2008 American Control Conference
Westin Seattle Hotel, Seattle, Washington, USA
June 11-13, 2008

Robust H_∞ SMC of Uncertain Switched Systems with Time Delay

Jie Lian, Georgi M. Dimirovski and Jun Zhao

Abstract— **This paper investigates the robust H_∞ sliding mode control problem for a class of uncertain switched delay systems. A single sliding surface is constructed such that the reduced-order equivalent sliding motion restricted to the sliding surface is completely invariant to all admissible uncertainties. For cases of known delay and unknown delay, the existence conditions of the sliding surface are proposed respectively. The corresponding hysteresis switching laws are designed to stabilize the sliding motion with H_∞ disturbance attenuation level γ. Furthermore, variable structure controllers are developed to drive the state of the switched system to reach the single sliding surface in finite time and remains on it thereafter. Finally, a numerical example is given to illustrate the effectiveness of the proposed design method.**

I. INTRODUCTION

The sliding mode control (SMC) has various attractive features such as fast response, good transient response and order-reduction [1-2]. It is also insensitive to variations in system parameters and external disturbances. Over the years, SMC has been widely applied to traditional linear or nonlinear systems [3-5]. On SMC of other systems rather than the traditional ones, there are also some exciting and significant results [6-8].

For switched systems, only a few research results in which the SMC technique is employed exist due to the complexity of control systems and the excess burden of the control synthesis and switching law design. The authors of [9] proposed a SMC method to make nominal switched systems exponentially stable. [10] proposed a variable structure control algorithm for a class of systems which can be stabilized by switching the system control input among a set of feedback control laws.

On the other hand, time-delay is often encountered in various industrial systems. Switched systems with time-delay are one of the most useful models and have strong engineering background such as power systems [11] and networked control systems [12]. However, very few results on switched delay systems have been reported. Sufficient conditions of asymptotical stability were established for switched linear delay systems under arbitrary and constructed switching signals respectively in [13]. [14] investigated the problem of delay-dependent common Lyapunov functions for switched linear delay systems, which established the relationship between the delay-dependent common Lyapunov functions and the common Lyapunov functions for corresponding switched systems without delays. The stabilization problem of arbitrary switched linear systems with unknown time varying delays was considered in [15]. For uncertain linear discrete-time switched systems with state delays, sufficient conditions of robust stability and stabilizability in terms of matrix inequalities and Riccati-like inequalities were given in [16]. Stability of a class of switched delay systems was shown in [17] by using a common Lyapunov functional method. [18] studied stability and L_2-gain for a class of switched delay systems via using average dwell time method. However, to the best of the authors' knowledge, there are no results for the SMC of switched delay systems in the current literature, which is indeed our motivation.

This paper considers the robust H_∞ SMC problem for a class of uncertain switched delay systems. A single sliding surface is constructed. For the delay-known case, a sufficient condition for the existence of the sliding surface is given and by using the information of current state and delay-state, a hysteresis switching law is designed to guarantee that the sliding motion is stabilizable with H_∞ disturbance attenuation level γ. For the delay-unknown case, a sufficient condition for the existence of the sliding surface is given by solving matrix inequalities, and the corresponding hysteresis switching law that only depend on the current state is designed. Variable structure controllers are developed for two cases such that the state of the system reaches the single sliding surface in finite time and remains on it.

In this paper, $\|\bullet\|$ denotes the Euclidean norm for a vector or the matrix induced norm for a matrix; N denotes the set of all nonnegative integers.

II. PROBLEM FORMULATION AND PRELIMINARIES

Consider the uncertain switched delay system of the form

$$\begin{aligned}\dot{x}(t) &= (A_\sigma + \Delta A_\sigma)x(t) + (A_{d\sigma} + \Delta A_{d\sigma})x(t-\tau)\\ &\quad + B(u_\sigma + U_\sigma(t)u_\sigma + f_\sigma(x,t)) + G_\sigma\omega(t), \qquad (1)\\ x(t) &= \varphi(t),\ t\in[-\tau,0],\ z(t) = C_\sigma x(t),\end{aligned}$$

where $x(t)\in R^n$ is the system state, $\sigma:[0,\infty)\to\Xi=\{1,2,\ldots,l\}$ is the piecewise constant switching signal,

This work was supported in part by Dogus University Fund for Science and the NSF of China under Grant 60574013.

Jie Lian and Jun Zhao are with Key Laboratory of Integrated Automation of Process Industry, Ministry of Education, Northeastern University, Shenyang, Liaoning, 110004, P.R. of China, and Jun Zhao is also with the Department of Information Engineering, Research School of Information Sciences and Engineering, The Australian National University, Canberra ACT 0200, Australia. lianjielj@163.com, zhaojun@ise.neu.edu.cn

Georgi M. Dimirovski is with Department of Computer Engineering, Dogus University, Kadikoy, TR-34722 Istanbul, Rep. of Turkey. gdimirovski@dogus.edu.tr

$u_i \in R^m$ is the control input of the $i-th$ subsystem, $\omega(t) \in R^h$ denotes the disturbance input which belongs to $L_2[0,\infty)$, $z(t) \in R^q$ is the controlled output, A_i, A_{di}, G_i, C_i, B are constant matrices with appropriate dimensions, $\varphi(t)$ is a differentiable vector-valued initial function on $[-\tau,0]$, ΔA_i and ΔA_{di} represent the system parameter uncertainties, $U_i(t)$ and $f_i(x,t)$ represent the input matrix uncertainty and nonlinearity of the system, respectively. The following assumptions are introduced.

Assumption 1. The input matrix B has full rank m and $m < n$.

Assumption 2. The parameter uncertainties can be represented and emulated as

$$[\Delta A_i \quad \Delta A_{di}] = \left[D_{1i}\Sigma_{1i}(t) \quad D_{2i}\Sigma_{2i}(t)\right]E\ ,\ i \in \Xi\ ,$$

where D_{1i}, D_{2i} and E are constant matrices with appropriate dimensions and the matrix E is right invertible, $\Sigma_{1i}(t)$ and $\Sigma_{2i}(t)$ are unknown matrices with Lebesgue measurable elements and satisfy $\Sigma_{1i}{}^{\mathrm{T}}\Sigma_{1i} \le I$, $\Sigma_{2i}{}^{\mathrm{T}}\Sigma_{2i} \le I$.

