2008 American Control Conference
Westin Seattle Hotel, Seattle, Washington, USA
June 11-13, 2008

Path Planning for Cooperative Time-Optimal Information Collection

Andrew T. Klesh, Pierre T. Kabamba and Anouck R. Girard

Abstract— **Motivated by cooperative exploration missions, this paper considers constant velocity, level flight path planning for Unmanned Air Vehicles (UAVs) equipped with range limited, omni-directional sensors. These active energy-based sensors collect information about objects of interest at rates that depend on the range to the objects according to Shannon's channel capacity equation, where the signal-to-noise ratio is governed by the radar equation. The mission of the UAVs is to travel through a given area and collect a specified amount of information about each object of interest while minimizing the total mission time. This information can then be used to classify the objects of interest. An optimal path planning problem is formulated where the states are the Cartesian coordinates of the UAVs and the amounts of information collected about each object of interest, the control inputs are the UAV heading angles, the objective function is the total mission time, and the boundary conditions are subject to inequality constraints that reflect the requirements of information collection. Necessary conditions for optimality are given, whose solutions yield extremal paths, and whose utilization highlights analytical properties of these extremal paths. The problem exhibits several limiting regimes, including the so-called Watchtower and the Multi-Vehicle Traveling Salesman Problem. These results are illustrated on several time-optimal cooperative exploration scenarios.**

I. INTRODUCTION

A. Overview

This paper is devoted to the problem of planning the paths of multiple aircraft for cooperative exploration of a given area. By exploration we mean collecting information about objects of interest with known locations, where information is understood in the classical sense of Shannon [1] as a "selection from a set." Although the information collected is to be used to classify the objects of interest (e.g., friend or foe, interesting or uninteresting, etc.), this paper focuses on information collection rather than classification. Information is collected by active onboard omni-directional sensors (e.g., radar, sonar) that read energy signals reflected off the objects of interest, implying a signal-to-noise ratio that decays as the reciprocal of the fourth power of the range.

The key idea of this work is to recognize and exploit the similarity between communication and exploration. Specifically, exploration can be viewed as a communication process where the object of interest is the transmitter, the sensor is the receiver, the sensing process is the noisy communication channel, and the sensed signal carries information about the object of interest. Consequently, the maximum rate at which the sensor can collect information about the object of interest is in fact the capacity [1] and depends on the signal-to-noise ratio of the channel through Shannon's equation.

Hence, the problem of exploration features the coupling between aircraft kinematics and information collection, which occurs through the signal-to-noise ratio of the sensor. Accounting for this coupling in the design of optimal paths for exploration is the main conceptual contribution of this paper.

B. Motivation

Unmanned Air Vehicles (UAVs) are increasingly used for dirty, dull or dangerous missions [2]. The most common use of UAVs is the collection of data for Intelligence, Surveillance and Reconnaissance missions. We are particularly interested in missions where multiple, possibly heterogeneous, cooperative UAVs are tasked with exploring a given area. An example of such a mission involves Mars exploration where objects of interest have been located by the Mars Global Surveyor. The collection of a specified amount of information is performed using UAVs, which requires planning paths. Moreover, it is often the case that gathering information quickly is of paramount importance (e.g., for tactical reasons in military operations or to avoid inclement weather on Mars). This motivates the time-optimal path planning problem formulated and solved in this paper.

C. Literature Review

A large body of research has been published in recent years about motion control and collaborative control of networked autonomous vehicles. Although an exhaustive overview of the state of the art is beyond the scope of this paper, a brief review of the most relevant literature is as follows.

Much of the collaborative control work focuses on formation maneuvering of multiple vehicles, such as satellite interferometry, gradient/environmental estimation using vehicle formations, etc. Moving vehicles as a formation simplifies path planning as a single path can be specified for the group [3].

