2008 American Control Conference
Westin Seattle Hotel, Seattle, Washington, USA
June 11-13, 2008

Uncertainty Reduction Through Active Disturbance Rejection

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***Abstract*—The theme of modern control is how to get around the unknowns, i.e. model uncertainties and disturbances, so that they do not degrade what is valued: stability and performance. That is, the unknowns are accepted as part of the system. Another option perhaps, proposed here, is to first make a frontal attack on the unknowns, to reduce their effects and then, only then, invoke the existing well-established methodology to deal with the remnants. In particular, it is shown that the amount of uncertainties can be reduced by way of active disturbance rejection, implemented in an inner loop to produce a well-behaved plant, which is then regulated by another controller in the outer loop. What's new here is a two degree of freedom design to deal with the unknowns: they are first actively estimated and rejected; then the remaining uncertainty, mostly in high frequency, is dealt with by, say, an H_∞ controller. The result is a hybrid H_∞-Active Disturbance Rejection Control (H-ADRC) strategy. A motion control scenario is used to illustrate how the new approach could benefit problem-solving in the real world.**

I. INTRODUCTION

The theory and practice of control system design have long had a symbiotic relationship. The former provides the insight and understanding; the latter the utility. The mathematics of feedback were discovered in classical control, whereas in modern control new controllers from mathematical models, or idealizations of physical plants, are synthesized. The utility of such design hinges upon, of course, the discrepancy between the real and modeled dynamics, also known as model uncertainty. This has been the focus of modern control theory for the last several decades.

Model uncertainty has been characterized in literature as structured and unstructured, reflecting the nature and degree of uncertainty. Structured uncertainty describes the unknowns in the parameters of an otherwise explicitly given mathematical model, often referred to as parametric uncertainty. Unstructured uncertainty, on the other hand, points to unknowns beyond those in the parameters in which case the dynamics, such as those at high frequency or those too complicated to describe, itself is neglected [1][2][3]. In the modern control paradigm, the uncertainty, regardless of its type, is separated from the nominal plant model and described in one of three general forms:

1) Multiplicative uncertainty:

$$G_p = G(1+W_I\Delta), \quad |\Delta| \le 1 \tag{1}$$

2) Inverse multiplicative uncertainty:

$$G_p = G\frac{1}{(1+W_I\Delta)}, \quad |\Delta| \le 1 \tag{2}$$

3) Additive uncertainty:

$$G_p = G + W_I\Delta, \quad |\Delta| \le 1 \tag{3}$$

where G_p represents the uncertain plant, G is the nominal plant model, Δ is the uncertainty whose magnitude is bounded by ±1, and W_I is the uncertain frequency weight which scales the uncertainty. The subscript I represents uncertainty at the input, but for Single Input Single Output (SISO) system, where this discussion is limited to for the sake of simplicity, the uncertainty at the input is equivalent to the uncertainty at the output.

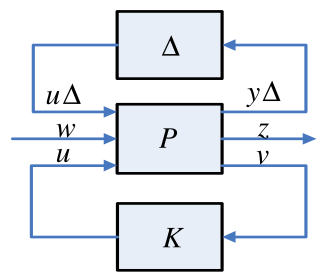

Figure 1. Δ-P-K Structure

In the modern control paradigm, the nominal plant model, uncertainty, and closed loop controller are organized into the *Δ-P-K* structure as shown in Figure 1 where: P is known as the general plant and written as:

$$\begin{bmatrix} y_\Delta \\ z \\ v \end{bmatrix} = P\begin{bmatrix} u_\Delta \\ w \\ u \end{bmatrix} \tag{4}$$

with inputs: u_Δ as uncertainty, w as external inputs, and u as plant input. The outputs of P are: y_Δ, the input to the uncertainty block above, z, the desired process variable to be minimized, and v, the tracking error which is the input to the controller.

