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Backstepping-based Landing Control of a RUAV using Tether Incorporating Flapping Correction Dynamics

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Abstract— **This paper presents a novel application of backstepping controller for landing of a Rotary wing UAV (RUAV) using a tether. This algorithm is an extension of a backstepping algorithm for general rigid bodies. In this paper, we will also present the detailed analysis of flapping correction dynamics by considering the practicality involved in the system. The performance of the proposed algorithm is demonstrated by simulation results. Future work is needed to implement the algorithm onboard RUAV platforms.**

I. INTRODUCTION

Rotary wing UAVs (RUAVs) with vertical take-off and landing capabilities have widely been used for military purposes. The hovering mode and flying at low speed makes RUAVs an ideal platform for reconnaissance missions to provide upper level information. These robots can also provide first hand information in rescue missions such as Katrina [1]. The overall objective of this research is the launch and recovery of RUAVs on moving platform. The method in this paper is proposed with a view to practical implementation on Eagle RUAV [2], [3, Fig. 1]. This research will also provide more enhanced multivariable control scheme for recovery operations of manned (full-scale) helicopters in high sea state to decrease the risk involved in safety of men and platform onboard ships.

An identified linear model for R50/ RMAX RUAVs is proposed using frequency domain identification methods [4]. A nonlinear RUAV model is augmented in [5] by considering only the first order effects due to a flybar and the flapping dynamics. A slightly different model is considered in [6], [7] by assuming that the main rotor blades are hinged directly from the hub and proposed new control variables, which are functions of the flapping angles and the main, tail rotor thrusts. The design of linear controller for RUAVs are proposed using LQG [8], H_2 [9], H_∞ [10], μ-synthesis [11] and dynamics inversion [12] methods.

The landing of a RUAV using tether as a guiding mechanism is proposed in [2] considers the direct control of the flapping angles. This is evident from the simulation results shown in [2] which is not suitable for implementation purposes and moreover the flapping angles are too far from their desired value. A selected literature review relating to the nonlinear control design techniques includes approximate input-output linearization [13], differential flatness [14], sliding mode [15], backstepping [16], [17], neural-network based controller [18], fuzzy control [19] and nonlinear H_∞ [20] control.

This paper presents a novel application of backstepping controller for landing control of a RUAV using tether. The innovation in this paper is the extension of the control algorithm in providing a *correction* for flapping dynamics. The flapping angles cannot be set directly because of the flapping and flybar dynamics. In this paper, a practical approach is presented to control the flapping dynamics indirectly. Moreover, the proposed algorithm is an extension of a backstepping algorithm for general rigid bodies, which holds good for the full envelop flight control of a RUAV.

The organization of the paper is as follows. In Section II, an overview of the nonlinear helicopter model is presented. In Section III, backstepping-based landing control of a RUAV using tether is discussed. This section also discusses a flapping correction dynamics to include in the proposed algorithm. The simulation results are given in Section IV followed by the conclusions of the paper.

II. RUAV MODEL

This section introduces the basic system blocks governing the complete dynamics of the RUAV. This model is based on the nonlinear rigid body dynamics [21], where forces and moments due to main rotor, tail rotor, fuselage and empennage are acting on the center of mass of the body. The position of the origin of the body is denoted by $\zeta = [x, y, z]^T$ in the inertial frame. Linear velocities along the axes of the body frame are given by $V = [u, v, w]^T$. The angular velocity expressed in the body frame is defined as $\omega = [p, q, r]^T$. The Euler angles denoted as $\eta = [\phi, \theta, \psi]^T$ established a kinematic relationship with the angular velocities $\dot{\eta} = \Pi\omega$, where Π is given by:

$$\Pi = \begin{bmatrix} 1 & \sin\phi\tan\theta & \cos\phi\tan\theta \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi\sec\theta & \cos\phi\sec\theta \end{bmatrix} \tag{1}$$

Assumption 1 *Euler angles are used in the model to represent the geometric coordinates. The representation in (1) has a geometric singularity at $\theta = \pm\frac{\pi}{2}$. It is assumed that the flight condition never reaches this singularity condition.*

The RUAV model is shown in Fig. 1. A brief description of each sub-system shown in Fig. 1 is given as follows:

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A. Rigid-Body Dynamics

The nonlinear rigid body dynamics in terms of the translational and rotation dynamics [16] is given by:

$$\dot{\zeta} = V \tag{2}$$
$$\dot{V} = -\omega \times V + \frac{f_b}{m} + \hat{g} \tag{3}$$
$$I\dot{\omega} = -\omega \times I\omega + M \tag{4}$$

where I is the inertia matrix and m is the mass of the body. The gravitational vector $\hat{g}$ is expressed in the body reference frame. Note that the external forces $f_b = [X, Y, Z]^T$ and moments $M = [L, R, N]^T$ are acting on the center of mass of the body due to main and tail rotor, fuselage and empennage.

