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Transient Deflection Performance Measures for Command Shapers

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Abstract— **Input shaping is a technique used to reduce command-induced vibration. Recently, a class of input shapers has been developed to limit both the residual vibration and the transient deflection of flexible systems. This paper evaluates two types of commands: pre-computed, deflection-limiting commands for a floating oscillator given a fixed move distance and real-time deflection-limiting command shapers for a mass under PD control. For these command shapers, transient performance is just as important as residual vibration. Therefore, the effect of modeling error on transient performance must be considered. Transient deflection sensitivity plots are presented to gauge the effectiveness of these commands in meeting their design goals. An evaluation of the effectiveness of traditional residual vibration performance measures to predict the transient performance robustness is also presented.**

I. Introduction

There has been a significant amount of work directed towards developing a variety of new command shaping techniques. Along with the development of these command shapers was the development of analysis tools to compare the performance of the shapers [1-6]. Recently new classes of command shapers that focus on the transient performance in addition to residual vibration have been introduced [7-10]. This paper investigates the applicability of the performance measure previously developed residual vibration analyses and introduces new tools for evaluating the transient performance of command shapers.

Input shaping works in real time by convolving the desired input with a sequence of impulses to produce a shaped input command [11-12]. Figure 1 shows an example of the input shaping process. Traditionally, the impulse amplitudes and time locations are determined by solving a set of constraint equations. The standard constraint equations consist of 1) residual vibration constraints, 2) robustness constraints, 3) impulse amplitude constraints and 4) time-optimality.

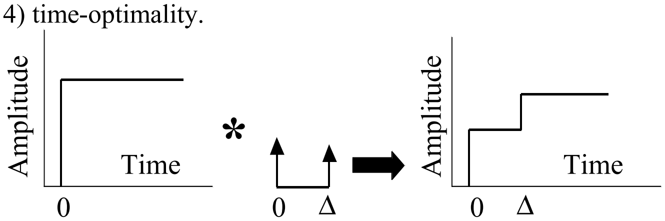

Fig. 1. Input Shaping Example.

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The constraint on residual vibration amplitude can be expressed as the ratio of residual vibration amplitude with shaping to that without shaping. The percentage vibration can be determined by using the expression for residual vibration of a second-order harmonic oscillator of frequency ω and damping ratio ζ. The vibration from a series of impulses is divided by the vibration from a single impulse to get the percentage vibration [13]:

$$V(\omega,\zeta) = e^{-\zeta\omega t_n}\sqrt{[C(\omega,\zeta)]^2 + [S(\omega,\zeta)]^2}\,, \tag{1}$$

where,

$$C(\omega,\zeta) = \sum_{i=1}^{n} A_i e^{\zeta\omega t_i}\cos\left(\omega\sqrt{1-\zeta^2}\,t_i\right), \tag{2}$$

and

$$S(\omega,\zeta) = \sum_{i=1}^{n} A_i e^{\zeta\omega t_i}\sin\left(\omega\sqrt{1-\zeta^2}\,t_i\right). \tag{3}$$

If $V(\omega,\zeta)$ is set equal to zero at the modeling parameters, (ω_m,ζ_m), then a shaper that satisfies the equation is called a Zero Vibration (ZV) shaper.

Singer and Seering developed a form of robust input shaping by setting the derivative with respect to the frequency of the residual vibration, Equation (1), equal to zero [12]. The resulting shaper is called a Zero Vibration and Derivative (ZVD) shaper. The improved robustness can be seen by plotting a shaper's sensitivity curve; its amplitude of vibration vs. normalized frequency. Figure 2 shows two such sensitivity curves for an undamped systems Notice that the ZVD Shaper results in low levels of vibration over a wider frequency range than the ZV Shaper.

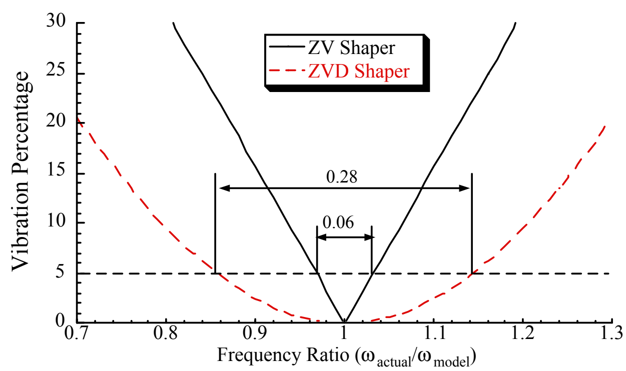

Fig. 2. Sensitivity Plot.

