{bmatrix} A_i(\rho_k) & 0 \\ 0 & \hat{A}_i(\rho_k) \end{bmatrix}, \\ \bar{B}_i(\rho_k) &= \begin{bmatrix} B_i(\rho_k) \\ \hat{B}_i(\rho_k) \end{bmatrix}, \\ \bar{C}_i(\rho_k) &= \begin{bmatrix} C_i(\rho_k) & -\hat{C}_i(\rho_k) \end{bmatrix}. \end{aligned}$$

To present the main objective of this paper more clearly, we also introduce the following definitions for switched linear systems, which are essential for the later development.

Definition 2: The equilibrium $x = 0$ of system (1) is exponentially stable under switching signal $\sigma(k)$ if there exist constants $K > 0, 0 < \beta < 1$ such that the solution $x(k)$ of the system satisfies $||x(k)|| \le K\beta^{(k-k_0)}||x(k_0)||, \forall k \ge k_0$.

Definition 3: Given scalars $\gamma > 0$, system (3) is said to be exponentially stable with a prescribed $l_2 - l_\infty$ error performance index γ if it is exponentially stable and, under zero initial condition, $\|e\|_\infty < \gamma \|u\|_2$ hold for all nonzero $u(k) \in l_2[0,\infty)$.

Therefore, the objective of this paper is to determine matrices $\left\{\hat{A}_i(\rho_k), \hat{B}_i(\rho_k), \hat{C}_i(\rho_k)\right\}$ of the parameterized reduced-order model, and find out admissible switching signals such that resulting model error system (3) is exponentially stable and has a guaranteed $l_2 - l_\infty$ error performance index.

Remark 2: Note that if we restrict $[\hat{A}_i(\rho_k),\ \hat{B}_i(\rho_k),\ \hat{C}_i(\rho_k)] \triangleq [\hat{A}_i, \hat{B}_i, \hat{C}_i]$ or select $[\hat{A}_i, \hat{B}_i, \hat{C}_i] \triangleq [\hat{A}, \hat{B}, \hat{C}]$ in (2), one will readily obtain the different non-parameterized reduced-order models with different conservatism and computational complexity.

Before ending this section, we present the following lemmas which are employed for further derivation.

Lemma 1: Consider the discrete-time switched system $x_{k+1} = f_{\sigma(k)}(x_k), \sigma(k) \in \mathcal{I}$ and let $0 < \alpha < 1, \mu > 1$ be given constants. Suppose that there exists $\mathcal{C}^1$ functions $V_{\sigma(k)} : \mathbb{R}^n \to \mathbb{R}$, $\sigma(k) \in \mathcal{I}$, and two class $\mathcal{K}_\infty$ functions β_1 and β_2 such that $\forall \sigma(k) = i \in \mathcal{I}$,

$$\beta_1(|x|) \le V_i(x) \le \beta_2(|x|) \tag{4}$$

$$\Delta V_i(x) \le -\alpha V_i(x) \tag{5}$$

and $\forall(\sigma(k_l) = i, \sigma(k_l - 1) = j) \in \mathcal{I} \times \mathcal{I}$, $i \ne j$,

$$V_i(x_{k_l}) \le \mu V_j(x_{k_l}), \tag{6}$$

then the system is globally asymptotically stable for any switching signals with the average dwell time

$$\tau_a \ge \tau_a^* = -\frac{\ln \mu}{\ln(1-\alpha)}. \tag{7}$$

Remark 3: The proof of Lemma 1 can be obtained simliar to Section 3.2 of [6]. Note that if we increase the value of μ, the existence likelihood of the multiple Lyapunov function for the system stability will be increased, which means that the stability of system can be ensured at the expense of increasing μ. In other words, for a given α, the system stability will directly depend on μ.

