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H_∞ Control for a Class of Non-Minimum-Phase Cascade Switched Nonlinear Systems

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***Abstract*— This paper is concerned with the H_∞ control problem for a class of non-minimum phase cascade switched nonlinear systems. The system under consideration is composed of two cascade-connected parts which are also switched systems. Sufficient conditions under which the H_∞ control problem is solvable under an arbitrary switching law are presented. The Common Lyapunov function and the switched state feedback controller are constructed explicitly based on the structure characteristics of the switched system. The corresponding closed-loop switched system under consideration is globally asymptotically stable and achieves an prescribed L_2-gain. The proposed method does not rely on the solutions of Hamilton-Jacobi equations.**

I. INTRODUCTION

H_∞ control theory has become a powerful tool to solve robust stabilization and disturbance attenuation problem. Many results about nonlinear H_∞ control have appeared[1, 2]. The nonlinear H_∞ control problem has been solved either based on the passivity theory, or based on the nonlinear version of classical bounded real lemma. Both methods require solving Hamilton-Jacobi equations, which imposes a formidable difficulty. Therefore, many attempts have been made towards designing nonlinear H_∞ controllers by solving reduced-order Hamilton-Jacobi equations or without the need of solving Hamilton-Jacobi equations by focusing on some special class of nonlinear systems. For example, "normal form" and backstepping technique are used in [3, 4].

On the other hand, there has been increasing interest in the analysis and synthesis of switched systems. The intricate intersection between continuous and discrete dynamics of switched systems has motivated a large and growing body of research work on a diverse array of issues, including the modeling, optimization, stability analysis, and control. Among which the stability issues have been a major focus in studying switched systems [5-7]. Stability of switched systems under arbitrary switching law is a desirable property which can be assured by a common Lyapunov function, because this property enables one to seek for other system performances by switching without changing stability. Except the common Lyapunov function method some other techniques are summarized in the recent books [8, 9].

The H_∞ property analysis of switched systems is a valuable issue deserving us pay more attention to among the growing body of research works that focuses on switched systems. All of the stability analysis method appeared up to now have been used in studying H_∞ property of the switched system although the research work is comparatively fewer [10~13]. [10] addressed the stabilization and L_2-gain analysis problem for a class of uncertain discrete-time switched systems with switched Lyapunov function technique. The disturbance attenuation problem was analyzed in [11] using average dwell-time method. [12] studied the robust H_∞ control problem for a class of switched linear systems with uncertainties using multiple Lyapunov function method. The H_∞ control problem for a class of cascade nonlinear minimum-phase switched systems was considered in [13] by constructing a common Lyapunov function.

In this paper, we shall address the H_∞ control problem for a class of non-minimum phase cascaded switched nonlinear systems with external disturbance input. The switched system under consideration is composed of two cascade-connected nonlinear parts which are also switching systems, and the controlled output is corrupted by the disturbance input but does not involve the control input. We aim at designing a switched state feedback controller such that the closed-loop switched system is uniformly globally asymptotically stable and the L_2-gain, from the disturbance input to the controlled output, is not larger than a given value under an arbitrary switching law. Motivated by [3,14], we assume that the first part of the switched system can be decomposed into

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two cascade-connected parts: the first decomposed part is uniformly input-to-state stable, and the other is uniformly asymptotically stabilizable. The control law and the common Lyapunov function are explicitly designed under these assumptions. The advantage of this paper with respect to other approaches is that the construction of the common Lyapunov function does not rely on the solutions of Hamilton-Jacobi equations.

