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LQG benchmarking for Economic Control Performance

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***Abstract*— Economic performance assessment of process control system has been a great interest to control engineers and academic researchers. In this paper, a novel approach for economic performance assessment of the constrained process control is presented. The method builds on steady-state economic optimization techniques and uses the LQG benchmark other than conventional minimum variance control (MVC) to estimate potential of reduction in variance. Combining the LQG benchmark directly with benefit potential of process control, both the economic benefit and the optimal operation condition can be obtained by solving the economic optimization problem. The proposed method is also illustrated by simulated examples where the economic potential due to improved process control is illustrated.**

I. INTRODUCTION

Economic performance assessment of process control has been an area of active research in process control community. Quantifying the economic benefit of existing process control is often based on the variance reduction for key process variables. A general approach for economic performance evaluation is to reduce the variance in controlled variable, which in turn shifts the process mean operating point closer to the operating constraint and thus results in better performance. Several performance assessment techniques have been proposed in the literature.

The notable work has been done by Martin [5], where a general framework for economic justification of APC applications is given . Using statistical analysis , an approach for analysis of variance reduction under various improved control operations is developed in [3]. Considering the probability constraints for the quality process variables, Zhao and Forbes (2003) proposed a structured procedure for economic benefits analysis using stochastic programming. According to steady-state model and back-off idea, an economic performance assessment of MPC using a linear matrix inequality approach has been developed [1].

It is assumed that improvement of process economic performance comes from variance reduction. The achievable variance reduction in process variables depends on dynamic control. The existing assessment methods are often concerned with variance reduction estimation with minimum variance controller (MVC) as the benchmark. Minimum variance control usually is not a desired control strategy in most practical situations since it demands excessive control action and has poor robustness. Thus, variance reduction estimation based on the MVC benchmark tends to obtain an overaggressive economic performance assessment for a practical control system. The LQG benchmark is an alternative benchmark, and it has been proven to be a more realistic one than MVC when evaluating process control system with constraints since it considers input variance as well as output variance.

In this work, based on the LQG benchmark, a model based technique to assess the economic performance of the constraint process control system is presented. The remainder of this paper is organized as follows: the general procedure for calculation of the LQG benchmark is briefly reviewed in section 2. The approach for economic performance assessment of process control using model-based optimization technique is discussed in section 3. In section 4, an algorithm to estimate the economic benefit based on the LQG benchmark is introduced. Simulations are presented in section 5, followed by concluding remarks in section 6.

II. LQG BENCHMARK

MVC appears not to be appropriate for performance assessment of constraint control systems such as MPC application since MVC does not explicitly take the control cost into account and is rarely implemented in practical situation. The LQG benchmark as an alternative benchmark has been proposed for performance assessment of control systems with consideration of the control action constraints. Using LQG benchmark, the achievable performance is given by a tradeoff curve as shown in Fig. 1, and this curve can be obtained from solving the LQG problem [7]. The LQG objective function is define as:

$$J(\lambda) = E(y_t^2) + \lambda E(u_t^2) \tag{1}$$

Calculation of the LQG benchmark is briefly reviewed in this section following the approach of [4].

Consider that the process is described by an ARMAX model:

$$\begin{aligned} y_t + a_1 y_{t-1} + \cdots + a_n y_{t-n} = b_1 u_{t-1} + \cdots + \\ b_n u_{t-n} + a_t + c_1 a_{t-1} + \cdots + c_n a_{t-n} \end{aligned} \tag{2}$$

The corresponding Kalman predictor can be written as

$$\begin{aligned} x_{t+1} &= A x_t + B u_t + K \alpha_t \\ y_t &= C x_t + \alpha_t \end{aligned} \tag{3}$$

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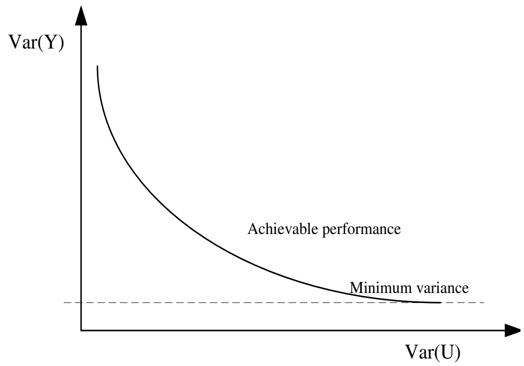