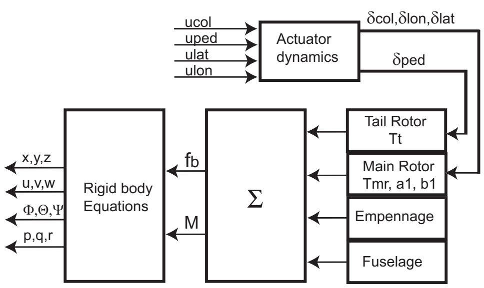

Fig. 1. Block diagram of a Rotary wing UAV dynamics

B. Main and tail rotor

In helicopters and RUAVs, the dominant response is due to the main and the tail rotor. The compilation of the forces and moments due to the main, tail rotor of a RUAV are given [22] as follows:

$$\begin{bmatrix} X_m \\ Y_m \\ Z_m \end{bmatrix} = \begin{bmatrix} -T_{mr}a_1 \\ T_{mr}b_1 + T_t \\ -T_{mr} \end{bmatrix} \tag{5}$$
$$\begin{bmatrix} L_m \\ R_m \\ N_m \end{bmatrix} = \begin{bmatrix} \frac{dL}{db_1}b_1 + Y_m.M_Z + Tt.T_Z \\ \frac{dM}{da_1}a_1 + X_m.M_Z \\ M_Q + Y_m.M_X + Tt.T_X \end{bmatrix} \tag{6}$$

where $\frac{dL}{db_1}, \frac{dM}{da_1}$ are the constant rolling and pitching moment hub stiffness. The main rotor coordinates M_X, M_Z and the tail rotor coordinates T_x, T_Z are with respect to the center of mass of the body. The same nonlinear expressions are used by the authors in [14] considering a small angle approximation. The main rotor torque M_Q can be computed as mentioned in [14].

Remark 1 There exists an algebraic relationship between the main, tail rotor thrusts (T_{mr}, T_t) and the servo actuator outputs $(\delta_{col}, \delta_{ped})$ [16, p. 1958] given by:

$$T_j = (w_j - v_{ij})\frac{\rho \Omega_j R_j^2 A_j B C_j}{4} \quad (j = mr, t) \tag{7}$$
$$v_{ij}^2 = \sqrt{(\frac{\hat{v}_j^2}{2})^2 + (\frac{T_j}{2\rho A_j})^2} - \frac{\hat{v}_j^2}{2} \tag{8}$$
$$w_{mr} = w + \frac{2}{3}\Omega R_{mr}\delta_{col}$$
$$w_t = v + r.T_x + p.T_z + \frac{2}{3}\Omega_t R_t \delta_{ped}$$

A first order identified flapping dynamics for the Eagle RUAV is given by:

$$\dot{a}_1 = -\frac{a_1}{\tau_f} + q + \frac{A_c}{\tau_f}c + \frac{A_{lon}}{\tau_f}\delta_{lon} \tag{9}$$
$$\dot{b}_1 = -\frac{b_1}{\tau_f} - p + \frac{B_d}{\tau_f}d + \frac{B_{lat}}{\tau_f}\delta_{lat} \tag{10}$$
$$\dot{c} = -\frac{c}{\tau_s} + q + \frac{C_{lon}}{\tau_s}\delta_{lon} \tag{11}$$
$$\dot{d} = -\frac{d}{\tau_s} - p + \frac{D_{lat}}{\tau_s}\delta_{lat} \tag{12}$$

where $\delta_{lat}, \delta_{lon}$ are servo actuator outputs and c, d are flybar flapping angles. The τ_f, τ_s are the main rotor flapping and the flybar time constants. The identified parameters $A_c, A_{lon}, B_d, B_{lat}, C_{lon}, D_{lat}$ are given in Table I.