II. SYSTEM DESCRIPTION AND PROBLEM STATEMENT

We address the H_∞ control problem for non-minimum-phase switched cascaded nonlinear systems of the form,

$$\begin{cases} \dot{\eta} = f_{\sigma(t)}(\eta, \xi) + p_{\sigma(t)}(\eta, \xi)w, \\ \dot{\xi} = a_{\sigma(t)}(\eta, \xi) + b_{\sigma(t)}(\eta, \xi)u_{\sigma(t)} + c_{\sigma(t)}(\eta, \xi)w, \\ y = h_{\sigma(t)}(\eta, \xi) + d_{\sigma(t)}(\eta, \xi)w, \end{cases} \tag{1}$$

where $\eta \in R^{n-d}$, $\xi \in R^d$ are the states, $w(t) \in L_2[0, \infty)$ is the external disturbance input, $y \in R^m$ is the controlled output, $\sigma(t) : [0, +\infty) \to \underline{P} = \{1, \cdots, N\}$ is the switching law, which is a piecewise constant function of time. In specific situations, the value $\sigma(t)$ at a given time t might just depend on t or $x(t)$, or both. $u_i \in R^d$ is the control input, $f_i(\cdot,\cdot)$, $p_i(\cdot,\cdot)$, $a_i(\cdot,\cdot)$, $b_i(\cdot,\cdot)$, $c_i(\cdot,\cdot)$, $h_i(\cdot,\cdot)$, $d_i(\cdot,\cdot)$ are smooth real functions for $i = 1, \cdots, N$, and $f_i(0,0) = 0$, $a_i(0,0) = 0$, $h_i(0,0) = 0$, $b_i(\eta, \xi)$ is nonsingular for $\forall(\eta(t)^T, \xi(t)^T)^T \in R^n$, $i = 1, \cdots, N$. The switching signal $\sigma(t)$ can be characterized by the switching sequence:

$$\begin{aligned} \Sigma \ = \ & \{(\eta_0^T, \xi_0^T)^T; (i_0, t_0), (i_1, t_1), \cdots, \\ & (i_n, t_n), \cdots, | \ i_n \in \underline{p}, \ n \in N\}. \end{aligned} \tag{2}$$

in which t_0 is the initial time, $(\eta_0^T, \xi_0^T)^T$ is the initial state. When $t \in [t_k, t_{k+1})$, $\sigma(t) = i_k$, that is, the i_kth subsystem is activated. Therefore, the trajectory $x(t)$ of the switched system (1) is the trajectory $x_{i_k}(t)$ of the i_kth subsystem when $t \in [t_k, t_{k+1})$. In addition, we assume that the state of the switched system (1) does not jump at the switching instants, i.e. the trajectory $x(t)$ is everywhere continuous.

Consider the switched systems described by equations of the form

$$\dot{x}(t) = f_{\sigma(t)}(x, d). \tag{3}$$

where $x(t) \in R^n$ is the state, $\sigma(t) : [0, \infty] \to \underline{P} = \{1, \ldots, N\}$ is the switching signal defined as in (1). d is a measurable locally bounded disturbance input.

Definition 1. The switched system (3) is said to be uniformly input-to-state stable if and only if there exists a proper, positive definite, and radially unbounded function $V(x)$ such that

$$\frac{\partial V(x)}{\partial x} f_i(x, d) \leq -\alpha(\|x\|) + \chi(\|d\|), \quad i \in \underline{P}. \tag{4}$$

for some class K_∞ functions $\alpha(\cdot)$ and $\chi(\cdot)$.

Throughout this paper, with abuse of terminology, we refer to the first equation of system (1) as the zero dynamics equation when $\xi = 0$. And we assume that the system (1) is stabilizable non-minimum phase and satisfies the following assumptions

Assumption 1. The η-part of the switched system (1) can be decomposed into the following two cascade-connected subsystems,

$$\begin{cases} \dot{\eta}_1 = f_{1\sigma(t)}(\eta_1, \eta_2, \xi) + p_{1\sigma(t)}(\eta_1, \eta_2, \xi)w, \\ \dot{\eta}_2 = f_{2\sigma(t)}(\eta_2, \xi), \end{cases} \tag{5}$$

Assumption 2. (i) For the η_1-subsystem there exist a proper, real-valued, and positive definite function $W_1(\eta_1)$, such that

$$\begin{aligned} & \frac{\partial W_1}{\partial \eta_1} [f_{1\sigma(t)}(\eta_1, \eta_2, \xi) + p_{1\sigma(t)}(\eta_1, \eta_2, \xi)w] \\ \leq \quad & -\alpha_1 \|\eta_1\|^2 + \gamma_1^2 \|w\|^2 + k_1(\eta_2, \xi), \end{aligned} \tag{6}$$

for some positive definite function $k_1(\eta_2, \xi)$ and some positive constants α_1, γ_1.