Many methods exist for solving the basic trajectory-planning problem [4]. However, not all of them solve the problem in its full generality. For instance, some methods require the workspace to be two-dimensional and the obstacles, if any, to be polygonal. Despite many external differences, the methods are based on few different general approaches: roadmap [4], [5], [6], cell decomposition [7], [8], [9], [5], [10], potential field [11], [12], [13] and probabilistic [14],

A. Klesh is a PhD Candidate, Aerospace Engineering, University of Michigan, 1320 Beal Ave, Ann Arbor, MI USA aklesh@umich.edu
P. Kabamba is a Professor, Aerospace Engineering, University of Michigan, 1320 Beal Ave, Ann Arbor, MI USA kabamba@umich.edu
A. Girard is an Assistant Professor, Aerospace Engineering, University of Michigan, 1320 Beal Ave, Ann Arbor, MI USA anouck@umich.edu

[15]. Optimal control approaches have also been studied in [16] and [17].

Although the current literature discusses various methods of planning optimal paths for multiple UAVs, no approach accounts for the coupling between UAV kinematics and information collection through the signal-to-noise ratio of the sensors. The current paper addresses this issue.

D. Original Contributions

This paper presents an integrated model of the aircraft kinematics and information collection, applicable to cooperative exploration, with the following original features. First, the rate at which a sensor collects information about an object of interest is specified by Shannon's channel capacity equation [1], which depends on the sensor's signal-to-noise ratio. Second, the signal-to-noise ratio decays as the reciprocal of the fourth power of the range to the object of interest, according to the radar equation [18]. This paper accounts for the coupling between aircraft kinematics and information collection.

Based on the integrated model, the problem of cooperative exploration for UAVs is formulated as an optimal path planning problem where the states are the Cartesian coordinates of the UAVs and the amounts of information collected about each object of interest, the control inputs are the UAV heading angles, the objective function is the total mission time, and the boundary conditions are subject to inequality constraints that reflect the requirements of information collection. The present paper studies this optimization problem and provides the following original contributions:

- The necessary conditions for optimality are derived using the integrated model.
- A numerical method is developed to generate optimal paths and infer their qualitative properties.
- The necessary conditions are used to prove analytically properties of the optimal paths.
- Several limiting regimes are identified for optimal paths.

E. Paper Outline

The remainder of the paper is as follows. In Section II, the integrated model is presented. In Section III, the optimization problem is formulated to minimize the total mission time of the UAVs. In Section IV, the model and problem are non-dimensionalized and in Section V the necessary conditions for optimality are derived. Section VI presents the numerical procedure used to generate optimal paths. Section VII presents properties of the optimal paths and identifies limiting cases. The results are illustrated through examples in Section VIII, while conclusions and future work are discussed in Section IX.

II. Modeling

The model consists of two parts: the aircraft kinematic model and the information collection model. In this paper, uppercase indicates unscaled while lowercase indicates scaled parameters.

A. The Aircraft Kinematic Model

The aircraft kinematic model is based upon the unicycle vehicle model [19]:

$$X'_i = V\cos\psi_i, 1 \le i \le n, \tag{1}$$

$$Y'_i = V\sin\psi_i, 1 \le i \le n, \tag{2}$$

where X_i and Y_i are the Cartesian coordinates of the ith aircraft, prime denotes time derivative, V is the velocity of the aircraft, ψ_i is the heading of the ith aircraft and n is the number of aircraft. For simplicity, V is assumed to be the same for all aircraft.

B. The Information Collection Model

We seek to explore a given area, by which we mean to collect a specified amount of information about each of m objects of interest, at known locations in the area. Without loss of generality, assume that the required amount of information is one bit for each object. To collect information, we use onboard active, energy-based sensors, e.g., radar.

The key idea of our work is to recognize and exploit the similarity between communication and exploration. According to [1], the maximum rate at which information can be transmitted over a noisy communication channel (i.e., the channel capacity) is:

$$\dot{I} = W\log_2(1 + \text{SNR}), \tag{3}$$

where W is the channel bandwidth and SNR is the signal-to-noise ratio.

Moreover, according to [18], a radar sensor located at Cartesian coordinates (X, Y) and observing an object at Cartesian coordinates (A, B) will provide a reading with signal-to-noise ratio of the form:

$$\text{SNR} = \frac{K^4}{((X-A)^2 + (Y-B)^2)^2}, \tag{4}$$

where the parameter K depends on the object.