Fig. 1. Optimal performance curve obtained through LQG benchmark

The optimal feedback control can be written as

$$\begin{aligned} \hat{x}_{t+1} &= (A - KC - BL)\hat{x}_t + K y_t \\ u_t &= -L\hat{x}_t \end{aligned} \tag{4}$$

where L is the optimal state feedback control gain and can be solved via a MATLAB LQR function. Combining the Kalman predictor with state feedback yields

$$\begin{bmatrix} x_{t+1} \\ \hat{x}_{t+1} \end{bmatrix} = \begin{bmatrix} A & -BL \\ KC & A-KC-BL \end{bmatrix} \begin{bmatrix} x_t \\ \hat{x}_t \end{bmatrix} + \begin{bmatrix} K \\ K \end{bmatrix} \alpha_t \tag{5}$$

Writing it in a compact form

$$X_{t+1} = A_{cl} X_t + B_{cl}\alpha_t \tag{6}$$

the variance of the input and output can be written as:

$$\operatorname{Var}(u_t) = \begin{bmatrix} 0 & -L \end{bmatrix} \operatorname{Var}(X_t) \begin{bmatrix} 0 \\ -L^T \end{bmatrix} \tag{7}$$

$$\operatorname{Var}(y_t) = \begin{bmatrix} C & 0 \end{bmatrix} \operatorname{Var}(X_t) \begin{bmatrix} C^T \\ 0 \end{bmatrix} + \operatorname{Var}(\alpha_t) \tag{8}$$

Define $P = Var(X_t)$, and P is the solution to the Lyapunov equation,

$$P = A_{cl} P A_{cl}^T + B_{cl} B_{cl}^T \tag{9}$$

For the MIMO system, the objective function of LQG control is written as

$$J(\lambda) = E\left[Y_t^T W Y_t\right] + \lambda E\left[U_t^T R U_t\right] \tag{10}$$

where the output weighting W is determined by the relative importance of the individual controlled variables, and the control weighting R is chosen according to the relative cost of individual control moves.

By varying λ, various LQG control solutions of $E\left[y_t^2\right]$ and $E\left[u_t^2\right]$ can be calculated. Then a tradeoff curve can be plotted from these solutions, and this curve provides a useful lower bound on the achievable performance of the controller in terms of both input and output variance.

III. Economic performance assessment with model-based optimization

Reduction of the variance of quality variables is identified as a key aspect in any attempt to increase benefit potential. The back-off method has been an effective way to deal with disturbances and uncertainty in control systems, and can be applied into the economic performance assessment of the run-time control system.

Based on the idea of back-off approach and the steady state optimization techniques in MPC, Xu (2007) proposed a method to evaluate economic performance of existing run-time MPC application. The economic performance assessment of MPC application problems can be transferred to the constrained optimization problems by applying the steady state model and the back-off strategy. Several different scenarios for MPC performance assessment have been described in the form of constrained quadratic optimization problems with consideration of variance reduction and tuning of constraints. Only the economic potential analysis is discussed here for the sake of brevity, and mathematical details can be found in [1].

Consider a $p \times m$ system with m inputs and p outputs, having steady-state process gain matrix K. It is assumed that $(\bar{y}_{i0}, \bar{u}_{j0})$ is the current operating point and $(\bar{y}_i, \bar{u}_j)$ is the optimal operating point. Then the quadratic economic objective function for the system is formulated as follows:

$$\begin{aligned} J = & \sum_{i=1}^{p} [b_i \times \bar{y}_i + a_i^2 \times (\bar{y}_i - y_{di})^2] + \\ & \sum_{j=1}^{m} [b_j \times \bar{u}_j + a_j^2 \times (\bar{u}_j - u_{dj})^2] \end{aligned} \tag{11}$$

where, b_i $(\text{respectively}, b_j)$ and a_i $(\text{respectively}, a_j)$ are the linear and quadratic coefficients for the i^{th} controlled variables (CVs) (respectively, the j^{th} manipulated variables (MVs)); y_{di} and u_{dj} are the i^{th} CVs and j^{th} MVs. The economic performance assessment may be transferred to the quadratic optimization problems as follows,

$$\min_{\bar{y}_i, \bar{u}_j} J \tag{12}$$

subject to:

$$\Delta\bar{y}_i = \sum_{j=1}^{m} [K_{ij} \times \Delta u_j] \tag{13}$$

$$\begin{aligned} (L_{yi} - r_{yi} \times y_{hoi} + 2 \times \alpha_{yi0} \times (1 + s_{yi})) \leq \bar{y}_i \leq \\ (H_{yi} + r_{yi} \times y_{hoi} - 2 \times \alpha_{yi0} \times (1 + s_{yi})) \end{aligned} \tag{14}$$

$$\begin{aligned} (L_{uj} - r_{uj} \times u_{hoj} + 2 \times \alpha_{uj0}) \leq \bar{u}_i \leq \\ (H_{uj} + r_{uj} \times u_{hoj} - 2 \times \alpha_{uj0}) \end{aligned} \tag{15}$$

where $i = 1, 2, \ldots, p$ and $j = 1, 2, \ldots, m$. Equation (13) means that the move $(\Delta\bar{y}_i, \Delta\bar{u}_i)$ must satisfy the steady-state relation of the system.

In the above two inequalities, L_{yi} and H_{yi} are the low limit and high limit of CV_i, L_{uj} and H_{uj} are the low limit and high limit of MV_j. σ_{yi0} and σ_{uj0} are the standard deviation of CV_i and MV_j under the base case operation where the base case refers to the existing operation. r_{yi} and r_{uj} are the user defined percentage of relaxation in the limit for CV_i and MV_j. y_{hoi} and u_{hoj} are the half constraint range of CV_i and MV_j. The standard deviations are set to be adjusted by the percentage of variability change s_{yi}.

Therefore, the benefit potentials under the various operation conditions are readily obtained via solution of the defined steady-state economic optimization problem.

IV. Economic performance assessment with LQG benchmark

There are two important problems that should be taken into account when assessing the economic performance of process control systems. One is how to determine the relationship between economic performance and process variance reduction, the other is the relationship between the input variance and the output variance. Based on the back-off approach and the steady-state model, the most economically attractive operating point, or the optimal operating point, is determined via the solution of a model based optimization problem with specified allowable constraint limit violation. Thus, the relationship between economic performance and the output variance is determined through the back-off approach.

The second issue can be considered as an optimal control problem that determines the relationship between input variance and output variance. Xu et al. (2007) solved this problem by using minimum energy covariance control. However, under the minimum energy covariance control, the relationship between the input variance and the output variance is not uniquely determined (i.e. an inequality relation rather than an equality relation). To this end, we consider the limit of control performance, represented by the LQG tradeoff curve. Several possible optimal benchmark controls have been identified from the tradeoff curve shown in Fig. 2, and each of them serves for a different control objective [2]. For example, the minimum energy controller is optimal in the sense that it offers minimum possible control effort, and indicates the maximum variance reduction of control action (MV). Minimum variance control offers minimum possible error and the maximum variance reduction of output variable (CV). While an LQG tradeoff controller has performance between the minimum variance control and minimum cost control, it offers a tradeoff between reduction of the output variance and the control action. In terms of input and output variances, LQG tradeoff provides limit of control performance, or Pareto optimal [7]. Each point of the tradeoff curve corresponds to an optimal control. In this work, the LQG benchmark is combined directly with benefit potential, which therefore leads to an achievable optimal benefit potential.

Given a process control system, the purpose of economic performance assessment can now be viewed as identifying the possibility of moving its operating point as close as possible to its optimal point with the consideration of likely disturbance and uncertainty. Given sufficient operating data,

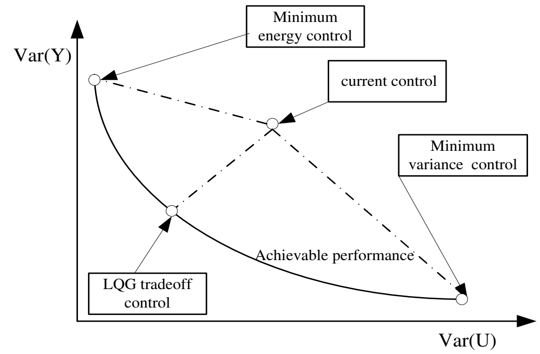

Fig. 2. Performance assessment benchmark with different control objective

the base case operation can be described as its current mean values and standard deviation. The optimal operation condition can be obtained by solving the economic steady state optimization problem subject to the current constraint limit settings and the input and output variability relation based on the LQG tradeoff curve. Generally, a reasonable percentage of constraint limit violation of controlled variables (CVs), say 5%, is allowed such that 95% of operation falls within the range of ± 2 times standard deviation [5]. Since manipulated variables usually represent the valve position of an actuator or the speed of rotation of a motor, constraint violation is not allowed in practice. Therefore, a more conservative back-off strategy is implemented on MVs than on CVs, and ± 3 times standard deviation is used for the analysis. Economic benefit potential can be determined by comparing optimal operation with base case operation. The problem formulation of optimal operation is described as follows.