(ii) For the η_2-subsystem there exist a switching-independent real valued function $\phi(\eta_2)$, a proper, and positive definite function $W_2(\eta_2)$, such that

$$\frac{\partial W_2}{\partial \eta_2} f_{2\sigma(t)}(\eta_2, \phi(\eta_2)) \leq -\alpha_2 W_2(\eta_2), \tag{7}$$

$$\|W_2(\eta_2)\| \geq \alpha_3 \|\eta_2\|^2. \tag{8}$$

for some positive constants α_2, α_3.

Assumption 3. The controlled output y of system (1) is of the form

$$y = h_i(\eta_2, \xi) + d_i(\eta, \xi)w, \qquad i \in \underline{P}. \tag{9}$$

where $h_i(\eta_2, \xi)$ are smooth functions with $h_i(0,0) = 0$, and $d_i(\eta, \xi)$ are uniformly bounded, i.e.

$$\|d_i(\eta, \xi)\| \leq \gamma_d. \qquad i \in \underline{P} \tag{10}$$

for some positive constant γ_d.

Remark 1. (i) of Assumption 2 indicates that the η_1-subsystem is uniformly input-to-state stable (see Definition 1). (ii) of Assumption2 indicates that the η_2-subsystem can be globally uniformly asymptotically stabilized by the switching-independent state feedback control law $\phi(\eta_2)$.

Remark 2. Assumption 1 is not conservative. In fact, even for non-switched linear system, similar assumptions is indeed necessary. Due to the complexity of switched systems, this assumption is reasonable. this can be seen from the fact that for a switched nonlinear system, asymptotic stability with zero input of the η_1-subsystem does not imply asymptotic stability when the input is not zero. Asymptotic stability with zero input of the η_1-subsystem and asymptotic stabilizability of the η_2-subsystem are not sufficient, in general. to guarantee the solution of the problem. Somewhat stronger conditions are needed. For the η_1-subsystem a form of uniformly input-to-state stability is required, while for the η_2-subsystem a particular form of uniformly stabilizability is needed.

Remark 3. The Assumption of the existence of a switching-independent state feedback $\phi(\eta_2)$ in (ii) of Assumption 2 is reasonable, this is a phenomena that does exist in switched systems.

Remark 4. It is easy to observe that the uniformly boundedness of $d_i(\eta,\xi)$ is necessary for the L_2-gain from the disturbance input w to the controlled output y to be bounded.

This paper addresses the following H_∞ control problem:

Given any constant $\gamma > \gamma_d$, design a switched state feedback controller $u_i = u_i(\eta,\xi)$, with $u_i(0,0) = 0$ for system (1), such that

(a) the corresponding closed-loop switched system (1) is globally uniformly asymptotically stable with $w = 0$.

(b) for some real-valued function $\beta : R^{n-d} \times R^d \to R$ with $\beta(0,0) = 0$

$$\int_0^\infty y^T(t)y(t)dt \le \gamma^2 \int_0^\infty w^T(t)w(t)dt + \beta(\eta_0,\xi_0), \quad (11)$$

holds for any initial condition $(\eta_0^T,\xi_0^T)^T \in R^n$. Where $w(t) \in L_2[0,+\infty)$.

III. MAIN RESULT

In this section, we derive sufficient conditions under witch the H_∞ control problem of system (1) under arbitrary switching law is solvable. And a nonlinear switched state feedback controller is also explicitly constructed.

Theorem 1 Given any constant $\gamma > \gamma_d$, if the disturbed switched nonlinear system (1) satisfies Assumptions 1∼3, then, the global H_∞ control problem for system (1) under arbitrary switching law is solvable.

