2008 American Control Conference
Westin Seattle Hotel, Seattle, Washington, USA
June 11-13, 2008

Robust Adaptive Inner-Loop Design for Vehicles with Uncertain Dynamics

Eugene Lavretsky[§]

***Abstract* – We present robust adaptive augmentation design for a class of 2nd order uncertain nonlinear cascaded systems. These dynamics generalize the models that are often used for the design of inner-loop flight controllers. The proposed control architecture augments a baseline dynamic inversion controller with a direct adaptive component and a variable structure system, (VSS). While the adaptive augmentation is designed to maintain tracking performance in the presence of the system uncertainties, the VSS component protects the system trajectories from leaving allowable subset in the system state space. The design is applied to construct angle-of-attack (AOA) command tracking system for short period dynamics of a fixed wing aircraft.**

I. Introduction

In this paper, we consider 2nd order uncertain dynamical systems in the *cascaded* form:

$$\begin{cases} \dot{x}_1 = F_1^0(x_1, z) + B_1 x_2 + f_1(x_1, z) \\ \dot{x}_2 = F_2^0(x_1, x_2, z) + \dot{x}_2^{cmd} + f_2(x_1, x_2, z) \end{cases} \tag{1.1}$$

where $x = (x_1 \quad x_2)^T$ is the system state vector, z is the known bounded external signal, $(F_1^0,\, F_2^0)$ are known state-dependent functions, B_1 is a known nonzero constant, $\dot{x}_2^{cmd}$ is the system control input, and $(f_1,\, f_2)$ are unknown, continuously differentiable functions that represent the system uncertainties.

The *control objective is bounded tracking* in the presence of the system uncertainties $(f_1,\, f_2)$. Specifically, the control goal is to design the control input $\dot{x}_2^{cmd}$ so that the system 1st state component x_1 tracks any given bounded time-varying command $x_1^{cmd}(t)$, *in the presence of the system uncertainties*, while keeping all the signals in the closed-loop system bounded, uniformly in time.

Our interest in considering this particular class of systems stems from flight control related applications, where inner-loop controllers for a fixed wing aircraft are often designed based on the so-called simplified models, [1–4]. The latter are in the form of (1.1) and represent the aircraft decoupled *fast* responses in pitch, roll, and yaw axes. The *1st control challenge* here is to design an inner-loop controller that maintains vehicle tracking performance in the presence of uncertain aerodynamic effects, actuator failures, and unknown environmental disturbances. This is accomplished using flight proven adaptive design methods from [1, 3].

[§]E. Lavretsky is with The Boeing Company, Huntington Beach, CA 92647, e-mail: eugene.lavretsky@boeing.com

This material is based upon work supported by the United States Air Force under Contracts No. FA9550-04-C-0047 and FA9550-07-C-0051.

Moreover, an aircraft flight controller must be designed to keep the vehicle dynamics in a pre-specified region of the corresponding state space. This region if often referred to as the operational flight envelope. For example, an AOA command tracking controller must include an AOA protection system, whose purpose is to maintain the aircraft AOA within a pre-specified range, outside of which a loss-of-control is expected. In essence, such an AOA controller would have to blend the two sub-systems, the AOA tracker and the AOA limiter, with only one or the other being active at any given time. Combining these 2 subsystems into a single inner-loop controller, while using theoretically justified design methods with performance and stability guarantees, constitutes the *2nd control challenge*. To address the latter, we will employ the design that was originally developed in [7]. Furthermore, often in real-world flight control applications, an inner-loop system must provide adequate damping in the presence of high order dynamics, such as the system structural modes, as well as other unmodelled effects. Towards that end, we pose the *3rd control challenge* which consists of adding damping to the system dynamics at low frequencies only, and without exciting the high frequency modes. Our proposed robust adaptive controller addresses and solves all of the 3 control challenges.

The rest of the paper is organized as follows. Section II presents the proposed robust adaptive control architecture. Online approximation of the system uncertainties is discussed in Section III. Sufficient conditions that guarantee bounded tracking and uniform ultimate boundedness of all signals in the corresponding closed-loop system are stated in Section IV. Based on these results, in Section V we perform AOA command tracking design, with flight envelope protection logic, and adaptive damping. This controller is constructed for short period dynamics of a fixed wing aircraft. The paper ends with conclusions that are given in Section VI.

II. Model Reference Control Architecture

We will employ a model reference based control design framework. The *reference model* is chosen to be 2nd order, with the desired damping ratio ξ and the natural frequency ω. This model is driven by a bounded possibly time-varying reference command, x_1^{cmd}.

$$x_1^m = \left[\frac{\omega^2}{s^2 + 2\,\xi\,\omega\, s + \omega^2}\right] x_1^{cmd} \tag{2.1}$$

Differentiating the first state component in (1.1) yields:

$$\ddot{x}_1 = \underbrace{\left[\frac{\partial F_1^0}{\partial x_1}\left(F_1^0 + B_1 x_2\right) + \frac{\partial F_1^0}{\partial z}\dot{z} + B_1 F_2^0\right]}_{f(x_1, x_2, z, \dot{z})} + B_1 \dot{x}_2^{cmd} + \underbrace{\left[\frac{\partial F_1^0}{\partial x_1} f_1 + \frac{\partial f_1}{\partial z}\dot{x}_1 + \frac{\partial f_1}{\partial z}\dot{z} + B_1 f_2\right]}_{d(x_1, x_2, z, \dot{z})} \tag{2.2}$$

or, equivalently

978-1-4244-2079-7/08/$25.00 ©2008 AACC.

$$\ddot{x}_1 = f(x_1, x_2, z, \dot{z}) + B_1 \dot{x}_2^{cmd} + d(x_1, x_2, z, \dot{z}) \tag{2.3}$$

Note that in (2.3), the function $f(x_1, x_2, z, \dot{z})$ and the constant B_1 are known, while $d(x_1, x_2, z, \dot{z})$ represents the unknown system uncertainty.

