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Performance Comparison of Robust Negative Input Shapers

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***Abstract*— Input shaping is a control method that limits motion-induced oscillation in vibratory systems by intelligently shaping the reference command. As with any control method, the robustness of input shaping to parameter variations and modeling errors is important. Input shaping has fundamental compromise between robustness and shaper duration, which is closely related to system rise time. For all shapers, greater robustness requires a longer duration shaper. However, if the shaper is allowed to have negative impulses, then the shaper duration may be shortened, at the expense of possible high mode excitation and a small decrease in robustness. This paper analyzes the compromise between shaper duration and robustness for several robust, negative input shapers. In addition, a formulation for Specified Negative Amplitude, Specified Insensitivity (SNA-SI) shapers is presented. These shapers provide a continuous spectrum of solutions for the duration/robustness trade-off. Experimental results from a portable bridge crane verify the theoretical predictions.**

I. Introduction

The control of flexible machines has been, and will continue to be, an important area of research. Input shaping is a control method that dramatically reduces motion-induced vibration [1], [2]. It accomplishes this vibration reduction by intelligently shaping the reference command such that the flexible modes of the system are not excited. To do this, a series of impulses, called an input shaper, is convolved with the original, unshaped, reference command to form the new shaped command. This process is shown with a two-impulse shaper and a step command in Fig. 1. Notice that the command rise time is increased by the duration of the impulse sequence, Δ.

Input shapers are generated using estimates of system natural frequencies and damping ratios. They are typically named according to the constraints used to form them. For example, the first shapers developed [1], [3], [4] sought to limit vibration to zero at the design frequency and damping ratio, and so are called Zero Vibration (ZV) shapers. Since then, additional constraints have been used to form input shapers that are robust to modeling errors [2], [5]–[8]. The fundamental compromise, however, is that the additional robustness afforded by these shapers comes at the cost of shaper duration. More robust shapers have longer durations, which has the effect of slowing command rise time. This fundamental compromise in input shaper design has been studied for input shapers containing only positive impulses [9]. This paper will present a study of input shapers containing negative impulses.

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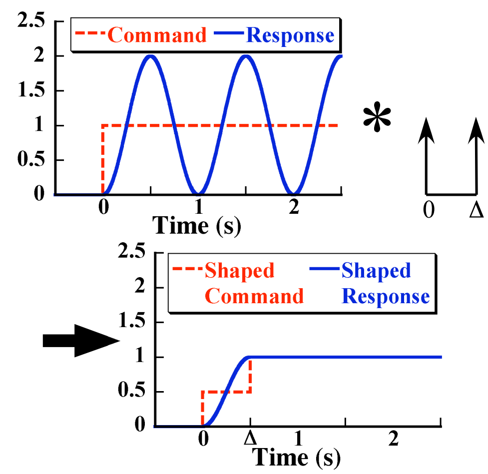


Fig. 1. The Input Shaping Process

A primary constraint in the formulation of input shapers is on residual vibration. The residual vibration of a second-order underdamped system from a series of n-impulses, expressed as a percentage of the vibration excited by a unity-magnitude impulse at time zero, is [2]:

$$V(\omega,\zeta) = e^{-\zeta\omega t_n}\sqrt{[C(\omega,\zeta)]^2+[S(\omega,\zeta)]^2} \tag{1}$$

where,

$$C(\omega,\zeta) = \sum_{i=1}^{n} A_i e^{\zeta\omega t_i}\cos(\omega t_i\sqrt{1-\zeta^2}) \tag{2}$$

$$S(\omega,\zeta) = \sum_{i=1}^{n} A_i e^{\zeta\omega t_i}\sin(\omega t_i\sqrt{1-\zeta^2}), \tag{3}$$

A_i and t_i are the i^{th} impulse amplitude and time location, $omega$ is the system frequency, and ζ is the damping ratio. A constraint on residual vibration can be formed by setting (1) less than or equal to a tolerable level of residual vibration at the modeled natural frequency and damping ratio.