Proof. For system (1), define the following coordinate transformation:

$$\eta = \eta, \qquad \zeta = \xi - \phi(\eta_2). \quad (12)$$

where $\phi(\eta_2)$ is as in (ii) of Assumption 2.

Applying this coordinate transformation and taking Assumption 1 into consideration, we can transform the η_2-subsystem into

$$\dot{\eta}_2 = \hat{f}_{2i}(\eta_2) + \tilde{f}_{2i}(\eta_2,\zeta)\zeta, \quad (13)$$

where $\hat{f}_{2i}(\eta_2) = f_{2i}(\eta_2,\phi(\eta_2))$, $\hat{f}_{2i}(0) = 0$. In view of (ii) of Assumption 2, (7) becomes

$$\frac{\partial W_2}{\partial \eta_2}\hat{f}_{2\sigma(t)}(\eta_2) \le -\alpha_2 W_2(\eta_2). \quad (14)$$

Using (1) and completing the squares, we obtain

$$\begin{aligned} & \|y\|^2 - \gamma^2\|w\|^2 \\ \le \quad & h_i^T(I + \frac{1}{\gamma_2^2}d_i^T d_i)h_i - w^T[(\gamma^2-\gamma_2^2)I - d_i^T d_i]w, \end{aligned}$$

It follows from (10) that

$$\|y\|^2 - \gamma^2\|w\|^2 \le (1+\frac{\gamma_d^2}{\gamma_2^2})\|h_i\|^2 - \gamma_3^2\|w\|^2, \quad (15)$$

where γ_d is as defined in Assumption 3, γ_2, γ_3 are two positive real constants satisfying $\gamma_3^2 = \gamma^2 - \gamma_2^2 - \gamma_d^2 > 0$.

For any positive constant $\tilde{\gamma} > \gamma_1$, from Assumption 3, we know that there exist positive definite functions $H_i(\eta_2,\zeta)$, $i \in \underline{P}$, such that

$$k_1(\eta_2,\xi) + \frac{\tilde{\gamma}^2}{\gamma_3^2}(1+\frac{\gamma_d^2}{\gamma_2^2})\|h_i\|^2 \le H_i(\eta_2,\zeta),$$

where γ_1, $k_1(\eta_2,\xi)$ are as defined in Assumption 1. Let $H(\eta_2,\zeta) = \max\{H_i : i \in \underline{P}\}$, we have

$$k_1(\eta_2,\xi)+\frac{\tilde{\gamma}^2}{\gamma_3^2}(1+\frac{\gamma_d^2}{\gamma_2^2})\|h_i\|^2 \le H(\eta_2,\zeta), \qquad i \in \underline{P}. \quad (16)$$

Since $H(\eta_2,\zeta)$ is also a positive definite function, we have $H(0,0) = 0$. Thus, $H(\eta_2,\zeta)$ can be decomposed as:

$$H(\eta_2,\zeta) = H_1(\eta_2) + H_2(\eta_2,\zeta)\zeta, \quad (17)$$

where $H_1(\eta_2) = H_1(\eta_2,0)$ and $H_1(0) = 0$.

Since $W_2(\eta_2)$ is positive definite and radially unbounded, there exists a class K_∞ function $k : R^+ \to R^+$, such that

$$H_1(\eta_2) + \|\eta_2\|^2 \le k(W_2(\eta_2)). \quad (18)$$

Define a K_∞ function $S : R^+ \to R^+$ as follows:

$$S(W_2) = W_2 \sup_{0\le t\le 1}\frac{dk(t)}{dt} + \int_{W_2}^{2W_2} k(t)dt. \quad (19)$$

From [4], we know that $S(W_2)$ satisfies:

$$S(W_2) \geq k(W_2), \qquad W_2\frac{dS(W_2)}{dW_2} \geq S(W_2). \tag{20}$$

Next, construct the composite common Lyapunov function W for system (1) as:

$$W(\eta_1,\eta_2,\xi) = W_1(\eta_1) + \frac{1}{\alpha_1}S(W_2(\eta_2)) + \frac{1}{2}\zeta^T\zeta, \tag{21}$$

where $W_1(\eta_1)$, α_1 are as defined in (i) of Assumption 2 and (ii) of Assumption 2 respectively.