Assumption 2 *It is assumed that the servo actuator dynamics of the Eagle RUAV is much faster than the main, tail rotor pitch control* $(\delta_{col}, \delta_{ped})$ *and the cyclic pitch controls* $(\delta_{lat}, \delta_{lon})$*.*

Apart from the dominant forces and moments due to the main and tail rotor, relative wind acting on the helicopter produces a certain amount of force due to the fuselage and vertical/horizontal wings. The forces and moments acting on a RUAV due to fuselage and empennage are given in next subsections.

C. Fuselage

The forces and moments due to fuselage are given as follows [23, p. 115]:

$$[X_{fs}, Y_{fs}, Z_{fs}]^T = \frac{\rho}{2}\left[F_{ax}\ u^2, F_{ay}\ v^2, F_{az}\ w^2\right]^T \tag{13}$$
$$[L_{fs}, R_{fs}, N_{fs}]^T = [0, 0, 0]^T \tag{14}$$

where, F_{ax} , F_{ay} and F_{az} are the coefficients of drag force relative to the center of gravity of the body.

D. Empennage

The forces and moments due to the vertical tail of the Eagle RUAV are as follows:

$$[X_v, Y_v, Z_v]^T = [0, F_{vt}, 0]^T \tag{15}$$
$$[L_v, R_v, N_v]^T = [F_{vt}\ vt_x, 0, F_{vt}\ vt_z]^T \tag{16}$$

where, F_{vt} is an aerodynamic force due to the vertical tail and given as follows:

$$F_{vt} = \frac{\rho}{2}V_a V_{area} u\,(v + v_{itr})$$

TABLE I

PARAMETERS OF THE EAGLE RUAV

Parameter	Description
m = 8.2 Kg	helicopter mass
I_{xx} = 0.23 Kg.m^2	Rolling moment of inertia
I_{yy} = 0.82 Kg.m^2	Pitching moment of inertia
I_{zz} = 0.4 Kg.m^2	Yawing moment of inertia
$M_x,\ M_z$ = 0, -0.284	Main rotor x, z axes w.r.t C.G.
$T_x,\ T_z$ = -0.915, -0.104	Tail rotor x, z axes w.r.t C.G.
$vt_x,\ vt_z$ = -0.845, 0.351	Vertical tail x, z axes w.r.t C.G.
K_β =270	Main rotor hub spring constant
$\tau_s,\ \tau_f$ = 0.226, 0.027 sec	Flybar, main rotor time constant
$A_c,\ B_d$= 0.152, 0.136 rad/ms	Bell-mixer derivatives
$A_{lon},\ B_{lat}$ = 0.19, 0.17 rad/ms	Stick to swash-plat gearings
$C_{lon},\ D_{lat}$ = 1.58, 1.02 rad/ms	Stick to swash-plate gearings

Remark 2 This paper considers a control problem for landing of a RUAV using a tether. The cable forces and moments acting on a RUAV needs to be considered in the complete system to account for any external perturbations.

E. Tether

In this paper, a landing approach is considered by using a tether, where one end of the tether is secured to the moving platform deck (fixed reference) and the other end is reeled out from the hovering RUAV. The cable forces and moments defined in the body reference frame are given by [2]:

$$\begin{aligned} [X_c, Y_c, Z_c]^T &= T\hat{l}(\varphi, \vartheta) && (17)\\ [L_c, R_c, N_c]^T &= r_{HC}\,[X_c, Y_c, Z_c]^T && (18) \end{aligned}$$

where T is a constant tension of the cable and φ, ϑ are the elevation and azimuth angles measured from a cable angle sensor attached to the helicopter. A position vector r_{HC} is from the center of mass of the body to the cable attachment point, expressed in the body reference frame. $\hat{l}(\varphi, \vartheta)$ is a vector pointing in the line of the cable. Note that the mass of the tether is negligible compared to the mass of the helicopter and thus neglected in the above equations.