Given the transcendental nature of (1) there are an infinite number of solutions. Additional constraints must be imposed to select a practical solution. To ensure the fastest solution possible, the time of the last impulse is typically minimized. Impulse amplitudes are also required to sum to one, which ensures the shaped command reaches the same set-point as the unshaped command. Additional impulse amplitude constraints are still required to keep the impulse amplitudes from being driven to positive and negative infinity by the minimum time constraint.

All positive shapers, such as the ZV shaper discussed above, constrain impulses to have positive amplitude. If negative impulse amplitudes are allowed, then there are two primary methods to constrain impulse amplitudes, Unity Magnitude and Specified Negative Amplitude. Allowing negative impulses has the primary advantage of decreasing the duration of the input shaper, speeding system response.

Unity Magnitude (UM) input shapers have impulse amplitudes that are constrained to be ± 1 [10]. If this constraint is applied, shaper impulses are also constrained to alternate sign and still sum to one. Additional constraints determine the full nature of the shaper. For example, a UM shaper designed for zero vibration at the design frequency, analogous to the ZV shaper, is called the UM-ZV Shaper.

Another amplitude constraint specifies the maximum negative impulse amplitude the input shaper may contain. This constraint has the form:

$$0 < A_i \leq 1 \qquad \text{when } i \text{ is odd} \tag{4}$$

$$A_i = -A_{max} \qquad \text{when } i \text{ is even,} \tag{5}$$

where A_{max} is the maximum negative amplitude allowed. Shapers formed using this amplitude constraint are called Specified Negative Amplitude (SNA) shapers [11].

The next section describes the most common measure of input-shaper robustness, Insensitivity. In Section III, several robust, negative input-shaping methods are presented. Section IV presents a new Specified Negative Amplitude, Specified Insensitivity (SNA-SI) shaper formulation. Section V analyzes the compromise between robustness and duration for each of these methods. In Section VI, the high-mode excitation characteristics of the input shapers are presented. Experimental results from a portable bridge crane verifying the theoretical predications are given in Section VII.

II. Sensitivity Curves and Insensitivity

One commonly used tool to examine the robustness of input shapers is the sensitivity curve. The natural frequency sensitivity curve for a ZV shaper is shown by the solid line in Fig. 2. The vertical axis is the residual vibration, (1), and the horizontal axis is the actual system natural frequency, ω, normalized by the modeled frequency, ω_m, used to design the input shaper. The curve indicates how residual vibration changes as a function of modeling errors in frequency.

One key measure of robustness derived from the sensitivity curve is *Insensitivity* [6], [12]. Insensitivity is the width of the sensitivity curve at a tolerable vibration level, V_{tol}, with respect to the parameter of interest. For example, Fig. 2 shows the ZV shaper has an Insensitivity at a V_{tol} of 5%, $I(5\%)$, of 0.06. A more robust shaper, labeled ZVD, will be discussed in Section III-A. Its sensitivity curve, also shown in Fig. 2, has an $I(5\%)$ of 0.29.

The large robustness provided by the ZVD shaper shown in Fig. 2 does not come without cost. This robust shaper is longer than the relatively non-robust ZV shaper. However, allowing a shaper to have negative impulses will shorten its duration. This presents the opportunity to compensate for increased duration from robustness constraints by designing input shapers that utilize negative impulses.

Fig. 2 also shows the sensitivity curves for Unity Magnitude ZV and ZVD shapers. The sensitivities of negative