This increase in robustness comes at a price. The ZVD shaper has a duration of one period of vibration, while the duration of the ZV Shaper is one-half period of vibration. Given that the rise time of the command is lengthened by the duration of the shaper, ZVD commands are slightly longer than ZV commands. A quantitative robustness measure known as Insensitivity (I) is defined to be the range of frequencies for which the shaper keeps the residual vibration below some acceptable level. For a tolerable vibration of 5%, the ZV shaper has an I of 0.06, while the ZVD shaper has an I of 0.28. These suppression ranges on labeled on Figure 2.

There are other performance measures in addition to the Insensitivity that are used to quantify robustness to modeling error. The Total Insensitivity measurement accounts for robustness to both natural frequency and damping ratio [1]. Kozak et al developed measurements that combine the residual vibration and settling time [5]. These two papers in particular highlight the limitations of using Insensitivity as the sole measure of robustness and provide motivation for the development of the performance measures introduced here.

In addition to reducing residual vibration, command shapers can be used improve the transient performance of flexible systems. Of interest here are pre-computed, deflection-limiting commands [7-9] that limit the transient deflection during rest-to-rest moves of the floating oscillator model shown in Figure 3 and Specified Deflection, Zero Vibration (SD-ZV) command shapers [10] that limit the transient deflection to a specified percentage of the unshaped command for the position input system shown in Figure 4.

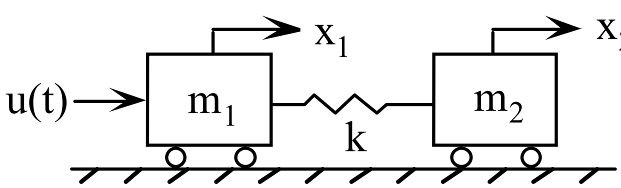

Fig. 3. Floating Oscillator Model.

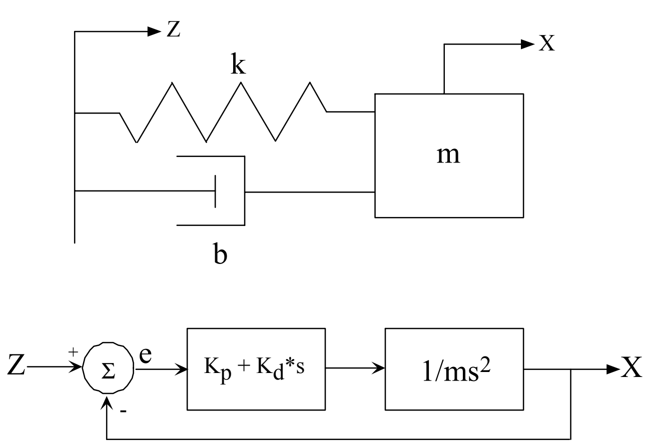

Fig. 4. Position Input System.

II. PRE-COMPUTED DEFLECTION-LIMITING COMMANDS

A. Residual Vibration

In traditional command shaping, the impulse amplitudes and time locations are only functions of the natural frequency and damping ratio [12]. When creating Analytic Deflection-Limiting commands [8], the impulse amplitudes and time locations are functions of the natural frequency, damping ratio, move distance and deflection limit. This complicates the determination of the robustness of these commands. Analytic Deflection-Limiting commands are a modification of Analytic On-Off commands [13] that create input profiles for vibration-free rest-to-rest moves without a constraint on transient deflection.

Figure 5 shows the 5% insensitivity of analytic commands as a function of move distance for the Analytic On-Off command and the Analytic Deflection-Limiting command whose deflection has been limited to 0.3 (60% of the On-Off command). Clearly there are move distances that are more robust than others and the robustness for a specific move distance varies with the deflection limit. For example, consider a move distance of 10 units. The insensitivity of the On-Off command is over six times that of the deflection-limiting command. However, for a move distance of 20 units, the insensitivity of the deflection-limited command is over six times that of the On-Off command.