The forces and moments given in (3)–(4) can be written in terms of the components given in (5)–(6) and (13)–(18).

$$\begin{aligned} X &= X_m + X_{fs} + X_v + X_c && (19)\\ Y &= Y_m + Y_{fs} + Y_v + Y_c && (20)\\ Z &= Z_m + Z_{fs} + Z_v + Z_c && (21)\\ L &= L_m + L_{fs} + L_v + L_c && (22)\\ R &= R_m + R_{fs} + R_v + R_c && (23)\\ N &= N_m + N_{fs} + N_v + N_c && (24) \end{aligned}$$

III. RUAV LANDING CONTROL

In this section, a backstepping-based landing control for a RUAV is presented with tether forces and moments as control inputs. A flapping correction dynamics is proposed in [17] using backstepping for a fully actuated system. In practice flapping and the flybar dynamics leads to an underactuated system which can be controlled by including a correction term proposed in this paper. The rigid-body dynamics from (2)–(4) and the flapping dynamics from (9)–(12) is given by:

$$\begin{aligned} \dot{\zeta} &= V && (25)\\ \dot{V} &= -\omega \times V + \frac{f_b}{m} + \hat{g} && (26)\\ I\dot{\omega} &= -\omega \times I\omega + M && (27)\\ \dot{a}_1 &= -\frac{a_1}{\tau_f} + q + \frac{A_c}{\tau_f}c + \frac{A_{lon}}{\tau_f}\delta_{lon} && (28)\\ \dot{b}_1 &= -\frac{b_1}{\tau_f} - p + \frac{B_d}{\tau_f}d + \frac{B_{lat}}{\tau_f}\delta_{lat} && (29)\\ \dot{c} &= -\frac{c}{\tau_s} + q + \frac{C_{lon}}{\tau_s}\delta_{lon} && (30)\\ \dot{d} &= -\frac{d}{\tau_s} - p + \frac{D_{lat}}{\tau_s}\delta_{lat} && (31) \end{aligned}$$

where forces f_b and moments M in (26)–(27) are given in terms of the four RUAV control inputs $(\delta_{col},\ \delta_{ped},\ \delta_{lat},\ \delta_{lon})$.

As a first step in backstepping, let the starting Lyapunov Function Candidate (LFC) be:

$$W_1(\zeta) = \frac{1}{2}\left(\zeta - \zeta_0\right)^T\left(\zeta - \zeta_0\right),$$

where ζ_0 is the desired position, then

$$\begin{aligned} \dot{W}_1 &= \left(\zeta - \zeta_0\right)^T\dot{\zeta}\\ &= \left(\zeta - \zeta_0\right)^T V && (32) \end{aligned}$$

If the velocity V is equal to V^d as defined below

$$V^d = -\alpha\left(\zeta - \zeta_0\right) \quad (33)$$

(note that $\alpha > 0$ is a scalar parameter which can be used to tune the output response) then $\dot{W}_1 \leq 0$. Let $z_1 \triangleq V - V^d$ be the 'error' or the difference between the desired and actual values of V.

The process of backstepping continues by having another LFC as follows:

$$W_2(\zeta, V) = \frac{1}{2}\left(\zeta - \zeta_0\right)^T\left(\zeta - \zeta_0\right) + \frac{1}{2}z_1^T z_1,$$

then

$$\dot{W}_2 = \left(\zeta - \zeta_0\right)^T\dot{\zeta} + z_1^T\dot{z}_1$$

Differentiating both sides of (33) and substituting it in the above equation we get:

$$\dot{W}_2 = \left(\zeta - \zeta_0\right)^T V + \alpha z_1^T V + z_1^T\dot{V} \quad (34)$$

From (26) and by expanding terms of (34) we get

$$\begin{aligned} \dot{W}_2 = {} & \underbrace{\left(\zeta - \zeta_0\right)^T V^d}_{\leq 0} + \left(\zeta - \zeta_0\right)^T z_1 + \alpha z_1^T z_1 + \alpha z_1^T V^d\\ & + z_1^T\left(S(V)\omega + \frac{f_b}{m} + \hat{g}\right) && (35) \end{aligned}$$

where $S(V)$ is the skew-symmetric matrix such that $S(V)\omega = V \times \omega = -\omega \times V$. If

$$S(V)\omega = -\alpha V - \left(\zeta - \zeta_0\right) - \beta\omega \quad (36)$$

and choosing f_b such that

$$\left\{ \begin{array}{ll} z_1^T \dfrac{f_b}{m} = \beta z_1^T \omega - z_1^T \hat{g} & z_1 \neq 0 \\ \frac{f_b}{m} = -\hat{g} + C_f |\hat{l}\,(\varphi, \vartheta)\,| & z_1 = 0 \end{array} \right\} \tag{37}$$