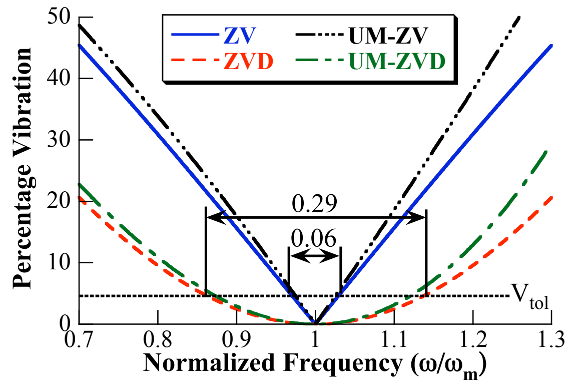


Fig. 2. Sensitivity Curves for (UM-)ZV and (UM-)ZVD Shapers

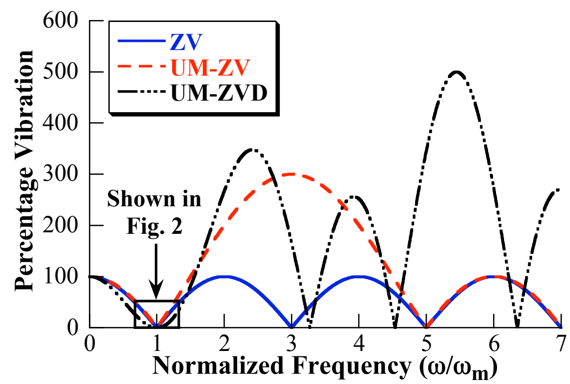


Fig. 3. ZV, UM-ZV, and UM-ZVD Sensitivity Curves

input shapers are similar to their positive counterparts near the design frequency. However, at frequencies much higher than the design frequency significant vibration can occur. Fig. 3 shows this effect for frequencies up to seven times the design frequency. Positive input shapers, like the one labeled ZV in Fig. 3, can never excite more oscillation than the unshaped case. However, negative shapers can [11], [13]. While it is unlikely that the estimate of the system natural frequency of the system will be incorrect by a factor of two or three, there may be unmodeled higher modes that can be excited. Additionally, higher modes that are known but do not significantly affect system response may be magnified such that they do so. Therefore, the effect of negative amplitude impulses on higher modes must be taken into account when evaluating negative input shaping techniques.

III. Robust, Unity Magnitude Methods

This section will discuss the Unity Magnitude analogies of several robust shaping methods. A detailed discussion of the positive forms of each can be found in [9].

A. Derivative Methods

The earliest form of robust input shaping was achieved by setting the derivative, with respect to the frequency, of the residual vibration equation (1) equal to zero [2].

If these same constraints are imposed, but impulse amplitudes are restricted to ± 1, then the UM-ZVD shaper results. The sensitivity curve for the UM-ZVD shaper was also shown in Fig. 2.

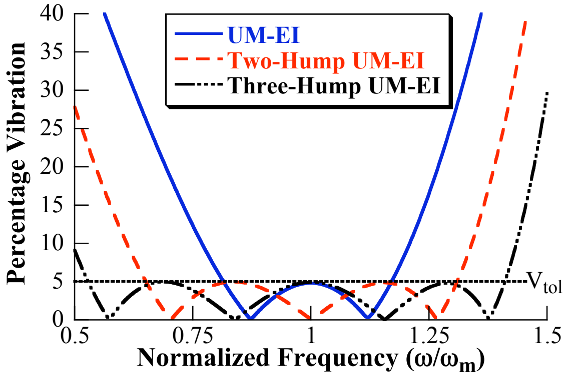


Fig. 4. Sensitivity Curves for UM-EI Method Shapers

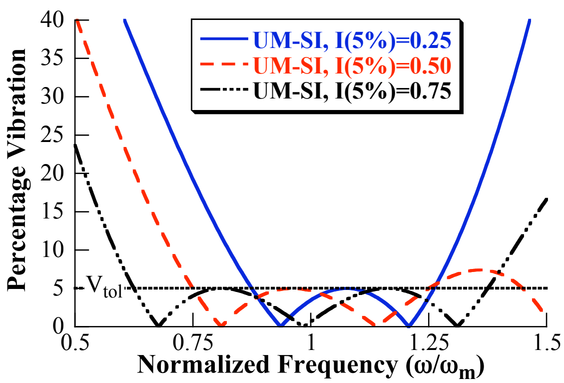


Fig. 5. Sensitivity Curves for Several UM-SI Shapers

B. Extra Insensitive (EI) Methods

The shapers discussed above have been formed using a constraint that there be zero residual vibration at the modeled frequency. However, even in systems for which a good model exists, there will be some modeling error and vibration will occur. This suggests that the zero vibration constraint should be relaxed to one in which residual vibration remains below some tolerable level, V_{tol} [6]. The first shaper utilizing this idea was the Extra Insensitive (EI) shaper. The Unity Magnitude analogy of this shaper also relaxes the zero vibration constraint to some tolerable level, V_{tol}. The sensitivity curve for the UM-EI shaper is shown in Fig. 4.