Assumption 3. There exist known nonnegative scalar-valued functions $\phi_i(x,t)$, $i \in \Xi$ such that $\left\|f_i(x,t)\right\| \le \phi_i(x,t)$.

Assumption 4. There exist known nonnegative constants ρ_i, $i \in \Xi$ such that $\left\|U_i(t)\right\| \le \rho_i < 1$ for all t.

Assumption 5. There exists a known nonnegative constant ϖ such that $\|\omega\| \le \varpi$.

Remark 1. Assumptions 1~5 are standard assumptions in the study of variable structure control.

We adopt the following notation from [19]. A switching sequence is expressed by

$$\Psi = \{x_0; (i_0,t_0), (i_1,t_1), \ldots, (i_j,t_j), \ldots, \big|\, i_j \in \Xi, j \in N\}\ . \quad (2)$$

In which t_0 is the initial time, x_0 is the initial state, (i_k,t_k) means that the i_k-th subsystem is activated for $[t_k,t_{k+1})$. Therefore, when $t \in [t_k,t_{k+1})$, the trajectory of the switched system (1) is produced by the i_k-th subsystem. For any $j \in \Xi$,

$$\begin{aligned}\Psi_t(j) = \{&[t_{j_1},t_{j_1+1}),[t_{j_2},t_{j_2+1}),\ldots,[t_{j_n},t_{j_n+1})\ldots,\\ &\sigma(t) = j, t_{j_k} \le t < t_{j_k+1}, k \in N\}\end{aligned} \quad (3)$$

denotes the sequence of switching time of the $j-th$ subsystem, in which the $j-th$ subsystem is switched on at t_{j_k} and switched off at t_{j_k+1}.

Let Γ be an $n \times n$ symmetric matrix satisfying

$$\Gamma = I - E^g E, \quad (4)$$

where E^g is the Moore-Penrose inverse of E.

We design the single sliding surface for the switched system (1) as

$$\zeta(t) = Sx(t) = B^{\mathrm{T}}(\Gamma X\Gamma + BYB^{\mathrm{T}})^{-1}x(t) = 0\ , \quad (5)$$

where X and Y are symmetric matrices which will be determined later.

Remark 2. The single sliding surface $\zeta(t) = Sx(t) = 0$ is designed such that the switched system (1) is asymptotically stable with an H_∞ norm bound based on the single Lyapunov function approach in the sliding surface. The purpose of designing the single sliding surface for the switched system is to reduce the reaching phase in which systems are sensitive to the uncertainties and the perturbations, and improve the transient performance and robustness.

To have a regular form of the system (1), we define nonsingular matrix G and an associated vector ξ as follows

$$G = \begin{bmatrix}\widetilde{B}^{\mathrm{T}} \\ S\end{bmatrix} = \begin{bmatrix}\widetilde{B}^{\mathrm{T}} \\ B^{\mathrm{T}}P^{-1}\end{bmatrix}, \quad (6)$$

where $\widetilde{B}$ is an orthogonal complement of the matrix B, $P = \Gamma X\Gamma + BYB^{\mathrm{T}}$ and

$$\xi(t) = \begin{bmatrix}\xi_1(t) \\ \xi_2(t)\end{bmatrix} = Gx(t) = \begin{bmatrix}\widetilde{B}^{\mathrm{T}} \\ B^{\mathrm{T}}P^{-1}\end{bmatrix}x(t) \quad (7)$$

with $\xi_1 \in R^{n-m}$, $\xi_2 = \zeta \in R^m$. Note that the matrix G is invertible. Indeed, it can be checked that

$$G^{-1} = \left[P\widetilde{B}(\widetilde{B}^{\mathrm{T}}P\widetilde{B})^{-1} \quad B(SB)^{-1}\right]. \quad (8)$$

By the state transformation (7), the system (1) is represented by the following regular form

$$\begin{aligned}\begin{bmatrix}\dot{\xi}_1(t) \\ \dot{\zeta}(t)\end{bmatrix} &= \begin{bmatrix}\overline{A}_{\sigma11} & \overline{A}_{\sigma12} \\ \overline{A}_{\sigma21} & \overline{A}_{\sigma22}\end{bmatrix}\begin{bmatrix}\xi_1(t) \\ \zeta(t)\end{bmatrix} + \begin{bmatrix}\overline{A}_{d\sigma11} & \overline{A}_{d\sigma12} \\ \overline{A}_{d\sigma21} & \overline{A}_{d\sigma22}\end{bmatrix}\begin{bmatrix}\xi_1(t-\tau) \\ \zeta(t-\tau)\end{bmatrix} \\ &\quad + \begin{bmatrix}0 \\ SB\end{bmatrix}(u_\sigma + U_\sigma(t)u_\sigma + f_\sigma(x,t)) + \begin{bmatrix}\widetilde{B}^{\mathrm{T}}G_\sigma \\ SG_\sigma\end{bmatrix}\omega(t), \\ z(t) &= C_\sigma P\widetilde{B}(\widetilde{B}^{\mathrm{T}}P\widetilde{B})^{-1}\xi_1(t) + C_\sigma B(SB)^{-1}\zeta(t), \\ \xi_1(t) &= \overline{\varphi}_1(t), t \in [-\tau,0],\ \zeta(t) = \overline{\varphi}_2(t), t \in [-\tau,0],\end{aligned} \quad (9)$$