(note that C_f is a positive constant parameter and can be determined to maintain a constant cable tension) then $\dot{W}_2 \leq 0$. Let the value of ω which makes $\dot{W}_2 \leq 0$ be defined as ω^d, i.e.,

$$\omega^d := [S(V) + \beta \mathcal{I}]^{-1} \left(-\alpha V - (\zeta - \zeta_0)\right) \tag{38}$$

where $\mathcal{I}$ is an identity matrix. For a suitable choice of β it can be seen that (38) can always be solved for ω. The process of backstepping continues by defining another 'error' $z_2 \triangleq \omega - \omega^d$, between the desired and actual values of ω. Substituting $V = z_1 + V^d$ and $\omega = z_2 + \omega^d$ in (26) we get:

$$\begin{aligned} \dot{V} &= -z_2 \times z_1 - z_2 \times V^d - \omega^d \times z_1 - \omega^d \times V^d \\ &\quad + \frac{f_b}{m} + \hat{g} \end{aligned} \tag{39}$$

Consider another LFC as follows:

$$W_3(\zeta, V, \omega) = \frac{1}{2} \left(\zeta - \zeta_0\right)^T \left(\zeta - \zeta_0\right) + \frac{1}{2} z_1^T z_1 + \frac{1}{2} z_2^T z_2,$$

then

$$\begin{aligned} \dot{W}_3 &= \left(\zeta - \zeta_0\right)^T z_1 + \left(\zeta - \zeta_0\right)^T V^d + \alpha z_1^T z_1 \\ &\quad + \alpha z_1^T V^d + z_1^T \dot{V} + z_2^T \dot{\omega} - z_2^T \dot{\omega}^d \end{aligned} \tag{40}$$

The time derivative of ω^d can be obtained by differentiating both sides of (38) written as follows:

$$\dot{\omega}^d = [S(V) + \beta \mathcal{I}]^{-1} \left(\left(S(\omega^d) - \alpha 1 \right) \dot{V} - V \right) \tag{41}$$

Substituting equations (37) and (39) into (40) we get:

$$\begin{aligned} \dot{W}_3 &= \left(\zeta - \zeta_0\right)^T V^d + \alpha z_1^T z_1 + z_1^T S(z_1) z_2 + z_1^T S(V^d) z_2 \\ &\quad + z_1^T S(z_1) \omega^d + z_2^T \dot{\omega} - z_2^T \dot{\omega}^d \end{aligned} \tag{42}$$

With the choice of f_b in (37), for

$$z_2^T (\dot{\omega} - \dot{\omega}^d + S^T(V^d) z_1) = 0, \tag{43}$$

$\dot{W}_3 \leq 0$. The equation (43) can be satisfied by choosing

$$M = I(\dot{\omega}^d - S^T(V^d) V) + \omega \times I \omega \tag{44}$$

(note that $S^T(V^d) V^d = 0$).

Remark 3 To numerically solve (37) and (44) for T_{mr}, T_t, a_1, b_1 a Gauss-Newton or Levenberg-Marquardt method is used.

Remark 4 The RUAV control inputs ($\delta_{col},\ \delta_{ped},\ \delta_{lat},\ \delta_{lon}$) are obtained in two steps. In the first step values for the desired main rotor flapping and the thrusts ($T_{mr}^d, T_t^d, a_1^d, b_1^d$) are obtained by solving for them from f_b and M given in (37) and (44). The second step is to obtain the $\delta_{col}, \delta_{ped}$ from (7)–(8) and $\delta_{lat}, \delta_{lon}$ by choosing the control as given below.

The flapping error dynamics form (28)–(31) is given by:

$$\dot{X}_e = A_e X_e + B_e \tilde{u} \tag{45}$$

where,

$$\begin{aligned} X_e &= [a_1 - a_1^d, b_1 - b_1^d, c - c^d, d - d^d, p, q]^T \\ \tilde{u} &= \left[\delta_{lat} - \delta_{lat}^d, \delta_{lon} - \delta_{lon}^d\right]^T \end{aligned}$$

The desired control inputs $\delta_{lon}^d, \delta_{lat}^d$ can be obtained by setting $\dot{a}_1, \dot{b}_1, \dot{c}, \dot{d} = 0$ in (28)–(31).