Combining (3) and (4), the information collection model is as follows. Assuming that all explorers contribute the rates of information collection additively, let I_j denote the amount of information that the n aircraft have cooperatively collected about the jth object of interest, $1 \le j \le m$. Then,

$$\dot{I}_j = W\sum_{i=1}^{n}\log_2\left(1 + \frac{K_j^4}{((X_i-A_j)^2 + (Y_i-B_j)^2)^2}\right), 1 \le j \le m, \tag{5}$$

where we assume that all radar sensing processes, viewed as communication channels, have the same bandwidth, and the parameters K_j depend on the jth object of interest, which is located at Cartesian coordinates (A_j, B_j).

III. Problem Formulation

The problem treated in this paper is motivated by the goal of minimizing the total mission time required for n UAVs to collect a specified amount of information about m objects of interest in a given area. The UAVs are equipped with active energy-based sensors.

The objects of interest are assumed isotropic in that the rate of information collection is independent of azimuth. We assume no redundant information is useful, i.e., only one bit of information about each object of interest is needed to accomplish the mission. Since the information is collected at the most optimistic rate possible, (5) becomes a lower bound on any rate of information collection. A conservative upper bound is found in [19]. Future work will consider reducing this upper bound based on our new information collection model.

The aircraft begin at an initial location with a free heading and have the objective of collecting at least one bit of information about each object in the area. Active sensors, based upon (5), are employed by each aircraft to collect information.

A. Dynamic Optimization Problem

The dynamic optimization problem is motivated by the requirement to minimize, with respect to the time-history of the heading angles, the total mission time, i.e.,

$$\min_{\psi_i(\cdot)} t_f, \tag{6}$$

$$\tag{7}$$

subject to (1)-(5) and boundary conditions:

$$X_i(0) \text{ given}, 1 \le i \le n, \tag{8}$$
$$Y_i(0) \text{ given}, 1 \le i \le n, \tag{9}$$
$$I_j(0) = 0, 1 \le j \le m, \tag{10}$$
$$X_i(t_f) \text{ free}, 1 \le i \le n, \tag{11}$$
$$Y_i(t_f) \text{ free}, 1 \le i \le n, \tag{12}$$
$$I_j(t_f) \ge 1, 1 \le j \le m. \tag{13}$$

IV. SCALING

If we assume that l_c and t_c are a characteristic length and time, respectively, we can rewrite (1) - (5) in nondimensional form. We begin by substituting $X_i = l_c x_i$, $Y_i = l_c y_i$ and $t = t_c \tau$ where x_i, y_i, and τ are nondimensional. Equations (1) - (5) become:

$$\dot{x}_i = v \cos\psi_i, \tag{14}$$
$$\dot{y}_i = v \sin\psi_i, \tag{15}$$
$$\dot{I}_j = \sum_{i=1}^{n} w_i \log_2(1 + \frac{k_j^4}{((x_i - a_j)^2 + (y_i - b_j)^2)^2}), 1 \le j \le m, \tag{16}$$

where the dot denotes derivatives with respect to nondimensional time, $v = \frac{t_c}{l_c} V$, $w = W t_c$, $k = \frac{K}{l_c}$, $a_j = \frac{A_j}{l_c}$ and $b_j = \frac{B_j}{l_c}$.

This nondimensional form allows for the scaling of solutions by changing the characteristic parameters.

V. OPTIMAL PATH PLANNING

In this section, we derive the necessary conditions for optimality, adapted from [20]. With states $[I_j, x_i, y_i]^T, 1 \le i \le n, 1 \le j \le m$ and control inputs $\psi_i, 1 \le i \le n$, the Hamiltonian is:

$$H = \sum_{j=1}^{m} \lambda_{I_j} \sum_{i=1}^{n} w_i \log_2(1 + \frac{k_j^4}{[(x_i - a_j)^2 + (y_i - b_j)^2]^2})$$
$$+ \sum_{i=1}^{n} \lambda_{x_i} v \cos(\psi_i) + \sum_{i=1}^{n} \lambda_{y_i} v \sin(\psi_i) + 1, \tag{17}$$

where λ_{x_i}, $\lambda_{y_i}, 1 \le i \le n$ and $\lambda_{I_j}, 1 \le j \le m$ are costate variables.

