2008 American Control Conference
Westin Seattle Hotel, Seattle, Washington, USA
June 11-13, 2008

Distributed parameter methods for moving sensor networks in unison

Michael A. Demetriou§

Index Terms— **Distributed Parameter Systems, Sensor Placement, Sensor Scheduling, Sensor Network, Coordinated Motion, Process Estimation, Spatially Distributed Sensors.**

Abstract— **We incorporate optimization results of spatially distributed systems to design the optimal spatiotemporal allocation of sensors moving in unison. By viewing the positioning of the mobile agents within a network as the discrete spatial representation of the density of an equivalent single mobile sensor, the problem of motion-in-unison is converted into the problem of obtaining the spatial distribution and motion of a single sensing device. The single mobile sensor is integrated within the process at which it interacts with, and then the optimal sensor distribution uses the covariance of the associate statistical estimator of the process as a performance metric. To minimize the computational and algorithmic complexity of viewing the optimal estimator with a variable lower time limit of a cost-to-go, we consider instead a hybrid equivalent, wherein the time horizon is decomposed into a sequence of disjoint integrals mimicking model predictive control.**

I. Introduction

There has been an increasing number of work on sensor networks in the last decade dealing with various aspects such as collision avoidance, obstacle avoidance, limited communications, and flocking, see for example [1], [2], [14] and references therein. Many works addressed specific issues both on the theoretical level as well as on the algorithmic and computational level. An interesting aspect of this work is to employ a mobile sensor network (or schedule a fixed sensor network) to improve state estimation. This becomes more involved when the sensors interact directly with the process at which they are obtaining information for. In particular, when a sensor network (whether fixed-in-space or mobile) provides information for systems governed by partial differential equations, such as reaction-diffusion equations, then the inclusion of the sensors in the system modeling is necessary and provides a natural interpretation of the role of the sensor in the spatial domain [9], [10], [11], [12], [13].

In this work, we consider the case of moving a spatially distributed sensor inside the spatial domain of definition of the process of interest. Such a process is governed by the 1D reaction-diffusion process that may model species concentration or temperature distribution. The basic idea is to move this single spatially distributed sensor in order to improve a state estimator. While the kinematics of mobile sensors have been included in many works addressing finite dimensional systems, such as [8], the current work on a class of infinite dimensional systems, considers only the positioning of the spatially distributed sensor within the spatial domain without explicitly incorporating the kinematics. However, it provides the abstract mathematical framework to view spatially distributed sensors that move within a spatial domain in order to improve state estimation and parameter identification. Spatially distributed in this manuscript is taken to mean that a given sensor is defined over a region of the spatial domain having a spatially varying weight. Such a spatial weight is termed as the spatial distribution of the sensing device. Such a sensor provides spatially distributed measurement on the state of the process.

In earlier work, the "motion" of a single spatially distributed sensor was utilized to improve state estimation for a class of distributed parameter systems [7]. Variations of that also included combined state estimation and detection of a moving source (intrusion detection) [4], [5], [6]. However, in this work we utilize such knowledge to the case of multiple sensors moving within the spatial domain to improve state estimation. However, consideration of the motion of N mobile sensors might lead to algorithmic and computational challenges. Instead, we consider the case wherein a set of N sensors, each with an identical spatial distribution, is moving *in unison* (string formation [14]) in order to provide collective information on the spatially distributed process. A related definition for moving in a coordinated fashion is flocking, but we avoid using such a term since each of the N individual sensors has a pre-assigned distance from the leader sensor and it assumed that each of the $N-1$ followers can maintain such a distance and orientation with respect to the leader. This leader is taken to be the centroid of the equivalent single-sensor spatial distribution. The remaining $N-1$ followers must maintain the distance from the centroid in order for the spatial distribution resulting by combining all N sensors to stay the same as the initial spatial distribution. So one now considers the motion of a single equivalent spatially distributed sensor. An immediate extension is to have each of the $N-1$ followers adjust their velocity in accordance to a weighted average of the difference of their velocity with those of the other sensors in the network, [3]. Such an average will not have the same form as that of the finite dimensional case but instead will involve a spatial average in the form of the first spatial moment.

The mathematical formulation of the PDE process is given in § II. The optimization of a single sensor position and spatial distribution are summarized in § III. Additionally, the optimization of the equivalent N sensors comprising the equivalent single sensor are also given along with a summary of the practical considerations regarding the optimization of a single equivalent sensor resulting from the combination of multiple sensors within a sensor network. The use of a single "moving" sensor for improvement of the state estimator is

§Department of Mechanical Engineering, Worcester Polytechnic Institute, Worcester, MA 01609-2880, USA, mdemetri@wpi.edu

presented in § IV and numerical results are presented in § V.