where

$$\overline{A}_{\sigma11} = \widetilde{B}^{\mathrm{T}}[A_\sigma + D_{1\sigma}\Sigma_{1\sigma}(t)E]P\widetilde{B}(\widetilde{B}^{\mathrm{T}}P\widetilde{B})^{-1},$$
$$\overline{A}_{\sigma12} = \widetilde{B}^{\mathrm{T}}[A_\sigma + D_{1\sigma}\Sigma_{1\sigma}(t)E]B(SB)^{-1},$$
$$\overline{A}_{d\sigma11} = \widetilde{B}^{\mathrm{T}}[A_{d\sigma} + D_{2\sigma}\Sigma_{2\sigma}(t)E]P\widetilde{B}(\widetilde{B}^{\mathrm{T}}P\widetilde{B})^{-1},$$
$$\overline{A}_{d\sigma12} = \widetilde{B}^{\mathrm{T}}[A_{d\sigma} + D_{2\sigma}\Sigma_{2\sigma}(t)E]B(SB)^{-1},$$
$$\overline{A}_{\sigma21} = S[A_\sigma + D_{1\sigma}\Sigma_{1\sigma}(t)E]P\widetilde{B}(\widetilde{B}^{\mathrm{T}}P\widetilde{B})^{-1},$$
$$\overline{A}_{\sigma22} = S[A_\sigma + D_{1\sigma}\Sigma_{1\sigma}(t)E]B(SB)^{-1},$$
$$\overline{A}_{d\sigma21} = S[A_{d\sigma} + D_{2\sigma}\Sigma_{2\sigma}(t)E]P\widetilde{B}(\widetilde{B}^{\mathrm{T}}P\widetilde{B})^{-1},$$
$$\overline{A}_{d\sigma22} = S[A_{d\sigma} + D_{2\sigma}\Sigma_{2\sigma}(t)E]B(SB)^{-1},$$
$$\overline{\varphi}_1(t) = \widetilde{B}^{\mathrm{T}}\varphi(t)\ ,\ \overline{\varphi}_2(t) = S\varphi(t)\ .$$

Then the sliding motion in the sliding surface $(\zeta(t) = \dot{\zeta}(t) = 0)$ can be described by following $(n-m)$ dimensional switched system

$$\begin{aligned}\dot{\xi}_1(t) &= \widetilde{B}^{\mathrm{T}}A_\sigma P\widetilde{B}(\widetilde{B}^{\mathrm{T}}P\widetilde{B})^{-1}\xi_1(t) + \widetilde{B}^{\mathrm{T}}A_{d\sigma}P\widetilde{B}(\widetilde{B}^{\mathrm{T}}P\widetilde{B})^{-1} \\ &\quad \times \xi_1(t-\tau) + \widetilde{B}^{\mathrm{T}}D_{1\sigma}\Sigma_{1\sigma}(t)EP\widetilde{B}(\widetilde{B}^{\mathrm{T}}P\widetilde{B})^{-1}\xi_1(t) \\ &\quad + \widetilde{B}^{\mathrm{T}}D_{2\sigma}\Sigma_{2\sigma}(t)EP\widetilde{B}(\widetilde{B}^{\mathrm{T}}P\widetilde{B})^{-1}\xi_1(t-\tau) + \widetilde{B}^{\mathrm{T}}G_\sigma\omega, \\ z(t) &= C_\sigma P\widetilde{B}(\widetilde{B}^{\mathrm{T}}P\widetilde{B})^{-1}\xi_1(t).\end{aligned} \quad (10)$$

By (4), we can easily obtain

$$EP\widetilde{B} = E[(1-E^g E)X(1-E^g E) + BYB^{\mathrm T}]\widetilde{B} = 0 .$$

Then the sliding motion (10) can be represented by the following form

$$\dot{\xi}_1(t) = \widetilde{B}^{\mathrm T} A_\sigma P\widetilde{B}(\widetilde{B}^{\mathrm T} P\widetilde{B})^{-1}\xi_1(t) + \widetilde{B}^{\mathrm T} A_{d\sigma} P\widetilde{B}(\widetilde{B}^{\mathrm T} P\widetilde{B})^{-1} \times \xi_1(t-\tau) + \widetilde{B}^{\mathrm T} G_\sigma \omega, \quad (11)$$

$$z(t) = C_\sigma P\widetilde{B}(\widetilde{B}^{\mathrm T} P\widetilde{B})^{-1}\xi_1(t).$$

Remark 3. We can see that by using the SMC method, the uncertainties ΔA_i ΔA_{di} and $f_i(x,t)$ disappear in the sliding motion (11) and the order of the considered system is reduced. Therefore we only need to study stabilization of the $(n-m)$ dimensional switched system (11) without uncertainties.

Definition 1. Given a constant $\gamma > 0$, the sliding motion (11) is said to be asymptotically stabilizable with H_∞ disturbance attenuation level γ via switching if there exists a Lyapunov functional $V(x)$ and a switching law $\sigma(t)$ such that under the switching law, it satisfies

i). the derivative of V along the trajectory of the system (11) with $\omega(t) = 0$ satisfies

$$L(t) = \dot{V}(t) < 0 .$$

ii). with zero-initial condition $\varphi(\theta) = 0,\ \theta \in [-\tau, 0]$, $\|z(t)\|_2 < \gamma\|\omega(t)\|_2$ holds for all nonzero $\omega \in L_2[0,\infty)$.

The objective in this paper is how to determine the sliding matrix S, design the switching law $\sigma(t)$ and variable structure controllers u_i such that

1). the $(n-m)$ dimensional sliding motion (11) restricted to the sliding surface (5) is asymptotically stabilizable with H_∞ disturbance attenuation level γ under the switching law $\sigma(t)$,

2). the state of the system (1) is driven towards the sliding surface (5) and stays there for all the future time.

We divide the design of the switched delay system (1) into the known and unknown time-delay cases. The correspondi-ng results will be described in the next sections.

III. Main Results

In this section, we give the design method. In general, variable structure control design methodology comprises two steps. First, the sliding surface is designed, so that the controlled system will yield the desired dynamic performance in the sliding surface. The second phase is to design the variable structure controller such that the trajectory of the system arrive the sliding surface and remain on the sliding surface for all subsequent time.