Given an $p \times m$ system, having steady-state process gain matrix K. $(\bar{y}_{i0}, \bar{u}_{j0})$ is defined as the current operating point and $(\bar{y}_i, \bar{u}_j)$ is the optimal operating point. Then the quadratic economic objective function in [1] is adopted in this work:

$$\begin{aligned} J \;=\; & \sum_{i=1}^{p} [b_i \times \bar{y}_i + a_i^2 \times (\bar{y}_i - y_{di})^2] + \\ & \sum_{j=1}^{m} [b_j \times \bar{u}_j + a_j^2 \times (\bar{u}_j - u_{dj})^2] \end{aligned} \tag{16}$$

where all notation are same as that for equation (11). The move $(\Delta y_i, \Delta u_j)$ must satisfy the steady state gain relation described by the following equations:

$$\begin{aligned} \Delta\bar{y}_i &= \sum_{j=1}^{m} [K_{ij} \times \Delta\bar{u}_j] \\ \bar{y}_i &= \bar{y}_{i0} + \Delta\bar{y}_i \\ \bar{u}_j &= \bar{u}_{j0} + \Delta\bar{u}_j \end{aligned} \tag{17}$$

Considering the allowable percentage of violation of constraints, the following inequalities must be satisfied:

$$L_{yi} + 2 \times \sigma_{yi} \leq \bar{u}_i \;\leq\; H_{yi} - 2 \times \sigma_{yi} \tag{18}$$

$$L_{uj} + 3 \times \sigma_{uj} \leq \bar{u}_i \quad \leq \quad H_{uj} - 3 \times \sigma_{uj} \qquad (19)$$

where σ_{yi} and σ_{uj} are the standard deviation of the i^{th} output variable and the j^{th} input variable, which can be determined by the LQG benchmark. The LQG tradeoff curve represents a relationship between the output variance and the input variance, which may be represented by a function as

$$\sigma_Y^2 \quad = \quad f\left(\sigma_U^2\right) \qquad (20)$$

There is no analytical solution to obtain tradeoff curve since the LQG control law can not be explicitly expressed in term of λ or its equivalence. As discussed in section 2, a series of LQG solutions of $\mathrm{Var}(u_t)$ and $\mathrm{Var}(y_t)$ can be calculated according to (7) and (8) by varying λ. Then we can obtain a function of input variance against output variance through numerical methods such as interpolation or regression. Spline interpolation technique is a simple and effective method to obtain the function. The standard deviation other than variance of variables is more relevant to the determination of the optimal operation condition based on a predefined back-off level. Thus, the same numerical method is used to determine the function of input standard deviation against output standard deviation,

$$\sigma_Y \quad = \quad f\left(\sigma_U\right) \qquad (21)$$

For MIMO systems, according to the LQG control law discussed in section 2, σ_Y and σ_U should satisfy:

$$\sigma_Y^2 \quad = \quad \sum_{i=1}^{p} w_i \sigma_{yi}^2 \qquad (22)$$

$$\sigma_U^2 \quad = \quad \sum_{j=1}^{m} r_j \sigma_{uj}^2 \qquad (23)$$

where $r_j, j = 1, \cdots, m$ and $w_i, i = 1, \cdots, n$ are the weighting coefficients of input and output variables, which are same as that in equation (10) where the weighting matrices are defined as diagonal matrices. Based on above analysis, the economic optimization problem for the benefit potential assessment of different scenarios can be transferred to the following form:

$$\min_{\bar{u}_j, \bar{y}_i, \sigma_{yi}, \sigma_{uj}} J \qquad \text{subject to} \quad (17) \sim (23) \qquad (24)$$

Equation (24) gives the achievable optimal operation condition for a process control system with the given economic objective and the steady-state model. For the base case operation, the economic objective function value is calculated by replacing $(\bar{y}_i, \bar{u}_j)$ with the current operating point $(\bar{y}_{i0}, \bar{u}_{j0})$ in (16), which is denoted as J_0. It is a value to be compared with for the calculation of economic potentials. In the following, we will discuss the economic potential calculations under different scenarios, following the notations used in [1].