Shapers that extend the EI shaper idea have a progressively more humps and are called Multi-Hump EI Shapers [14]. Unity Magnitude equivalents exist for these shapers too. The sensitivity curves for the Two-Hump UM-EI and Three-Hump UM-EI shapers are shown in Fig. 4. The price for increased robustness gained for Two and Three-Hump UM-EI shapers is a corresponding increase in shaper duration.

C. Specified Insensitivity (SI) Methods

Specified Insensitivity (SI) shapers are formed by generating constraint equations to match the desired level of robustness [15]. Similarly, UM-SI shapers can be designed to have any level of robustness, but with impulses amplitudes of ± 1. Shaper impulse times are generated using optimization routines. Fig. 5 shows the sensitivity curves for UM-SI shapers designed for several different five-percent Insensitivities.

Additional advantages to UM-SI shapers are that they can be designed for any level of tolerable vibration and for unsymmetric Insensitivities. This fact can be particularly useful when a system is known to deviate in one direction more frequently or to a greater extent than in the other.

IV. Specified Negative Amplitude-Specified Insensitivity (SNA-SI) Methods

The faster system motion provided by UM shapers comes at the cost of increased actuator demands and possible high-mode excitation. Specified Negative Amplitude (SNA) shapers provide a method to reduce actuator demands and high-mode excitation, while retaining the benefits of negative impulses [11]. To date, SNA shapers have been developed that fulfill zero-vibration (ZV) and zero vibration and derivative (ZVD) constraints for a specified maximum negative impulse amplitude [11]. To create a shaper that has minimum duration for a given Insensitivity and maximum negative impulse amplitude, SNA and SI constraints can be combined to create an SNA-SI shaper.

An additional constraint is needed to ensure the shaped command remains within the bounds established by the unshaped command. This constraint limits the running sum of impulse amplitudes to between zero and one:

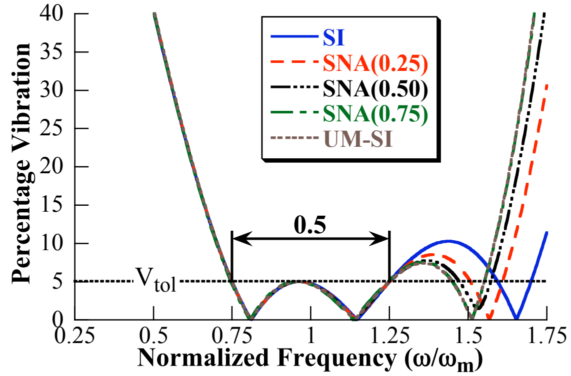
$$0 \leq \sum_{i=1}^{k} A_i \leq 1 \quad k = 1, \ldots, n \tag{6}$$

where A_i is the i^{th} impulse amplitude, k is the current impulse, and n is the number of shaper impulses.

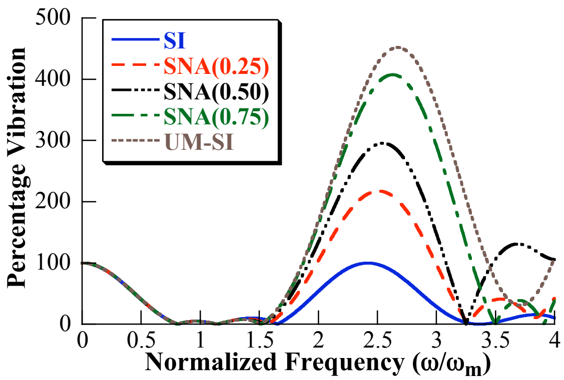
SNA-SI shapers can be considered a general form of SI shaping. If the maximum negative amplitude allowed is zero, resulting in only positive impulses, then the positive SI shaper results. If the maximum negative impulse amplitude is set to one, then UM-SI shapers result. In this paper, the maximum negative amplitude allowed is indicated in parenthesis. An SNA-SI shaper with a specified negative amplitude of 0.25 is indicated by SNA(0.25).