A. τ is a Known Constant

In this subsection, the system (1) with a known time-delay τ is considered.

The following theorem shows that the system (1) in the sliding surface (5) is robustly asymptotically stabilizable with H_∞ disturbance attenuation level γ via switching.

Theorem 1. Given a constant $\gamma > 0$, the sliding motion (11) based on the sliding surface (5) is stabilizable with H_∞ disturbance attenuation level γ via switching, if there exist symmetric matrices X, Y matrix $Q_0 > 0$ and scalars $\alpha_i \ge 0$, $\sum_{i=1}^{l}\alpha_i = 1$ satisfying the following inequalities

$$\Gamma X\Gamma + BYB^{\mathrm T} > 0,$$

$$\begin{bmatrix} \widetilde{B}^{\mathrm T}\overline{W}\widetilde{B} + Q_0 & \widetilde{B}^{\mathrm T}\overline{A}_d\Gamma X\Gamma\widetilde{B} \\ \widetilde{B}^{\mathrm T}\Gamma X\Gamma\overline{A}_d^{\mathrm T}\widetilde{B} & -Q_0 \end{bmatrix} < 0, \quad (12)$$

where $\overline{W} = \overline{A}\Gamma X\Gamma + \Gamma X\Gamma\overline{A}^{\mathrm T} + \Gamma X\Gamma\widetilde{C}^{\mathrm T}\widetilde{C}\Gamma X\Gamma + \gamma^{-2}\widetilde{G}\widetilde{G}^{\mathrm T}$, $\overline{A} = \sum_{i=1}^{l}\alpha_i A_i$, $\overline{A}_d = \sum_{i=1}^{l}\alpha_i A_{di}$, $\widetilde{C} = \left[\sqrt{\alpha_1}C_1, \dots, \sqrt{\alpha_l}C_l\right]$, $\widetilde{G} = \left[\sqrt{\alpha_1}G_1, \dots, \sqrt{\alpha_l}G_l\right]$.

Proof. Denote $W_i = A_i\Gamma X\Gamma + \Gamma X\Gamma A_i^{\mathrm T} + \Gamma X\Gamma C_i^{\mathrm T}C_i\Gamma X\Gamma + \gamma^{-2} \times G_iG_i^{\mathrm T}$, $i \in \Xi$.

We define regions

$$\Omega_i = \left\{ \begin{bmatrix} \xi_1(t) \\ \xi_1(t-\tau) \end{bmatrix} \middle| \begin{bmatrix} (\widetilde{B}^{\mathrm T}P\widetilde{B})^{-1}\xi_1(t) \\ (\widetilde{B}^{\mathrm T}P\widetilde{B})^{-1}\xi_1(t-\tau) \end{bmatrix}^{\mathrm T} \times \begin{bmatrix} \widetilde{B}^{\mathrm T}W_i\widetilde{B} + Q_0 & \widetilde{B}^{\mathrm T}A_{di}\Gamma X\Gamma\widetilde{B} \\ \widetilde{B}^{\mathrm T}\Gamma X\Gamma A_{di}^{\mathrm T}\widetilde{B} & -Q_0 \end{bmatrix} \times \begin{bmatrix} (\widetilde{B}^{\mathrm T}P\widetilde{B})^{-1}\xi_1(t) \\ (\widetilde{B}^{\mathrm T}P\widetilde{B})^{-1}\xi_1(t-\tau) \end{bmatrix} < 0, i \in \Xi \right\}. \quad (13)$$

Obviously, $\bigcup_{i\in\Xi}\Omega_i = R^{2(n-m)} \setminus \{0\}$.

The hysteresis switching law for the sliding motion (11) is designed as follows

$$\sigma(0) = \min\arg\{\Omega_i \mid \hat{\xi}_1(0) \in \Omega_i\},$$

for $t > 0$,

$$\sigma(t) = \begin{cases} i, & \text{if } \hat{\xi}_1(t) \in \Omega_i \text{ and } \sigma(t^-) = i, \\ \min\arg\{\Omega_k \mid \hat{\xi}_1(t) \in \Omega_k\}, & \text{if } \hat{\xi}_1(t) \notin \Omega_i \text{ and } \sigma(t^-) = i, \end{cases} \quad (14)$$

where $\hat{\xi}_1(t) = [\xi_1^{\mathrm T}(t), \xi_1^{\mathrm T}(t-\tau)]^{\mathrm T}$.

We first verify the stabilization of the sliding motion (11) with $\omega = 0$. Take symmetric positive-define matrices P_1, Q and choose the Lyapunov functional candidate

$$V = \xi_1^{\mathrm T}(t)P_1\xi_1(t) + \int_{t-\tau}^{t}\xi_1^{\mathrm T}(\theta)Q\xi_1(\theta)d\theta . \quad (15)$$

Then the derivative of the Lyapunov functional (15) along the trajectory of the system (11) with $\omega = 0$ is

$$\dot{V} = \begin{bmatrix} \xi_1(t) \\ \xi_1(t-\tau) \end{bmatrix}^{\mathrm T} \begin{bmatrix} \widetilde{A}_{\sigma 11}^{\mathrm T}P_1 + P_1\widetilde{A}_{\sigma 11} + Q & P_1\widetilde{A}_{d\sigma 11} \\ \widetilde{A}_{d\sigma 11}^{\mathrm T}P_1 & -Q \end{bmatrix} \begin{bmatrix} \xi_1(t) \\ \xi_1(t-\tau) \end{bmatrix}$$

$$= \begin{bmatrix} P_1\xi_1(t) \\ P_1\xi_1(t-\tau) \end{bmatrix}^{\mathrm T} \begin{bmatrix} P_1^{-1}\widetilde{A}_{\sigma 11}^{\mathrm T} + \widetilde{A}_{\sigma 11}P_1^{-1} + P_1^{-1}QP_1^{-1} & \widetilde{A}_{d\sigma 11}P_1^{-1} \\ P_1^{-1}\widetilde{A}_{d\sigma 11}^{\mathrm T} & -P_1^{-1}QP_1^{-1} \end{bmatrix} \times \begin{bmatrix} P_1\xi_1(t) \\ P_1\xi_1(t-\tau) \end{bmatrix}, \quad (16)$$