Consider a *dynamic inversion based controller* in the form:

$$\dot{x}_2^{cmd} = B_1^{-1}\left(\ddot{x}_1^m - f - K_D(\dot{x}_1 - \dot{x}_1^m) - K_P(x_1 - x_1^m) - K_I\left(\frac{x_1(t) - x_1^m(t)}{s}\right) - v\right) \tag{2.4}$$

where $\frac{x_1(t) - x_1^m(t)}{s} \triangleq \int_0^t (x_1(\tau) - x_1^m(\tau))\, d\tau$, and v denotes an augmentation component, defined later in (2.7). Let $e_1 = x_1 - x_1^m$ be the *tracking error* signal. Substituting (2.4) into (2.3), results in the closed-loop tracking error dynamics:

$$\ddot{e}_1 = -K_D\,\dot{e}_1 - K_P\,e_1 - K_I\int_0^t e_1(\tau)\,d\tau + \underbrace{d + K_D\,f_1}_{D(x_1, x_2, z, \dot{z})} - v \tag{2.5}$$

Introduce $\bar{x} = (x_1 \quad x_2 \quad z \quad \dot{z})^T$. Then (2.5) can be written as:

$$\ddot{e}_1 = -K_D\,\dot{e}_1 - K_P\,e_1 - K_I\int_0^t e_1(\tau)\,d\tau + D(\bar{x}) - v \tag{2.6}$$

At this time, control signal v is defined to *approximate / dominate* the system uncertainties on-line.

$$v \triangleq (1 - \bar{\gamma}(\bar{x}))\hat{D}(\bar{x}) + \bar{\gamma}(\bar{x})v_{sc} + w_{ad} \tag{2.7}$$

where $\hat{D}(\bar{x}) = \hat{D}(x_1, x_2, z, \dot{z})$ is the on-line *adaptive approximator*, w_{ad} is the so-called *adaptive damping* term to be defined later using a rate lead-lag filter, and v_{sc} represents the *switching component* of the control law. In addition, $\bar{\gamma}(\bar{x})$ is the so-called *modulation function*. This is a continuous state-dependent map which allows the controller to smoothly transition between the switching and the adaptive modes of operation. Construction of the modulation function is performed next.

Let Ω represent a compact region of approximation for the adaptive component $\hat{D}$, and let $\Omega_\delta \subset \Omega$ be its compact subset. The modulation function $\bar{\gamma}$ is defined as:

$$\bar{\gamma} = \begin{cases} 0, & \bar{x} \in \Omega_\delta \\ 1, & \bar{x} \notin \Omega \\ 0 < \bar{\gamma} < 1, & \bar{x} \in \Omega - \Omega_\delta \end{cases} \tag{2.8}$$

It provides *continuous* transition from the adaptive component $\hat{D}$ in (2.7) to the switching component v_{sc}, if and when the vector $\bar{x}$ leaves the subset Ω_δ, but before it reaches the boundary of Ω. The sufficiently small parameter δ defines the width of the annulus region $\Omega - \Omega_\delta$.

The main goal of the adaptive component in (2.7) is to cancel / dominate the uncertainty $D(\bar{x})$ in (2.5) by using its on-line estimated value $\hat{D}(\bar{x})$, for all $\bar{x} \in \Omega_\delta$. With (2.7), tracking error dynamics take the form:

$$\ddot{e}_1 == -K_D\,\dot{e}_1 - K_P\,e_1 - K_I\int_0^t e_1(\tau)\,d\tau - (1 - \bar{\gamma})\underbrace{(\hat{D} - D)}_{e_D} - \bar{\gamma}(v_{sc} - D) - w_{ad} \tag{2.9}$$

where e_D is the *uncertainty estimation error*. Detailed design of the switching component v_{sc} will be presented later in the paper.

Using pole placement, baseline *PID feedback gains* are chosen to enforce the desired 2nd order dynamics (2.1), augmented by integrated tracking error. The feedback gains are:

$$K_D = 2\xi\omega + k_1, \quad K_P = \omega(\omega + 2\xi k_1), \quad K_I = \omega^2 k_1 \tag{2.10}$$

Substituting (2.10) into (2.9) yields:

$$\ddot{e}_1 = -(2\xi\omega + k_1)\dot{e}_1 - \omega(\omega + 2\xi k_1)e_1 - \omega^2 k_1\int_0^t e_1(\tau)\,d\tau - (1 - \bar{\gamma})e_D - \bar{\gamma}(v_{sc} - D) - w_{ad} \tag{2.11}$$

Regrouping the terms in (2.11) results in:

$$\ddot{e}_1 + k_1\dot{e}_1 = -2\xi\omega(\dot{e}_1 + k_1 e_1) - \omega^2\left(e_1 + k_1\int_0^t e_1(\tau)\,d\tau\right) - (1 - \bar{\gamma})e_D - \bar{\gamma}(v_{sc} - D) - w_{ad} \tag{2.12}$$

Introduce the so-called *filtered tracking error*:

$$e_1^f = e_1 + k_1\int_0^t e_1(\tau)\,d\tau \tag{2.13}$$

Using (2.12) and (2.13), *filtered tracking error dynamics* can be written as:

$$\ddot{e}_1^f = -2\xi\omega\dot{e}_1^f - \omega^2 e_1^f - (1 - \bar{\gamma})e_D - \bar{\gamma}(v_{sc} - D) - w_{ad} \tag{2.14}$$

As seen from (2.14), the term $2\xi\omega\dot{e}_1^f$ provides *baseline damping* to the error dynamics. Often, in real-world applications, the value of the baseline damping term is optimized such that maximum possible damping is achieved in the presence of high order dynamics, such as the system structural modes, as well as other unmodelled effects. In order to add *extra* damping, without exciting the high frequency modes, the former is introduced into the system at low frequencies only. Towards this end, define:

$$\eta = \left(\frac{a}{s + a}\right)\dot{e}_1^f = \left(\frac{s\,a}{s + a}\right)e_1^f \tag{2.15}$$

and choose the *adaptive damping term* as:

$$w_{ad} = \left(K_D^f + \hat{k}_D^f\right)\eta \tag{2.16}$$

where K_D^f represents the baseline damping gain, $\hat{k}_D^f$ is the adaptive *incremental* damping gain, and $a > 0$ is the desired crossover frequency, above which the incremental damping must resort back to its baseline value. Relation (2.15) can be written in state-space form.