- **Ideal operation scenario:** In this scenario, the disturbance effect is not considered and a nominal steady-state operation is assumed. There is no back-off due to the disturbance and the constraint limits are kept unchanged. The solution of (24) results in an ideal operation point $(\bar{y}_{Ii}, \bar{u}_{Ij})$ and corresponding objective function is denoted as J_I. Then, the ideal economic potential ΔJ_I can be calculated by

$$\Delta J_I = J_I - J_0 \qquad (25)$$

- **Existing variability scenario:** In this scenario, the present level of disturbance is taken into account, and no action is taken to reduce the variability of the output variables. Thus, the existing economic potential is obtained by shifting mean value only. The resultant optimal operating point is denoted as $(\bar{y}_{Ei}, \bar{u}_{Ej})$, and corresponding objective function as J_E, which is calculated by replacing σ_{yi} in equation (18) with existing standard deviation σ_{yi0} in the QP problem (24). Thus, the existing economic potential δJ_E can be defined as following:

$$\Delta J_E = J_E - J_0 \qquad (26)$$

- **Reducing variability scenario:** In this scenario, the increased economic benefit comes from the variability reduction on controlled variables based on the LQG trade-off. With variability reduction, the back-off can also be reduced, which allows further mean values shifting in the direction of the optimal operation point. The optimal operation point is usually located on the constraint limit. The mean shift reduces the distance between the actual operating point and the optimal point, and thus gives rise to increased economic beneficial. The optimal operating point is denoted as $(\bar{y}_{Vi}, \bar{u}_{Vj})$, and corresponding objective function as J_V, which can be calculated via the solution of the QP problem (24). Thus, the optimal economic potential by reducing variability ΔJ_V can be defined as following:

$$\Delta J_V = J_V - J_0 \qquad (27)$$

The economic performance assessment of the process control can be done using the information obtained by performing the optimizations discussed above. Two economic performance indices, the existing economic performance index (η_E) and the best achievable economic performance index (η_B), are used to assess the economic performance potential of process control.

$$\eta_E = \frac{\Delta J_E}{\Delta J_I} \qquad (28)$$

$$\eta_B = \frac{\Delta J_V}{\Delta J_I} \qquad (29)$$

It is obvious that $0 \leq \eta_E, \eta_B \leq 1$. Comparing η_E with η_B, the following inequality holds $0 \leq \eta_E \leq \eta_B \leq 1$. A positive value of (η_B) means that this economic potential could be achieved by reducing the variability through LQG control, while the economic potential given by a positive value of (η_E) could be actually achieved by simply moving the operating point to the optimal one.

V. CASE STUDIES

In this section, two simulation examples are performed to demonstrate the effectiveness of the proposed approach in assessing the economic performance of process control systems. The calculations for both examples are based upon closed-loop data sets containing 3000 observations of the controlled and manipulated variables generated using Matlab/Simulink.

A. SISO system

In this example the proposed algorithm is applied to a system described by the following linear transfer functions for the plant G_p, and disturbance model G_d,

$$\begin{aligned} Y_t &= G_p U_t + G_d \alpha_t \\ &= \frac{0.6299z^{-1}}{1-0.8899z^{-1}} U_{t-2} + \frac{1-0.8z^{-1}}{1-0.8899z^{-1}} \alpha_t \end{aligned} \quad (30)$$

where $\{\alpha_t\}$ is a normally distributed white noise sequence of mean 0 and variance 1. The economic objective chosen for this problem is to maximize the output while satisfying the following constraints:

$$\begin{aligned} -5 \le u(k) \le 5 \\ -10 \le y(k) \le 10 \end{aligned}$$

By setting the economic objective function as $J = -2\bar{y}$, the optimal operation condition can be obtained by solution of the optimization problem as follows:

$$\begin{aligned} &\min_{\bar{y},\bar{u},\sigma_y,\sigma_u} -2\bar{y} \\ &(\bar{y} - \bar{y}_0) = 5.72\,(\bar{u} - \bar{u}_0) \\ &Y_{Lk} + 2 \times \sigma_y \le \bar{y} \le Y_{Hk} - 2 \times \sigma_y \\ &U_{Lk} + 3 \times \sigma_u \le \bar{u} \le U_{Hk} - 3 \times \sigma_u \\ &\sigma_y = spline(\sigma_{ui0}, \sigma_{yi0}, \sigma_u) \end{aligned} \quad (31)$$