where $\widetilde{A}_{\sigma 11} = \widetilde{B}^{\mathrm{T}} A_{\sigma} P\widetilde{B}(\widetilde{B}^{\mathrm{T}} P\widetilde{B})^{-1}$, $\widetilde{A}_{d\sigma 11} = \widetilde{B}^{\mathrm{T}} A_{d\sigma} P\widetilde{B}(\widetilde{B}^{\mathrm{T}} P\widetilde{B})^{-1}$. Take the matrices $P_1 = (\widetilde{B}^{\mathrm{T}} P\widetilde{B})^{-1}$, $Q = (\widetilde{B}^{\mathrm{T}} P\widetilde{B})^{-1} Q_0 (\widetilde{B}^{\mathrm{T}} P\widetilde{B})^{-1}$, then we have $\dot{V} < 0$. By the single Lyapunov function method, the sliding motion (11) with $\omega = 0$ based on the sliding surface (5) is asymptotically stable under the switching law (14).

In the following, we show that the overall L_2-gain from ω to z is less than or equal to γ in the single sliding surface (5). Under zero initial condition, without loss of generality, for $\forall T \geq t_0 = 0$, assume $T \in [t_k, t_{k+1})$ for some k. Now we introduce

$$J = \int_0^T (\|z\|^2 - \gamma^2 \|\omega\|^2) dt . \quad (17)$$

According to the switching sequence (2), when $T \in [t_k, t_{k+1})$, we have

$$\begin{aligned} J &= \sum_{j=0}^{k-1} \left(\int_{t_j}^{t_{j+1}} (\|z\|^2 - \gamma^2 \|\omega\|^2 + \dot{V}(t)) dt - (V(t_{j+1}) - V(t_j)) \right) \\ &\quad + \int_{t_k}^{T} (\|z\|^2 - \gamma^2 \|\omega\|^2 + \dot{V}(t)) dt - (V(T) - V(t_k)) \\ &= \sum_{j=0}^{k-1} \left(\int_{t_j}^{t_{j+1}} (\|z\|^2 - \gamma^2 \|\omega\|^2 + \dot{V}(t)) dt \right) \\ &\quad + \int_{t_k}^{T} (\|z\|^2 - \gamma^2 \|\omega\|^2 + \dot{V}(t)) dt - V(T). \end{aligned} \quad (18)$$

Note that

$$\begin{aligned} &\|z\|^2 - \gamma^2 \|\omega\|^2 + \dot{V}(t) \\ &= \begin{bmatrix} P_1 \xi_1(t) \\ P_1 \xi_1(t-\tau) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} \widetilde{B}^{\mathrm{T}} W_{i_j} \widetilde{B} + Q_0 & \widetilde{B}^{\mathrm{T}} (A_{di_j} \Gamma X \Gamma) \widetilde{B} \\ \widetilde{B}^{\mathrm{T}} (\Gamma X \Gamma A_{di_j}^{\mathrm{T}}) \widetilde{B} & -Q_0 \end{bmatrix} \begin{bmatrix} P_1 \xi_1(t) \\ P_1 \xi_1(t-\tau) \end{bmatrix} \\ &- (\gamma^{-1} G_{i_j}^{\mathrm{T}} \widetilde{B} P_1 \xi_1(t) - \gamma \omega)^{\mathrm{T}} (\gamma^{-1} G_{i_j}^{\mathrm{T}} \widetilde{B} P_1 \xi_1(t) - \gamma \omega) < 0. \end{aligned} \quad (19)$$

Therefore, $J < 0$ for $\forall \omega \in L_2[0, \infty)$. That is $\|z(t)\|_2 < \gamma \|\omega(t)\|_2$. This completes the proof.

Next, the result of controller design of reaching motion is given.

Theorem 2. Suppose that (12) is feasible and the single sliding surface is given by (5). Then the state of the system (1) can enter the sliding surface in finite time, and subsequently remains on it by employing the following variable structure controllers

$$\begin{aligned} u_i &= -(SB)^{-1} (SA_i x(t) + SA_{di} x(t-\tau)) - \frac{(SB)^{-1}}{1-\rho_i} \{ \rho_i \|SA_i x(t)\| \\ &\quad + \rho_i \|SA_{di} x(t-\tau)\| + \|SD_{1i}\| \|Ex(t)\| + \|SD_{2i}\| \|Ex(t-\tau)\| \\ &\quad + \|SB\| \phi_i(x,t) + \|SG_i\| \varpi + \mu_1 \} sign(\zeta), i \in \Xi, \end{aligned} \quad (20)$$

where μ_1 is a positive scalar.

Proof. Consider the following Lyapunov function candidate

$$V(\zeta) = \frac{1}{2} \zeta^{\mathrm{T}} \zeta . \quad (21)$$

Its derivative along the trajectory of the system (1) is given

$$\begin{aligned} \dot{V} &\leq \zeta^{\mathrm{T}} \{ SA_i x(t) + SA_{di} x(t-\tau) + SB[u_i + U_i(t) u_i] \} \\ &\quad + \|SD_{1i}\| \|Ex(t)\| \|\zeta\| + \|SD_{2i}\| \|Ex(t-\tau)\| \|\zeta\| \\ &\quad + \|SB\| \|\zeta\| \phi_i(x,t) + \|SG_i\| \|\zeta\| \varpi. \end{aligned} \quad (22)$$

Applying the variable structure controllers (20) into the inequality (22) results in $\zeta^{\mathrm{T}} \dot{\zeta} \leq -\mu_1 \|\zeta\|$. Hence the state of the system (1) will reach the single sliding surface (5) in finite time and subsequently remains on it. This completes the proof.

B. τ is an Unknown Constant

When time-delay τ is an unknown constant, the switching law (14) and the controllers (20) are not applicable. We assume the time-delay is an unknown, but it is bounded by the known constant $\bar{\tau}$.