$$\dot{\eta} = -a(\eta - \dot{e}_1^f) \tag{2.17}$$

Augmenting the error dynamics (2.14) with (2.17), the system *extended error dynamics* can be written as:

$$\underbrace{\begin{pmatrix} \dot{e}_1^f \\ \ddot{e}_1^f \\ \dot{\eta} \end{pmatrix}}_{\dot{e}_f} = \underbrace{\begin{pmatrix} 0 & 1 & 0 \\ -\omega^2 & -2\xi\omega & -K_D^f \\ 0 & a & -a \end{pmatrix}}_{A_{ref}} \underbrace{\begin{pmatrix} e_1^f \\ \dot{e}_1^f \\ \eta \end{pmatrix}}_{e_f} - \underbrace{\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}}_{b_{ref}} \left((1 - \bar{\gamma})e_D + \bar{\gamma}(v_{sc} - D) + \hat{k}_D^f\,\eta\right) \tag{2.18}$$

where $e_f = (e_1^f \quad \dot{e}_1^f \quad \eta)^T$ represents the *extended filtered tracking error vector*. Next, conditions must be found such that matrix A_{ref} becomes Hurwitz. The matrix characteristic polynomial can be computed as follows:

$$\det\left(\lambda I - A_{ref}\right) = \begin{vmatrix} \lambda & -1 & 0 \\ \omega^2 & \lambda + 2\xi\omega & K_D^f \\ 0 & -a & \lambda + a \end{vmatrix} \tag{2.19}$$
$$= \lambda^3 + \lambda^2\left(2\xi\omega + a\right) + \lambda\left(2\xi\omega a + K_D^f a + \omega^2\right) + \omega^2 a$$

In order for the characteristic polynomial in (2.19) to have all its roots in the left half plane, it is *sufficient* to impose the following relations:

$$\begin{cases} a > 0, \quad K_D^f > -2\xi\omega - \dfrac{\omega^2}{a} \\ \omega^2 a < \left(2\xi\omega a + K_D^f a + \omega^2\right)\left(2\xi\omega + a\right) \end{cases} \tag{2.20}$$

Remark 2.1

The 1st inequality in (2.20) is already satisfied, since a defines the desired crossover frequency for the rate filter in (2.17). The 2nd condition in (2.20) places a lower bound on the value of the baseline rate damping gain. It is clear that the inequality is satisfied for any *positive* gain K_D^f. Finally, the 3rd inequality follows from the previous two.

Remark 2.2

If the time derivative of the filtered tracking error $\dot{e}_1^f(t)$ is driven to become small then the original tracking error signal $e_1(t)$ will also become small. This statement directly follows from the definition in (2.13). In fact, the latter can be written as:

$$\dot{e}_1 = -k_1 e_1 + \dot{e}_1^f \tag{2.21}$$

Consequently, if there exists T such that $\left|\dot{e}_1^f(t)\right| \le \varepsilon_1$ for all $t \ge t_0 + T$, then as $t \to \infty$

$$\left|e_1(t)\right| \le e^{-k_1(t-t_0-T)}\left|e_1(t_0)\right| + \frac{\varepsilon_1}{k_1}\left(1 - e^{-k_1(t-t_0-T)}\right) \to \frac{\varepsilon_1}{k_1} \tag{2.22}$$

Remark 2.3

If the filtered tracking error $e_1^f(t)$ is driven to become small then the original tracking error signal $e_1(t)$ will also become small. Using (2.21), yields:

$$e_1(t) = e^{-k_1(t-t_0)}\left(e_1(t_0) - e_1^f(t_0)\right) + e_1^f(t) - k_1\int_{t_0}^{t} e^{-k_1(t-\tau)} e_1^f(\tau)\,d\tau \tag{2.23}$$

Consequently, if there exists T such that $\left|e_1^f(t)\right| \le \varepsilon_1$ for all $t \ge T + t_0$, then as $t \to \infty$

$$\left|e_1(t)\right| \le e^{-k_1(t-t_0-T)}\left|e_1(t_0+T) - e_1^f(t_0+T)\right| + \varepsilon_1\left(2 - e^{-k_1(t-t_0-T)}\right) \to 2\varepsilon_1 \tag{2.24}$$

In other words, if $\left|e_1^f(t)\right| \le \varepsilon_1$ then as $t \to \infty$ the following asymptotic relation takes place:

$$\left|e_1(t)\right| \le 2\varepsilon_1 + \mathrm{o}(1) \tag{2.25}$$

III. On-line Uncertainty Approximation

On-line approximation of the uncertain function $D(\bar{x})$ in (2.5) is performed on a compact $\bar{x}$–region Ω, and using linear-in-parameters artificial neural network (NN), with radial basis functions (RBF) in its inner-layer, [6]. The on-line function approximation is:

$$\hat{D}(\bar{x}) = \hat{\theta}_D^T \Phi_D(\bar{x}) \tag{3.1}$$

where $\hat{\theta}_D$ is the on-line estimated vector of parameters and $\Phi_D(\bar{x})$ represents fixed RBF regressor vector of the corresponding dimension. It is assumed that the uncertainty can be approximated by an RBF NN, within a prescribed tolerance, and on the compact $\bar{x}$–region Ω:

$$D(\bar{x}) = \left(\theta_D^*\right)^T \Phi_D(\bar{x}) + \varepsilon_D(\bar{x}) \tag{3.2}$$

In (3.2), θ_D^* is the true unknown constant parameter and ε_d is the unknown bounded approximation error:

$$\left|\varepsilon_D(\bar{x})\right| \le \varepsilon_D^{\max} \tag{3.3}$$

Subtracting (3.2) from (3.1), the on-line *function approximation error* can be expressed in terms of the on-line *parameter estimation error*:

$$e_D \triangleq \hat{D} - D = \underbrace{\left(\hat{\theta}_D - \theta_D^*\right)}_{\Delta\theta_D}^T \Phi_D - \varepsilon_D = \Delta\theta_D^T \Phi_D - \varepsilon_D \tag{3.4}$$

Substituting (3.4) into (2.18), *closed-loop filtered tracking error dynamics* can be derived.

$$\dot{e}_f = A_{ref}\, e_f - b_{ref}\left(\left(1-\bar{\gamma}\right)\left(\Delta\theta_D^T \Phi_D - \varepsilon_D\right) + \bar{\gamma}\left(v_{sc} - D\right) + \hat{k}_D^f \eta\right) \tag{3.5}$$