In view of (12), (13) and Assumption 1, we can calculate the time derivative of W as follows:

$$\begin{aligned}\dot{W} \;=\; & \frac{\partial W_1}{\partial \eta_1}\big[f_{1i}(\eta_1,\eta_2,\xi) + p_{1i}(\eta_1,\eta_2,\xi)w\big] + \frac{1}{\alpha_2}\frac{dS(W_2)}{dW_2} \\ & \cdot\frac{\partial W_2}{\partial \eta_2}\big[\hat{f}_{2i}(\eta_2) + \tilde{f}_{2i}(\eta_2)\zeta\big] + \zeta^T\dot{\zeta},\end{aligned} \tag{22}$$

Moreover,

$$\begin{aligned}\dot{\zeta} \;=\; & a_i(\eta,\xi) + b_i(\eta,\xi)u_i + c_i(\eta,\xi)w \\ & -\frac{\partial\phi(\eta_2)}{\partial\eta_2}f_{2i}(\eta_2,\xi),\end{aligned} \tag{23}$$

Substituting (23) into (22), and taking Assumption 2, (14), (18), and (20) into consideration, we obtain

$$\begin{aligned}\dot{W} \;\leq\; & -\alpha_1\|\eta_1\|^2 + \gamma_1^2\|w\|^2 + k_1(\eta_2,\xi) - \frac{dS(W_2)}{dW_2}W_2(\eta_2) \\ & +\frac{1}{\alpha_2}\frac{dS(W_2)}{dW_2}\frac{\partial W_2}{\partial\eta_2}\tilde{f}_{2i}(\eta_2)\zeta + \zeta^T\big[a_i(\eta,\xi) + b_i(\eta,\xi)u_i \\ & +c_i(\eta,\xi)w - \frac{\partial\phi(\eta_2)}{\partial\eta_2}f_{2i}(\eta_2,\xi)\big] \\ \leq\; & -\alpha_1\|\eta_1\|^2 + \gamma_1^2\|w\|^2 + k_1(\eta_2,\xi) - k(W_2) \\ & +\zeta^T\Big[a_i(\eta,\xi) + b_i(\eta,\xi)u_i - \frac{\partial\phi(\eta_2)}{\partial\eta_2}f_{2i}(\eta_2,\xi) \\ & +\frac{1}{\alpha_2}\Big(\frac{dS(W_2)}{dW_2}\frac{\partial W_2}{\partial\eta_2}\tilde{f}_{2i}(\eta_2)\Big)^T + c_i(\eta,\xi)w\Big] \\ \leq\; & -\alpha_1\|\eta_1\|^2 + \gamma_1^2\|w\|^2 + k_1(\eta_2,\xi) - H_1(\eta_2) - \|\eta_2\|^2 \\ & +\zeta^T\Big[a_i(\eta,\xi) + b_i(\eta,\xi)u_i - \frac{\partial\phi(\eta_2)}{\partial\eta_2}f_{2i}(\eta_2,\xi) \\ & +\frac{1}{\alpha_2}\Big(\frac{dS(W_2)}{dW_2}\frac{\partial W_2}{\partial\eta_2}\tilde{f}_{2i}(\eta_2)\Big)^T + c_i(\eta,\xi)w\Big],\end{aligned} \tag{24}$$