The problem becomes that of finding a controller K such that the performance specifications are met and, more importantly, the closed-loop system remains stable for all possible uncertainties given in equations (1) to (3). A typical solution is H_∞ design where the robustness is attained based on the small-gain theorem, with the premise that the uncertainty is small [8]. μ-synthesis offers another solution with a given uncertainty weight function, but the resulting controller is often of a high order and may be difficult to implement [3][8]. Both methods are clearly limited in the amount of uncertainties they can handle, which then poses the question of whether the amount of uncertainty can be reduced first before robust control is applied.

The active disturbance rejection paradigm [9][13][14] provides an alternative to model-based design. Its central premise is that certain unknowns in physical systems, including both dynamics and disturbances, can be estimated from the input-output data and compensated for in real time, thus transforming a highly uncertain system into a well-behaved one. To this end, several disturbance observer techniques have been proposed, including the unknown input observer (UIO) [6][7], perturbation observer (PoB) [5], the disturbance observer (DoB) [4], and the extended state observer (ESO) [9]-[15] as a few examples.

The differences in these disturbance observers can be seen in terms of 1) the amount of modeling information required; 2) representation: transfer function or state space; 3) observer gains: linear or nonlinear. Of all the observers, the ESO appears to require the least amount of modeling information; is implemented in the state space form, which offers better numerical properties; and can employ both linear or nonlinear gains for maximum performance benefits. It is for these reasons that the Active Disturbance Rejection Control (ADRC) employs the ESO as its core, although other disturbance observers can also be viewed as special cases of ADRC.

The objective of this research is to combine the active disturbance rejection ideas with the modern robust control methodology to form a powerful one-two punch in making a control system truly robust. Instead of passively coping with uncertainties as constraints in design, a pro-active stance is taken in first attempting to reduce the amount of uncertainty through ADRC and then applying the robust control paradigm to deal with the remnants. The paper is organized as follows. The main idea of uncertainty reduction is presented in Section II, followed by the design for the canonical plant in section III. Section IV compares the traditional ADRC controller against the hybrid H_∞ -Active Disturbance Rejection Control (H-ADRC) controller, followed by a robust stability analysis in Section V. Finally, concluding remarks are included in section VI.

II. Uncertainty Reduction

Any real physical plant contains uncertainties, including both the external disturbance and unknown dynamics. To deal with the latter, the main approach in control theory consists of three steps: 1) determine the mathematical model as accurately as possible, leaving the smallest amount of modeling uncertainty as possible; 2) determine the bound of model uncertainty, mostly in frequency domain; 3) use the uncertainty bound as a design constraint to find a solution that is a compromise between robust stability and performance. Parallel to this approach, ADRC asks a different question: can the total uncertainty, including both types mentioned above, be reduced first, leaving the feedback control loop to deal with a system that is rather certain and deterministic?

Perhaps without realizing it, disturbance observers are different answers to this question. Although most were designed to estimate and cancel external disturbances, these disturbance observers, as shown below, all have the additional benefit of reducing model uncertainty. If possible, for the time being, ignore the differences and concentrate on the commonalities among these disturbance observers, which can be reduced to the form of Figure 2, using the transfer function metaphor. Here Q is a noise filter, $P_f = G^{-1}Q$, and G_p is the perturbed plant which may be in any of the forms of equations (1) to (3).

The original intent of the disturbance observer design is to estimate the external disturbance, d, and cancel it such that the new plant from u_0 to y is disturbance free. Such characteristics have been well established in practice and analysis. What is of interest here is the effect such a disturbance observer has on the uncertain dynamics. In particular, in the absence of d, is the model uncertainty reduced in Figure 2? That is, if the plant G_p is of the form of equation (1), (2), or (3), is there less uncertainty in the transfer function from u_0 to y? From Figure 2:

$$G_{yu_0} = \frac{G_p}{1 - Q + G_p P_f} \tag{5}$$

or, if P_f is replaced by $G^{-1}Q$:

$$G_{yu_0} = \frac{G_p G}{Q(G_p - G) + G} \tag{6}$$

which was first shown in the analysis of the DoB [4].