IV. Parameter Adaptation and Closed-Loop System Dynamics

Choose a symmetric positive-definite matrix $Q > 0$ and solve the following algebraic Lyapunov equation:

$$P A_{ref} + A_{ref}^T P = -Q \tag{4.1}$$

Since A_{ref} is Hurwitz, the Lyapunov equation has a unique positive-definite symmetric solution P. Use the latter to form a Lyapunov function candidate:

$$V\left(e_f, \Delta\theta_D\right) = e_f^T P e_f + \Delta\theta_D^T \Gamma_D^{-1} \Delta\theta_D + \gamma_D^{-1}\left(\hat{k}_D^f\right)^2 \tag{4.2}$$

where symmetric positive-definite matrix Γ_D and positive scalar γ_D will be used to define the *rates of adaptation*. Differentiating (4.2) along the trajectories of the system (3.5) trajectories, yields:

$$\begin{aligned} \dot{V} &= -e_f^T Q e_f + 2\left(1-\bar{\gamma}\right) e_f^T P b_{ref}\left(-\Delta\theta_D^T \Phi_D + \varepsilon_D\right) \\ &-2 e_f^T P b_{ref}\, \hat{k}_D^f \eta + 2\Delta\theta_D^T \Gamma_D^{-1} \dot{\hat{\theta}}_D + 2\gamma_D^{-1} \hat{k}_D^f \dot{\hat{k}}_D^f - 2\bar{\gamma}\left(v_{sc} - D\right) e_f^T P b_{ref} \end{aligned} \tag{4.3}$$

Regrouping the terms further yields:

$$\begin{aligned} \dot{V} &= -e_f^T Q e_f + 2 e_f^T P b_{ref}\left(1-\bar{\gamma}\right)\varepsilon_D + 2\bar{\gamma}\left(D - v_{sc}\right)\hat{e}_f^T P b_{ref} \\ &+2\Delta\theta_D^T\left(-\Phi_D\left(1-\bar{\gamma}\right)\hat{e}_f^T P b_{ref} + \Gamma_D^{-1}\dot{\hat{\theta}}_D\right) + 2\hat{k}_D\left(-\hat{e}_f^T P b_{ref}\,\eta + \gamma_D^{-1}\dot{\hat{k}}_D^f\right) \end{aligned} \tag{4.4}$$

In order to make the time derivative $\dot{V}$ in (4.4) to be negative outside of a compact $\left(e, \Delta\theta_D, \hat{k}_D^f\right)$–subset of Ω, choose the following parameter adaptation laws:

$$\begin{cases} \dot{\hat{\theta}}_D = \Gamma_D \operatorname{Proj}\left(\hat{\theta}_D, \Phi_D \underbrace{e_f^T P b_{ref}}_{\bar{e}_f(t)}\left(1-\bar{\gamma}\right)\right) \\ \dot{\hat{k}}_D^f = \gamma_D \operatorname{Proj}\left(\hat{k}_D^f, \eta \underbrace{e_f^T P b_{ref}}_{\bar{e}_f(t)}\right) \end{cases} \tag{4.5}$$

In (4.5), Proj denotes the Projection Operator, which forces the adaptive parameters to evolve in a pre-specified compact $\left(\theta_D, \hat{k}_D^f\right)$–region, [8]. Furthermore, it is easy to see that

$$\overline{e}_f(t) \triangleq e_f^T P b_{ref} = p_{12}\, e_1^f + p_{22}\, \dot{e}_1^f + p_{23}\, \eta \tag{4.6}$$

Similar to [7], the switching component of the controller is defined as:

$$v_{sc} = K_{sc}\, \mathrm{sgn}\, \overline{e}_f(t) = K_{sc}\, \mathrm{sgn}\left(p_{12}\, e_1^f + p_{22}\, \dot{e}_1^f + p_{23}\, \eta\right) \tag{4.7}$$

where K_{sc} is a sufficiently large positive constant gain. Substituting (4.5), (4.6) and (4.7) into (4.4), yields

$$\dot{V} = -e_f^T Q e_f + 2\overline{\gamma}\left(D - K_{sc}\, \mathrm{sgn}\, \overline{e}_f(t)\right)\overline{e}_f(t) + 2\overline{e}_f(t)(1-\overline{\gamma})\varepsilon_D \tag{4.8}$$

Recall that the modulation function $\overline{\gamma}$ is defined as in (2.8). Consider three distinct cases.

Case a). If $\overline{x} \notin \Omega$ then $\overline{\gamma} = 1$ and choosing

$$K_{sc} \geq D_{\max} \tag{4.9}$$

relation (4.8) becomes:

$$\begin{aligned} \dot{V} &= -e_f^T Q e_f + 2\left(D - K_{sc}\, \mathrm{sgn}\, \overline{e}_f(t)\right)\overline{e}_f(t) \\ &= -e_f^T Q e_f + 2\left(D\, \mathrm{sgn}\, \overline{e}_f(t) - K_{sc}\right)\left|\overline{e}_f(t)\right| \\ &\leq -\lambda_{\min}(Q)\left\|e_f\right\|^2 + 2\underbrace{\left(D_{\max} - K_{sc}\right)}_{<0}\left|e_f^T P b_{ref}\right| \leq -\lambda_{\min}(Q)\left\|e_f\right\|^2 \end{aligned} \tag{4.10}$$

Consequently, the tracking error will decay until the system state enters the region of approximation Ω.

Case b). If $\overline{x} \in \Omega_\delta$ then according to (2.8) $\overline{\gamma} = 0$, and therefore

$$\dot{V} = -e_f^T Q e_f + 2 e_f^T P b_{ref}\, \varepsilon_D \leq -\lambda_{\min}(Q)\left\|e_f\right\|^2 + 2\left\|e_f\right\|\left\|P b_{ref}\right\|\varepsilon_D^{\max} \tag{4.11}$$

where $\lambda_{\min}(Q)$ is the minimum eigenvalue of Q, $\varepsilon_D^{\max} \triangleq \max_{\overline{x}\in\Omega} \varepsilon_D(\overline{x})$. Also note that because of the Projection Operator, norms of the parameter estimation errors will stay uniformly bounded, that is:

$$\left\|\Delta\theta(t)\right\| \leq \Delta\theta_{\max} < \infty \quad \wedge \quad \left\|\hat{k}_D^f(t)\right\| \leq \left(\hat{k}_D^f\right)_{\max} < \infty \tag{4.12}$$

where $\left[\Delta\theta_{\max}, \left(\hat{k}_D^f\right)_{\max}\right]$ are the parameter bounds.