Fig. 6 compares several SNA shapers designed with differing specified negative amplitudes and $I(5\%) = 0.5$. Fig. 6(a) shows the sensitivity curves for these shapers near the modeled frequency. One can see that the sensitivity curves for these shapers are nearly identical for frequencies inside the insensitivity range and lower. At higher frequencies, shown in Fig. 6(b), SNA-SI shapers with higher maximum negative impulse amplitudes display larger amounts of high-mode excitation. This, along with increased actuator requirements, is the penalty for the decrease in rise time afforded by the larger negative impulse amplitudes.

SNA-SI shapers provide the shortest duration shaper for a given Insensitivity and maximum negative impulse amplitude. As such, they provide the controls engineer with a method to design a shaper that best meets the given requirements on robustness, high-mode excitation, actuator limits, and system rise time.



(a) Near the Modeled Frequency



(b) Over a Larger Range of Frequencies

Fig. 6. Sensitivity Curves for SI, SNA, and UM-SI Shapers for I(5%)=0.5

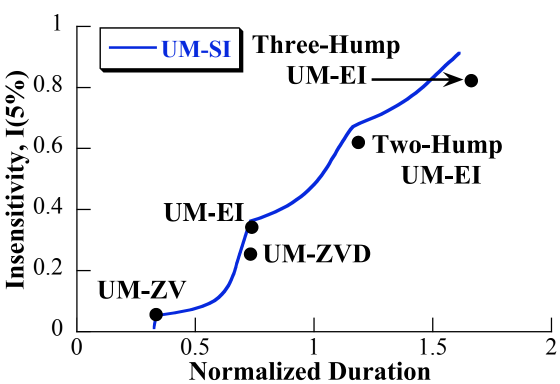


Fig. 7. I(5%) as a Function of Shaper Duration

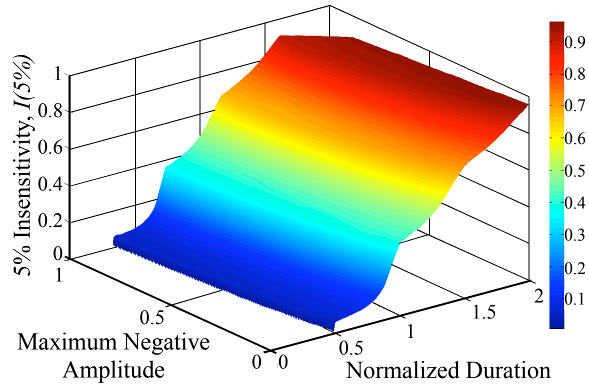


Fig. 8. Insensitivity as a Function of Shaper Duration and Maximum Negative Amplitude

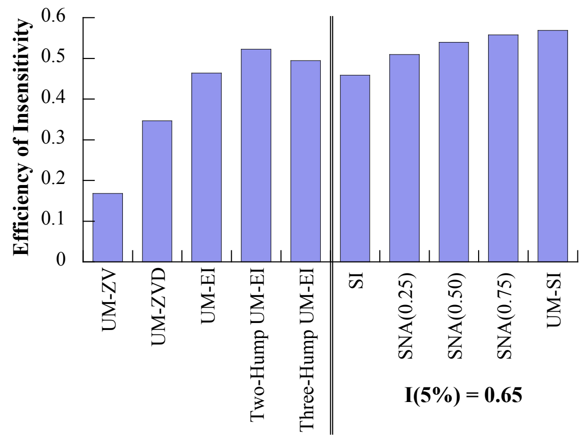


Fig. 9. Efficiency of Insensitivity

V. Shaper Insensitivity versus Shaper Duration

For both positive and negative robust input shapers, shaper duration increases with Insensitivity. The Insensitivity for a given shaper duration, however, differs between methods. This tradeoff is well documented for positive shapers [9]. This section will provide an analysis of this compromise for shapers containing negative impulses. To ensure that this analysis is system independent, the shaper duration is normalized by the damped natural period of the system.