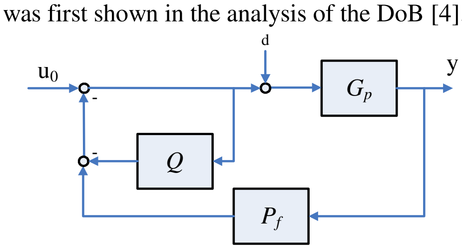

Figure 2. Equivalent Block Diagram of Various Disturbance Observers

Now, consider a plant with multiplicative uncertainty as written in equation (1), substituting (1) into (6) results in the transfer function of:

$$G_{yu_0} = G\left(1 + \frac{(1-Q)W_I\Delta}{1 + QW_I\Delta}\right) \tag{7}$$

and as Q approaches unity, assuming that noise is negligible, (7) reduces to:

$$G_{yu_0} \approx G \tag{8}$$

which demonstrates that, under ideal conditions, the model uncertainty is completely removed by the disturbance observer! Of course it is unrealistic to believe that such feat can be pulled off in practice, as previous researchers have demonstrated; it is nonetheless an important discovery, the connection between the external disturbance removal and the model uncertainty reduction. And this is not limited to multiplicative uncertainty.

Consider a system with inverse multiplicative uncertainty, equation (2), and an active disturbance rejection technique designed around the plant as shown in Figure 2. The transfer function of the inner plant becomes:

$$G_{yu_0} = \frac{G}{1 + (1-Q)W_I\Delta} \tag{9}$$

and when $Q \approx 1$, (9) reduces to:

$$G_{yu_0} \approx G \tag{10}$$

The same may be shown for additive uncertainty where the plant in (3) is substituted into (6) which results in:

$$G_{yu_0} = \frac{G(G+W_I\Delta)}{G+QW_I\Delta} \tag{11}$$

and when $Q \approx 1$, (11) reduces to:

$$G_{yu_0} \approx G \tag{12}$$

These results demonstrate that the disturbance observers have the effect of reducing the amount of uncertainty in a plant, *forcing* it to behave like the nominal transfer function G.

It is here that ADRC takes one more bold step: making G a cascaded integral plant of order to the real plant, regardless of its dynamics. That is, in the ADRC framework, both the external disturbance and internal dynamics are estimated and canceled, leaving the feedback control loop to deal only with a simple cascaded integral plant. A general nonlinear, time-varying second order plant will be used as an illustration.

A. Uncertainty Reduction via Active Disturbance Rejection

For the purpose of illuminating the idea of active disturbance rejection and evaluate its potential in uncertainty reduction, a second order plant with unity gain is selected here:

$$\ddot{y} = f(\dot{y}, y, t, d) + u \tag{13}$$

where $f(\cdot)$, generally unknown, represents the nonlinear, time-varying dynamics, and the effect of external disturbance, d. A unity gain is chosen for the sake of simplicity. A conventional approach would start with modeling, i.e. obtaining the approximate mathematical expression of $f(\cdot)$, upon which the control design would follow. The key idea of ADRC is to target $f(\cdot)$ as a general disturbance to be estimated and rejected (canceled) and, if successful, reduce the problem to the control of a double integral plant. That is, if f from equation (13) can be fairly estimated as $\hat{f}$, the control law:

$$u = -\hat{f} + u_0 \tag{14}$$

reduces the plant in (13) to:

$$\ddot{y} \approx u_0 \tag{15}$$

thus transforming a nonlinear, unknown, and time-varying plant to a well behaved, easy to control one.