II. Mathematical formulation

We consider a 1D diffusion process whose (distributed) state is to be estimated

$$\frac{\partial}{\partial t}x(t,\xi) = \alpha_1 \frac{\partial^2}{\partial \xi^2}x(t,\xi) + \alpha_2 \frac{\partial}{\partial \xi}x(t,\xi) + \alpha_3 x(t,\xi) + b_1(\xi)w(t) + b_2(\xi)u(t). \tag{1}$$

The state of the process is denoted by $x(t,\xi)$ and represents its value at time t and spatial position ξ within the spatial domain Ω, which is assumed to be $\Omega = [0,\ell] \subset \mathbb{R}$. The constants $\alpha_1, \alpha_2, \alpha_3$ are the parameters of the associated elliptic operator and $b_1(\xi)$ and $b_2(\xi)$ denote the spatial distributions of the disturbances and control inputs, respectively. Associated with (1), are the appropriate boundary conditions $x(t,0) = x(t,\ell) = 0$, and initial conditions $x(0,\xi) = x_0(\xi)$.

For process measurements, it is assumed that N mobile sensing agents are available, each supplying information on the state $x(t,\xi)$ over a small spatial region within Ω. The motivation for employing mobile sensors stems from a desire to improve the state estimator. However, instead of coordinating N mobile sensors, we consider the case of guiding a *single equivalent sensor* for such improved state estimation. Such an equivalent single sensor represents the *collective* information from the N individual sensors found as their weighted sum.

Regarding sensor measurements, we mention two different types: *spatially averaged* and *spatially distributed*. For the *spatially averaged* sensor, the information on the state $x(t,\xi)$ is averaged over a small spatial region within the domain Ω and may be weighted by a function, as for example

$$y_{averaged}(t) = \int_0^\ell f(\xi)x(t,\xi)\,d\xi,$$

where the function $f(\xi)$ is the spatial distribution of the sensing device and which acts as the weight in the spatial averaging of the state $x(t,\xi)$. The output $y(t)$ is a scalar function that depends on time only. In the *spatially distributed* sensor, the output is a weighted function of the state $x(t,\xi)$, distributed over a small region within Ω. For example,

$$y_{distributed}(t,\xi) = g(\xi)x(t,\xi),$$

provides an output that is distributed over the domain of definition of the function $g(\xi)$, and which is spatially distributed.

For simplicity, we first consider the case of fixed-in-space spatially distributed sensors resulting

$$y_i(t) = c_i(\xi)x(t,\xi), \quad i = 1,2,\ldots,N, \tag{2}$$

where the spatial function $c_i(\xi)$ denotes the spatial distribution of the i^{th} spatially distributed sensor and produces a spatially weighted state over the domain (support) of the function $c_i(\xi)$. To incorporate the location of the centroid of the i^{th} sensing device, the above spatial distribution is redefined and rewritten as $c(\xi;\xi_i)$ denoting the i^{th} device at the spatial location ξ_i. Hence the function $c_i(\xi)$ is a simplified notation of $c(\xi;\xi_i)$. When we subsequently consider mobile, the notation for $c(\xi;\xi_i(t))$ reduces to $c_{i(t)}(\xi)$. It is assumed that the network of spatially distributed sensors occupies a small continuous region P within the spatial domain Ω. The problem now is how to distribute (or locate) the N spatially distributed sensors within the spatial domain. This is simply the problem of sensor positioning. Figure 1 depicts various cases of spatial distributions of two sensors (left column) and the equivalent spatial distribution of a single sensor (right column) found by combining the two distributions with equal weights. The individual sensors are positioned within the region $P = [0, 0.2]$.

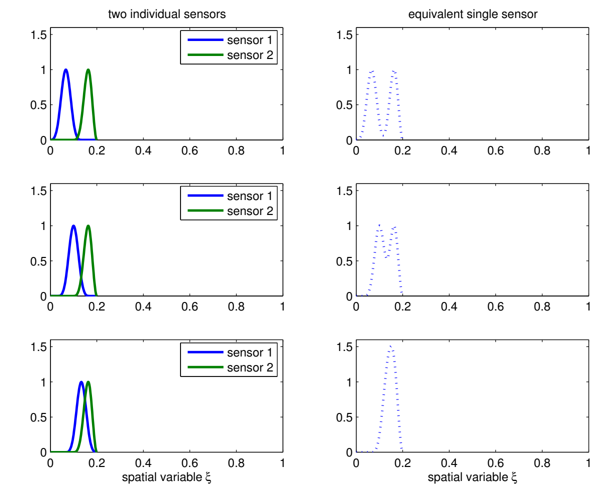


Fig. 1. Examples of spatial distribution of two individual sensors and the spatial distribution of the equivalent sensor.