Using (4.11), Uniform Ultimate Boundedness (UUB) [5] of the closed-loop system trajectories can now be established. Towards that end, define the following compact subset in the e_f – region:

$$S_r \triangleq \left\{\left\|e_f\right\| \leq r \triangleq \frac{2\left\|P b_{ref}\right\|\varepsilon_D^{\max}}{\lambda_{\min}(Q)}\right\} \tag{4.13}$$

Define minimal level set $\Omega_b = \left\{e_f^T P e_f \leq b\right\}$ that contains S_r. Since

$$\lambda_{\min}(P)\left\|e_f\right\|^2 \leq e_f^T P e_f \leq \lambda_{\max}(P)\left\|e_f\right\|^2 \tag{4.14}$$

then choosing

$$b = \lambda_{\max}(P)\, r^2 \tag{4.15}$$

implies that for all $\left\|e_f\right\| \leq r$

$$e_f^T P e_f \leq \lambda_{\max}(P)\left\|e_f\right\|^2 \leq \lambda_{\max}(P)\, r^2 = b \tag{4.16}$$

Hence, the set S_r is contained in the level set Ω_b.

Suppose that all *initial* values of the filtered tracking error $e_f(t_0)$ start in a compact set $S_R \triangleq \left\{\left\|e_f\right\| \leq R\right\}$. Let $\Omega_B = \left\{e_f^T P e_f \leq B\right\}$ be the maximal level set which belongs to S_R. In order to maintain closed-loop system stability, a specific relation between the boundaries for the sets Ω_b, Ω_B, S_r, and S_R must be imposed. These sets will be used to prove that the closed-loop system trajectories are UUB. Graphical representation of the four sets is given in Figure 3.1.

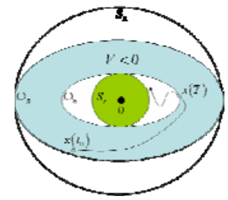

Figure 3.1: UUB Sets

Choose:

$$B = \lambda_{\min}(P)\, R^2 \tag{4.17}$$

Then if $e_f^T P e_f \leq B$ then using (4.14) yields:

$$\lambda_{\min}(P)\left\|e_f\right\|^2 \leq e_f^T P e_f \leq B = \lambda_{\min}(P)\, R^2 \tag{4.18}$$

Consequently $\left\|e_f\right\| \leq R$, that is the filtered tracking error is in S_R. Because of (4.11) and (4.13), the time derivative $\dot{V}$ is negative *outside* of S_r. Consequently, the filtered tracking error e_f will enter level set Ω_b in finite time, and will remain in the set from then on. Consequently, the closed-loop system trajectories are UUB.

Case c). If $\overline{x} \in \Omega - \Omega_\delta$ then both the adaptive and the switching components of the controller are active. In this case, using (4.8) one gets

$$\begin{aligned} \dot{V} &= -e_f^T Q e_f + 2\overline{\gamma}\left(D - K_{sl}\, \mathrm{sgn}\, \overline{e}_f(t)\right)\overline{e}_f(t) + 2\overline{e}_f(t)(1-\overline{\gamma})\varepsilon_D \\ &\leq -\lambda_{\min}(Q)\left\|e_f\right\|^2 + 2\left[\overline{\gamma}\left(D_{\max} - K_{sl}\right) + (1-\overline{\gamma})\varepsilon_D^{\max}\right]\left|e_f^T P b_{ref}\right| \\ &\leq -\lambda_{\min}(Q)\left\|e_f\right\|^2 + 2(1-\overline{\gamma})\left\|e_f\right\|\left\|P b_{ref}\right\|\varepsilon_D^{\max} \end{aligned} \tag{4.19}$$

Since by definition $0 \leq \overline{\gamma} \leq 1$

$$\begin{aligned} \dot{V} &\leq -\lambda_{\min}(Q)\left\|e_f\right\|^2 + 2(1-\overline{\gamma})\left\|e_f\right\|\left\|P b_{ref}\right\|\varepsilon_D^{\max} \\ &\leq -\lambda_{\min}(Q)\left\|e_f\right\|^2 + 2\left\|e_f\right\|\left\|P b_{ref}\right\|\varepsilon_D^{\max} \end{aligned} \tag{4.20}$$

Similar to Case b), one can show that the system trajectories will enter the sub-region Ω_δ in finite time, after which the Case c) conditions take place.

The three cases prove UUB of the closed-loop system trajectories. Moreover, due to the use of the Projection Operator in (4.5), all the estimated parameters $\hat{\theta}(t)$ are bounded. Hence the tracking problem is solved.

The corresponding *total explicit model following control* signal can be written using relations (2.4), (2.7), (2.16), (3.1), and (4.7).

$$\begin{aligned} \dot{x}_2^{cmd} = {} & \underbrace{B_1^{-1}\left(\ddot{x}_1^m - f(\overline{x}) - K_D\left(\dot{x}_1 - \dot{x}_1^m\right) - K_P\left(x_1 - x_1^m\right) - K_I\left(\frac{x_1(t) - x_1^m(t)}{s}\right)\right)}_{\text{Baseline Dynamic Inversion Controller}} \\ & -\left(1-\overline{\gamma}(\overline{x})\right)\underbrace{B_1^{-1}\,\hat{\theta}_D^T(t)\,\Phi_D(\overline{x})}_{\text{Adaptive Augmentation}} - \underbrace{\overline{\gamma}(\overline{x})}_{\text{Modulation Function}}\ \underbrace{B_1^{-1} K_{sc}\, \mathrm{sgn}\left(p_{12}\, e_1^f + p_{22}\, \dot{e}_1^f\right)}_{\text{Switching Component}} \\ & \underbrace{-B_1^{-1}\left(K_D^f + \hat{k}_D^f\right)\eta}_{\text{Additional Rate Damping}} \end{aligned} \tag{4.21}$$