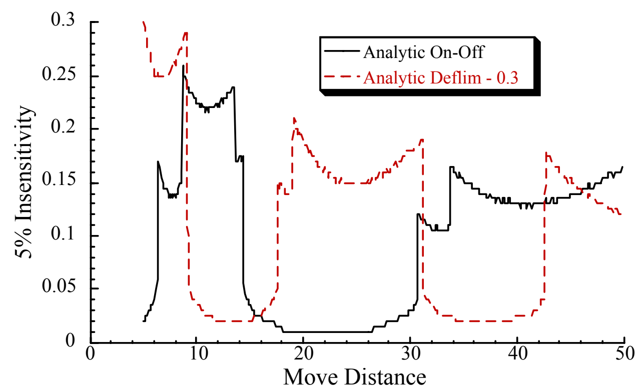

Fig. 5. Residual Vibration Robustness.

Equation 1 gives the ratio of the vibration resulting from a shaped command (series of impulses) to that of the unshaped command (single impulse). However, for rest-to-rest moves comparing the residual vibration to that of a single impulse does not take into account the effect of move distance. The unshaped command should move the system the required distance, something a single impulse cannot do. For the benchmark system shown in Figure 3, a bang-bang command is used as the unshaped, baseline command to which the new commands will be compared.

There is one difficulty with using a bang-bang command as the baseline command. Unlike a single impulse, which will always induce a non-zero residual vibration, there exists some move distances for which a bang-bang command will result in zero vibration [14]. The switch time for a bang-bang command can be found by

$$t_2 = \sqrt{x_d / \alpha} \tag{5}$$

where α is the force-to-mass ratio. A bang-bang command will induce zero vibration when

$$t_2 = nT \tag{6}$$

where n is a positive integer and T is the period of vibration. Combining (5) and (6) gives the move distances for which the vibration will be zero

$$x_d = \alpha(nT)^2 \tag{7}$$

For the benchmark system with α=0.5 and T=4.425 seconds (m_1=m_2=k=1), the first three move distances for which a bang-bang command will have zero vibration are 9.868, 39.472 and 88.811. If the residual vibration percentage for rest-to-rest commands is defined as the vibration of a shaped command to that of a bang-bang command, then these move distances will yield undefined percent vibration given the zero residual vibration from the bang-bang command. Figure 6 shows the position response resulting from a bang-bang command for a move distance of 5, 10 and 15 units. There is considerable residual vibration for the moves of 5 and 15 units. There is very little vibration for the 10-unit move given its close proximity to the first zero vibration move distance of 9.868. However, Figure 7 shows that even for the case when the residual vibration is nearly zero, there is still a large amount a transient deflection. An Analytic ON-OFF command would produce a deflection of only 0.5 units.

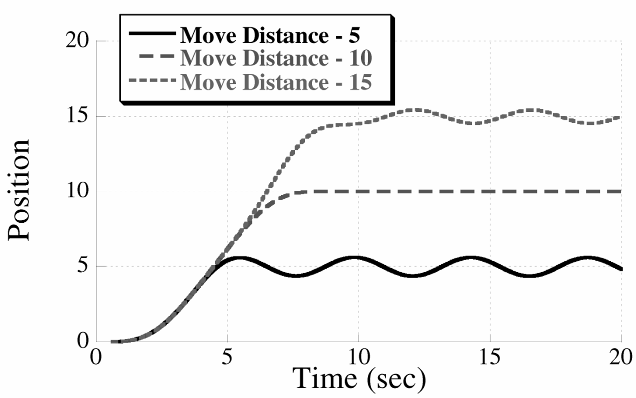

Fig. 6. Position Responses for Bang-Bang Commands.

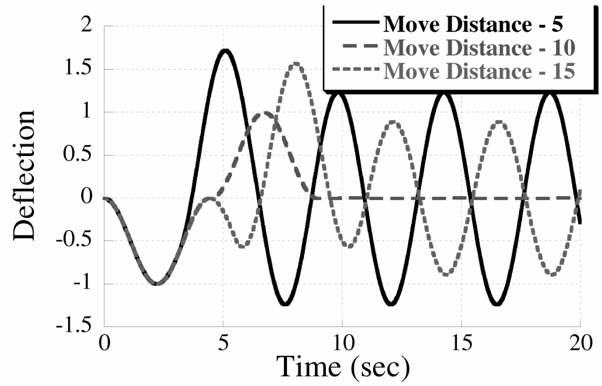

Fig. 7. Deflection Responses for Bang-Bang Commands.

