2008 American Control Conference
Westin Seattle Hotel, Seattle, Washington, USA
June 11-13, 2008

Workspace Analysis of Planar and Spatial Redundant Cable Robots

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Abstract— **This paper presents analysis of workspace of planar and spatial redundant cable robots by using two analytical approaches. The first one is based on linear algebra. It can include the upper limits for tension in cables as an important factor in optimum design of cable robots in order to find the largest possible workspace. The second approach is based on a variant of Bland's pivot rule; by virtue of this method we leave out the use of successive determinates to compute workspace resulting in less computation time. Both approaches provide all of poses (positions/orientations) reaching by the cable robot end-effector for all types of cable robots with any number of redundancy.**

I. Introduction

CABLE robots are a type of robotic manipulators which has recently attracted researchers' interest for large workspace manipulation tasks. The cable robots are relatively simple in form, with multiple cables attached to a mobile platform or an end-effector. The cable robots are appealing because of their structural simplicity, high stiffness, and high exerted wrench-to-weight ratio. A major disadvantage of cable robots is that a cable can only exert tension. Some researchers have incorporated workspace limits based on the cable interference, but these workspace limits are determined either experimentally [1] or numerically [2]. In most studies, the static equilibrium workspace (SEW) has been found numerically via "brute force" methods, where the entire task space is discretized and exhaustively searched to find the statically reachable poses [3]-[6]. Two exceptions are presented in [7] and [5],where the boundaries of the SEW are defined analytically, but both of the formulations rely on special manipulator geometries. Bosscher and Ebert-Uphoff presented an analytical method for computation of workspace [8]; however, while their work also involved characterizing the space spanned by available wrench vectors of the robot, they focused on whether this wrench-closure workspace encompasses a given bounded set of possible loads. Despite the attractiveness of this type of analysis, it is computationally difficult to find the workspace in most of complicated systems. This is due to the possible interactions between the bounded loads and the boundaries of the available wrench set.

Manuscript received March 26, 2008.

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Stump and Kumar presented two other analytical methods for computation of workspace [9]. When a cable robot has more than one redundant cable; these methods do not find all poses belonging to the workspace because of using determinates for computation of boundary of workspace which takes a lot of time. Also none of them considers any limits on the tension of cables which is very important in real applications.

In this paper, two analytical approaches are proposed in order to find the workspaces of planar and spatial cable robots with any number of redundancy. The first one is based on linear algebra. It can include tension limits for cables. The second approach, which is based on a variant of Bland's pivot rule [10], needs much less computation time compared with other methods including the first proposed approach. It is worth nothing that the second one can not compute workspace with tension limits.

I. KINEMATIC MODELING OF CABLE ROBOTS

Consider an end-effector (i.e., the moving platform) held in place by n-cables, as shown in figure 1. P_i and B_i are two attaching points of the ith cable on the end-effector and the base, respectively. $\mathbf{a}_i$ represents the position vector of B_i in the base frame, $\boldsymbol{b}_i$ represents the position vector of the cable connection in the end-effector frame. Therefore, $\boldsymbol{T}_i=\boldsymbol{a}_i-R\times\boldsymbol{b}_i-\boldsymbol{c}$ is the vector representing the length of each cable, $\vec{T}_i / \|\vec{T}_i\|$ is direction of the force along each cable, c is the position vector of mass center of end-effector parameterized by x and y for planar systems and by x,y and z for spatial systems, and R is the rotation matrix between the two frames. The equations of equilibrium for this system with respect to the end-effector frame can be written as

$$W^{m*n} f^{n*1} = D^{m*1} \tag{1}$$

where n is the number of cables, and m is 3 or 6 for planar and spatial robots, respectively. w_i, ith column of the matrix W, is the unit-length wrench vector in the form:

$$w_i = \frac{1}{\|T_i\|}\begin{bmatrix} T_i \\ (R\,b_i)\times T_i \end{bmatrix} \tag{2}$$

f is the vector of cable tensions and Vector D is the load wrench which represents external forces and torques in the following form:

$$D = \begin{bmatrix} f_{ext} \\ W_{ext} \end{bmatrix} \tag{3}$$

Major drawback for cable robots is that cables can only exert tension. Therefore, the statement of the constrained problem can be considered as:

$$W\,f = D\,, f \geq 0 \tag{4}$$

II. WORKSPACE ANALYSIS

The workspace of a cable robot is defined as the set of all poses (positions/orientations) to which the end-effector of the manipulator can physically reach while the force feasibility conditions and perhaps some additional constraints are satisfied. For the theoretical development, we propose two approaches to analytically find the workspace of the system: one is based on linear algebra with consideration of tension limit of each cable; and the other one is based on a variant of Bland's pivot rule [10]. Both analytical methods provide all of poses that satisfy equation (4).