From (15), (16), and (17), it follows that

$$\begin{aligned}\frac{\tilde{\gamma}^2}{\gamma_3^2}(\|y\|^2 - \gamma^2\|w\|^2) \leq\; & \frac{\tilde{\gamma}^2}{\gamma_3^2}(1 + \frac{\gamma_d^2}{\gamma_2^2})\|h_i\|^2 - \tilde{\gamma}^2\|w\|^2 \\ \leq\; & H_1(\eta_2) + H_2(\eta_2,\zeta)\zeta - k_1(\eta_2,\xi) \\ & -\tilde{\gamma}^2\|w\|^2.\end{aligned} \tag{25}$$

Combining (24) and (25), gives

$$\begin{aligned}& \dot{W} + \frac{\tilde{\gamma}^2}{\gamma_3^2}(\|y\|^2 - \gamma^2\|w\|^2) \\ \leq\; & -\alpha_1\|\eta_1\|^2 - \gamma_4^2\|w\|^2 - \|\eta_2\|^2 + \zeta^T\Big[a_i(\eta,\xi) \\ & +b_i(\eta,\xi)u_i + c_i(\eta,\xi)w - \frac{\partial\phi(\eta_2)}{\partial\eta_2}f_{2i}(\eta_2,\xi) \\ & +H_2^T(\eta_2,\zeta) + \frac{1}{\alpha_2}\Big(\frac{dS(W_2)}{dW_2}\frac{\partial W_2}{\partial\eta_2}\tilde{f}_{2i}(\eta_2)\Big)^T\Big] \\ \leq\; & -\alpha_1\|\eta_1\|^2 - \|\eta_2\|^2 + \zeta^T\Big[a_i(\eta,\xi) + b_i(\eta,\xi)u_i \\ & -\frac{\partial\phi(\eta_2)}{\partial\eta_2}f_{2i}(\eta_2,\xi) + \frac{1}{\alpha_2}\Big(\frac{dS(W_2)}{dW_2}\frac{\partial W_2}{\partial\eta_2}\tilde{f}_{2i}(\eta_2)\Big)^T \\ & +H_2^T(\eta_2,\zeta)\Big] - \gamma_4^2\|w\|^2 + \zeta^Tc_iw \\ \leq\; & -\alpha_1\|\eta_1\|^2 - \|\eta_2\|^2 + \zeta^T\Big[a_i(\eta,\xi) + b_i(\eta,\xi)u_i \\ & -\frac{\partial\phi(\eta_2)}{\partial\eta_2}f_{2i}(\eta_2,\xi) + \frac{1}{\alpha_2}\Big(\frac{dS(W_2)}{dW_2}\frac{\partial W_2}{\partial\eta_2}\tilde{f}_{2i}(\eta_2)\Big)^T \\ & +H_2^T(\eta_2,\zeta)\Big] - (\gamma_4w - \frac{1}{\gamma_4}c_i^T\zeta)^T(\gamma_4w - \frac{1}{\gamma_4}c_i^T\zeta) \\ & +\frac{1}{\gamma_4^2}\zeta^Tc_ic_i^T\zeta \\ \leq\; & -\alpha_1\|\eta_1\|^2 - \|\eta_2\|^2 + \zeta^T\Big[a_i(\eta,\xi) + b_i(\eta,\xi)u_i \\ & -\frac{\partial\phi(\eta_2)}{\partial\eta_2}f_{2i}(\eta_2,\xi) + \frac{1}{\alpha_2}\Big(\frac{dS(W_2)}{dW_2}\frac{\partial W_2}{\partial\eta_2}\tilde{f}_{2i}(\eta_2)\Big)^T \\ & +\frac{1}{\gamma_4^2}c_ic_i^T\zeta + H_2^T(\eta_2,\zeta)\Big],\end{aligned}$$

where $\gamma_4^2 = \tilde{\gamma}^2 - \gamma_1^2$.