A plot of the robustness measure using the bang-bang as the unshaped command, as opposed to a single impulse, is shown in Figure 8 for a move distance of 5 units. For this move distance, Analytic Deflection-Limiting command whose deflection has been limited to 0.3 is much more robust than that of the Analytic ON-OFF command. However, for the move distance of 15 units shown in Figure 9, the opposite is true. The relative levels of robustness when using a bang-bang input as the standard unshaped command follow the trends shown in Figure 5. For example, for a move distance of 5 units, the deflection-limited command has traditional residual vibration Insensitivity approximately 15 times larger than the zero-vibration command. When using the bang-bang command as the unshaped command, the deflection-limited command has traditional residual vibration Insensitivity approximately 4 times larger than the zero-vibration command. Similar trends exist across all move distances and deflection limits. Therefore, while using the traditional residual vibration robustness measure does not take into account the fact that the move distance is a constraint as well as the natural frequency, its results follow the same trends as the robustness measure that does account for move distance and it avoids cases where the robustness would be undefined.

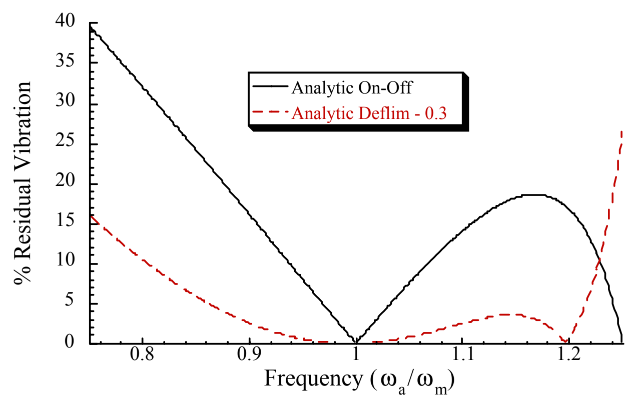

Fig. 8. Residual Vibration (Compared to Bang-Bang) for Move Distance = 5.

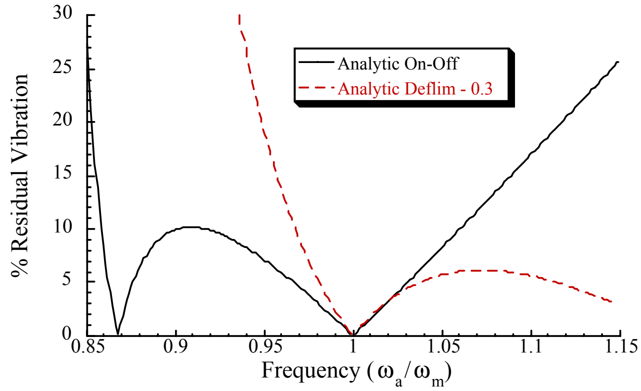

Fig. 9. Residual Vibration (Compared to Bang-Bang) for Move Distance = 15.

B. Transient Deflection

It may not be entirely appropriate to use residual vibration reduction as the primary measure of robustness for commands whose major goal is limiting transient deflection. At the very least, percent residual vibration does not tell the entire story. One alternative is to use the percent by which the deflection exceeds the desired deflection limit. That is, we should use a measure of how modeling errors affect the transient performance.

Figure 10 shows the deflection in the presence of modeling errors for a move distance of 5 units and a deflection limit of 0.3. When the actual frequency is higher than the model frequency, then the deflection remains below the limit. When the actual frequency is lower than the model frequency, then the deflection exceeds the limit. For this particular case, underestimation of the natural frequency does not cause the system to exceed the deflection limit.