A. Linear Algebra

1. m=n case

Given *m=n* cables in *n*-dimensional space and a load wrench vector *D,* the equilibrium equation has the following form

$$W\,f = D$$

A square system of equations which can be solved by Cramer's rule is

$$f_i = \frac{H_i}{H_0} \tag{5}$$

where

$$H_0 = \det([w_1\; w_2 \ldots. w_n]) \tag{6}$$

and for *i=1, 2 , …, n,* and

$$H_i = \det([w_1\; w_2 \ldots w_{i-1}\; D\; w_{i+1} \ldots w_n]) \tag{7}$$

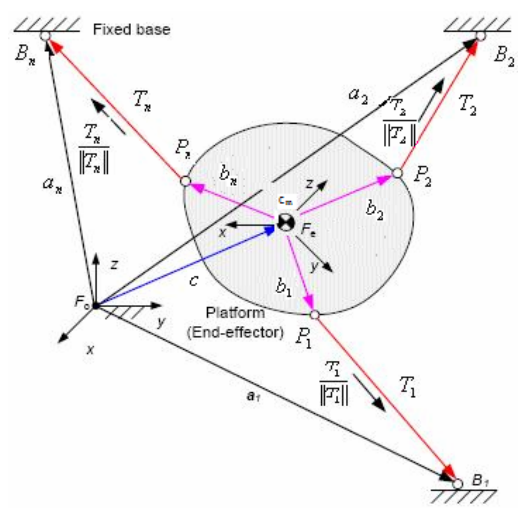


Fig.1. General kinematics of a cable robot

Therefore, there exist positive tensions if and only if the determinants $H_0, H_1, \ldots, H_m$ have the same sign, i.e.

$$H_0\, H_i > 0 \qquad \text{for i} = 1, 2, \ldots, n$$

2. *m>n case*

Equation *W f=D* is solvable if and only if *rank(W)=rank([W,D])* and this condition holds if and only if *rank(W)=m* with

$$W = [w_1\; w_2 \ldots. w_n]$$

We can rearrange the matrix columns so that the first *m* columns are linearly independent, then:

$$W' = [w_1'\; w_2' \ldots. w_m' \ldots. w_n'] \tag{8}$$

$$W'\,f' = D \tag{9}$$

Matrix A is defined as:

$$A = [W' \,|\, D]$$

We compute reduced row echelon form (Hoffman *et al.,* 1971) of the matrix *A* as:

$$U_A = \left[I^{m*m}\,,\; Q^{m*(n-m)}\,, D'\right]$$

where I^{m*m} is *m*m* identity matrix.

Equation $W\,f = D$ is equivalent of:

$$[I\,, Q]\,f' = D'$$

Solution of the above equation is:

$$f' = f_p' + f_h'$$

where

$$f_h' = x_1 \begin{Bmatrix} -Q(:,1) \\ 1 \\ 0 \\ \vdots \\ 0 \end{Bmatrix} + \ldots + x_{n-m} \begin{Bmatrix} -Q(:,n-m) \\ 0 \\ 0 \\ \vdots \\ 1 \end{Bmatrix} \tag{10}$$

and

$$f_p' = \begin{Bmatrix} D' - y_1\, Q(:,1) - \ldots - y_{n-m}\, Q(:,n-m) \\ y_1 \\ \vdots \\ y_{n-m} \end{Bmatrix} \tag{11}$$

Using equation (10) and (11);

$$f' = \begin{Bmatrix} D' \\ 0 \\ \vdots \\ 0 \end{Bmatrix} + (x_1 + y_1) \begin{Bmatrix} -Q(:,1) \\ 1 \\ \vdots \\ 0 \end{Bmatrix} + \ldots.. + (x_{n-m} + y_{n-m}) \begin{Bmatrix} -Q(:,n-m) \\ 0 \\ \vdots \\ 0 \end{Bmatrix}$$

which can also be shown in the following compact from:

$$f' = -B\,Y + C,$$

where

$$B = \begin{bmatrix} -Q^{m*(n-m)} \\ I^{(n-m)*(n-m)} \end{bmatrix}, C = \begin{bmatrix} D'^{m*1} \\ 0^{(n-m)*1} \end{bmatrix}, \; Y = \begin{bmatrix} x_1 + y_1 \\ \vdots \\ x_{n-m} + y_{n-m} \end{bmatrix}$$