In this problem formulation, we have no control constraints.

The state equations, derived from (17), are:

$$\dot{I}_j = \sum_{i=1}^{n} w_i \log_2(1 + \frac{k_j^4}{((x_i - a_j)^2 + (y_i - b_j)^2)^2}), 1 \le j \le m, \tag{18}$$

$$\dot{x}_i = v \cos(\psi_i), 1 \le i \le n, \tag{19}$$
$$\dot{y}_i = v \sin(\psi_i), 1 \le i \le n. \tag{20}$$

The costate equations are:

$$\dot{\lambda}_{I_j} = 0, 1 \le j \le m, \tag{21}$$
$$\dot{\lambda}_{x_i} = \sum_{j=1}^{m} \frac{4k_j^4 w_i (x_i - a_j)\lambda_{I_j}}{((x_i - a_j)^2 + (y_i - b_j)^2)^3 \Delta_j}, 1 \le i \le n, \tag{22}$$
$$\dot{\lambda}_{y_i} = \sum_{j=1}^{m} \frac{4k_j^4 w_i (y_i - b_j)\lambda_{I_j}}{((x_i - a_j)^2 + (y_i - b_j)^2)^3 \Delta_j}, 1 \le i \le n, \tag{23}$$

where $\Delta_j = (1 + \frac{k_j^4}{((x_i - a_j)^2 + (y_i - b_j)^2)^2}), 1 \le j \le m$.

The first-order optimality conditions are:

$$0 = v\lambda_{y_i} \cos(\psi_i) - v\lambda_{x_i} \sin(\psi_i), 1 \le i \le n. \tag{24}$$

The boundary conditions for this problem are:

$$I_j(0) = 0, 1 \le j \le m \tag{25}$$
$$I_j(t_f) \ge 1, 1 \le j \le m, \tag{26}$$
$$\lambda_{I_j}(t_f) = \text{free if } I_j = 1, 1 \le j \le m, \tag{27}$$
$$\lambda_{I_j}(t_f) = 0 \text{ if } I_j > 1, 1 \le j \le m, \tag{28}$$
$$\lambda_{x_i}(t_f) = 0, 1 \le i \le n, \tag{29}$$
$$\lambda_{y_i}(t_f) = 0, 1 \le i \le n. \tag{30}$$

We will refer to the flight paths that satisfy the first order necessary conditions (18)-(24) as *extremal paths*. Further simplification of these necessary conditions is provided in Appendix A.

VI. THE DISCRETIZATION PROCEDURE

To obtain numerical approximations of optimal paths, we discretize the problem as follows. For a chosen integer $p \ge 1$, we subdivide the interval $[t_o, t_f]$ into p subintervals $[t_o, t_1], [t_1, t_2], ..., [t_{p-1}, t_f]$ of equal duration. In each subinterval we assume that the control inputs are constant, i.e., $(\psi_i(t)) = (\psi_{i_g}), t \in [t_g, t_{g+1}]$, where the parameters $\psi_{i_g}, 0 \le g \le p-1$, are unknown.

We treat the parameters ψ_{i_g}, $0 \le g \le p-1$, and t_f as inputs to a nonlinear optimization problem. As an initial choice, in all subintervals we choose $\psi_{i_g} = 0$ and $t_f = t_o + T_M$ where T_M is a maximum duration allowed. Constraints upon this problem are imposed from the boundary conditions (25)-(30). From (6), the objective function is the total mission time. We then numerically solve for optimal flight paths using the MATLAB® Optimization Toolbox function *fmincon* and the ordinary differential equation solver *ode45*. We will call this strategy the discretization method.