Remark 5 The flapping angles can be estimated by using (28)–(31) and the following rotor moment formulation [5] given by:

$$\dot{p} = K_\beta b_1 \tag{46}$$

$$\dot{q} = -K_\beta a_1 \tag{47}$$

Consider a system LFC including the states of the flapping error dynamics given by:

$$W(\zeta, z_1, z_2, X_e) = W_3 + X_e^T P X_e, \tag{48}$$

where P is a positive definite matrix. Let us define $\frac{f_b}{m} = U_v = U_v^d + \tilde{U}_v$ and $M = U_m = U_m^d + \tilde{U}_m$ in (37) and (44), where $\tilde{U}_v$ and $\tilde{U}_m$ are the *difference* terms in the control signals due to *error* in the flapping angles. The time derivative of (48), along the system trajectories is given as follows:

$$\begin{aligned} \dot{W} &= \dot{W}_3 + z_1^T \tilde{U}_v + z_2^T \tilde{U}_m + X_e^T \left(A_e^T P + A_e P\right) X_e \\ &\quad + \tilde{u} \left(B_e^T P + B_e^T P^T\right) X_e \end{aligned} \tag{49}$$

where $\dot{W}_3$ is the same as in (42) and $z_1^T \tilde{U}_v + z_2^T \tilde{U}_m$ is due to the *difference* terms.

Remark 6 It is shown in the previous analysis that if $U_v = U_v^d$ and $U_m = U_m^d$ then $\dot{W}_3 \leq 0$ in (42).

$\dot{W}$ can shown to be non-positive by considering three separate cases:

1) *X_e is equal to zero*: In that case both $\tilde{U}_v$ and $\tilde{U}_m$ are zero because the actual flapping angles are at their desired values. The control signals U_v^d and U_m^d will make $\dot{W} = 0$.
2) *X_e is non-zero and $B_e^T P + B_e^T P^T$ is not orthogonal to X_e*: In this case choose

$$\tilde{u} = K X_e \tag{50}$$

where K is a fixed gain matrix. Choose K in (50) such that $(A_e + B_e K)^T P + P(A_e + B_e K) \leq 0$. Substituting (50) in (49) we get:

$$\begin{aligned} \dot{W} &= \dot{W}_3 + z_1^T \tilde{U}_v + z_2^T \tilde{U}_m + \\ &\quad \underbrace{X_e^T \left[\left(A_e + B_e K\right)^T P + P \left(A_e + B_e K\right) \right] X_e}_{\leq 0 \ for \ suitable \ K} \end{aligned}$$

It is possible to make the $\dot{W}_3+z_1^T\tilde{U}_v+z_2^T\tilde{U}_m$ term non-positive by choosing control as given in Proposition 1 below.

3) $\left(B_e^T P + B_e^T P^T\right) X_e = 0$: In this case the proposition 1 can be used to introduce an additional main, tail rotor thrusts to set $\dot{W} = 0$.

Proposition 1 *If* $\left(B_e^T P + B_e^T P^T\right) X_e = 0$ *then the main and tail rotor thrusts* $T_i^d = T_i + \tilde{T}_i;\ i := mr, t$ *can be chosen such that* $\dot{W} = 0$*, provided* $z_1(1) + M_Z z_2(2) \neq 0$*.*

Remark 7 The idea here is to choose the main, tail rotor thrusts (T_{mr}, T_t) to make $\dot{W} = 0$. The system is underactuated by 2 and there are no control inputs available due to higher DOF. If $z_1(1) + M_Z z_2(2)$ is zero, then there are no control inputs available to provide the cyclic pitch controls.

Proof: In (49), we have

$$\tilde{U}_v = \begin{bmatrix} -T_{mr}^d\tilde{a}_1 - \tilde{T}_{mr}a_1^d - \tilde{T}_{mr}\tilde{a}_1 \\ T_{mr}^d\tilde{b}_1 + \tilde{T}_{mr}b_1^d + \tilde{T}_{mr}\tilde{b}_1 + \tilde{T}_t \\ -\tilde{T}_{mr} \end{bmatrix}$$
$$\tilde{U}_m = \begin{bmatrix} \frac{dL}{db_1}\tilde{b}_1 + \sigma_3 M_Z + \tilde{T}_t T_Z \\ \frac{dM}{da_1}\tilde{a}_1 - \sigma_4 M_Z \\ \tilde{T}_t T_X \end{bmatrix}$$

where,

$$\sigma_3 = T_{mr}^d\tilde{b}_1 + \tilde{T}_{mr}b_1^d + \tilde{T}_{mr}\tilde{b}_1 + \tilde{T}_t$$
$$\sigma_4 = T_{mr}^d\tilde{a}_1 + \tilde{T}_{mr}a_1^d + \tilde{T}_{mr}\tilde{a}_1$$