Remark 4.1

Using (2.15), the damping term in (2.16) can be written as:

$$w = \left(K_D^f + \hat{k}_D^f\right)\frac{s\,a}{(s+a)}\frac{(s+k_1)}{s}\left(x_1 - x_1^m\right) = \left(K_D^f + \hat{k}_D^f\right)\underbrace{\boxed{k_1 \boxed{\frac{\left(\frac{s}{k_1}+1\right)}{\left(\frac{1}{a}s+1\right)}}}}_{G(s)\,=\,\text{Lead-Lag Filter}}\left(x_1 - x_1^m\right) \tag{4.22}$$

If the adaptation rate γ_D in (4.5) is set to zero then the extra damping term becomes part of the baseline controller. Thus, the total control command can be reformulated as:

$$\begin{aligned} \dot{x}_2^{cmd} = & \underbrace{B_1^{-1}\left(\ddot{x}_1^m - f(\bar{x}) - \left(\overbrace{\boxed{K_D\,s + K_P + \frac{K_I}{s}}}^{\text{PID Controller}} + K_D^f\,\overbrace{\boxed{G(s)}}^{\text{Lead-Lag}}\right)\left(x_1 - x_1^m\right)\right)}_{\text{Baseline Dynamic Inversion Controller}} \\ & -\left(1-\bar{\gamma}(\bar{x})\right)\underbrace{B_1^{-1}\,\hat{\theta}_D^T(t)\,\Phi_D(\bar{x})}_{\text{Adaptive Augmentation}} - \underbrace{\bar{\gamma}(\bar{x})}_{\substack{\text{Modulation}\\ \text{Function}}}\;\underbrace{B_1^{-1}\,K_{sc}\,\mathrm{sgn}\left(p_{12}\,e_1^f + p_{22}\,\dot{e}_1^f\right)}_{\text{Switching Component}} \\ & -\underbrace{B_1^{-1}\,\hat{k}_D^f\,G(s)\left(x_1 - x_1^m\right)}_{\text{Adaptive Rate Damping}} \end{aligned} \tag{4.23}$$

From (4.23), it follows that the *total control signal* is comprised of the *five major terms*: a) baseline dynamic inversion controller, b) adaptive augmentation, c) switching component, d) modulation function, and e) adaptive damping.

V. DESIGN EXAMPLE: ANGLE OF ATTACK TRACKING

In this section, we apply the developed robust adaptive design methodology to construct AOA command tracking system for a fixed wing aircraft, whose short period dynamics, with lift and pitching moment uncertainties, can be written in the cascaded form (1.1), [1, 4].

$$\begin{cases} \dot{\alpha} = -\tilde{L}_\alpha\,\alpha + Q_{grav} + q + \Delta L(\alpha) \\ \dot{q} = M(\alpha) + M_q\,q + M_{IC} + \dot{q}_{cmd} + \Delta M(\alpha, q) \end{cases} \tag{5.1}$$

where α is the aircraft AOA, q is the angular pitch rate, $\tilde{L}_\alpha$ is the known lift curve slope, Q_{grav} is the known gravity term, $\Delta L(\alpha)$ is the lift force uncertainty, $M(\alpha)$ is the known pitching moment, M_q is the known constant pitch damping, M_{IC} is the known pitching moment increment due to inertial cross-coupling effects, $\dot{q}_{cmd}$ is the commanded pitch acceleration (control input), and finally $\Delta M(\alpha, q)$ represents the pitching moment uncertainty.

The AOA desired / reference model dynamics is chosen in the form of (2.1):

$$\alpha_m = \left[\frac{\omega^2}{s^2 + 2\,\xi\,\omega\,s + \omega^2}\right]\alpha_{cmd} \tag{5.2}$$

Note, that in addition to the known quantities in (5.1), the system dynamics contain uncertainties in the aircraft lift force $\Delta L(\alpha)$ and in the pitching moment $\Delta M(\alpha, q)$. According to (4.22), the rate damping term is chosen as:

$$w = \frac{\left(K_D^f + \hat{k}_D^f\right)}{\left(\frac{s}{a}+1\right)}\left(q - q^m\right) = \underbrace{\boxed{K_D^f\,\frac{a}{(s+a)}\left(q - q^m\right)}}_{\text{Baseline}} + \underbrace{\boxed{\hat{k}_D^f(t)\,\frac{a}{(s+a)}\left(q - q^m\right)}}_{\text{Adaptive}} \tag{5.3}$$

The baseline portion $\dot{q}_{cmd}^{bl}$ of the total pitch acceleration command $\dot{q}_{cmd}$, with the damping term included, can now be written as:

$$\dot{q}_{cmd}^{bl} = \dot{q}_m - M(\alpha) - M_q\,q - M_{IC} + \left[2\,\xi\,\omega + K_D^f\,\frac{a}{(s+a)}\right]\left(q_m - q\right) + \omega^2\,\frac{\left(q_m - q\right)}{s} \tag{5.4}$$

where $q_m \triangleq \dot{\alpha}_m + \tilde{L}_\alpha\,\alpha_m - Q_{grav}$ is the reference model pitch rate signal. The pitch rate error can be written in terms of the AOA tracking error as:

$$q_m - q = \left(s + \tilde{L}_\alpha\right)\left(\alpha_m - \alpha\right) \tag{5.5}$$

In order to perform adaptive / switching augmentation design, we compare (5.1) with the generic cascaded dynamics (1.1).

$$\begin{cases} x_1 = \alpha, \quad x_2 = q, \quad z = \left(Q_{grav} \quad M_{IC}\right)^T, \quad \dot{x}_2^{cmd} = \dot{q}_{cmd} \\ F_1 = -\tilde{L}_\alpha\,\alpha + Q_{grav}, \quad B_1 = 1, \quad F_2 = M(\alpha) + M_q\,q + M_{IC} \\ f_1 = \Delta L(\alpha), \quad f_2 = \Delta M(\alpha, q) \end{cases} \tag{5.6}$$