The above partial differential equation along with the expression for spatially distributed measurements may be viewed in an abstract framework in terms of an evolution equation in appropriate infinite dimensional (Hilbert) space. We let $\mathcal{X}$ denote a Hilbert space with inner product $\langle\cdot,\cdot\rangle$ and corresponding induced norm $|\cdot|$. Additionally, the interpolating space $\mathcal{V}$ is a Banach space with norm $\|\cdot\|$, and is assumed to be embedded densely and continuously in $\mathcal{X}$ [15]. The conjugate dual of $\mathcal{V}$ is denoted by $\mathcal{V}^*$ and $\|\cdot\|_*$ denotes the usual operator norm on $\mathcal{V}^*$. Then $\mathcal{V} \hookrightarrow \mathcal{X} \hookrightarrow \mathcal{V}^*$. Now consider the operator $\mathcal{A} : \mathcal{V} \to \mathcal{V}^*$ which satisfies boundedness and coercivity conditions. For the specific PDE in (1), the operator is $\mathcal{A}\phi = \alpha_1 \frac{d^2\phi}{d\xi^2} + \alpha_2 \frac{d\phi}{d\xi} + \alpha_3\phi$, $\phi \in \mathrm{Dom}(\mathcal{A})$. We define the disturbance operator $\mathcal{B}_1 : \mathbb{R} \to \mathcal{V}^*$ and control input operator $\mathcal{B}_2 : \mathbb{R} \to \mathcal{V}^*$ via $\mathcal{B}_1 w(t) = b_1(\xi)w(t)$ and $\mathcal{B}_2 u(t) = b_2(\xi)u(t)$. Therefore, the PDE in (1) is written as

$$\dot{x}(t) = \mathcal{A}x(t) + \mathcal{B}_1 w(t) + \mathcal{B}_2 u(t), \quad x_0 \in \mathcal{X}. \tag{3}$$

The proposed method to place the N spatially distributed sensors within the region P, is to find the optimal spatial distribution of a single spatially distributed sensor constrained within the region P, and then approximate such a distribution by N test functions. Each of these test functions corresponds to the spatial distribution of the N sensors. The resulting Fourier coefficients of the approximation will be directly related to the position of the N sensing devices.

The optimization problem now is decomposed into two parts: (i) how to find the optimal distribution of a *single* *sensor* and (ii) how to find the position of the N sensors that best approximate the spatial distribution of the single optimal sensor distribution found in (i). However, such an optimization problem might be infeasible, since one is usually given the spatial distribution of a given sensing device. Thus, the above problems can be redefined and combined into a single part: (i)$'$ how to find the optimal distribution of a single sensor subject to the fact that the N individual sensors have a prescribed spatial distribution. This will also provide the position of the N sensing devices within the domain of definition P of the equivalent single sensor.

III. Optimization of sensor distribution and location

Before we proceed with the coordinated motion of the sensor network, we describe the case in which one first finds the optimal location and optimal distribution of a single fixed-in-space sensor that is distributed over the region P. Such a constrained optimization relies on the optimization of the observability measure, i.e. find the sensor position (centroid of sensor distribution) that maximizes the system's observability. This is made possible by optimizing the location-plus-distribution-parameterized observability gramian.

A. Optimization of spatial distribution and spatial location of a single fixed-in-space sensor

Earlier approaches on sensor optimization focused on the optimal position of a sensing device within the spatial domain, with the spatial distribution assumed given. The prevalent reason for such a justifiable assumption was that the sensor distribution was already a priori given by the devices used. No freedom for choosing the spatial distribution was incorporated as this was not realistic or even feasible - one would never be able to design a sensing device with a prescribed spatial distribution. However, one may be able to combine a finite number of sensing devices with a prescribed spatial distribution and consider them as an equivalent single sensing device with a spatial distribution equal to the weighted sum of the individual sensors' spatial distributions. This can be viewed as a way of strengthening the information (signal) from a given group of sensing devices by grouping them together in certain orientation. An approximation to a sensing device with a uniform-in-space spatial distribution (i.e. boxcar function) is the grouping of sensors with spatial delta (pointwise) distributions. This is illustrated in Figure 2(a).

When N point sensors are placed next to each other with equal spacing between them, then their combination (interpolation) can be viewed as equivalent to a single distributed sensor with a spatial distribution given by the boxcar function Figure 2(a). However, when three different clusters of point sensors are grouped together, the resulting interpolated spatial distribution for an equivalent single sensor has a non-constant distribution, see Figure 2(b), where the equivalent spatial distribution of a single sensor has a W-shape.

Here both the distribution and location (positioning of the subregion P within Ω) will be found in an optimal way. The i^{th} distributed output may be written as $y_i(t,\xi) = c_i(\xi)x(t,\xi)$. When the finite dimensional approximation $x(t,\xi) = \sum_{j=1}^{n} x_j(t)\phi_j^n(\xi)$ is used, then we can assume the expansion $y_i(t,\xi) = \sum_{j=1}^{n} y_{ij}(t)\phi_j^n(\xi)$, where $y_{ij}(t),\ j = 1,\dots,n$ are the Fourier expansion coefficients for the i^{th} sensor. At the same time