VII. Analytical Properties of Time-Optimal Flight Paths

A set of objects of interest is considered isolated if there is never a location where one aircraft can simultaneously collect information about several objects at a high rate. Objects of interest are considered visible when an aircraft can remain at a distance from the object and still receive information about it at a high rate. In terms of the non-dimensional equations (14) - (16), visibility and isolation are quantified by the parameters w and k_j. Specifically, the non-dimensional distance r_j can be defined as

$$1 = \log \frac{w_i k_j^4}{r_j^4}, \tag{31}$$

where after solving for r_j,

$$r_j = \frac{k_j}{\sqrt[4]{2^{\frac{1}{w_i}} - 1}}, \tag{32}$$

and is such that a radar located at distance r_j from the jth object of interest collects information about that object at a rate of one bit per non-dimensional unit of time. The jth object is visible if the distance r_j is large. The set of objects of interest is isolated if the m disks, centered at Cartesian coordinates (a_j, b_j) and with radii $r_j, 1 \leq j \leq m$, respectively, do not intersect.

We can use the necessary conditions for optimality (18)-(24) to prove the following properties of extremals.

Proposition 1: *If the objects of interest are isolated, then the optimal flight paths consist of sequences of straight lines (far from the objects of interest) connected by short turns (near the objects of interest).*

Proof: See Appendix B.

Corollary 1: *If in addition to being isolated, the objects of interest are poorly visible, then the problem becomes a multi-vehicle traveling salesman problem (MTSP) [21].*

Proof: See Appendix B.

Proposition 2: *When the visibility of all the objects of interest approaches infinity, $t_f \to 0$ and the lengths of paths traveled by the UAVs approach zero.*

Proof: See Appendix C.

Proposition 1 implies that, in the isolated case, each aircraft essentially "visits" a sequence of objects of interest, flying straight paths between them. This is the traveling salesman problem (TSP).

We call the situation described by Proposition 2 the Watchtower case.

VIII. Examples

Figure 1 presents an optimal flight path for a single UAV exploring an area with three objects of interest. Each object is designated by an x on the figure. The small circles have radii r_j defined in Equation (32). In this example, the UAV does not fly over the objects of interest, but only approaches each of them and then turns away towards the next. Thus, the total optimal path consists of straight flights and short turns.

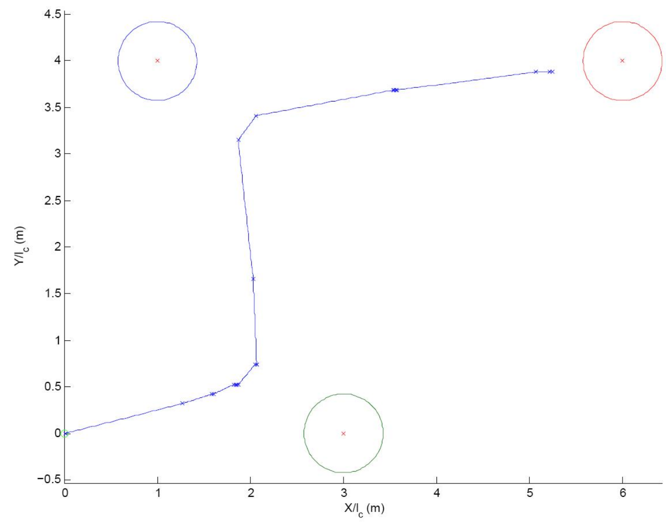

Fig. 1. Optimal flight path for a single UAV

In the TSP limiting case, all of the objects of interest have a very low visibility. Figure 2 presents the same objects of interest as in Figure 1, but each object is barely visible. Here the UAVs must virtually reach the target before enough information is collected and similarities to typical TSPs exist.