The following theorem shows that the system (1) in the sliding surface (5) is robustly asymptotically stabilizable with H_∞ disturbance attenuation level γ via switching.

Theorem 3. Given a constant $\gamma > 0$, the sliding motion (11) based on the sliding surface (5) is stabilizable with H_∞ disturbance attenuation level γ via switching, if there exist symmetric matrices X, Y, a positive number ε, matrix $Q_1 > 0$ and scalars $\beta_i \geq 0$, $\sum_{i=1}^{l} \beta_i = 1$ satisfying the following inequalities

$$\begin{aligned} &\Gamma X \Gamma + BYB^{\mathrm{T}} > 0, \\ &\widetilde{B}^{\mathrm{T}} (\hat{A} \Gamma X \Gamma + \Gamma X \Gamma \hat{A}^{\mathrm{T}} + \Gamma X \Gamma \hat{C}^{\mathrm{T}} \hat{C} \Gamma X \Gamma + \gamma^{-2} \hat{G} \hat{G}^{\mathrm{T}}) \widetilde{B} \\ &\quad + \varepsilon Q_1 + (\widetilde{B}^{\mathrm{T}} \hat{A}_d \Gamma X \Gamma \widetilde{B}) \varepsilon^{-1} R^{-1} (\widetilde{B}^{\mathrm{T}} \hat{A}_d \Gamma X \Gamma \widetilde{B})^{\mathrm{T}} < 0, \end{aligned} \quad (23)$$

where $\hat{A} = \sum_{i=1}^{l} \beta_i A_i$, $\hat{C} = \left[\sqrt{\beta_1} C_1, \ \ldots, \ \sqrt{\beta_l} C_l \right]$, $\hat{G} = \left[\sqrt{\beta_1} G_1, \ \ldots, \ \sqrt{\beta_l} G_l \right]$, $\hat{A}_d = \left[\sqrt{\beta_1} \widetilde{B}^{\mathrm{T}} A_{d1} \Gamma X \Gamma \widetilde{B}, \ldots, \sqrt{\beta_l} \widetilde{B}^{\mathrm{T}} \times A_{dl} \Gamma X \Gamma \widetilde{B} \right]$ $R = diag\{Q_1, \ldots, Q_1\}$.

Proof. We define regions

$$\begin{aligned} \Phi_i = &\{ \xi_1(t) | \xi_1^{\mathrm{T}}(t) \{ \widetilde{B}^{\mathrm{T}} (A_i \Gamma X \Gamma + \Gamma X \Gamma A_i^{\mathrm{T}} + \Gamma X \Gamma C_i^{\mathrm{T}} C_i \Gamma X \Gamma \\ &+ \gamma^{-2} G_i G_i^{\mathrm{T}}) \widetilde{B} + \varepsilon Q_1 + (\widetilde{B}^{\mathrm{T}} A_{di} \Gamma X \Gamma \widetilde{B}) \varepsilon^{-1} Q_1^{-1} (\widetilde{B}^{\mathrm{T}} A_{di} \Gamma X \Gamma \widetilde{B})^{\mathrm{T}} \} \\ &\times \xi_1(t) < 0, i \in \Xi \}. \end{aligned} \quad (24)$$

Obviously, $\bigcup_{i \in \Xi} \Phi_i = R^{(n-m)} \setminus \{0\}$.

The hysteresis switching law for the system (11) is designed as follows

$$\sigma(0) = \min \arg \{ \Phi_i | \xi_1(0) \in \Phi_i \},$$

for $t > 0$,

$$\sigma(t) = \begin{cases} i, & if\ \xi_1(t) \in \Phi_i\ and\ \sigma(t^-) = i, \\ \min \arg \{ \Phi_k | \xi_1(t) \in \Phi_k \}, & if\ \xi_1(t) \notin \Phi_i\ and\ \sigma(t^-) = i. \end{cases} \quad (25)$$

We first verify the stabilization the system (11) with $\omega = 0$. Take symmetric positive-define matrix P_2, and define the Lyapunov-Krasovskii functional

$$V(t) = \xi_1^{\mathrm{T}} P_2 \xi_1 + \int_{t-\tau}^{t} \xi_1^{\mathrm{T}}(\theta) P_2 \varepsilon Q_1 P_2 \xi_1(\theta) d\theta , \quad (26)$$

where Q_1 satisfies (23). The derivative of (26) along the trajectory of the sliding motion (11) is

$$\begin{aligned}\dot{V}(t) &= \xi_1^{\mathrm T}(t)\{(\widetilde{B}^{\mathrm T}A_\sigma P\widetilde{B}(\widetilde{B}^{\mathrm T}P\widetilde{B})^{-1})^{\mathrm T}P_2 + P_2(\widetilde{B}^{\mathrm T}A_\sigma P\widetilde{B} \\ &\quad\times(\widetilde{B}^{\mathrm T}P\widetilde{B})^{-1}) + P_2\varepsilon Q_1P_2\}\xi_1(t) + 2\xi_1^{\mathrm T}(t)P_2\widetilde{B}^{\mathrm T}A_{d\sigma}P\widetilde{B} \\ &\quad\times(\widetilde{B}^{\mathrm T}P\widetilde{B})^{-1}\xi_1(t-\tau) - \xi_1^{\mathrm T}(t-\tau)P_2\varepsilon Q_1P_2\xi_1(t-\tau).\end{aligned} \tag{27}$$

Note that

$$\begin{aligned}&2\xi_1^{\mathrm T}(t)P_2\widetilde{B}^{\mathrm T}A_{d\sigma}P\widetilde{B}(\widetilde{B}^{\mathrm T}P\widetilde{B})^{-1}\xi_1(t-\tau) \\ &\le \xi_1^{\mathrm T}(t)P_2\widetilde{B}^{\mathrm T}A_{d\sigma}P\widetilde{B}(\widetilde{B}^{\mathrm T}P\widetilde{B})^{-1}P_2^{-1}\varepsilon^{-1}Q_1^{-1}P_2^{-1}(\widetilde{B}^{\mathrm T}P\widetilde{B})^{-1} \\ &\quad\times\widetilde{B}^{\mathrm T}PA_{d\sigma}^{\mathrm T}\widetilde{B}P_2\xi_1(t) + \xi_1^{\mathrm T}(t-\tau)P_2\varepsilon Q_1P_2\xi_1(t-\tau).\end{aligned} \tag{28}$$