$$y_i(t,\xi) = c_i(\xi)x(t,\xi) = c_i(\xi)\sum_{j=1}^{n} x_j(t)\phi_j^n(\xi)$$

and therefore, when the above are viewed in weak form

$$\int_0^{\ell} y_i(t,\xi)\phi_k^n(\xi)\,d\xi = \int_0^{\ell} c_i(\xi)\sum_{j=1}^{n} x_j(t)\phi_j^n(\xi)\phi_k^n(\xi)\,d\xi,$$

for $k = 1,\dots,n$, produce $My_i(t) = M_c^i x(t)$ where
$y_i(t) = \begin{bmatrix} y_{i1}(t) & \dots & y_{in}(t) \end{bmatrix}'$, $x(t) = \begin{bmatrix} x_1(t) & \dots & x_n(t) \end{bmatrix}'$
and M and M_c^i denote the mass and weighted mass matrices $[M]_{jk} = \int_0^{\ell}\phi_j(\xi)\phi_k(\xi)\,d\xi$, $[M_c^i]_{jk} = \int_0^{\ell} c_i(\xi)\phi_j(\xi)\phi_k(\xi)\,d\xi$, with $j,k = 1,\dots,n$. Therefore, the coefficients of the i^{th} distributed sensor are given by $y_i(t) = \left(M^{-1}M_c^i\right)x(t) = C^i x(t)$, with $C^i \triangleq \left(M^{-1}M_c^i\right)$. The matrix C^i, and more specifically the matrix M_c^i, contains all the information on the spatial distribution and location of the i^{th} distributed sensor. To optimize the location and distribution $c_i(\xi)$ is equivalent to optimizing an appropriate observability measure with respect to the parametrization of the matrix M_c^i. Towards that, we assume that the spatial distribution of a single distributed sensor $c_i(\xi)$ can be expressed as a weighted sum of some approximating functions. We denote by $\varphi_j^m(\xi),\ j = 1,\dots,m$ the spatial distributions of these m functions. Then the problem of optimizing the spatial distribution and centroid of a distributed sensor that is defined in a region P spanned by the $\varphi_j^m(\xi)$'s, reduces to locating the m functions in the region P. Thus $c_i(\xi) = \sum_{j=1}^{m} s_j^i\varphi_j^m(\xi)$. It should be noted that the individual shaping functions $\varphi_j^m(\xi)$ incorporate information on their location within P as well. When the above expansion is utilized in the expression for the Fourier coefficients of the distributed output signal, then

$$\begin{aligned} y_i(t) &= M^{-1}\Big(\sum_{k=1}^{m} s_k^i M_{ck}^i\Big)x(t) \\ &= \Big(\sum_{k=1}^{m} s_k^i M^{-1}M_{ck}^i\Big)x(t) = \Big(\sum_{k=1}^{m} s_k^i C_{ck}^i\Big)x(t), \end{aligned} \tag{4}$$

where the m matrices M_{ck}^i are given by

$$[M_{ck}^i]_{IJ} = \int_0^{\ell}\varphi_k^m(\xi)\phi_I^n(\xi)\phi_J^n(\xi)\,d\xi, \quad I,J = 1,\dots,n,$$

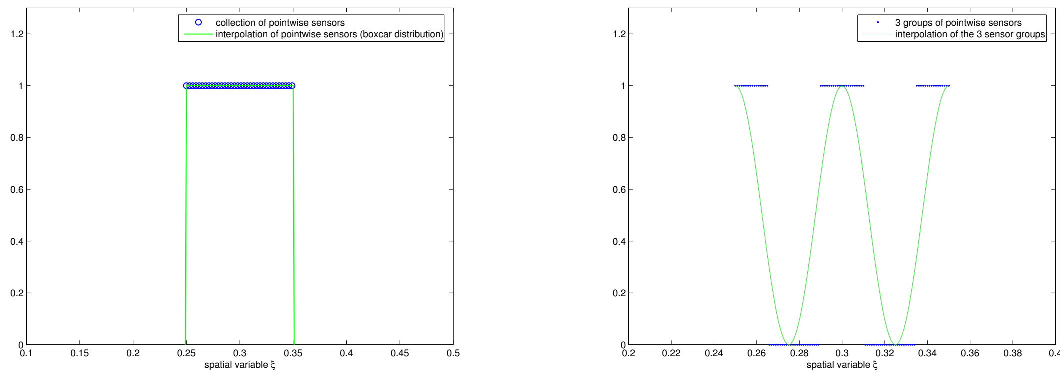
and the finite dimensional representation of the single spatially distributed sensor is given by

$$C_k^i = \Big(\sum_{k=1}^{m} s_k^i C_{ck}^i\Big) = \Big(\sum_{k=1}^{m} s_k^i M^{-1}M_{ck}^i\Big).$$

When the plant in (1) is approximated by the above and viewed in variational form, the following system emerges

$$\begin{aligned} \dot{x}(t) &= Ax(t) + B_1 w(t) + B_2 u(t), \\ y_i(t) &= C_k^i x(t), \quad i = 1,\dots,N. \end{aligned} \tag{5}$$

One way to find the optimal sensor location and distribution



(a) grouping of many point sensors equally spaced (blue dots) and equivalent interpolated spatial distribution of a single sensor having a boxcar distribution.