Design the state feedback controller as:

$$\begin{aligned}u_i = b_i^{-1}(\eta,\xi)\Big[& -a_i(\eta,\xi) + \frac{\partial\phi(\eta_2)}{\partial\eta_2}f_{2i}(\eta_2,\xi) - H_2^T(\eta_2,\zeta) \\ & -\frac{1}{\alpha_2}\Big(\frac{dS(W_2)}{dW_2}\frac{\partial W_2}{\partial\eta_2}\tilde{f}_{2i}(\eta_2)\Big)^T - \frac{1}{\gamma_4^2}c_ic_i^T\zeta - \zeta\Big].\end{aligned} \tag{26}$$

Thus, we have,

$$\dot{W} + \frac{\tilde{\gamma}^2}{\gamma_3^2}(\|y\|^2 - \gamma^2\|w\|^2) \leq -\alpha_1\|\eta_1\|^2 - \|\eta_2\|^2 - \|\zeta\|^2, \tag{27}$$

which means

$$\dot{W} + \frac{\tilde{\gamma}^2}{\gamma_3^2}(y^Ty - \gamma^2w^Tw) \leq 0, \qquad \forall t \geq 0. \tag{28}$$

For $\forall T \geq 0$, we let $t_j \leq T \leq t_{j+1}$ for some j. Integrating both sides of (28) from $t_0 = 0$ to T yields

$$\begin{aligned}& \int_0^T \big[\dot{W} + \frac{\tilde{\gamma}^2}{\gamma_3^2}(y^Ty - \gamma^2w^Tw)\big]dt \\ =\; & \int_0^{t_1} \big[\dot{W} + \frac{\tilde{\gamma}^2}{\gamma_3^2}(y^Ty - \gamma^2w^Tw)\big]dt + \int_{t_1}^{t_2} \big[\dot{W} + \frac{\tilde{\gamma}^2}{\gamma_3^2} \\ & \cdot(y^Ty - \gamma^2w^Tw)\big]dt + \ldots \\ & + \int_{t_i}^{T} \big[\dot{W} + \frac{\tilde{\gamma}^2}{\gamma_3^2}(y^Ty - \gamma^2w^Tw)\big]dt\end{aligned}$$

$$= \int_0^{t_1} \dot{W}dt + \int_{t_1}^{t_2} \dot{W}dt + \ldots + \int_{t_j}^{T} \dot{W}dt + \frac{\tilde{\gamma}^2}{\gamma_3^2}\int_0^T (y^T y - \gamma^2 w^T w)dt$$
$$= W(T) - W(0) + \frac{\tilde{\gamma}^2}{\gamma_3^2}\int_0^T (y^T y - \gamma^2 w^T w)dt$$

which is equivalent to

$$\int_0^T y^T y \le \gamma^2 \int_0^T w^T w dt + \frac{\gamma_3^2}{\tilde{\gamma}^2} W(0), \quad \forall (\eta_0^T, \xi_0^T)^T \neq 0.$$

where $W(T) = W(W_1(\eta_1(T)), S(W_2(\eta_2(T))), \zeta(T))$, $W(0) = W(W_1(\eta_1(0)), S(W_2(\eta_2(0))), \zeta(0))$. Therefore, (10) holds under arbitrary switching law.

When $w = 0$, it follows from (27) that

$$\dot{W} \le -\alpha_1\|\eta_1\|^2 - \|\eta_2\|^2 - \|\zeta\|^2. \quad (29)$$

This implies that W is a common Lyapunov function of system (1), and thus the global asymptotic stability of system (1) with (26) with $w = 0$ under arbitrary switching law follows.

IV. EXAMPLE

Consider the switched system with the following two subsystems

$$\begin{cases} \dot{\eta} = -\eta^3 + \xi, \\ \dot{\xi} = \xi^2 + u_1 + \eta\xi^2 w, \\ y = \eta + w\sin\xi, \end{cases} \qquad \begin{cases} \dot{\eta} = -\eta - \eta\xi^2 \sin^2\eta, \\ \dot{\xi} = \eta^2 + u_2 + \xi^2 w, \\ y = \eta + w\cos\xi, \end{cases} \quad (30)$$

where $\eta \in R$, $\xi \in R$, $w \in R$, $\underline{P} = \{1, 2\}$. Let $\gamma = 2$, we will design a switched state feedback controller for switched system (30) such that the closed-loop system is uniformly globally asymptotically stable and the L_2-gain from the disturbance input w to the controlled output y is not larger than 2.