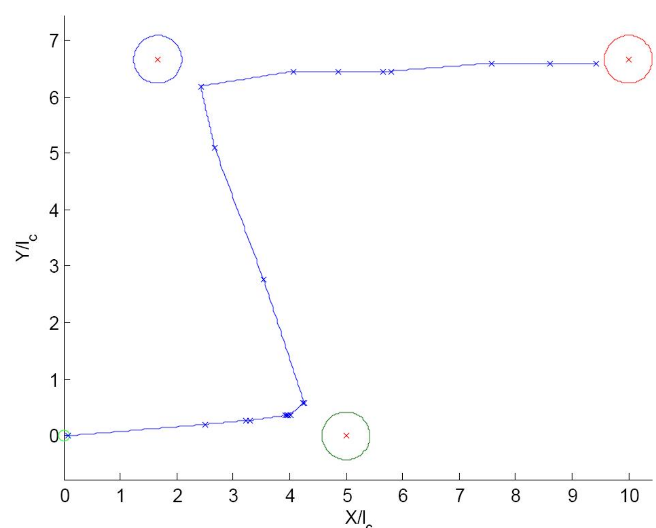

Fig. 2. TSP limiting case - Optimal flight path for a single UAV

Figure 3 demonstrates a hybrid example: one object of interest is relatively visible while two others are barely visible. The resulting optimal path is a gentle curve near the visible object and a typical TSP touring path towards the other two objects.

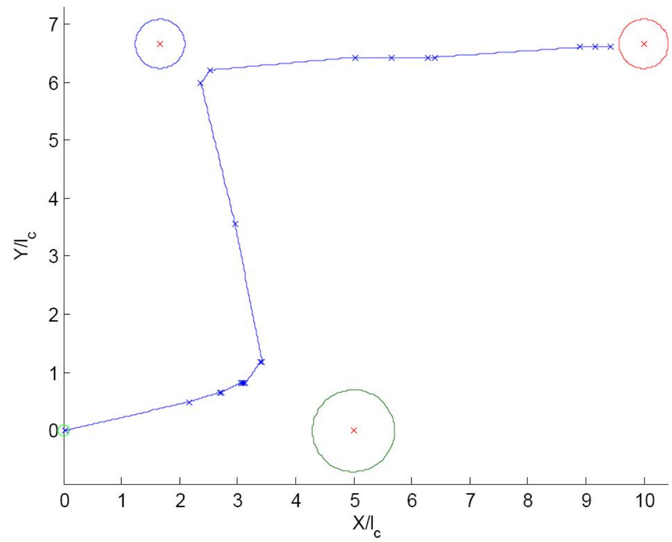

Fig. 3. Optimal flight path for a single UAV with varying target visibility

Figure 4 presents the same objects of interest as in Figure 1, but with two UAVs flying a cooperative exploration mission. Here the UAVs do not approach the objects as close as in Figure 1 and share the information collected.

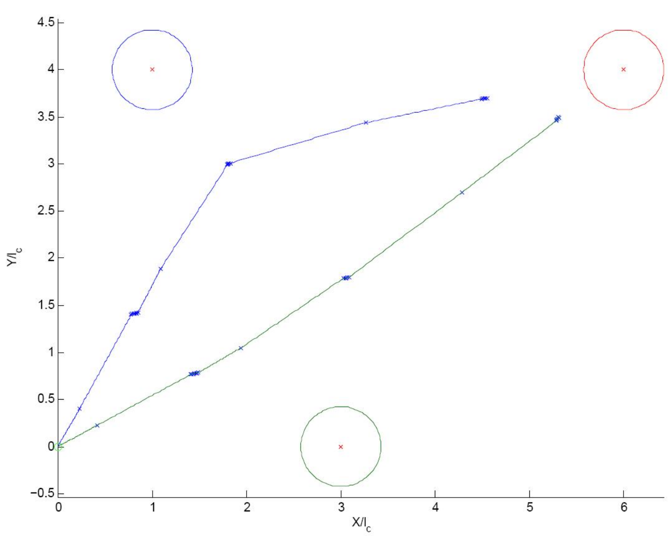

Fig. 4. Optimal flight path for cooperative UAVs

IX. Conclusions

This paper has presented a new information-based formulation for optimal exploration. The problem of optimal path planning is phrased in terms of Shannon's communication theory, the radar equation and aircraft kinematics. This formulation exploits the similarity between communication and exploration. We have presented necessary conditions for optimality with multiple objects of interest and multiple aircraft. These necessary conditions have been solved using a discretization technique, which highlight the qualitative nature of extremals.

In future work, non-isotropic objects of interest and time-varying dynamic situations will be considered as well as larger numbers of targets and aircraft. Non-heterogenous aircraft will also be discussed. In a time-varying dynamic situation, the objects of interest may move and attempt to evade identification. The framework considered may be extended to these situations.