(b) combining three different clusters of point sensors (blue dots) and the equivalent interpolated spatial distribution of a single sensor.

Fig. 2. Combination of point sensors in different configurations and their equivalent interpolated spatial distributions for a single sensor.

for the system, is to enhance its observability. This is done by parameterizing the observability gramian by the parameterized sensor location and optimize the resulting gramian with respect to the location that makes the system "more" observable. Thus, the optimal distribution and location is

$$c_i^{opt}(\xi) = \arg\max \|S(\xi;\xi_i)\| \qquad (6)$$

where $S(\xi;\xi_i)$ denotes the location-parameterized gramian $S(\xi;\xi_i)A' + AS(\xi;\xi_i) = -(C_k^i)'C_k^i$.

B. Spatial orientation of N fixed-in-space sensors within subdomain P

Once the spatial distribution and location of a single sensor is found, then one may find the equivalent distribution of N individual sensors with a prescribed spatial distribution. This is in essence an interpolation of a given spatial function by a set of given test functions; the test functions are the spatial distributions of the sensing devices. In this case, the spatial distribution of each of the sensing devices is $c_i(\xi)$ and thus the N dimensional output is given by

$$\begin{aligned} y(t,\xi) &= \begin{bmatrix} y_1(t,\xi) & \ldots & y_N(t,\xi) \end{bmatrix}' \\ &= \begin{bmatrix} c_1(\xi)x(t,\xi) & \ldots & c_N(t,\xi)x(t,\xi) \end{bmatrix}'. \end{aligned}$$

Following the earlier results for a single sensor, the finite dimensional representation of the N dimensional vector of distributed outputs is given by

$$\begin{aligned} y(t,\xi) &= \begin{bmatrix} \left(\sum_{k=1}^m s_k^1 C_{ck}^1\right)' & \ldots & \left(\sum_{k=1}^m s_k^N C_{ck}^N\right)' \end{bmatrix}' x(t) \\ &= \begin{bmatrix} (C_k^1)' & \ldots & (C_k^N)' \end{bmatrix}' x(t). \end{aligned}$$

The above may then be used to parameterize the centroid-which would determine the location of the sensor within the spatial domain-with respect to the centroids. Enhancing the observability of the resulting system would then provide the optimal location of the N sensors. However, this may lead to many algorithmic and computation issues. Since this is beyond the scope of this work, we simply pose the optimization problem and direct the reader to already established results that deal with positioning of multiple sensing devices in systems governed by PDEs.

We denote the N dimensional vector of the centroids of the sensing devices by $\vec{\xi} = \begin{bmatrix} \xi_1 & \xi_2 & \ldots & \xi_N \end{bmatrix}'$ and parameterize the $N \times n$ output matrix by $\vec{\xi}$. Then the optimal location is given by

$$\vec{\xi}^{opt} = \arg\max \|S(\xi;\vec{\xi})\|$$

where $S(\xi;\vec{\xi})$ is the sensor centroid-parameterized observability gramian $S(\xi;\vec{\xi})A' + AS(\xi;\vec{\xi}) = -(C_k^i(\vec{\xi}))'C_k^i(\vec{\xi})$.

C. Practical considerations: finding the optimal distribution of a single sensor by combining N individual sensors with prescribed spatial distributions

In this case, one combines the N distributed outputs into a single distributed output. In terms of the spatial distributions of the sensing devices, this is expressed as

$$c_{equiv}(\xi) = \sum_{i=1}^N \alpha_i c_i(\xi) \qquad (7)$$

where $c_i(\xi)$ denote the spatial distributions of each of the N sensing devices defined over a spatial region P, and the problem now is to find the optimal weights α_i such that the observability of the resulting system is enhanced. By adopting the notation in § III-A, we denote by C^i the finite dimensional representation of each of the N sensing devices and thus the above becomes $C_{equiv}(\vec{\alpha}) = \sum_{i=1}^N \alpha_i C^i$. Therefore, the optimization becomes

$$\vec{\alpha}^{opt} = \arg\max \|S(\xi;\vec{\alpha})\| \qquad (8)$$

with $\vec{\alpha} = [\alpha_1 \ldots, \alpha_N]$, where $S(\xi;\vec{\alpha})$ is the sensor weight-parameterized observability gramian $S(\xi;\vec{\alpha})A' + AS(\xi;\vec{\alpha}) = -C'_{equiv}(\vec{\alpha})C_{equiv}(\vec{\alpha})$. One may impose constraints on the weights of the form $\sum_{i=1}^N \alpha_i \le N$ and $\alpha_i \in \{0,1\}$ to ensure that the equivalent sensor is comprised of individual sensors with either full strength ($\alpha_i = 1$), or zero strength ($\alpha_i = 0$).