As positive exertion condition, tension of each cable is constrained by:

$$f' \geq 0$$

$$\begin{bmatrix} Q \\ -I \end{bmatrix} Y \leq \begin{bmatrix} D' \\ 0 \end{bmatrix}$$

$$B\,Y \leq C\,, \tag{12}$$

If we consider tension limit of each cable, then:

$0 \le f' \le f'_{\max}$

$0 \le -B\,Y + C \le f'_{\max}$

If robot has one redundant cable, we define $B_- = \{i \mid B_i < 0\}$, $B_0 = \{i \mid B_i = 0\}$ and $B_+ = \{i \mid B_i > 0\}$. The inequality of (12) is satisfied if and only if one of the conditions below holds:

1) $i \in B_-$ *or* $i \in B_0$

if $i \in B_0$ *then* $C(i) \ge 0$

2) $i \in B_+$ *or* $i \in B_0$

if $i \in B_0$ *then* $C(i) \ge 0$

3) $\max\limits_{\forall i \in B_-} \left(\dfrac{C(i)}{B(i)}\right) \le \min\limits_{\forall i \in B_+} \left(\dfrac{C(i)}{B(i)}\right)$ *and*

if $i \in B_0$ *then* $C(i) \ge 0$

Also, when a cable robot has one or more redundant cables, one may show equivalence of equation (12) with $E(Y)$ being zero where

$$E(Y) = (C - B\,Y)^T \,(C - B\,Y - \left|C - B\,Y\right|), \qquad (13)$$

Since equation (13) has a positive value, if minimum of equation (13) is zero the corresponding pose belongs to the workspace. When we consider tension limit in each cable, not only equation (13) should be zero, but also the following equation must be zero.

$$E2(Y) = (f'_{\max} - C + B\,Y)^T \,(f'_{\max} - C + B\,Y - \left|f'_{\max} - C + B\,Y\right|), \qquad (14)$$

Equations (13, 14) can be solved by non linear programming (NLP) method.

B. Variant of Bland's pivot rule

We can rearrange the matrix columns, so that the first m columns of W are linearly independent, then:

$$\left[W', W''\right] f = D \qquad (15)$$

where

$rank(W') = m$

Now equation (15) can be expanded to

$$W'\,f' + W''\,f'' = D \qquad (16)$$

We may multiply both sides of equation (16) by $(W')^{-1}$

$f' + (W')^{-1}\,W''\,f'' = (W')^{-1}\,D$

$f' \ge 0 \Rightarrow$

Therefore

$$(W')^{-1}\,W''\,f'' \le (W')^{-1}\,D$$
$$f'' \ge 0 \qquad (17)$$

We now have a complete description of the algorithm which is called the b-rule, for solving a problem in the form of (17) as [10]:

Step 1: Introduce m slack variables $f''_{n-m+1},\ldots,f''_n$ and use these as the basis (left-hand side):

$$f''_{n-m+i} = ((W')^{-1}\,D)_i - \sum_{j=1}^{n-m} ((W')^{-1}\,W'')_{ij}\; f''_j$$

$i = 1,\ldots,m$

Step 2: Set the cobasic (right-hand side) variable to zero. Find the smallest index of the basic (left-hand side) variables which receive a negative value. If there is none, terminate the algorithm with a feasible solution.

Step 3: Find the cobasic variable in the equation chosen in Step 2 that has the smallest index and a positive coefficient. If there is none, terminate as the problem is infeasible. The coefficients of the slack variables give a certificate of infeasibility. Otherwise, solve this equation for the variable, and substitute in all of the other equations. Go to Step 2.

It can be proved that the algorithm described above terminates in a finite number of steps [10]. This method, in addition to computing all poses that satisfy equation (4), is less time consuming in as compared to the linear algebra and the direct analysis [9] methods.

III. SIMULATION

In this article, analytical techniques are developed to find the workspace wherein all cables are under positive tension while exerting all possible wrenches. This is called the static workspace. It is assume that velocities and accelerations on the end-effector are small and thus the device may be controlled in a pseustatic manner. We will find the workspace for two types of cable robots, planar and spatial, containing redundant cables by the two methods presented in the previous section. In addition, the workspace is computed when tension limit for each cable is included.

We first consider a planar cable robot with two redundant cables (cable robot 1). Dimensions of this cable robot are listed in Table 1.