In this case, baseline PID feedback gains are:

$$\begin{cases} K_{\dot{\alpha}} \triangleq K_D = 2\,\xi\,\omega + k_1 \\ K_\alpha \triangleq K_P = \omega\left(\omega + 2\,\xi\,k_1\right) \\ K_\alpha^I \triangleq K_I = \omega^2\,k_1 \end{cases} \tag{5.7}$$

and the integrator pole k_1 is:

$$k_1 = \tilde{L}_\alpha \tag{5.8}$$

Using (2.13), the *filtered tracking error signal*, gives

$$e_1^f = \frac{(s+k_1)}{s}e_1 = \frac{\left(s+\tilde{L}_\alpha\right)}{s}\left(\alpha - \alpha_m\right) = \frac{q - q_m}{s} \tag{5.9}$$

Hence, the *filtered tracking vector* is:

$$e_f = \begin{pmatrix} e_1^f & \dot{e}_1^f & \frac{a}{(s+a)}\dot{e}_1^f \end{pmatrix}^T = \begin{pmatrix} \frac{q - q_m}{s} & q - q_m & \frac{a}{(s+a)}\left(q - q_m\right) \end{pmatrix}^T \tag{5.10}$$

The regressor vector Φ_D is chosen to depend on AOA only. Then parameter adaptation laws are written based on (4.5).

$$\begin{cases} \dot{\hat{\theta}}_D = \Gamma_D\ \mathrm{Proj}\left(\hat{\theta}_D,\, \Phi_D(\alpha)\begin{pmatrix} \frac{q-q_m}{s} & q - q_m & \frac{a}{(s+a)}\left(q - q_m\right)\end{pmatrix} P \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}(1-\bar{\gamma})\right) \\ \dot{\hat{k}}_D^f = \gamma_D\ \mathrm{Proj}\left(\hat{k}_D^f,\, \eta\begin{pmatrix} \frac{q-q_m}{s} & q - q_m & \frac{a}{(s+a)}\left(q - q_m\right)\end{pmatrix} P \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}\right) \end{cases} \tag{5.11}$$

where $P = P^T > 0$ is the unique positive definite symmetric solution of the algebraic Lyapunov equation (4.1) with the Hurwitz reference model matrix A_{ref} as specified in (2.18).

In summary, using (2.16) and (3.1), *adaptive pitch acceleration command* takes the form:

$$\dot{q}_{cmd}^{ad} = \hat{\theta}_D^T(t)\,\Phi_D(\alpha) + \hat{k}_D^f(t)\,\frac{a}{(s+a)}\left(q - q_m\right) \tag{5.12}$$

Also relation (4.7) defines the *switching component* of the pitch acceleration command:

$$\dot{q}_{cmd}^{sc} = K_{sl}\,\mathrm{sgn}\left(p_{12}\,e_1^f + p_{22}\,\dot{e}_1^f + p_{23}\,\eta\right) \tag{5.13}$$

Note that the switching gain K_{sc} needs to be chosen to dominate the system uncertainty:

$$D = \frac{\partial F_1}{\partial x_1}f_1 + \frac{\partial f_1}{\partial z}\dot{z} + B_1\,f_2 = -\tilde{L}_\alpha\,\Delta L(\alpha) + \Delta M(\alpha, q) \tag{5.14}$$

In other words,

$$K_{sc} \geq \max_{\substack{\alpha_{\min} \leq \alpha \leq \alpha_{\max} \\ q_{\min} \leq q \leq q_{\max}}} \left|\Delta M(\alpha, q) - \tilde{L}_\alpha\,\Delta L(\alpha)\right| \triangleq D_{\max} \tag{5.15}$$

In (5.15), the subset $\left(\alpha_{\min} \quad \alpha_{\max}\right)\times\left(q_{\min} \quad q_{\max}\right)$ represents the aircraft operational AOA / pitch rate envelope.

For simplicity, let's assume that the pitching moment uncertainty depends on AOA only, that is $\Delta M = \Delta M(\alpha)$. Then the switching mode gain must be chosen such that

$$K_{sc} \geq \max_{\alpha_{\min} \leq \alpha \leq \alpha_{\max}} \left| \Delta M(\alpha) - \tilde{L}_{\alpha} \Delta L(\alpha) \right| \triangleq D_{\max} \tag{5.16}$$

Let $\Omega = \begin{pmatrix} \alpha_{\min}^{bp} & \alpha_{\max}^{bp} \end{pmatrix}$ denote AOA approximation region, where $\alpha_{\min}^{bp} \geq \alpha_{\min}$ and $\alpha_{\max}^{bp} \leq \alpha_{\max}$ are the AOA minimum and maximum break points, correspondingly. Piece-wise linear *modulation function* $\bar{\gamma}(\alpha)$ can be easily written to satisfy (2.8):

$$\bar{\gamma}(\alpha) = \begin{cases} 1, & \alpha \leq \alpha_{\min}^{bp} - \delta \\ \dfrac{\alpha_{\min}^{bp} - \alpha}{\delta}, & \alpha_{\min}^{bp} - \delta < \alpha \leq \alpha_{\min}^{bp} \\ 0, & \alpha_{\min}^{bp} < \alpha < \alpha_{\max}^{bp} \\ \dfrac{\alpha - \alpha_{\max}^{bp}}{\delta}, & \alpha_{\max}^{bp} < \alpha < \alpha_{\max}^{bp} + \delta \\ 1, & \alpha \geq \alpha_{\max}^{bp} + \delta \end{cases} \tag{5.17}$$

Figure 5.1 shows a sketch of the modulation function.