The success of this active disturbance rejection approach to control design hinges upon the timely and accurate estimation of f. To this end, the extended state observer (ESO) is introduced. If (13) is written in state space form and augment the state vector with f as an extra, or extended state, then:

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= x_3 + u \\ \dot{x}_3 &= \dot{f} \end{aligned} \tag{16}$$

and the state observer of which, the ESO, can be constructed as:

$$\begin{aligned} \dot{z} &= Az + Bu + L_c(y - \hat{y}) \\ \hat{y} &= Cz \end{aligned} \tag{17}$$

where:

$$\begin{aligned} A &= \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \\ C &= \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}^T, \quad L_c = \begin{bmatrix} l_1 \\ l_2 \\ l_3 \end{bmatrix} \end{aligned} \tag{18}$$

and L_c is the gain vector to be selected. For the ease of tuning, it was suggested [10] that the observer be parameterized by the observer bandwidth, ω_o, such that its characteristic equation is:

$$\lambda(s) = \lambda^3 + l_1\lambda^2 + l_2\lambda + l_3 = (\lambda + \omega_o)^3 \tag{19}$$

Note that the ESO can be converted from state space form to transfer function form in the form of Figure 2 with:

$$Q = \frac{\omega_o^3}{(s+\omega_o)^3} \tag{20}$$

$$P_f = \frac{s^2\omega_o^3}{(s+\omega_o)^3} \tag{21}$$

as shown in [12]. A detailed comparison of the ESO and other observers, such as DoB and UIO, is beyond the scope of this paper. It suffices to say that the ESO offers distinct advantages in 1) numerical efficiency in implementation; 2) ease of tuning through parameterization; 3) requiring the least amount model information.

To demonstrate the effectiveness of the ESO in uncertainty reduction, consider a second order plant with a nominal transfer function of:

$$G = \frac{200}{s(s+3)} \tag{22}$$

for which the unknown dynamics is characterized by the weight, adopted from [8], in the form of:

$$W_{ud} = \frac{\tau s + r_0}{\frac{\tau}{r_\infty}s + 1} \tag{23}$$

where r_0 is the modeling error in steady state, r_∞ is an uncertainty scalar at high frequency, and τ^{-1} is the frequency at which the system is completely unknown. For this example, assume that there is 100% modeling error at steady state, $r_0 = 1$, the frequency at which the system is unknown is $0.1Hz$, or $\tau^{-1} = 0.2\pi$, and r_∞ is chosen randomly as $r_\infty = 5$. The perturbed plant is of the form:

$$G_p = G(1 + W_{ud}\Delta), \quad |\Delta| \le 1 \tag{24}$$

The magnitude plot of the perturbed plant is depicted in Figure 3.

The amount of uncertainty reduction by the ESO is shown in Figure 4. Bode plots of the transfer function from u_0 to y in Figure 2 are shown for different observer bandwidths, ω_o. Clearly, the quality of uncertainty reduction is directly correlated to the bandwidth: the higher the ω_o, the closer the compensated plant is to the ideal double integral plant. From Figure 4 it is concluded that the plant from u_0 to y is reduced to a pure double integrator with very small error up to the frequency of $0.1\omega_o$. That is, the control design problem is reduced to dealing with a pure double integral plant at or below the frequency of $0.1\omega_o$.

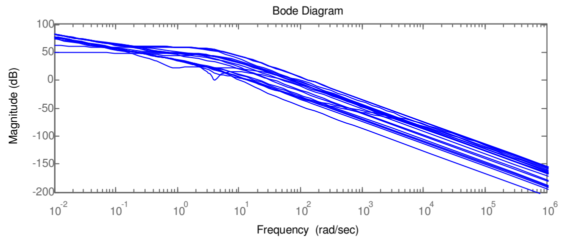

Figure 3. Magnitude Plot of the Perturbed Plant

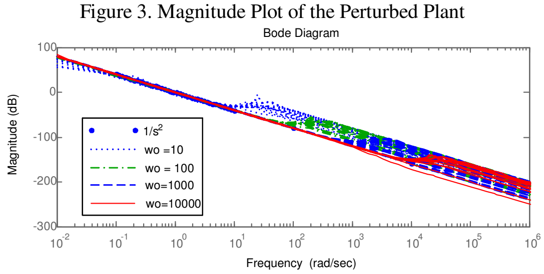

Figure 4. Magnitude Plot of the Compensated Plant

III. CONTROL DESIGN FOR A CONANICAL PLANT

Now turn to the problem of designing a front end controller for the compensated plant from u_0 to y. Depending on how high an ω_o is practically attainable, the robustness problem may or may not have to be dealt with. When ω_o can be made sufficiently higher than the control loop bandwidth, a PD controller for the double integral plant will suffice. When this is not the case, to deal with the uncertainty above the $0.1\omega_o$ frequency is where, perhaps, an opportunity exists to take advantage of the vast progress made in robust control over the last several decades.