If matrix $\left(B_e^T P + B_e^T P^T\right) X_e$ is zero then the remaining terms from (49) are given by:

$$\dot{W} = \dot{W}_3 + z_1^T\tilde{U}_v + z_2^T\tilde{U}_m + X_e^T\left(A_e^T P + A_e P\right) X_e$$

In the above equation, choose P a positive definite such that $\left(A_e^T P + A_e P\right)$ is negative definite. The remaining terms are given by:

$$\Delta\dot{W} = \dot{W}_3 + z_1^T\tilde{U}_v + z_2^T\tilde{U}_m \qquad (51)$$

where Δ denotes the remaining terms from (49). Substituting in the above equation we get:

$$\Delta\dot{W} = \dot{W}_3 - \tilde{T}_{mr}\left(\chi_1^T z_1 + \chi_2^T z_2\right) + \tilde{T}_t\left(\chi_3^T z_1 + \chi_4^T z_2\right) + \chi_5^T z_1 + \chi_6^T z_2$$

where,

$$\chi_1 = \begin{bmatrix} a_1^d + \tilde{a}_1 \\ -b_1^d - \tilde{b}_1 \\ 1 \end{bmatrix} \chi_2 = \begin{bmatrix} (-b_1^d - \tilde{b}_1)M_Z \\ (a_1^d + \tilde{a}_1)M_Z \\ 0 \end{bmatrix}$$

$$\chi_3 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \chi_4 = \begin{bmatrix} M_Z + T_Z \\ 0 \\ T_X \end{bmatrix} \chi_5 = \begin{bmatrix} -\tilde{a}_1 T_{mr}^d \\ \tilde{b}_1 T_{mr}^d \\ 0 \end{bmatrix}$$

$$\chi_6 = \begin{bmatrix} \tilde{b}_1(\frac{dL}{db_1} + T_{mr}^d M_Z) \\ \tilde{a}_1(\frac{dM}{da_1} - T_{mr}^d M_Z) \\ 0 \end{bmatrix}$$

The main and the tail rotor thrusts can be chosen in the following manner to make $\Delta\dot{W} = 0$ in (51).

1) When $\chi_1^T z_1 + \chi_2^T z_2 \neq 0$, choose

$$-\tilde{T}_{mr} = \frac{-\chi_5^T z_1 - \chi_6^T z_2}{\chi_1^T z_1 + \chi_2^T z_2} \text{ and } \tilde{T}_t = 0. \qquad (52)$$

2) When $\chi_3^T z_1 + \chi_4^T z_2 \neq 0$, choose

$$-\tilde{T}_t = \frac{-\chi_5^T z_1 - \chi_6^T z_2}{\chi_3^T z_1 + \chi_4^T z_2} \text{ and } \tilde{T}_{mr} = 0. \qquad (53)$$

One of the above two conditions is always true when $z_1(1) + M_Z\ z_2(2) \neq 0$.

Above it is shown that it is possible to find $\tilde{\delta}_{lat}, \tilde{\delta}_{lon}$ even when $\left(B_e^T P + B_e^T P^T\right) X_e = 0$. The above results are novel and can be used in other control applications for the underactuated mechanical systems.

IV. Simulation

The controller flow chart is shown in Fig. 2. A simulation is done for the case where the initial position $\zeta = [-5.0, 0, -3.2]^T$ and the desired position $\zeta_0 = [-5.0, 0, -2.2]^T$. The simulation results with controller parameters $\alpha = 20$, $\beta = 20$ for landing control of the Eagle RUAV are shown in Fig. 3 and Fig. 4. The simulation results shows an acceptable practical values of the RUAV control inputs suitable for the implementation purposes.

V. Conclusion

In this paper, the backstepping-based landing controller is presented including the flapping correction dynamics. The tether is used for landing application and as a guidance mechanism for safe landing operation of RUAVs. This paper also presents a detailed analysis of flapping dynamics correction, essential for control implementation purposes. Future work is needed to implement the algorithm onboard platforms.