Table 1. Dimensions of cable robot 1

Position vector	X	Y	Position vector	X	y
$\mathbf{a_1}$	6	2.5	$\mathbf{b_1}$	0.4	0
$\mathbf{a_2}$	6	5	$\mathbf{b_2}$	0.1236	0.3804
$\mathbf{a_3}$	0	5	$\mathbf{b_3}$	-0.3236	0.2351
$\mathbf{a_4}$	0	0	$\mathbf{b_4}$	-0.3236	-0.2351
$\mathbf{a_5}$	6	0	$\mathbf{b_5}$	0.1236	-0.3804

We may also include a load wrench due to gravity in the problem statement. Therefore, the load wrench D for the planar cable robot will be:

$D = mg\,\left[0 \;\; 1 \;\;\; x\right]^T$

The result of applying variant of Bland's pivot rule to this robot is represented in figure 2.

The workspace of cable robot 1 computed by linear algebra method when maximum tension in each cable is limit, is shown in figure 3.

It can be seen that the workspace for f_{max}=*50N*, *180N* are subregion of the workspace without tension limits. By increasing f_{max} the subregion becomes larger. But there is a limit for f_{max}. It means for f_{max} larger than a special value (279.65N for this planar cable robot), which can be found by trial and error, the workspace can not become larger than workspace without any tension limits.

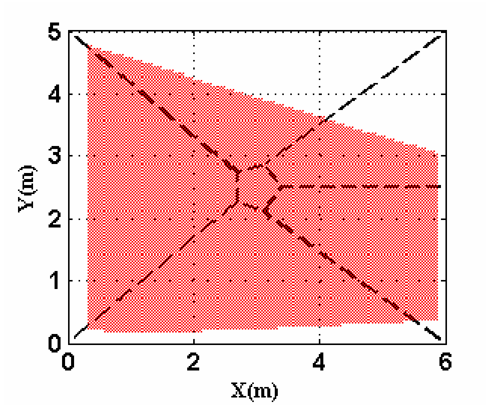


Fig 2. Workspace when the end-effector's orientation is zero, by variant of Bland's pivot rule, cable robot 1.

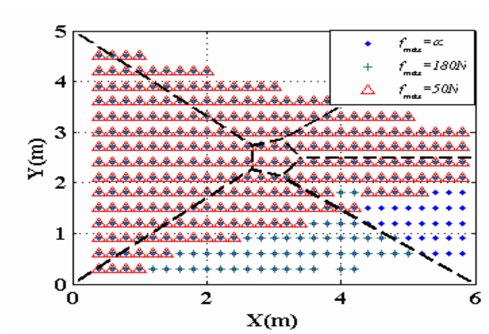


Fig 3.Workspace when the end-effector's orientation is zero (Δ points: f_{max}=$50N$, + points: f_{max}=$180N$, • points: without tension limit), by linear algebra, cable robot 1.

The resulting workspace of cable robot 1 shown in figure 4, for different θ (orientation angle), is descretized into slices here to better illustrate the interior shape of the workspace.

Workspace of cable robot 1 found by direct analysis method [9] is shown in figure 5.

It can be observed that the first two methods have computed the complete workspace in compare to the direct analysis method.

Now, we compare required computation time among the three analytical methods for cable robot 1 by a computer with CPU Duo 3GHz, as shown in Table 2.

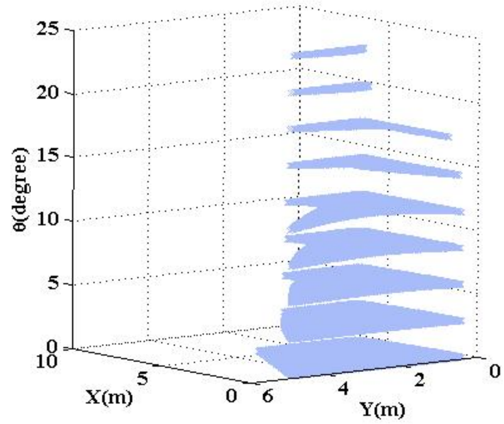


Fig 4. Workspace when the end-effector's orientation is variable, cable robot 1.

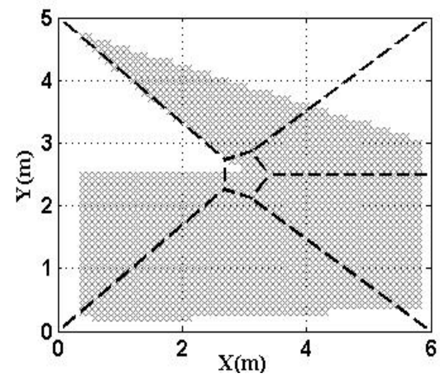


Fig 5. Workspace when the end-effector's orientation is zero, by direct analysis method, cable robot 1.