Lemma 2: Consider model error system (3) and let $\alpha > 0$, $\gamma > 0$ and $\mu > 1$ be given constants. If there exist matrix functions $P_i(\rho_k) > 0$, $\forall i \in \mathcal{I}$ such that

$$\begin{bmatrix} -P_i(\rho_{k+1}) & P_i(\rho_{k+1})\bar{A}_i(\rho_k) & \begin{matrix}[P_i(\rho_{k+1}) \\ \times \bar{B}_i(\rho_k)]\end{matrix} \\ * & -(1-\alpha)P_i(\rho_k) & 0 \\ * & * & -I \end{bmatrix} < 0 \tag{8}$$

$$\begin{bmatrix} P_i(\rho_k) & \bar{C}_i^T(\rho_k) \\ * & \gamma^2 I \end{bmatrix} > 0 \tag{9}$$

$$P_i(\rho_k) - \mu P_j(\rho_k) \leq 0 \tag{10}$$

then system (3) is exponentially stable and has an $l_2 - l_\infty$ error performance γ over the entire parameter set for any switching signals with the ADT satisfying (7).

Remark 4: In Lemma 2, the desired $l_2 - l_\infty$ performance index for the underlying system in the paper is achieved by setting $\gamma = \max\{\gamma_i\}_{i\in\mathcal{I}}$, where γ_i corresponds to the performance index for each subsystem. The proof of Lemma 2 can be readily obtained using Theorem 1 in [8] and Lemma 3 in [16].

III. Model Reduction

The following theorem presents sufficient conditions for the existence of an $l_2 - l_\infty$ reduced-order model in the form (2).

Theorem 3: Consider switched linear system (1) and let $\alpha > 0$, $\gamma > 0$ and $\mu > 1$ be given scalars. Then, an admissible $l_2 - l_\infty$ reduced-order model in the form (2) exists if there exist matrices $\bar{P}_{1i}(\rho_k) > 0$, $\bar{P}_{3i}(\rho_k) > 0$, and matrices $\bar{P}_{2i}(\rho_k)$, $R_i(\rho_k)$, $S_i(\rho_k)$, T_i, $\check{A}_i(\rho_k)$, $\check{B}_i(\rho_k)$, $\check{C}_i(\rho_k)$, $\forall i \in \mathcal{I}$ such that the following parameterized LMIs hold:

$$\begin{bmatrix} \Lambda_{11}^i & \Lambda_{12}^i & R_i^T(\rho_k)A_i(\rho_k) & E\check{A}_i(\rho_k) & \Lambda_{15}^i \\ * & \Lambda_{22}^i & S_i^T(\rho_k)A_i(\rho_k) & \check{A}_i(\rho_k) & \Lambda_{25}^i \\ * & * & -(1-\alpha)\bar{P}_{1i}(\rho_k) & -(1-\alpha)\bar{P}_{2i}(\rho_k) & 0 \\ * & * & * & -(1-\alpha)\bar{P}_{3i}(\rho_k) & 0 \\ * & * & * & * & -I \end{bmatrix} < 0 \tag{11}$$

$$\begin{bmatrix} \bar{P}_{1i}(\rho_k) & \bar{P}_{2i}(\rho_k) & C_i^T(\rho_k) \\ * & \bar{P}_{3i}(\rho_k) & -\check{C}_i^T(\rho_k) \\ * & * & \gamma^2 I \end{bmatrix} > 0 \tag{12}$$

$$\begin{bmatrix} \begin{matrix}[\bar{P}_{1i}(\rho_k) \\ -\mu R_i^T(\rho_k) - \mu R_i(\rho_k)]\end{matrix} & \begin{matrix}[\bar{P}_{2i}(\rho_k) \\ -\mu S_i(\rho_k) - \mu E T_i]\end{matrix} & R_i^T(\rho_k) & ET_j \\ * & \bar{P}_{3i}(\rho_k) - \mu T_i^T - \mu T_i & S_i^T(\rho_k) & T_j \\ * & * & -\mu^{-1}\bar{P}_{1j}(\rho_k) & -\mu^{-1}\bar{P}_{2j}(\rho_k) \\ * & * & * & -\mu^{-1}\bar{P}_{3j}(\rho_k) \end{bmatrix} \leq 0 \tag{13}$$