The system (30) is already in the form of (1). Note that this switched system contains only the η_2-subsystem, thus, $W_1 = 0$, $\alpha_1 = 0$, $\gamma_1 = 0$, $k_1(\eta, \xi) = 0$ in (i) of Assumption 2, and (ii) of Assumption 2 are satisfied with the $\phi(\eta_2) = \eta^3 - \eta$, the Lyapunov function $W_2 = \frac{1}{2}\eta^2$, $\alpha_2 = 2$, $\alpha_3 = 0.5$. Moreover, Assumption 3 are satisfied with $\gamma_d = 1$. i.e. Assumption $1 \sim 3$ hold. Therefore, Theorem 1 is applicable, then, we will apply theorem 1 to design a suitable switched state feedback controller for switched system (30).

Taking $\gamma_2 = 1$, $\gamma_3 = \sqrt{3}$, $\tilde{\gamma} = 1$, we can obtain

$$H(\eta_2,\ \xi) = \eta, \quad H_1(\eta_2) = \eta^2, \quad H_2(\eta_2, \xi) = 0,$$
$$k(W_2) = 4W_2, \quad S(W_2) = 4W_2 + 6W_2^2,$$

and $W = \eta^2 + \frac{3}{4}\eta^4 + \frac{1}{2}\xi^2$ as the common Lyapunov function. From (26), the switched state feedback controller can be taken as:

$$\begin{aligned} u_1 &= -\xi(\xi+2) - \eta(3\eta^4 + \eta^2 + 3) + \eta^2\xi(\eta^3\xi^3 - \eta\xi^3 \\ &\quad -\xi^4 + 3), \\ u_2 &= -\xi(\xi^4 + 1) - \eta^2(\eta^3\sin^2\eta + 2\eta + 1) - (3\eta^3\xi^2 \\ &\quad -\eta\xi^2 - 2\eta^2\xi + 2\eta^3 - 3\eta^4\xi - 3\eta^7)\sin^2\eta \\ &\quad +\eta\xi^4(\eta^2 - 1), \end{aligned}$$

Let the initial state $(\eta_0^T, \xi_0^T)^T = (-1.5, -2)$, Figure 1 shows the the state response of the closed-loop switched system (28) with the forgoing designed state feedback under an arbitrary chosen switching law when $w = \sin t$, which indicate that the feasibility of our result.

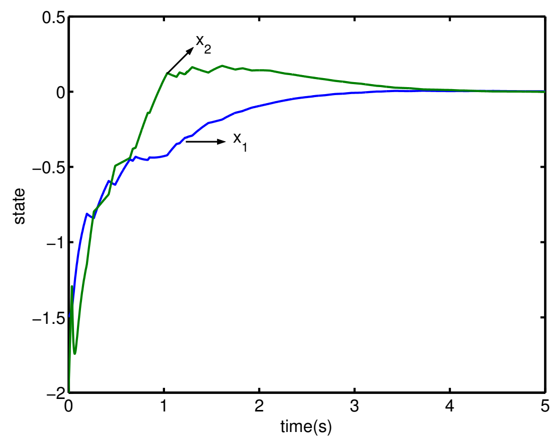

Fig. 1. The state response of the switched system (30)

V. CONCLUSIONS

In this paper, we have studied the H_∞ control problem for a class of non-minimum phase switched cascaded nonlinear systems with external disturbances. Sufficient conditions for the solvable of the H_∞ control problem under arbitrary switching law are presented. The common Lyapunov function is designed explicitly. An numerical example is given to illustrate the applicability of the method proposed. The H_∞ controllers designed according to the method proposed in this paper successfully stabilize the switched nonlinear cascaded system and meanwhile maintain an acceptable H_∞ disturbance attenuation level of the closed-loop system. The situation that when the state feedback in (ii) of Assumption 2 is switching-depend maintains for further study.

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