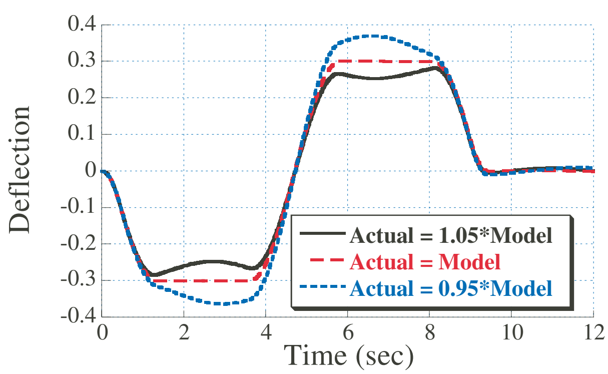

Fig. 10. Deflection Response of 60% Deflection-Limited Command, Move Distance – 5 Units.

However, this trend does not hold for all cases. Figure 11 shows the deflection in the presence of modeling errors for a move distance of 10 units and a deflection limit of 0.3. Here, both overestimation and underestimation of the natural frequency will result in commands that exceed the deflection limit. For all move distances, overestimating the natural frequency by as little as 2% will lead to commands that exceed the deflection limit by 10%. Underestimating the natural frequency has such dramatic effects on transient performance in only the 10-unit move, while such errors have little effect on the 5-unit move. Recall that on Figure 5 the 5-unit move was much more robust, in a residual vibration sense, than the 10-unit move. Figure 12 shows a Transient Deflection Sensitivity Plots for various move distances with a deflection limit of 0.3. These plots elucidate how modeling errors effect the transient deflection. Robust commands (in the residual vibration sense) such as those for move distances of 5 or 20 display a one-sided transient deflection robustness. While the residual vibration measure does not take into account the deflection limits, it does give a qualitative representation of the relative transient deflection robustness of different commands. However, it should be noted the transient deflection robustness is much smaller than the residual vibration (both relative to a single impulse and the Bang-Bang command). So while the Insensitivity measurement may be used to compare commands, it is not a good indicator of the overall robustness of the command.

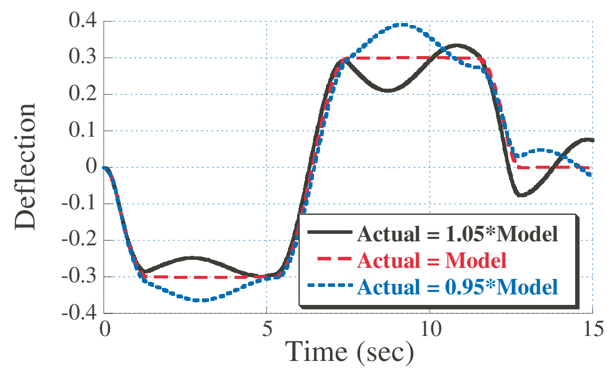

Fig. 11. Deflection Response of 60% Deflection-Limited Command, Move Distance - 10 Units.

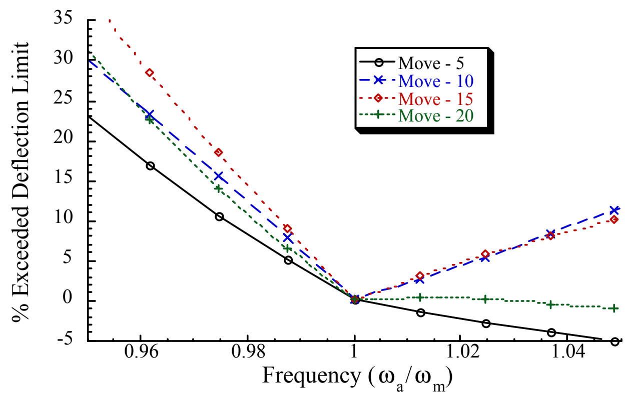

Fig. 12. Transient Deflection Sensitivity.

III. Real-Time Trajectory Tracking

Specified-Deflection, Zero-Vibration command shapers are designed to both eliminate residual vibration and reduce the transient deflection of the mass-spring-damper system with position input (mass under PD Control) shown in Figure 4. These Specified-Deflection shapers limit the transient deflection to a predetermined ratio of the baseline deflection resulting from an arbitrary reference command [10]. Unlike the commands presented in the previous sections, SD-ZV commands can be implemented in real-time. Because they are designed independent of the unshaped command, they cannot limit the deflection to a specific value. They can only limit the deflection to a certain percentage of the unshaped command.