where

$$\begin{aligned} \Lambda_{11}^i &\triangleq \bar{P}_i(\rho_{k+1}) - R_i^T(\rho_k) - R_i(\rho_k), \\ \Lambda_{12}^i &\triangleq \bar{P}_{2i}(\rho_{k+1}) - S_i(\rho_k) - ET_i, \\ \Lambda_{22}^i &\triangleq \bar{P}_{3i}(\rho_{k+1}) - T_i^T - T_i \\ \Lambda_{15}^i &\triangleq R_i^T(\rho_k)B_i(\rho_k) + E\check{B}_i(\rho_k), \\ \Lambda_{25}^i &\triangleq S_i^T(\rho_k)B_i(\rho_k) + \check{B}_i(\rho_k), \\ E &\triangleq \begin{bmatrix} I & 0 \end{bmatrix}^T, I \in \mathbb{R}^q \end{aligned}$$

Then, there exists a parameterized reduced-model such that the corresponding model error system (3) is exponentially stable with an guaranteed $l_2 - l_\infty$ performance index γ for any switching signals with the ADT satisfying (7). Furthermore, if a feasible solution to above LMIs exists, then an admissible $l_2 - l_\infty$ reduced-order model in the form of (2) are given by

$$\begin{bmatrix} \hat{A}_i(\rho_k) & \hat{B}_i(\rho_k) \\ \hat{C}_i(\rho_k) & 0 \end{bmatrix} \triangleq \begin{bmatrix} T_i^{-1} & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} \check{A}_i(\rho_k) & \check{B}_i(\rho_k) \\ \check{C}_i(\rho_k) & 0 \end{bmatrix} \tag{14}$$

Proof: By Lemma 2, system (3) is exponentially stable with a prescribed $l_2 - l_\infty$ error performance index γ if (8)-(10) hold.

Then, consider an arbitrary matrix function $G_i(\rho_k), \forall i \in \mathcal{I}$ of compatible dimensions, which satisfies the inequalities

$$\begin{aligned} (P_i(\rho_{k+1}) - G_i(\rho_k))^T P_i^{-1}(\rho_{k+1}) & \\ \times (P_i(\rho_{k+1}) - G_i(\rho_k) &\geq 0 \\ (P_j(\rho_k) - G_i(\rho_k))^T P_j^{-1}(\rho_k) & \\ \times (P_j(\rho_k) - G_i(\rho_k) &\geq 0 \end{aligned}$$

Thus we have

$$\begin{aligned} P_i(\rho_{k+1}) - G_i(\rho_k) - G_i^T(\rho_k) &\geq \\ -G_i^T(\rho_k)P_i^{-1}(\rho_{k+1})G_i(\rho_k) & \\ P_j(\rho_k) - G_i(\rho_k) - G_i^T(\rho_k) &\geq \\ -G_i^T(\rho_k)P_j^{-1}(\rho_k)G_i(\rho_k) & \end{aligned}$$

Therefore, if one has

$$\begin{aligned} P_i(\rho_k) - \mu\left[G_i(\rho_k) + G_i^T(\rho_k)\right. & \\ \left. -G_i^T(\rho_k)P_j^{-1}(\rho_k)G_i(\rho_k)\right] &\leq 0 \end{aligned} \tag{15}$$

then (10) is satisfied. Also, if the following inequality holds

$$\begin{bmatrix} P_i(\rho_{k+1}) - G_i(\rho_k) - G_i^T(\rho_k) & G_i^T(\rho_k)\bar{A}_i(\rho_k) & G_i^T(\rho_k)\bar{B}_i(\rho_k) \\ * & -(1-\alpha)P_i(\rho_k) & 0 \\ * & * & -I \end{bmatrix} < 0 \quad (16)$$

it leads to

$$\begin{bmatrix} -G_i^T(\rho_k)P_i^{-1}(\rho_{k+1})G_i(\rho_k) & G_i^T(\rho_k)\bar{A}_i(\rho_k) & G_i^T(\rho_k)\bar{B}_i(\rho_k) \\ * & -(1-\alpha)P_i(\rho_k) & 0 \\ * & * & -I \end{bmatrix} < 0$$

Performing a congruence transformation in the last inequality to above formula via $diag\{G_i^{-1}(\rho_k)P_i(\rho_{k+1}), I, I\}$ yields (8) (note that $G_i(\rho_k)$ is invertible, if it satisfies (16)).