Fig. 7 shows the 5% Insensitivity, $I(5\%)$, of various Unity Magnitude shapers as a function of normalized shaper duration. The UM-SI shaper is plotted as a line, as it can be designed to have any desired level of Insensitivity. It is the minimum duration shaper for any given Insensitivity. Other shapers discussed in this paper are also shown on the plot. One point of interest is that the UM-EI shapers do not exactly correspond to nodes on the UM-SI shaper curve, as they do for positive shapers.

This plot may be extended to include SNA-SI shapers by plotting the maximum negative amplitude allowed on the third axis, as shown in Fig. 8. One can see that the same general trends from Fig. 7 continue for all levels of negative impulse amplitude. Note that this plot provides a graphical representation of the maximum 5% Insensitivity possible given a shaper duration and maximum negative amplitude. The decrease in shaper duration as larger negative impulses are allowed is also illustrated.

In order to further quantify the compromise between insensitivity and shaper duration, the *Efficiency of Insensitivity* is used [9]. The Efficiency of Insensitivity is the Insensitivity of a shaper divided by its normalized duration. Higher numbers indicate that a shaper achieves its robustness more efficiently, in terms of shaper duration. Fig. 9 shows the Efficiency of Insensitivity for $V_{tol} = 5\%$ for various shapers. On the left side of the plot are Unity Magnitude shapers. To the right of the vertical line are SI, SNA-SI, and UM-SI shapers designed for $I(5\%) = 0.65$. Note that the Efficiency of Insensitivity increases as higher magnitude negative impulses are allowed. This indicates that allowing negative impulses increases the efficiency with which robustness is achieved.

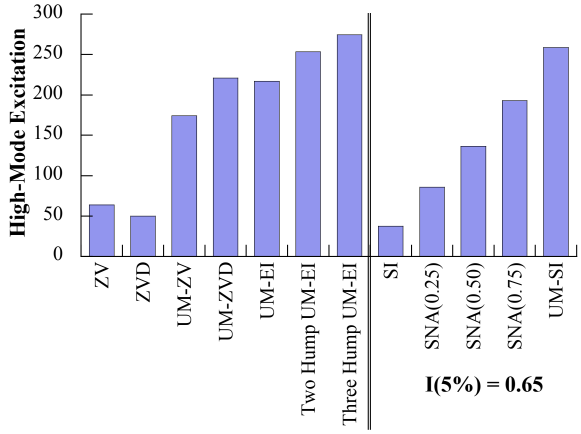


Fig. 10. Average High-Mode Excitation

VI. High-Mode Excitation

One price for the decrease in shaper duration and increase in Efficiency of Insensitivity afforded by negative impulses is high-mode excitation. To characterize the high-mode excitation caused by an input shaper, the average value of the sensitivity curve between 2 and 10 times the design frequency is used. The high-mode excitation for the shapers discussed in this paper is shown in Fig. 10. Notice that the two positive shapers (ZV and ZVD) average less than 100%. This is expected as the maximum amount of vibration they excite is 100% of the unshaped case. The remaining shapers to the left of the vertical solid line are Unity Magnitude shapers. All these shapers have high-mode excitation averages above 100%. Note that more robust shapers, which require a higher number of impulses, have higher values of average high-mode excitation. This is also expected as the maximum high-mode excitation increases with the number of impulses [11].

To the right of the solid vertical line are the same SI, SNA-SI, and UM-SI shapers from Fig. 9. In Fig. 9, it was shown that increasing the maximum allowed negative impulse improved the Efficiency of Insensitivity. In Fig. 10, the cost of this increased efficiency is shown, as average high-mode excitation increases with maximum negative impulse amplitude.