A. *Residual Vibration*

Figures 13 & 14 show the sensitivity plots of SD-ZV shapers designed for a deflection ratio of 0.4 and 0.3, respectively. For the 0.4 deflection ratio (40% of the unshaped command's deflection) case shown in Figure 13, the Insensitivities are centered about the modeling frequency. While for the 0.3 deflection ration case in Figure 14, the Insensitivities are asymmetric about the modeling frequency. The asymmetry increases as the damping ratio increases. In general, the more the deflection is limited and the more damping in the system, the more the robustness plot will be asymmetric.

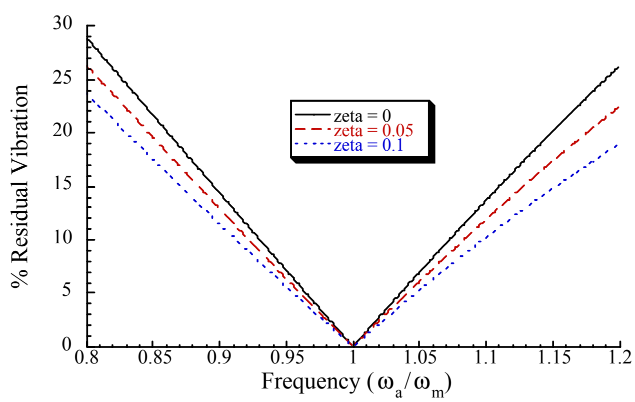

Fig. 13. SD-ZV Sensitivity Plots (SD = 0.4).

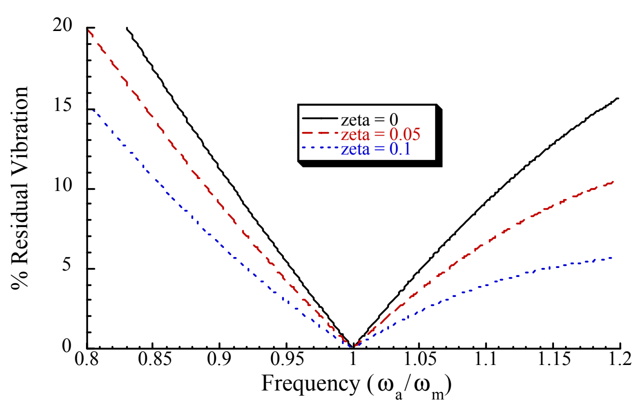

Fig. 14. SD-ZV Sensitivity Plots (SD = 0.3).

B. *Transient Deflection*

To investigate the effect the asymmetric sensitivity plot has on transient performance, SD-ZV shapers were developed for a system with f=1 Hz and ζ=0.1. Simulations were conducted using a trapezoidal position command consisting of a 2-unit 1.5 second rise, a 2 second dwell, and a 2-unit 1.5 second return. The deflection profiles in the presence of modeling error for deflection ratios of 0.4 and 0.3 are shown in Figures 15 & 16, respectively. Modeling error affects the transient deflection for the SD-ZV commands in a way similar to the trends found for the pre-computed deflection-limiting commands: underestimation of the natural frequency is leads to much higher transient deflection than overestimating the natural frequency. For both the SD-0.4 and SD-0.3 commands, underestimation by 10% leads to deflections that exceed the limit by nearly 25%. The residual vibration resulting from the modeling error is less than 10%. The transient deflection is much more sensitive to modeling errors than the residual vibration.

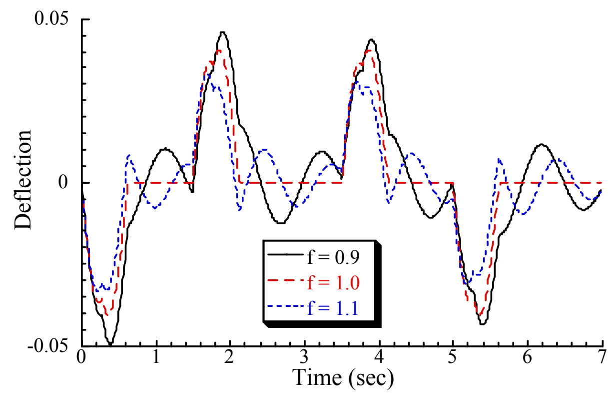

Fig. 15. Deflection Profile for SD = 0.4.