In addition, by Schur complement, (15) is equivalent to

$$\begin{bmatrix} P_j(\rho_k) - \mu G_i(\rho_k) - \mu G_i^T(\rho_k) & G_i^T(\rho_k) \\ * & -\mu^{-1}P_j(\rho_k) \end{bmatrix} \leq 0. \quad (17)$$

Now, let us show that conditions (11)-(13) ensure, respectively, that (16), (9) and (17) are satisfied. Firstly, if (11) holds, we have $\bar{P}_{3i}(\rho_{k+1}) - T_i^T - T_i < 0$, thus we can infer that $T_i^T + T_i > 0$, which implies that T_i is nonsingular. Then, one can always find nonsingular matrices G_{3i} and G_4 satisfying $T_i = G_4^T G_{3i}^{-1} G_4, \forall i \in \mathcal{I}$. Now, introduce the following matrices related to G_{3i} and G_4:

$$\begin{aligned} V_i &\triangleq \begin{bmatrix} I & 0 \\ 0 & G_{3i}^{-1}G_4 \end{bmatrix}, \\ G_i(\rho_k) &\triangleq \begin{bmatrix} R_i(\rho_k) & S_i(\rho_k)G_4^{-1}G_{3i} \\ G_4E^T & G_{3i} \end{bmatrix}. \end{aligned}$$

Performing a congruence transformation to (11)-(13) via $diag\{V_i^{-1}, V_i^{-1}, I\}$, $diag\{V_i^{-1}, I\}$ and $diag\{V_i^{-1}, V_j^{-1}\}$, respectively, and defining matrix functions

$$\begin{aligned} P_i(\rho_k) &\triangleq V_i^{-T}\bar{P}_i(\rho_k)V_i^{-1} \\ &= V_i^{-T}\begin{bmatrix} \bar{P}_{1i}(\rho_k) & \bar{P}_{2i}(\rho_k) \\ * & \bar{P}_{3i}(\rho_k) \end{bmatrix} V_i^{-1} \end{aligned}$$

$$\begin{aligned} &\begin{bmatrix} \hat{A}_i(\rho_k) & \hat{B}_i(\rho_k) \\ \hat{C}_i(\rho_k) & 0 \end{bmatrix} \\ \triangleq &\begin{bmatrix} G_4^{-T} & 0 \\ * & I \end{bmatrix} \begin{bmatrix} \check{A}_i(\rho_k) & \check{B}_i(\rho_k) \\ \check{C}_i(\rho_k) & 0 \end{bmatrix} \begin{bmatrix} G_4^{-1}G_{3i} & 0 \\ * & I \end{bmatrix} \end{aligned} \quad (18)$$

we obtain (16), (9) and (17).

Meanwhile, from (18), we know that an admissible reduced-order model for the underlying system can be obtained setting

$$\begin{aligned} \hat{A}_i(\rho_k) &= G_4^{-T}\check{A}_i(\rho_k)G_4^{-1}G_{3i}, \ \hat{B}_i(\rho_k) = G_4^{-T}\check{B}_i(\rho_k), \\ \hat{C}_i(\rho_k) &= \check{C}_i(\rho_k)G_4^{-1}G_{3i} \end{aligned} \quad (19)$$

Now, denote the reduced-order model transfer function from $u(k)$ to $e(k)$ by

$$T(\mathbf{z}) = \hat{C}_i(\rho_k)(\mathbf{z}I - \hat{A}_i(\rho_k))^{-1}\hat{B}_i(\rho_k)$$