VII. Experimental Comparison of Robust Shapers

To rigorously test the various shaping methods, representative shapers from each method were experimentally evaluated using the portable bridge crane shown in Fig. 11. The portable bridge crane has a workspace of approximately 1m$\times$1m$\times$1.6m. The overhead bridge and trolley are driven using Siemens synchronous AC servo motors attached to timing belts that provide motion in the x and y directions. The motors are controlled using a Siemens PLC using Proportional-plus-Integral (PI) Control with feedback from motor-mounted encoders. The crane is also equipped with a vision system to measure payload position.

Input shapers were designed for a system natural frequency of 0.74 Hz and zero damping, corresponding to a suspension length of approximately 0.46m (18 in). The frequency was varied by changing the suspension cable length. Eight shapers were evaluated using suspension cable lengths from approximately 0.38-1.57m (15-62 in).

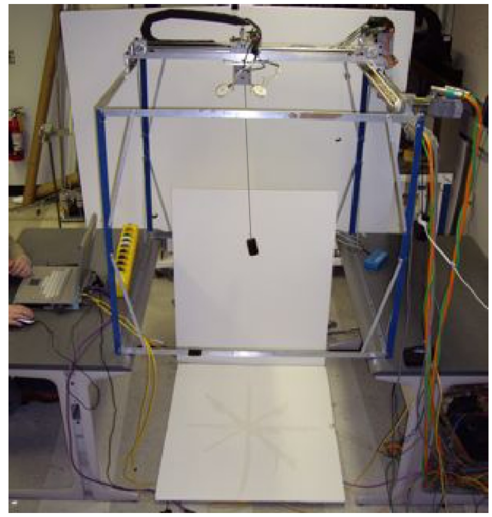


Fig. 11. Portable Bridge Crane

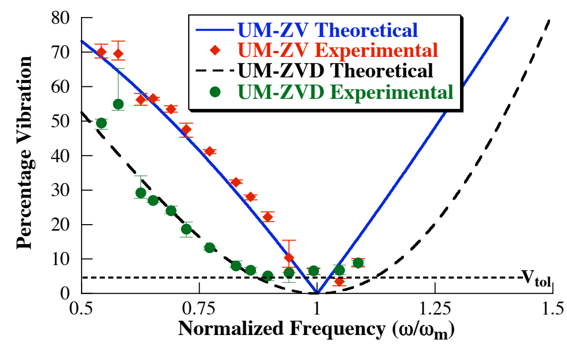


Fig. 12. Experimental Sensitivities of UM-ZV and UM-ZVD Shapers

Fig. 12 shows the theoretical and experimental sensitivity curves for the UM-ZV and UM-ZVD shapers. The experimental points are the average of three trials, with error bars indicating the minimum and maximum values of vibration for each set. For both the UM-ZV and UM-ZVD, the experimental results closely match those predicted by theory. Both of these shapers are designed to provide zero vibration at the design frequency. However, neither shaper achieves this theoretical minimum. This further motivates the use of tolerable vibration methods. Fig. 13 shows the theoretical and experimental sensitivity curves for two such shapers, the UM-EI and Two-Hump UM-EI. The sensitivity curves for these two shapers also closely match the theoretical predictions.

Figs. 14–17 show the theoretical and experimental sensitivity curves for a series of SNA-SI shapers, beginning with the UM-SI. Each shaper is designed for $I(5\%) = 0.5$ and the experimental results closely match the theoretical predictions.