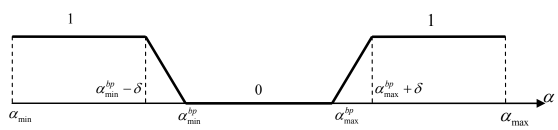

Figure 5.1: Piece-wise linear modulation function

Remark 5.1

A *multi-dimensional* modulation function $\bar{\gamma}(\bar{x})$ can be created as follows. Suppose that $\bar{x}_0$ is a center point of the sphere $\Omega_R = \left\{ \|\bar{x} - \bar{x}_0\| \leq R \right\}$. Also suppose that the set Ω_R represents the approximation region for RBF-s. Let $\delta > 0$ be a small positive constant and define $\Omega_{R-\delta} = \left\{ \|\bar{x} - \bar{x}_0\| \leq R - \delta \right\}$. The modulation function is defined as:

$$\bar{\gamma} = \begin{cases} 0, & \bar{x} \in \Omega_{R-\delta} \\ 0 \leq \bar{\gamma}(\bar{x}) \leq 1, & \bar{x} \in \Omega_R - \Omega_{R-\delta} \\ 1, & \bar{x} \notin \Omega_R \end{cases} \tag{5.18}$$

Formally, the modulation function can be written as:

$$\bar{\gamma}(\bar{x}) = \max\left[0,\ \min\left[1,\ 1 + \frac{\|\bar{x} - \bar{x}_0\| - R}{\delta} \right] \right] \tag{5.19}$$

In order to see that (5.19) implies (5.18) it is sufficient to simply sketch $\bar{\gamma}(\bar{x})$ versus $\|\bar{x} - \bar{x}_0\|$. Figure 5.2 shows the data.

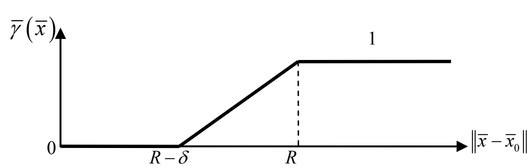

Figure 5.2: Multi-dimensional modulation function

Using (5.19) yields three relations that formally prove the validity of the modulation function choice.

$$\bar{\gamma}(\bar{x}) = 0 \Leftrightarrow 1 + \frac{\|\bar{x} - \bar{x}_0\| - R}{\delta} \leq 0 \Leftrightarrow \|\bar{x} - \bar{x}_0\| \leq R - \delta \Leftrightarrow \bar{x} \in \Omega_{R-\delta}$$

$$\bar{\gamma}(\bar{x}) = 1 \Leftrightarrow 1 + \frac{\|\bar{x} - \bar{x}_0\| - R}{\delta} \geq 1 \Leftrightarrow \|\bar{x} - \bar{x}_0\| \geq R \Leftrightarrow \bar{x} \notin \Omega_R$$

$$0 \leq \bar{\gamma}(\bar{x}) \leq 1 \Leftrightarrow 0 \leq 1 + \frac{\|\bar{x} - \bar{x}_0\| - R}{\delta} \leq 1 \Leftrightarrow R - \delta \leq \|\bar{x} - \bar{x}_0\| \leq R \Leftrightarrow \bar{x} \in \Omega_{R-\delta}$$

Remark 5.2

Suppose that $\bar{x} = \begin{pmatrix} \alpha & \dot{\alpha} \end{pmatrix}$ and $\bar{x}_0 = \begin{pmatrix} \alpha_0 & 0 \end{pmatrix}$. Choose L_1–weighted norm with the weights set to $\alpha_{\max}$ and $\dot{\alpha}_{\max}$. Set $R = 1$, $\delta = 0.1$ and write the corresponding 2–dimensional modulation function.

$$\bar{\gamma}(\alpha, \dot{\alpha}) = \max\left[0,\ \min\left[1,\ 1 + \frac{\dfrac{|\alpha - \alpha_0|}{\alpha_{\max}} + \dfrac{|\dot{\alpha}|}{\dot{\alpha}_{\max}} - 1}{0.1} \right] \right]$$

In this case

$$\Omega_1 = \left\{ \frac{|\alpha - \alpha_0|}{\alpha_{\max}} + \frac{|\dot{\alpha}|}{\dot{\alpha}_{\max}} \leq 1 \right\}, \quad \Omega_{0.9} = \left\{ \frac{|\alpha - \alpha_0|}{\alpha_{\max}} + \frac{|\dot{\alpha}|}{\dot{\alpha}_{\max}} \leq 0.9 \right\}$$

In summary, *total pitch acceleration command* consists of the five terms: a) the baseline dynamic inversion command $\dot{q}_{cmd}^{bl}$, b) the adaptive augmentation $\dot{q}_{cmd}^{ad}$, c) the switching component $\dot{q}_{cmd}^{sc}$, d) the AOA modulation function $\bar{\gamma}(\alpha)$, and e) the adaptive damping term $\dot{q}_{cmd}^{D}$.

$$\dot{q}_{cmd} = \dot{q}_{cmd}^{bl} - \left(1 - \bar{\gamma}(\alpha)\right) \dot{q}_{cmd}^{ad} - \bar{\gamma}(\alpha) \dot{q}_{cmd}^{sc} + \dot{q}_{cmd}^{D}$$

VI. CONCLUSIONS

Motivated by flight control applications, in this paper we presented robust adaptive control design augmentation of a baseline dynamic inversion controller. In order to protect the system trajectories from leaving an allowable subset in the system state space, a VSS component was added. Also, a frequency-dependent adaptive damping term was incorporated into the system. The proposed design was applied to construct AOA command tracking system for short period dynamics of a fixed wing aircraft.

REFERENCES

[1] K.A. Wise, E. Lavretsky, and N. Hovakimyan, "Adaptive Control of Flight: Theory, Applications, and Open Problems," In Proceedings of American Control Conference, Minneapolis, MN, 2006.

[2] A. Young, C. Cao, N. Hovakimyan, E. Lavretsky, "An Adaptive Approach to Nonaffine Control Design for Aircraft Applications," AIAA-2006-6343, In Proceedings of AIAA Guidance, Navigation and Control Conference, Keystone, CO, 2006.

[3] E. Lavretsky, K.A. Wise, "Adaptive Flight Control for Manned / Unmanned Military Aircraft," In Proceedings of American Control Conference, Portland, OR, 2005.

[4] B. Stevens, F. Lewis. Aircraft control and simulation, Wiley, New York, 1992.

[5] H.K. Khalil. Nonlinear Systems, 3rd Edition, Prentice Hall Inc., 2002.

[6] S. Haykin, Neural Networks: A Comprehensive Foundation, 2nd edition, Prentice Hall Inc., 1999.

[7] R. Sanner and J.J. Slotine, "Gaussian networks for direct adaptive control," IEEE Transactions on Neural Networks, 3(6), pp. 837–864, 1992.

[8] J. B. Pomet and L. Praly. Adaptive nonlinear regulation: estimation from the Lyapunov equation. IEEE Trans. Autom. Contr., 37(6), pp. 729-740, 1992.