A PI controller is chosen to regulate the control loop and the base case operation is defined as $\left[\begin{array}{cccc} \bar{u}_0 & \bar{y}_0 & \sigma_{u0} & \sigma_{y0} \end{array}\right]^T$, which is estimated as $\left[\begin{array}{cccc} -0.022 & 0.382 & 0.632 & 2.212 \end{array}\right]^T$ based on simulation results. According to the solution to the QP problem (24), the optimal operation condition under reducing variability scenario is $\left[\begin{array}{cccc} 0.466 & 2.936 & 0.468 & 1.956 \end{array}\right]^T$. On the other hand, the economic potentials of other scenarios are: $\Delta J_I = -8.14$, $\Delta J_E = -2.43$, and $\Delta J_V = -5.11$. Accordingly, the existing economic performance index and the best achievable economic performance index are calculated as $\eta_E = 29\%$ and $\eta_B = 64\%$. This means that 35% of the ideal potential benefits $(\eta_B - \eta_E)$ is possibly achieved by further variability tuning through advanced control.

The calculated ΔJ_V is the benefit potential that could be achieved if the LQG control is implemented. This benefit potential can be verified by replacing the existing PI controller with a designed LQG controller in the control system. With the same simulation condition, the realized economic potential is calculated as $\Delta J_{ver} = -4.86$, which is about 59% of ideal economic potential. It means that the calculated economic potential is indeed achieved by the control system upgrading. The base and the optimal operation condition in term of standard deviation is shown in Fig. 3.

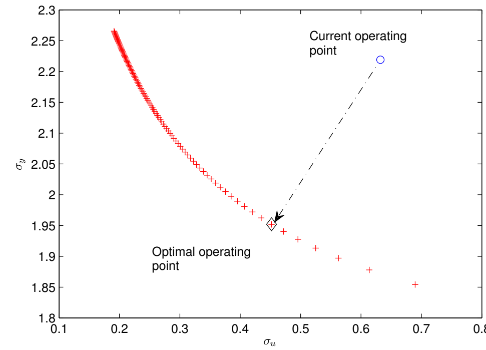

Fig. 3. Comparison of the base and the optimal operation condition in term of standard deviation

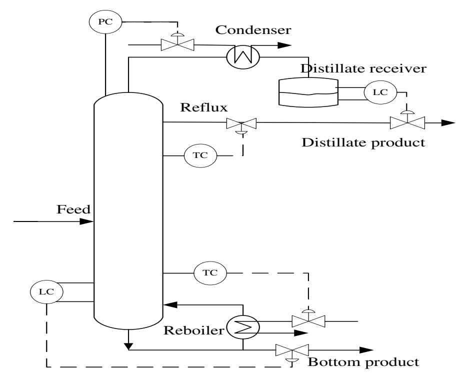

Fig. 4. Schematic diagram of the separation process

B. MIMO Case

The proposed economic performance assessment method is further tested in another case study involving control-optimization application [6]. The process is a binary separation process shown in Fig. 4, which has two manipulated variables, two controlled variables, and one disturbance variable. The manipulated variables are reflux flow rate u_1 and vapor boil up rate u_2, the output variables are the distillate product y_1 and bottom product y_2, and the disturbance variable is feed flow rate d.

The input-output transfer function model of the process is

$$\left[\begin{array}{c} y_1 \\ y_2 \end{array}\right] = \left[\begin{array}{cc} \frac{4e^{-5s}}{14s+1} & \frac{2e^{-3s}}{22s+1} \\ \frac{-1e^{-2s}}{25s+1} & \frac{5}{27s+1} \end{array}\right] \left[\begin{array}{c} u_1 \\ u_2 \end{array}\right] + \left[\begin{array}{c} \frac{-0.5e^{-1s}}{15s+1} \\ \frac{0.1e^{-3s}}{20s+1} \end{array}\right] d \quad (32)$$

An MPC controller is implemented in this distillation process with the following parameters: prediction horizon as $P = 20$, control horizon as $M = 4$; weighting coefficients matrix as $W = diag\,(10, 1)$, and input weighting matrix as $R = diag\,(3, 3)$. The MPC design problem has the following