A. Parameterized Proportional Derivative Controller

For the ideal double integral plant, a parameterized PD controller [10] is proposed with one tuning parameter, the controller bandwidth ω_c. The design goal is to make the closed loop transfer function, from the reference r to output y:

$$\frac{y}{r} = \frac{\omega_c^2}{(s+\omega_c)^2} \tag{25}$$

With the ESO providing the estimated states, the PD control law for the double integral plant of (15):

$$u_0 = k_p(r - z_1) + k_d z_2 \tag{26}$$

with the gains of:

$$k_p = \omega_c^2 \quad k_d = 2\omega_c \tag{27}$$

This is a common ADRC controller configuration [10], [13], and [15] that is simple and effective, allowing intuitive tuning on the fly based on the well-known fact that an increase in the control bandwidth results in a more aggressive closed loop system. With the observer bandwidth set as a multiple of the controller bandwidth, the entire system is tuned by adjusting the control bandwidth. Such simplicity is very attractive to practitioners. To ensure stability robustness at high frequency, it may be required to turn to a more advanced control design methodology.

B. H_∞-ADRC Control

To cope with model uncertainties, H_∞ design is a predominant solution in the literature. In this section this design is applied to the compensated plant of (15) in the hope of enhancing the robustness of ADRC against the uncertainties that could not completely be estimated and rejected by the ESO. In the H_∞ formulation, this compensated plant is rewritten as:

$$\dot{x} = Ax + B_1 w + B_2 u \tag{28}$$

$$z = C_1 x + D_{11} w + D_{12} u \tag{29}$$

$$y = C_2 x + D_{21} w + D_{22} u \tag{30}$$

where for this system:

$$A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \quad B_1 = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \quad B_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$
$$C_1 = \begin{bmatrix} 1 & 0 \end{bmatrix} \quad D_{11} = \begin{bmatrix} 0 & 0 \end{bmatrix} \quad D_{12} = [0] \tag{31}$$
$$C_2 = \begin{bmatrix} -1 & 0 \end{bmatrix} \quad D_{21} = \begin{bmatrix} 1 & 0 \end{bmatrix} \quad D_{21} = [0]$$

Using γ-iteration, the optimal H_∞ solution may be determined. More realistically, a suboptimal H_∞ controller may be obtained by specifying a desired γ and a tolerance, or accuracy.

For this system, the H_∞ design solution was determined iteratively using the Matlab Toolbox in the following manner: 1) an initial γ was selected; 2) Matlab was used to determine whether a corresponding controller K exists; 3) if it did, then γ was reduced and step 2 is repeated for the new γ. This process continued until the γ was reduced to the point where a controller did not exist. At this point, the lowest γ corresponds to the optimal controller:

$$\dot{x} = Ax + Be \tag{32}$$

$$u_0 = Cx + De \tag{33}$$

where:

$$A = \begin{bmatrix} -0.4966 & 1 \\ -1.636\times10^4 & -192.7 \end{bmatrix} \quad B = \begin{bmatrix} -0.3651 \\ 0.09066 \end{bmatrix} \tag{34}$$
$$C = \begin{bmatrix} -1.463\times10^6 & -1.724\times10^4 \end{bmatrix} \quad D = [0]$$

where e is the tracking error between the reference, r, and the estimated position from the ESO, z_1, or the actual position, y, depending on which signal is fed back to the controller, and u_0 is the input to the compensated plant shown in Figure (4). This hybrid H_∞ and ADRC design is denoted as H-ADRC.