Substituting the matrices ($\hat{A}_i(\rho_k)$, $\hat{B}_i(\rho_k)$, $\hat{C}_i(\rho_k)$) in (19) and considering $T_i = G_4^T G_{3i}^{-1} G_4$, we have

$$\begin{aligned} T(\mathbf{z}) &= \check{C}_i(\rho_k)G_4^{-1}G_{3i}(\mathbf{z}I - G_4^{-T}\check{A}_i(\rho_k)G_4^{-1}G_{3i})^{-1} \\ &\quad \times G_4^{-T}\check{B}_i(\rho_k) \\ &= \check{C}_i(\rho_k)(\mathbf{z}I - T_i^{-1}\check{A}_i(\rho_k))^{-1}T_i^{-1}\check{B}_i(\rho_k) \end{aligned}$$

which implies that an admissible reduced-order model can be given by (14), this completes the proof. □

Remark 5: From (7), it is easily seen that the ADT in the solved switching signals will be not less than τ_a^*. Then, one actually need to specify the minimal μ for a given system decay degree α for the underlying systems, which is analogous to the *delay-dependent* issues in time-delay systems to determine the delay bounds. Therefore, the so-called *μ-dependent* idea, proposed in [14], is adopted here for the underlying system, and the results obtained with this concept will be less conservative than the ones within the "*μ-independent"* framework such as those based on switched Lyapunov function [2] or global Lyapunov function.

Remark 6: Conditions (11)-(13) are formulated in terms of a set of parameterized LMIs, which involve not only matrix variables but also the scalar γ^2. Therefore, the scalar can be optimized by a *μ-dependent* convex optimization problem for a fixed system decay degree as follows.

Problem 1:

Min δ subject to (11)-(13), $\forall i \in \mathcal{I}$, with $\delta = \gamma^2$
over $\bar{P}_{1i}(\rho_k)$, $\bar{P}_{3i}(\rho_k)$, $\bar{P}_{2i}(\rho_k)$, $R_i(\rho_k)$, $S_i(\rho_k)$, T_i, $\check{A}_i(\rho_k)$, $\check{B}_i(\rho_k)$, $\check{C}_i(\rho_k)$

The minimum error performance index is then obtained setting $\gamma = \sqrt{\delta^*}$, where δ^* is the optimal value of δ, and the matrices of the corresponding reduced model are given by (14).

As shown in the LPV literature [13], by choosing appropriate basis functions $\{f_l(\rho_k)\}_{l=1}^{n_f}$, the matrix functions $\mathcal{Y}_i(\rho) = \{\bar{P}_{1i}(\rho_k), \bar{P}_{3i}(\rho_k), \bar{P}_{2i}(\rho_k), R_i(\rho_k), S_i(\rho_k), T_i, \check{A}_i(\rho_k), \check{B}_i(\rho_k), \check{C}_i(\rho_k)\}$ in above convex problem can be represented as:

$$\mathcal{Y}_i(\rho) = \sum_{l=1}^{n_f} f_l(\rho_k)\mathcal{Y}_i^l \quad (20)$$

where $f_l(\rho_k)$ and n_f can be chosen by designers according to the dependence structure in system (1), and consequently, $\mathcal{Y}_i^l = \{\bar{P}_{1i}^l, \bar{P}_{2i}^l, \bar{P}_{3i}^l, R_i^l, S_i^l, \check{A}_i^l, \check{B}_i^l, \check{C}_i^l\}$ serves as the corresponding decision variables in *Problem 1*. Also, one can utilize the gridding technique to eliminate the dependence on the parameter vector ρ_k in the parameterized LMIs emerging in LPV systems (see [13] for more details).

IV. NUMERICAL EXAMPLE

Consider the following discrete-time switched linear systems consisting of two subsystems with time-varying parameters:

$$
\begin{aligned}
A_1 &= \begin{bmatrix} 0.78 & 0.82 & -0.50 & 0.31 \\ 0.19 & -0.18 & 0.74 & -0.23+0.12\rho_k \\ -0.23 & -0.19 & -0.15 & -0.46 \\ -0.66 & 0.82 & 0.11 & 0.78 \end{bmatrix}, \\
B_1 &= \begin{bmatrix} 0.74 \\ 0.70 \\ 0.62 \\ -0.31 \end{bmatrix}, \\
A_2 &= \begin{bmatrix} 0.77 & 0.73 & -0.45 & 0.28 \\ 0.18 & -0.04 & 0.67 & -0.21 \\ -0.21 & -0.18 & -0.07 & -0.42+0.15\rho_k \\ -0.60 & 0.74 & 0.11 & 0.78 \end{bmatrix}, \\
B_2 &= \begin{bmatrix} 0.74 \\ 0.70 \\ 0.55 \\ -0.31 \end{bmatrix}, \\
C_1 &= C_2 = \begin{bmatrix} 0.47 & 0.59 & 0.51 & -0.24 \end{bmatrix},
\end{aligned}
$$

where $\rho_k = \cos(0.2\pi k)$ are the time-varying measurable parameters.