Acknowledgements

This work was supported in part by the Air Force under grant number FA 8650-07-2-3744.

Appendix A: Simplification of the Necessary Conditions

For analysis, we can simplify the necessary conditions by expanding $\log(\cdot)$ with a Taylor series if we assume that the SNR are all small. This simplified Hamiltonian expression becomes:

$$H = \sum_{j=1}^{m} \lambda_{I_j} \sum_{i=1}^{n} w_i 1 + \frac{k_j^4}{[(x_i - a_{x_j})^2 + (y_i - a_{y_j})^2]^2} + \sum_{i=1}^{n} \lambda_{x_i} v \cos(\psi_i) + \sum_{i=1}^{n} \lambda_{y_i} v \sin(\psi_i) + 1 \tag{33}$$

The simplified conditions become:

$$0 = v\lambda_{y_i} \cos(\psi_i) - v\lambda_{x_i} \sin(\psi_i), 1 \le i \le n, \tag{34}$$

$$\dot{I}_j = \sum_{i=1}^{n} \frac{w_i k_j^4}{((x_i - a_j)^2 + (y_i - b_j)^2)^2}, 1 \le j \le m, \tag{35}$$

$$\dot{x_i} = v\cos(\psi_i), 1 \le i \le n, \tag{36}$$

$$\dot{y_i} = v\sin(\psi_i), 1 \le i \le n, \tag{37}$$

$$\dot{\lambda_{I_j}} = 0, 1 \le j \le m, \tag{38}$$

$$\dot{\lambda_{x_i}} = \sum_{j=1}^{m} \frac{4k_j^4 w_i (x_i - a_j)\lambda_{I_j}}{((x_i - a_j)^2 + (y_i - b_j)^2)^3}, 1 \le i \le n,$$

$$\dot{\lambda_{y_i}} = \sum_{j=1}^{m} \frac{4k_j^4 w_i (y_i - b_j)\lambda_{I_j}}{((x_i - a_j)^2 + (y_i - b_j)^2)^3}, 1 \le i \le n, \tag{39}$$

Appendix B: Proof of Proposition 1 and Corollary 1

B.1 Proof of Proposition 1

Assume that the objects of interest are isolated, i.e., every location in the area to be explored is either far from all the objects of interest or close to only one of them.

Whenever a UAV is far from all objects of interest, the right hand sides of (22) and (23) are negligible, implying that the costates correponding to the Cartesian coordinates of that UAV are approximately constant. Then (24) implies that the heading angle of that UAV is also approximately constant, implying that its flight path approximates a straight line.

When a UAV approaches one of the objects of interest, the above argument no longer applies and the UAV may turn. However, the turn will be relatively short because it is predicated upon closeness to the object of interest. Once the UAV leaves the vicinity of that object, it resumes an approximately straight flight, as per the above argument.

B.2 Proof of Corollary 1

In the TSP case, the information rate of each object approaches zero, through a low w or k or high separation distance between the object of interest and the aircraft. If we assume that $\frac{wk_j^4}{((x_i-a_j)^2+(y_i-b_j)^2)^2} = 0$, then λ_{x_i} and λ_{y_i} are constant and equal to zero as is required by their final conditions. The optimality condition relates the ratio of λ_{x_i} and λ_{y_i} to the heading angle, ψ_i. Thus, the heading angle remains constant and the aircraft flies straight.

It is optimal in this case for the aircraft to visit each object of interest individually before switching to another target.

APPENDIX C: PROOF OF PROPOSITION 2

In the Watchtower case, each object's information rate approaches infinity, either through a high bandwidth, w, or gain, k, or low separation distance between the object of interest and the aircraft. We assume that the denominator of (5) is never zero, that is, the aircraft is always away from any object of interest. As $k \to \infty$, the right hand side of (5) also approaches infinity. For any final time of flight, no matter how short, a k_j can always be found that is large enough that, within the alloted time, enough information can be collected to satisfy the boundary conditions. This means that the aircraft do not need to move to collect information in the Watchtower case. This argument is valid for all headings.

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