In the remainder, it is assumed that one is given N individual sensors with prescribed spatial distributions $c_i(\xi)$ and that the equivalent sensor is found by constraining the weights α_i to take values in the set $\{0,1\}$ (i.e. α_i is either zero or one), and that the above method for finding these weights by enhancing the observability of the system is

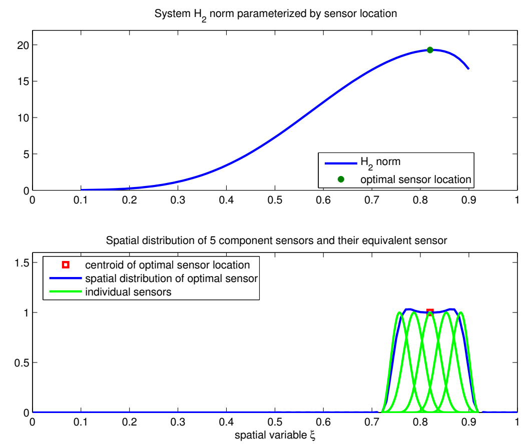


Fig. 3. Optimal sensor location by optimization of the H_2 system norm and spatial distribution of optimally located sensor.

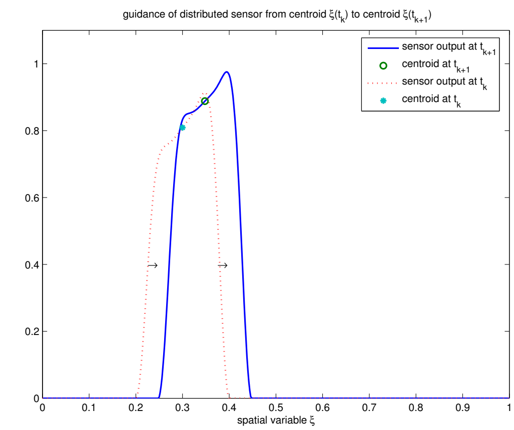


Fig. 4. Scheduling of a "moving" sensor from centroid position $\xi_i(t_k)$ to position $\xi_i(t_{k+1})$ using spatially distributed output estimation error $\varepsilon(t;\xi_i)$.

used. To consider the motion of the single equivalent sensor, it is further assumed that the spatial distribution does not change, i.e. the relation of the individual sensors to each other (i.e. the weights α_i) are fixed initially and are the same for all times, as in the case of a sensor that does not move. The only varying parameter is simply the centroid (location) of these sensors. As an example, we consider $N=5$ and set $\alpha_i = 1$, $i = 1,\ldots,5$. These five sensors would cover 20% of the spatial domain $[0,1]$. The observability gramian was parameterized by the single equivalent sensor resulting from the addition of these 5 sensors. Therefore, the interval $[0.1, 0.9]$[1] was divided into 100 equidistant points and each one serves as the candidate centroid of the single sensor. The observability gramian was found for each of these 100 points and the resulting H_2 norm of the system $(A, B_1, C_{equiv}(\vec{\alpha}))$ was evaluated and plotted in Figure 3a.[2] In Figure 3b, the spatial distribution of the optimal location of the single sensor, resulting from the maximum of the H_2 norm, is depicted along the prescribed spatial distributions of the 5 individual sensors.

IV. State estimation with a single "moving" sensor

It is henceforth assumed that one has a single distributed sensor to obtain information on the process. This translates to a time varying sensor centroid $\xi_i(t)$. For a sensor with a moving centroid, the process is described by

$$\begin{aligned}\dot{x}(t) &= \mathcal{A}x(t) + \mathcal{B}_1 w(t) + \mathcal{B}_2 u(t)\\ y(t;\xi_i(t)) &= \mathcal{C}(\xi_i(t))x(t) + \mathcal{D}(\xi_i(t))v(t)\end{aligned} \tag{9}$$

where $\mathcal{D}(\xi_i(t))$ denotes the operator associated with the spatial distribution of the measurement noise, and $v(t)$ its associated temporal component. The state estimator is

$$\dot{\widehat{x}}(t) = \mathcal{A}\widehat{x}(t) + \mathcal{B}_2 u(t) + \mathcal{L}(\xi_i(t))\left(y(t;\xi_i(t)) - \mathcal{C}(\xi_i(t))\widehat{x}(t)\right) \tag{10}$$

where $\widehat{x}(0) = \widehat{x}_0$ with $\widehat{x}(0) \neq x(0)$. The associated *state estimation error* $e(t) \triangleq x(t) - \widehat{x}(t)$ is governed by

$$\begin{aligned}\dot{e}(t) &= \Big(\mathcal{A} - \mathcal{L}(\xi_i(t))\mathcal{C}(\xi_i(t))\Big)e(t) + \mathcal{B}_1 w(t)\\ &\qquad - \mathcal{L}(\xi_i(t))\mathcal{D}(\xi_i(t))v(t)\\ e(t) &= x(0) - \widehat{x}(0).\end{aligned} \tag{11}$$