Note that in H-ADRC controller there is only one tuning parameter, the observer bandwidth. For the feedback control loop, H-ADRC can not be tuned, but only redesigned for different design specifications. This inconvenience can be alleviated somewhat by the fact that since the H_∞ controller is always designed for the double integral plant in this case, it can be designed off-line, according to various requirements, and stored in a look up table to be selected by the users. Of course, tuning on the fly, as in the PD solution, is not an option.

IV. SIMULATION AND HARDWARE VERIFICATION

In this section the simple plant model will be simulated in Matlab to demonstrate the similarity of H-ADRC to ADRC. Both controllers have been implemented on a simple motion control testbed to compare the two controllers.

A. Simulation Comparison

For comparison purposes, both the ADRC and H-ADRC controllers are applied in simulation to the plant of equation

(22). Both are tuned for approximately the same settling time. The ADRC controller has controller gains of ω_c = 65,

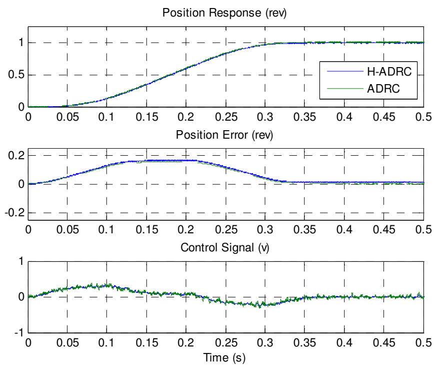

Figure 5. Comparison Between ADRC and H-ADRC

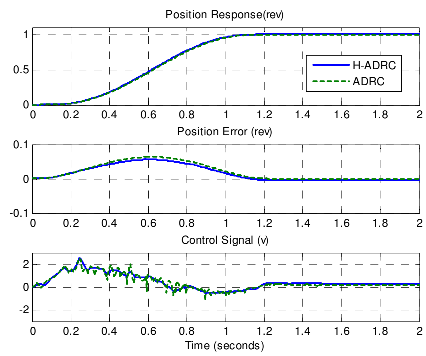

Figure 6. Hardware Results Between ADRC and H-ADRC

ω_o = 325, while the H-ADRC controller uses the same ω_o = 325. The ADRC and H-ADRC system responses are shown in Figure 5, which shows that each controller produces a response with approximately the same settling time, noise level in the control signal, and approximately same position error. In other words, performance wise, the two controllers are roughly the same.

B. Hardware Results

To verify the simulation results, the two controllers are further tested in hardware, implemented in c and tested on the Education Controls Products (ECP) Model 220 with a sampling rate of 1 kHz. The ECP Model 220 is a torsional system that contains two motors and three plates, one that attaches to the drive motor, one that attaches to the disturbance motor which injects a disturbance to the system, and one wheel that is the load itself. The system is approximately a 3:1 torque increaser, and contains weights that may be added to the system to change the amount of inertia in the system. The system has a linear time invariant model of:

$$\ddot{y} = -1.41\dot{y} + 23.2u \tag{35}$$

In this case, the estimate of b is chosen to be $\hat{b}$ = 24 for both ADRC and H-ADRC. The H-ADRC controller has an observer bandwidth of 75 while the ADRC controller has a controller bandwidth of 50 and observer bandwidth of 100. Figure 6 shows the difference between H-ADRC and ADRC controllers. In Figure 6, the H-ADRC and ADRC controllers are shown to have similar system responses, approximately the same settling time and steady state error. The difference between the two closed loop systems is the amount of noise that is present in the control signal. Furthermore, if the observer bandwidth of H-ADRC is increased to be the same as that in ADRC, the two systems will have approximately the same amount of noise in the control signal, reflecting the same observation in simulation. That is, performance wise, the two controllers are very similar.