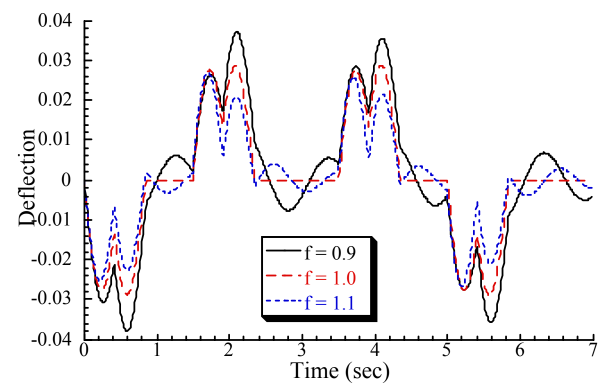

Fig. 16. Deflection Profile for SD = 0.3.

Figure 17 shows the transient deflection sensitivity for the SD-0.4 and SD-0.3 (ζ=0.1) commands. It is clear that any underestimation of the natural frequency results in commands that exceed the deflection limit while an overestimation of the natural frequency by 40-60% are easily handled. However, the residual vibration resulting from that level of modeling error would greatly exceed the toleration limit.

Figure 18 shows a sensitivity plot for errors in the modeled damping ratio. Here, an underestimation of damping ratio (ζ greater than 0.1 on the x-axis) is well tolerated while an overestimation of the damping ratio (ζ less than 0.1 on the x-axis) leads to commands that exceed the deflection limit. Overall, the transient deflection is much more sensitive to modeling errors than the residual vibration.

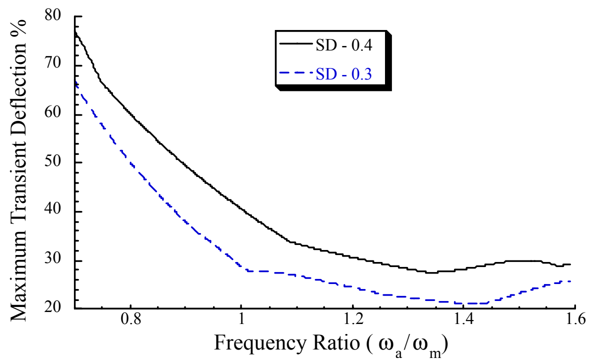

Fig. 17. Transient Deflection Sensitivity Plot.

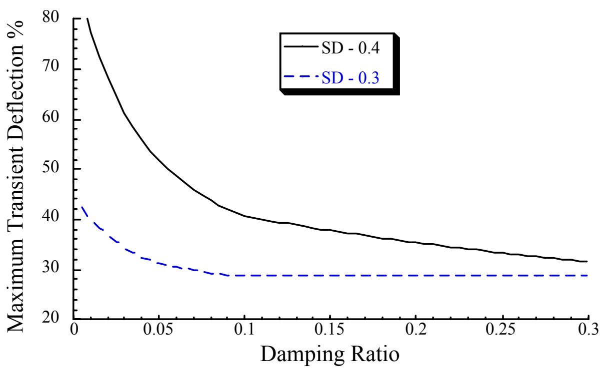

Fig. 18. Transient Deflection Sensitivity Plot for Damping Ratio Error.

IV. CONCLUSION

The transient deflection for pre-computed, rest-to-rest deflection-limiting commands and real-time specified-deflection commands are extremely sensitive to modeling errors. Errors that lead to small amounts in residual vibration often have transient deflections that greatly exceed the design specifications. Traditional measures for residual vibration robustness do not take transient deflection into consideration. However, the residual vibration measures do give a qualitative comparison between the transient deflection robustness of different commands. That is, while no clear conclusions about the magnitude of the transient robustness of a command can be made by looking at the magnitude of the residual vibration robustness of that command, a command with a higher level of residual vibration robustness than another tends to have higher transient deflection robustness also. To fully quantify the transient deflection robustness, transient deflection sensitivity plots (with respect to natural frequency and damping ratio) are used. In general, overestimation of the natural frequency or underestimation damping ratio will lead to much higher transient deflections than underestimation of the natural frequency or overestimation of the damping ratio.

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