Substituting the right side of the inequality (28) into (27), we have

$$\begin{aligned}\dot{V}(t) &\le \xi_1^{\mathrm T}(t)P_2\{P_2^{-1}(A_\sigma P\widetilde{B}(\widetilde{B}^{\mathrm T}P\widetilde{B})^{-1})^{\mathrm T} \\ &\quad+(A_\sigma P\widetilde{B}(\widetilde{B}^{\mathrm T}P\widetilde{B})^{-1})P_2^{-1} + \varepsilon Q_1 + \widetilde{B}^{\mathrm T}A_{d\sigma}P\widetilde{B}(\widetilde{B}^{\mathrm T}P\widetilde{B})^{-1} \\ &\quad\times P_2^{-1}\varepsilon^{-1}Q_1^{-1}P_2^{-1}(\widetilde{B}^{\mathrm T}P\widetilde{B})^{-1}\widetilde{B}^{\mathrm T}PA_{d\sigma}^{\mathrm T}\widetilde{B}\}P_2\xi_1(t).\end{aligned} \tag{29}$$

Choose $P_2 = (\widetilde{B}^{\mathrm T}P\widetilde{B})^{-1}$, we get $\dot{V}(t) < 0$. By the single Lyapunov function method, the sliding motion (11) with $\omega(t) = 0$ based on the sliding surface (5) is asymptotically stable under the switching law (25).

In the following, we show that the overall L_2-gain from ω to z is less than or equal to γ in the single sliding surface. Similar to the proof of Theorem 1, we introduce

$$J = \int_0^T (\|z\|^2 - \gamma^2\|\omega\|^2)dt\,. \tag{30}$$

According to the switching sequence (2), when $T \in [t_k, t_{k+1})$, we have

$$\begin{aligned}J &= \sum_{j=0}^{k-1}\left(\int_{t_j}^{t_{j+1}}(\|z\|^2 - \gamma^2\|\omega\|^2 + \dot{V}(t))dt\right) \\ &\quad+ \int_{t_k}^{T}(\|z\|^2 - \gamma^2\|\omega\|^2 + \dot{V}(t))dt - V(T).\end{aligned} \tag{31}$$

Note that

$$\|z\|^2 - \gamma^2\|\omega\|^2 + \dot{V}(t) < 0\,. \tag{32}$$

Therefore, $J < 0$ for $\forall\omega \in L_2[0,\infty)$. That is $\|z(t)\|_2 < \gamma\|\omega(t)\|_2$. This completes the proof.

Next, the result of controller design of reaching motion is given.

Theorem 4. Suppose that (23) is feasible and the single sliding surface is given by (5). Then the state of the system (1) can enter the sliding surface in finite time, and subsequently remain on it by employing the following variable structure controllers

$$\begin{aligned}u_i &= -(SB)^{-1}SA_ix(t) - \frac{(SB)^{-1}}{1-\rho_i}\{\rho_i\|SA_ix(t)\| + \lambda\|SA_{di}\|\|x(t)\| \\ &\quad+\|SD_{1i}\|\|Ex(t)\| + \lambda\|SD_{2i}\|\|E\|\|x(t)\| + \|SB\|\phi_i(x,t) \\ &\quad+\|SG_i\|\varpi + \mu_2\}sign(\zeta), i \in \Xi,\end{aligned} \tag{33}$$

where μ_2 is a positive scalar.

Proof. It follows from the Razumikin theorem [20] that for any solution $x(t+\theta)$ of (1) there exist a constant $\lambda > 1$ such that

$$\|x(t+\theta)\| \le \lambda\|x(t)\|, -\bar{\tau} \le \theta \le 0\,. \tag{34}$$

Consider the following Lyapunov function candidate

$$V(\zeta) = \frac{1}{2}\zeta^{\mathrm T}\zeta\,. \tag{35}$$

Its derivative along the trajectory of the system (1) is

$$\begin{aligned}\dot{V} &\le \zeta^{\mathrm T}\{SA_ix(t) + SB[u_i + U_i(t)u_i]\} + \|SA_{di}\| \\ &\quad\times\|x(t-\tau)\|\|\zeta\| + \|SD_{1i}\|\|Ex(t)\|\|\zeta\| + \|SD_{2i}\|\|E\| \\ &\quad\times\|x(t-\tau)\|\|\zeta\| + \|SB\|\|\zeta\|\phi_i(x,t) + \|SG_i\|\|\zeta\|\varpi.\end{aligned} \tag{36}$$

Applying the variable structure controllers (33) to the inequality (36) results in $\zeta^{\mathrm T}\dot{\zeta} \le -\mu_2\|\zeta\|$. Hence the state of the system (1) will reach the single sliding surface (5) in finite time and subsequently remains on it. This completes the proof.

IV. Examples

In this section, we present a numerical example to demonstrate the effectiveness of the proposed design method.