V. ROBUST STABILITY AND TRADE-OFFS

The real benefit of the H$_\infty$ design over the PD design in ADRC turns out to be in robustness. In both cases, the plant is reduced to a nominal pure double integrator with an uncertain weight that is a function of the observer bandwidth:

$$W_I = \frac{(\frac{1}{0.1\omega_o})s + 0.01}{(\frac{1}{0.1\omega_o(5)})s + 1} \tag{36}$$

That is, the target of control design is approximately a pure double integral plant in the frequency range from DC up to the frequency of $0.1\omega_o$, beyond which there is significant dynamic uncertainty. This is of course the result of the uncertainty reduction shown in section II. This particular type of uncertainty is known as multiplicative uncertainty and the perturbed plant is written as shown in equation (1). The general robust stability condition is:

$$\bar{\sigma}(M(j\omega)) < 1, \quad \forall \omega \tag{37}$$

where M is the $N_{y\Delta u\Delta}$ transfer function from the N-Δ structure. An alternative robust stability condition for a SISO system with multiplicative uncertainty:

$$\|TW_I\|_\infty \le 1 \tag{38}$$

where T is the complimentary sensitivity function [8]. The inverse of $\|T\, W_I\|_\infty$ is deemed as the robust stability bound, which describes the tolerance in the amount uncertainty while system stability is still assured. Therefore the robust stability bound must be greater than 1 to guarantee robust stability.

For the ADRC and H-ADRC controllers tested in simulation above, the robust stability bound is found to be 0.4 for the former and 7.2 for the latter. This clearly shows that, given the same performance, the robustness of H-ADRC is superior than that of ADRC. For ADRC to meet the robust stability condition, the controller bandwidth needs to be detuned, leading to a less desired performance. On the other hand, there is, of course, a cost and trade-off associated with H-ADRC.

In Figure 5, the observer bandwidth for both ADRC and H-ADRC are equal, and the system response, position error, and control signal are approximately the same. However, in the hardware test, Figure 6, the H-ADRC observer bandwidth is smaller than the ADRC observer bandwidth, resulting in a control signal that is less noisy, but approximately the same settling time and position error. This difference shows that by decreasing the observer bandwidth the noise in the control signal is decreased. The real difference between these two controllers lies in the front-end controller. The parameterized PD design allows for the ability to easily change the controller bandwidth and quickly adjust aggressiveness of the system to suit the operational needs. This can be very advantageous, for example, in servo systems where the crossover frequency should be maintained as high as possible but must be limited to avoid excitation of mechanical resonance. The disadvantage of this design, as shown above, is the lack of robust stability for large uncertainties in the system at higher frequencies.

The H_∞ design, on the other hand, guarantees minimization of the worst case error, resulting in improved stability robustness. Performance wise, it is similar to the original ADRC with a PD controller, as shown in both simulation and hardware tests. The main disadvantage of this controller is its rigidity, or the lack of flexibility to be tuned for different operation conditions, and the inability to change the aggressiveness of the system by adjusting the controller bandwidth. One possible remedy is to design many H_∞ controllers for the double integral plant off line and put them in a look up table to be switched in and out, according the change in needs. This is of course far more complex than tuning the PD controller using a single parameter, ω_c.

VI. CONCLUSION

In this paper it is demonstrated for the first time that the uncertainty stemming from both the external disturbance and the unknown internal dynamics, which is the subject of intense research efforts in the last few decades, can be greatly reduced through active disturbance rejection. Accordingly, it is demonstrated that control of uncertain system can be carried out in two steps: 1) reducing the uncertain plant, via active disturbance rejection, to a class of cascaded integral plants; and 2) design the front end controller for these compensated plants. This paper shows quantitatively how much uncertainty reduction can be achieved, which, not surprisingly, is proportional to the bandwidth of the disturbance observer. Furthermore, once the uncertain plant is reduced to a cascaded integral one, both PD and H_∞ design can be applied to control it. Through a comparison of the two controllers in both simulation and hardware tests, it is concluded that they are similar in performance but drastically different in robustness and ease of tuning. In particular, the H_∞ design achieves better robustness at the cost of ease of tuning. Further research is needed to conduct a comprehensive study on how to make the controller both robust and easy to tune.

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