VIII. Conclusion

This paper presented an overview of various robust negative input shapers, with emphasis on the compromise between shaper duration and robustness. It also presented a

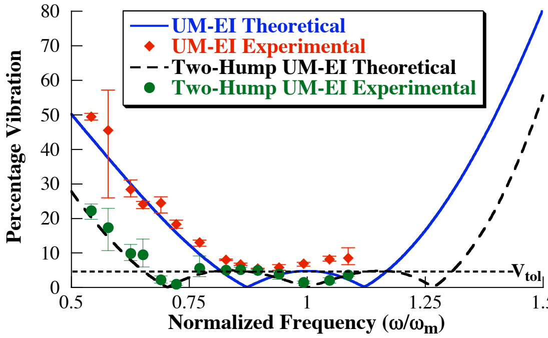


Fig. 13. Experimental Sensitivities of UM-EI and Two-Hump UM-EI Shapers

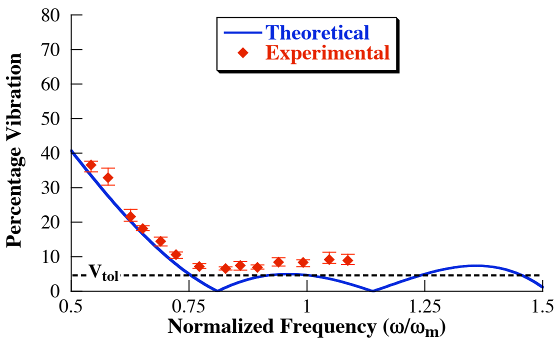


Fig. 14. Experimental Sensitivity of UM-SI $[I(5\%) = 0.5]$ Shaper

new formulation for Specified Negative Amplitude, Specified Insensitivity (SNA-SI) shapers, which provide a continuous spectrum for the duration/robustness trade-off. The Efficiency of Insensitivity was calculated for the various robust negative input shapers. For SNA-SI shapers, it was shown that allowing higher amplitude negative impulses increases the efficiency at which Insensitivity is achieved at the cost of increased high-mode excitation. Experimental results from a portable bridge crane verified the theoretical predictions.

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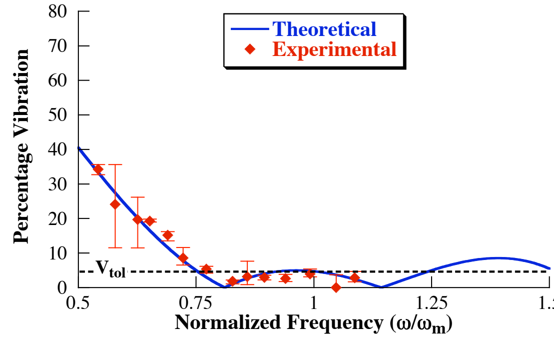

Fig. 15. Experimental Sensitivity of SNA(0.25) $[I(5\%) = 0.5]$ Shaper

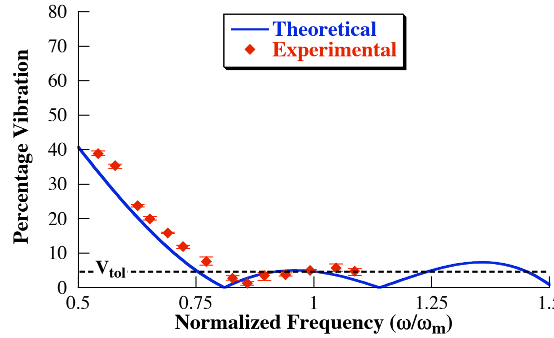

Fig. 16. Experimental Sensitivity of SNA(0.50) $[I(5\%) = 0.5]$ Shaper

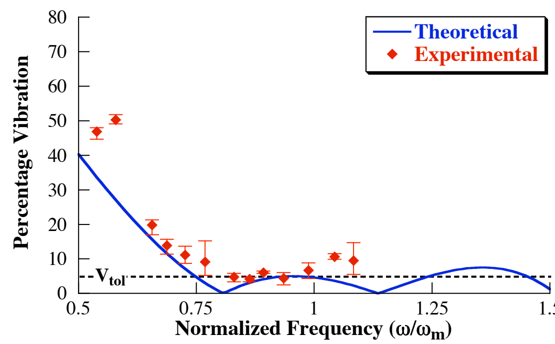

Fig. 17. Experimental Sensitivity of SNA(0.75) $[I(5\%) = 0.5]$ Shaper

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