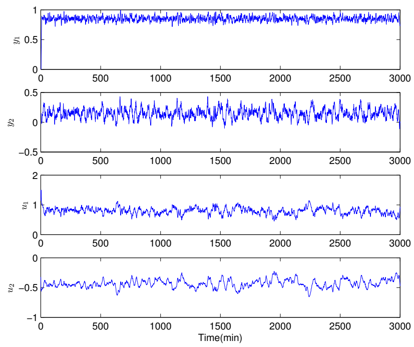

Fig. 5. Base case operation of simulated separation process

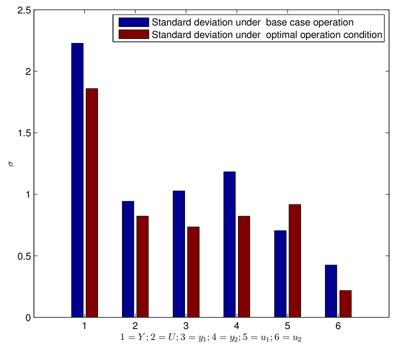

Fig. 6. Standard deviation under base and optimal operation condition

TABLE I

RESULTS OF ECONOMIC PERFORMANCE VALUATION PROBLEM

scenarios	optimal operation point				economic potentials	
	y_1	y_2	u_1	u_2	calculated	verified
ideal	0.796	0.129	2.124	1.882	0.221	0.201
existing	0.217	0.385	1.655	2.071	0.052	0.047
reducing	0.470	0.170	2.301	1.552	0.131	0.115

formulation

$$\begin{aligned} \underset{\substack{u_{1,k+j},u_{2,k+j} \\ j=0,\ldots,M}}{Min} & \sum_{i=1}^{P}\sum_{l=1}^{2}[w_{yl}(y_{l,k+i}-y_{l,ss})]+ \\ & \sum_{j=0}^{M-1}\sum_{l=1}^{2}[w_{ul}(u_{l,k+j}-u_{l,ss})] \end{aligned} \tag{33}$$

subject to

$$\begin{aligned} -0.5 \le y_{1,k+j} \le 1, &\quad 1 \le j \le P \\ -0.5 \le y_{2,k+j} \le 0.5, &\quad 1 \le j \le P \\ -5 \le u_{1,k+j} \le 5, &\quad 1 \le j \le M-1 \\ -5 \le u_{2,k+j} \le 5, &\quad 1 \le j \le M-1 \\ -0.03 \le \Delta u_{1,k+j} \le 0.03, &\quad 1 \le j \le M-1 \\ -0.03 \le \Delta u_{2,k+j} \le 0.03, &\quad 1 \le j \le M-1 \end{aligned} \tag{34}$$

where $y_{1,ss} = 0.95$, $y_{2,ss} = 0.05$, $u_{1,ss} = 3.95$ and $u_{2,ss} = 2.19$ are the nominal steady state values. By simulation on this MPC application, the base case operation with given constraints limits is shown in Fig. 5.

The economic objective function is set as the maximization of the controlled variable y_1. According to the steady-state economic potentials analysis discussed in section 4, the economic benefits and optimal operation conditions under different scenarios are calculated and verified in Table. I. the existing economic performance index and the best achievable economic performance index are calculated as $\eta_E = 23.5\%$ and $\eta_B = 59.3\%$, which means that 23.5% and 59.3% of ideal economic potential can be achieved by the mean shifting only and the further controller tuning respectively. According to the calculated optimal operation condition, the best achievable economic potential ΔJ_V can be achieved when reducing the standard deviation of y_1,y_2,u_1 and u_2 by 28.4%,10.5%,-22.4%,48.7% respectively, as shown in Fig. 6.

The calculated economic potentials ΔJ_E, ΔJ_E and ΔJ_V are verified by setting the setpoint as the corresponding optimal operation point in MPC application. From Table I we can see that the realized economic potentials are close to those of calculated ones. Simulated results once again show that realized economic potentials agree with those calculated ones, which demonstrates the feasibility of the proposed approach for economic performance assessment of process control.

VI. CONCLUSION

An economic performance assessment algorithm based on the LQG benchmark is developed to evaluate the benefit potentials in this study. The LQG benchmark offers a more realistic tradeoff between the variability reduction in the output and the input variables. Based on the LQG trade-off curve as well as the back-off strategy, the economic performance assessment problems under different scenarios are formulated as the constrained optimization problems. The economic potential as well as the optimal operation condition can be obtained via solution of the formulated optimization problem. Two case studies show the feasibility of the proposed algorithm.

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