Due to the nature of the sensing device, the *output estimation error* $\varepsilon(t;\xi_i(t)) \triangleq y(t;\xi_i(t)) - \mathcal{C}(\xi_i(t))x(t)$ is also spatially distributed and *is an available signal*. The output estimation error provides distributed information throughout the spatial support of the sensing device, i.e. over $[\xi_i(t) - \Delta\xi, \xi_i(t) + \Delta\xi]$. Specifically, the spatial support of the single moving distributed sensor is given by $P = [\xi_i(t) - \Delta\xi, \xi_i(t) + \Delta\xi]$, where $\Delta\xi$ is the one-half spatial support of the sensing device. The proposed moving sensor guidance policy for improved state estimation is based on a gradient scheme, whereby one considers the distributed output estimation error over a given time interval, and chooses the next sensor position to the location of the largest value of the distributed error. An indirect way to consider delays due to the motion of a mobile sensor, is to assume that over a given time interval, the sensor can move a maximum of a distance $\Delta\xi$ from its current position. We consider the time instances $t_0 + k\Delta t$, $k = 0,1,2,\ldots$. The proposed mobile sensor centroid is

$$\xi_i(t_{k+1}) = \arg \max_{\xi_i(t_k)-\Delta\xi \leq \xi \leq \xi_i(t_k)+\Delta\xi} |\varepsilon(t;\xi_i(t_k))|, \tag{12}$$

which basically finds the maximum value of the spatially distributed output estimation error over the domain $[\xi(t_k) - \Delta\xi, \xi(t_k) + \Delta\xi]$ of the current sensor position $\xi(t_k)$ and moves the sensor to that maximum. Figure 4 depicts a scenario with the current single sensor centroid $\xi(t_k) = 0.3$ and subsequent sensor centroid $\xi(t_{k+1}) = 0.348$, with $\Delta\xi = 0.1$.

V. Numerical results

The PDE in (1) was simulated using $n = 100$ linear elements in $\Omega = [0,1]$ and an initial condition $x(0,\xi) =$

[1] the centroid of a single equivalent sensor is at the center of the spatial support which occupies 20% of the spatial domain; therefore the left endpoint of the centroid is at $\xi = 0.1$ with the sensor distributed over $[0, 0.2]$ and the right endpoint is at $\xi = 0.9$ with the sensor distributed over $[0.8, 1]$.

[2] The optimization method used here was essentially exhaustive search due to the low-dimensionality of the problem. In more elaborate geometries and higher dimensions (2D or 3D) a more sophisticated optimization method must be considered to make the proposed scheme computationally viable.

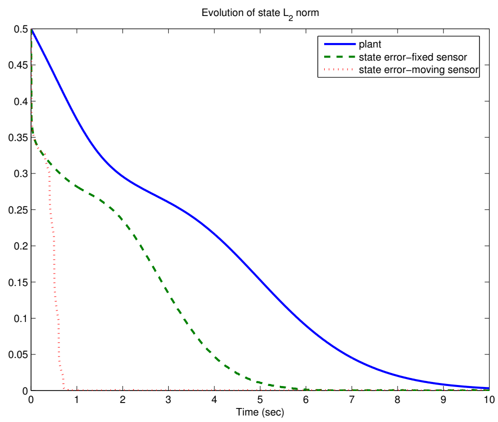


Fig. 5. L_2 norm evolution for state error with fixed sensor (dashed), state error with mobile sensor network (dotted). and for state (solid).

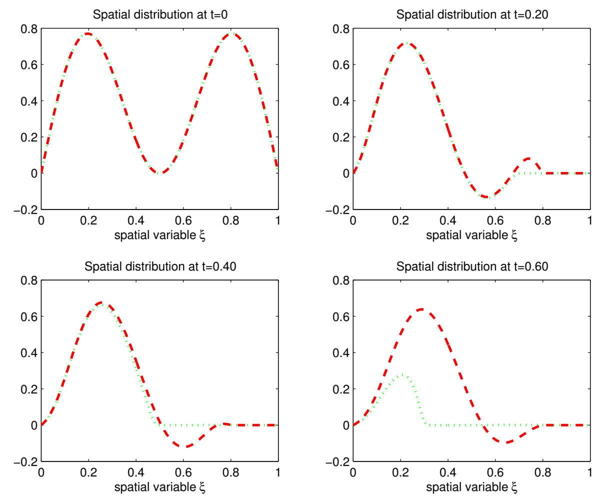


Fig. 6. Spatial distribution of state error $e(t,\xi)$ with fixed sensor (dashed) and state error with mobile sensor network (dotted) at different times.