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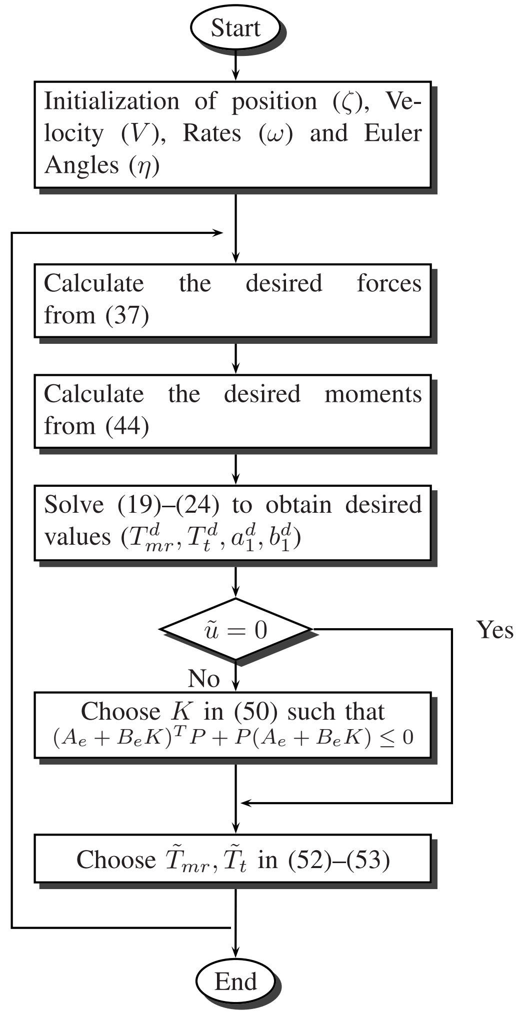

Fig. 2. Backstepping-based controller flow chart for the Eagle RUAV. The flow chart explains a step-wise procedure to obtain the RUAV control inputs. The controller uses a novel result to provide a correction for the flapping dynamics due to higher DOF system.

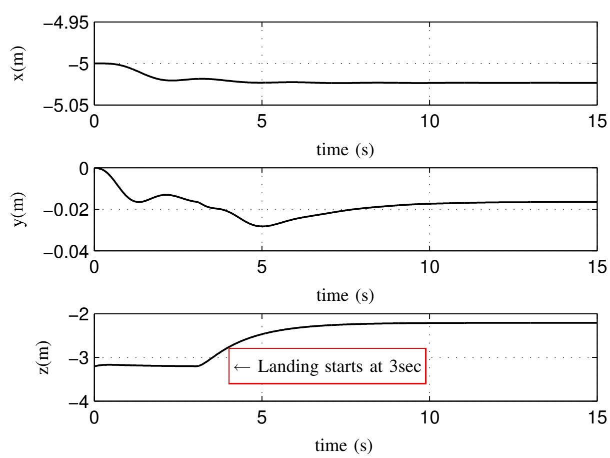

Fig. 3. Cartesian state $\zeta = [x, y, z]^T$ of the Eagle RUAV computed at sampling time 0.02 seconds. The landing operation starts after 3 seconds of the simulation time.

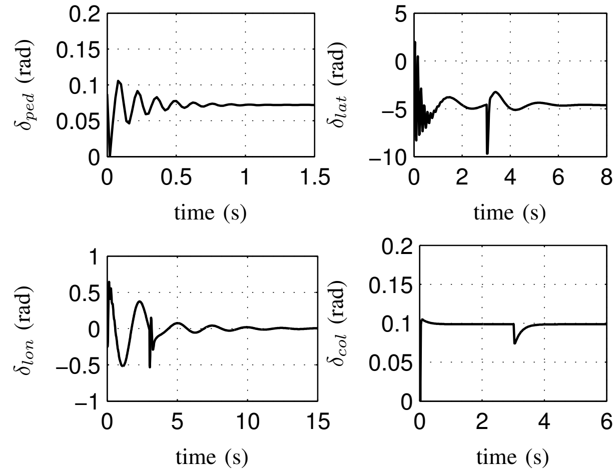

Fig. 4. Control inputs of the Eagle RUAV computed at sampling time 0.02 seconds

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