Our purpose is to design a parameterized reduced-order model in the form of (2) and find out the admissible ADT switching signals for the above switched system such that the resulting model error system is exponentially stable and has a guaranteed $l_2 - l_\infty$ performance for a given decay degree α.

According to the structure of the parameter dependence in the above system, we choose the basic functions in (20) as follows

$$f_1(\rho_k) = 1,\ \ f_2(\rho_k) = \cos(0.2\pi k)$$

Gridding the parameter space of ρ_k with 10 uniform grid, assigning $\alpha = 0.1$ and varying μ, and solving *Problem 1*, we obtain different ADT τ_a^* and different optimal $l_2 - l_\infty$ performance indices γ^*, as shown in Table 1. It is clear that γ^* depends on μ for a given system decay degree α, and we obtain the minimal $\mu = 1.08$ such that the underlying system can achieve the $l_2 - l_\infty$ performance indices. Note that the larger μ corresponds to the smaller γ^*, but the corresponding longer ADT are demanded in the system.

In addition, an admissible reduced model can be also obtained by solving *Problem 1*. For instance, the desired 2nd-order reduced model corresponding to $\mu = 1.40$ with the accessible parameters $\rho_k = \cos(0.2\pi k)$ is determined as:

TABLE I

μ-*dependent* OPTIMAL γ^* FOR GIVEN $\alpha = 0.1$

μ	1.07	1.08	1.25	1.40	1.60
τ_a^*	0.64	0.73	2.11	3.19	4.46
γ^*	$infeasible$	3.33	2.79	2.78	2.78

$$
\begin{aligned}
A_{F1} &= \begin{bmatrix} 0.95 & 0.86 \\ -0.10 & -0.02 \end{bmatrix} + \rho_k \begin{bmatrix} -0.19 & -0.50 \\ -0.17 & 0.49 \end{bmatrix}, \\
B_{F1} &= \begin{bmatrix} -0.14 \\ -1.58 \end{bmatrix} + \rho_k \begin{bmatrix} -10.5 \\ 10.9 \end{bmatrix} \\
C_{F1} &= \begin{bmatrix} -0.58 & -0.60 \end{bmatrix} + \rho_k \begin{bmatrix} -0.04 & -0.05 \end{bmatrix}
\end{aligned} \tag{21}
$$

$$
\begin{aligned}
A_{F2} &= \begin{bmatrix} 0.74 & 0.69 \\ 0.08 & 0.13 \end{bmatrix} + \rho_k \begin{bmatrix} -0.18 & -0.21 \\ -0.16 & 0.18 \end{bmatrix}, \\
B_{F2} &= \begin{bmatrix} -0.51 \\ -0.90 \end{bmatrix} + \rho_k \begin{bmatrix} -1.34 \\ 1.27 \end{bmatrix} \\
C_{F2} &= \begin{bmatrix} -0.62 & -0.65 \end{bmatrix} + \rho_k \begin{bmatrix} -0.02 & -0.03 \end{bmatrix}
\end{aligned} \tag{22}
$$

Furthermore, consider the input signal

$$u(k) = 0.1\exp(-0.03k)\sin(0.02\pi k)$$

and apply the solved reduced-order model (21)-(22), Figures 1 and 2 show the output trajectories of the original system and $2nd$-order reduced model for given two different switching signals (both are with $\tau_a = 4 > 3.19$ for $\mu = 1.40$); Figure 3 and 4 present the output errors between original system and the reduced-order system. It is clearly observed from the simulation curves that for given energy bounded input $u(k)$, the model error system is stable against time-varying parameters under different switching signals, which thereby implies that the designed reduced-order model is feasible and effective.