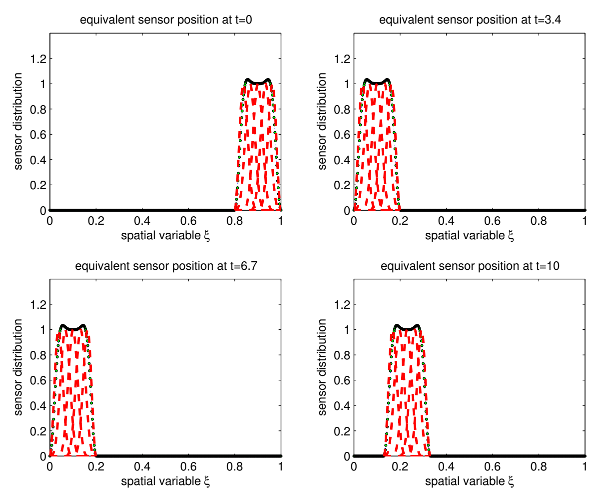


Fig. 7. Spatial distribution of equivalent single sensor (green) and mobile sensor network composed of 5 individual agents (red), at different times.

$\sin(2\pi\xi)\cos(\pi\xi)$ and $\widehat{x}(0,\xi)=0$. The coefficients of the elliptic operator were $\alpha_1=0.005, \alpha_2=-0.15, \alpha_3=-0.003$. For simplicity, $u=0$ and the spatial distribution of disturbances was $b_1(\xi)=\sin(\pi\xi)\sin(\pi\xi/2)$. A gaussian noise with zero mean and variances 10^{-3} and 10^{-5} were used for the process and measurement noise.

The state estimator given by (10) along with the single spatially distributed sensor scheduling policy (12) was implemented for the above system described by (9). The evolution of the state error norm when using a single equivalent distributed sensor that is mobile (red dotted line) and a single equivalent distributed sensor that is fixed (green dashed line) in the position $\xi(t)=0.9$, $\forall t>0$ are depicted in Figure 5. Additionally, the L_2 norm of the process state is included for comparison. It is observed that the norm of the estimation error converges to zero (in norm) faster when the equivalent single sensor is allowed to be moved within the spatial domain. Similarly, Figure 6 depicts the state estimation error $e(t,\xi)$ vs the spatial variable at different time instances. Finally, the single sensor distribution along with the 5 individual mobile sensors are depicted in Figure 7 for different time instances.

References

[1] J. Baillieul and P. J. Antsaklis, *Control and communication challenges in networked real-time systems*, 95 (2007), pp. 9–28.

[2] C. G. Cassandras and W. Li, *Sensor networks and cooperative control*, European Journal of Control, 11 (2005), pp. 436–463.

[3] F. Cucker and S. Smale, *Emergent behavior in flocks*, IEEE Transactions on Automatic Control, 52 (2007), pp. 852–862.

[4] M. A. Demetriou, *Power management of sensor networks for detection of a moving source in 2-D spatial domains*, in Proceedings of the American Control Conference, Minneapolis, MN, June 2006.

[5] ———, *Detection and containment policy of moving source in 2-d diffusion processes using sensor/actuator network*, in Proceedings of the European Control Conference, Kos, Greece, July 2-5 2007.

[6] ———, *Process estimation and moving source detection in 2-D diffusion processes by scheduling of sensor networks*, in Proceedings of the American Control Conference, New York, NY, July 11-13 2007.

[7] I. Hussein and M. A. Demetriou, *Estimation of distributed processes using mobile spatially distributed sensors*, in Proceedings of the American Control Conference, New York, NY, July 11-13 2007.

[8] R. Olfati-Saber, *Distributed tracking for mobile sensor networks with interaction-driven mobility*, in Proceedings of the American Control Conference, New York, NY, July 11-13 2007.

[9] Z. Song, Y. Chen, J. Liang, and D. Uciński, *Optimal mobile sensor motion planning under nonholonomic constraints for parameter estimation of distributed systems*, in Proceedings of the IEEE/RSJ Intl. Conference on Intelligent Robots and Systems, 2-6 August 2005.

[10] D. Uciński, *Sensor network design for parameter estimation of distributed systems using nonsmooth optimality criteria*, in Proc. of the 44th IEEE Conference on Decision and Control, Spain.

[11] D. Uciński, *Optimal Measurement Methods for Distributed Parameter System Identification*, CDC Press, Boca Raton, 2004.

[12] D. Uciński and Y. Chen, *Time-optimal path planning of moving sensors for parameter estimation of distributed systems*, in Proc. of the 44th IEEE Conf. on Decision and Control, Spain.

[13] D. Uciński and J. Korbicz, *Path planning for moving sensors in parameter estimation of distributed systems*, in Proceedings of the 1st workshop on Robot motion and control, June 1999, pp. 273–278.

[14] R. Vidal, O. Shakernia, and S. Sastry, *Following the flock: distributed formation control with omnidirectional vision-based motion segmentation and visual servoing*, IEEE Robotics & Automation Magazine, (2004), pp. 14–20.

[15] J. Wloka, *Partial Differential Equations*, Cambridge University Press, Cambridge, 1987.