Consider the following uncertain switched delay system

$$\begin{aligned}\dot{x}(t) &= (A_\sigma + \Delta A_\sigma)x(t) + (A_{d\sigma} + \Delta A_{d\sigma})x(t-\tau) \\ &\quad+ B(u_\sigma + U_\sigma(t)u_\sigma + f_\sigma(x,t)) + G_\sigma\omega(t), \\ x(t) &= \varphi(t),\ t \in [-\tau, 0], z(t) = C_\sigma x(t),\end{aligned} \tag{37}$$

where $\sigma(t) \in \Xi = \{1,2\}$, τ is an unknown constant and $\tau \le 2$,

$$A_1 = \begin{bmatrix}0.2, 1, -0.5\\ 0.5, 1,\ 0.5\\ 0, 0, -0.5\end{bmatrix},\ A_2 = \begin{bmatrix}-0.1, 1, 0.2\\ 1,\ 1,\ -1\\ 05,\ 1, -0.5\end{bmatrix},\ A_{d1} = \begin{bmatrix}1, 0.5, -0.5\\ -0.5, 0, 0.5\\ 1,\ 1, -0.5\end{bmatrix},$$

$$A_{d2} = \begin{bmatrix}0.1,\ 1,\ 0\\ 0, 0.5, -0.5\\ 0,\ 1,\ 0.5\end{bmatrix},\ B = \begin{bmatrix}0\\ 1\\ -0.5\end{bmatrix},\ G_1 = \begin{bmatrix}0.1\\ 0\\ 0.2\end{bmatrix},\ G_2 = \begin{bmatrix}0.1\\ 0\\ 0.1\end{bmatrix},$$

$C_1 = C_2 = [1,0,0]$, $Z_1 = Z_2 = 0$, $f_1 = f_2 = 0$, the parameter uncertainties $\Delta A_i = D_{1i}\Sigma_{1i}(t)E$, $\Delta A_{di} = D_{2i}\Sigma_{2i}(t)E$, where $D_{11} = D_{12} = [0,1,0]^{\mathrm T}$, $D_{21} = D_{22} = [1,0,0]^{\mathrm T}$, $E = [1\ \ 1\ \ 0]$, $\Sigma_{1i} = v_{1i} \in [-1,1]$, $\Sigma_{2i} = v_{2i} \in [-1,1]$.

Take the combination coefficients $\beta_1 = 0.4$, $\beta_2 = 0.6$, constants $\varepsilon = 0.1$, $\mu_2 = 1.5$ and the disturbance attenuation level is given by $\gamma = 1$.

By solving inequality (23), we can obtain the following solutions

$$X = \begin{bmatrix}9474,\ 2737.4,\ 5033.2\\ 2737.4, -3998.8, 5033\\ 5033.2, 5033,\ \ 0.3\end{bmatrix}\ Q_1 = \begin{bmatrix}0.5108,\ -0.1198\\ -0.1198, 1.8125\end{bmatrix},$$

$$Y = 65.153\,.$$

Refer to [2], to regulate value of the sliding function; we take the single sliding function

$$\hat{\zeta}(t) = N\zeta(t)\,, N = 100\,.$$

The single sliding function is

$$\hat{\zeta}(t) = \begin{bmatrix}1.5348 & 1.5348 & 0\end{bmatrix}x(t)\,. \tag{38}$$

According to (33), the controllers are

$$
\begin{aligned}
u_1 &= -0.7x_1(t) - 2x_2(t) - (1.4141\|x(t)\| \\
&\quad + \|x_1(t) + x_2(t)\| + 1.5)sign(\hat{\zeta}(t)), \\
u_2 &= -0.9x_1(t) - 2x_2(t) + 0.8x_3(t) - (3.1685\|x(t)\| \\
&\quad + \|x_1(t) + x_2(t)\| + 1.5)sign(\hat{\zeta}(t)).
\end{aligned} \tag{39}
$$

Following the proposed design method in Section 3.2, the initial state is $x_0 = [1,2,-1]^{\mathrm{T}}$. It is easy to verify that the conditions of Theorem 3 and 4 are satisfied.

The switching law is

$$
\sigma(t) = \begin{cases} 1, if\,(x(0) \in \Phi_1) or (x(t) \in \Phi_1 and \sigma(t^-) = 1) \\ \quad or (x(t) \notin \Phi_2 and \sigma(t^-) = 2), \\ 2, if\,(x(0) \notin \Phi_1) or (x(t) \in \Phi_2 and \sigma(t^-) = 2) \\ \quad or (x(t) \notin \Phi_1 and \sigma(t^-) = 1), \end{cases} \tag{40}
$$

where $\Phi_1 = \{x(t) | x^{\mathrm{T}}(t) \begin{bmatrix} -0.1444 & -0.0503 & -0.1006 \\ -0.0503 & 0.0182 & 0.0364 \\ -0.1006 & 0.0364 & 0.0728 \end{bmatrix} x(t) < 0\}$,

$$
\Phi_2 = \{x(t) | x^{\mathrm{T}}(t) \begin{bmatrix} 0.0341 & 0.003 & 0.006 \\ 0.003 & -0.0368 & -0.0736 \\ 0.006 & -0.0736 & -0.1472 \end{bmatrix} x(t) < 0\}.
$$

The simulation result for the switched system (37) with unknown time-delay τ is shown in Fig. 1.

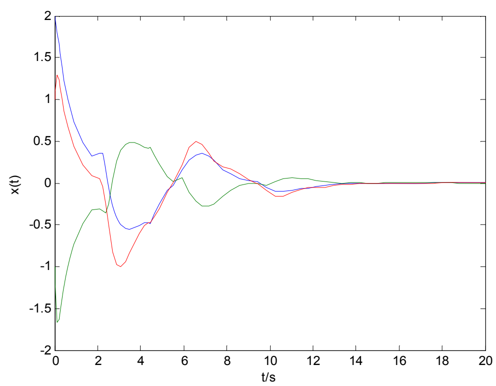

Fig. 1. The state responses of the switched system (37)

The state of the system (37) in the closed-loop is shown in Fig. 1. It is clearly seen that the closed-loop system of the switched system (37) with the designed controllers (39) and the switching law (40) is robustly asymptotically stable.

V. Conclusion

In this paper, the problem of robust H_∞ sliding mode variable structure control has studied for a class of uncertain switched delay systems. The single sliding surface has been constructed. The existence conditions of the sliding surface have been proposed for delay-known and delay-unknown cases, respectively. The corresponding hysteresis switching laws and variable structure controllers have been developed such that the resulting closed-loop system is robust stabilizable with H_∞ disturbance attenuation level γ and completely invariant to all admissible uncertainties in the sliding surface.

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