V. CONCLUSIONS

The model reduction problem is studied for a class of discrete-time switched linear parameter varying systems under average dwell time switching. A parameterized reduced model is designed and the corresponding existence conditions of such reduced-order models are derived via LMI formulation. By solving a convex optimization problem, the minimal ADT and the desired reduced model can be obtained for a given decay degree to ensure that the resulting model error system is exponentially stable and has a guaranteed $l_2 - l_\infty$ error performance. A numerical example is provided to show the effectiveness and applicability of the developed reduced model.

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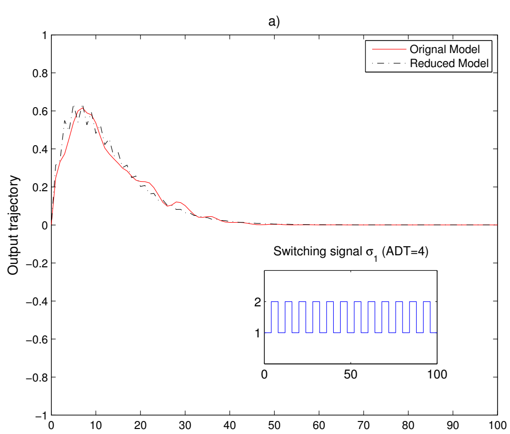

Fig. 1. Output trajectories of the original system and 2nd-order reduced model for given switching signal 1

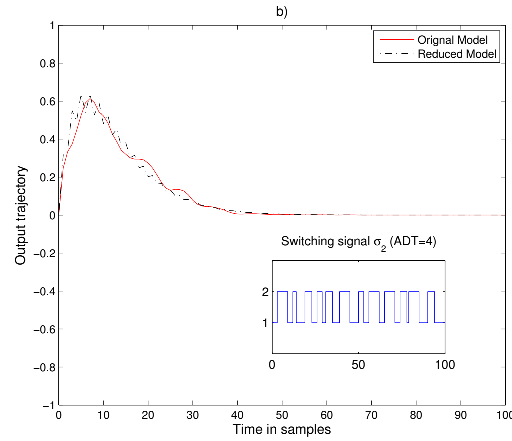

Fig. 2. Output trajectories of the original system and 2nd-order reduced model for given switching signal 2

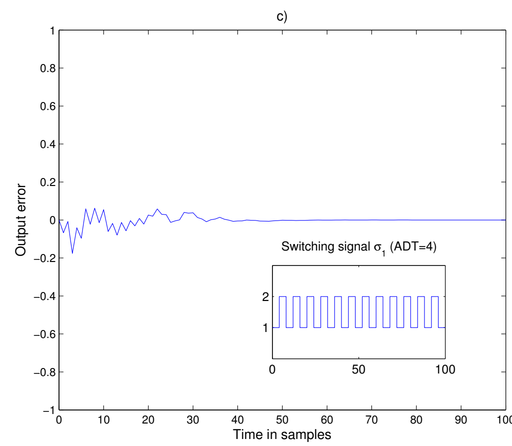

Fig. 3. Output errors between original system and the reduced-order system for given switching signal 1

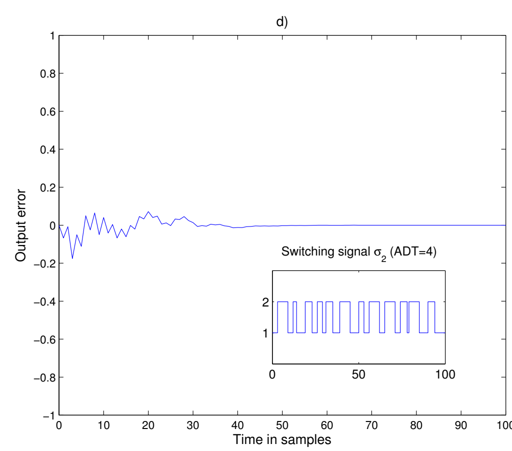

Fig. 4. Output errors between original system and the reduced-order system for given switching signal 2