Table 2. Comparison among time efficiency of the three methods for workspace determination

Method	Time(sec)
Direct Analysis	122
Linear Algebra	121
Variant of Bland's pivot rule	16

The variant of Bland's pivot rule has less time spent for computation of workspace compared to the other methods; but, a major advantage of linear algebra method is it's capability to include tension limit for each cable.

As the second example, we consider a spatial cable robot with a redundant cable (cable robot 2). Dimensions of this cable robot are listed in Table 3.

Table 3. Dimensions of cable robot 2

Position vector	X(m)	Y(m)	Z(m)
$\mathbf{a}_1$	-0.5	0	1
$\mathbf{a}_2$	-0.5	0	1
$\mathbf{a}_3$	0.5	0	1
$\mathbf{a}_4$	0.5	0	1
$\mathbf{a}_5$	-0.2	1	1
$\mathbf{a}_6$	-0.2	1	1
$\mathbf{a}_7$	0.4	1	1

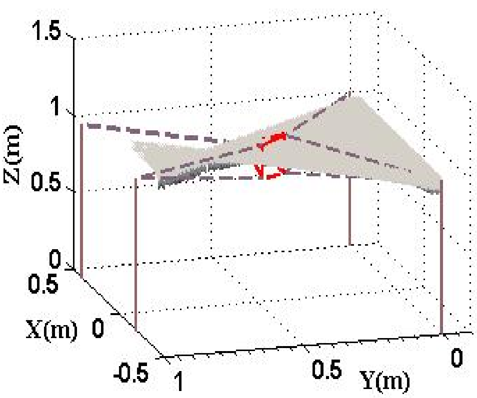
Table 3. Dimensions of cable robot 2, continued

Position vector	x(m)	y(m)	z(m)
$\mathbf{b}_1$	0	0	-0.125
$\mathbf{b}_2$	0	0	0.125
$\mathbf{b}_3$	0	0	0.125
$\mathbf{b}_4$	0	0	-0.125
$\mathbf{b}_5$	-0.0625	0.1	-0.10825
$\mathbf{b}_6$	-0.0625	0.1	0.10825
$\mathbf{b}_7$	0.125	0.1	0

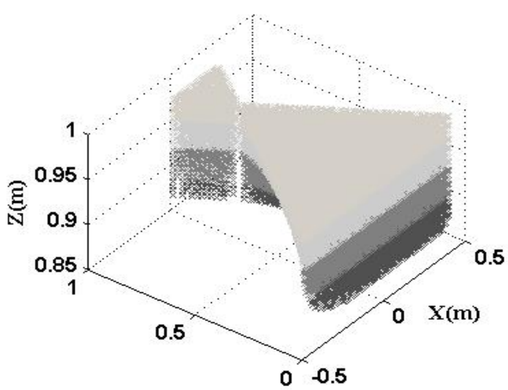
The load wrench, *D,* for the spatial robot will be:

$$D = mg\begin{bmatrix}0 & 0 & 1 & y & -x & 0\end{bmatrix}^T$$

The result obtained by applying variant of Bland's pivot rule is presented in figure 5.



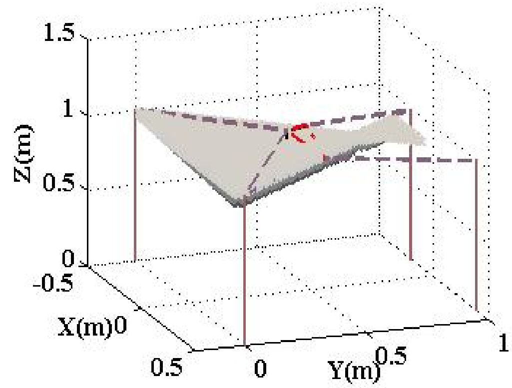
(a)



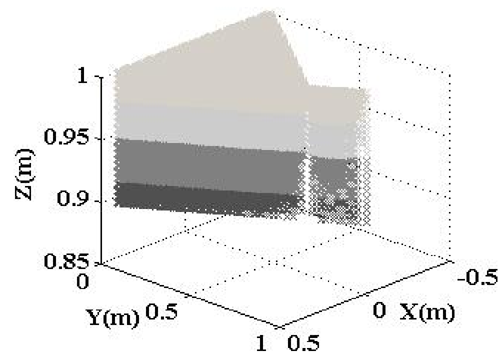
(b)

Fig 6. The computed workspace for the cable robot 2 when the end-effector's orientation is fixed by variant of Bland's pivot rule; (a) the overall view, and (b) the detail view of the same workspace.

The workspace of cable robot 2 computed by linear algebra method is shown in figure 7.



(a)



(b)

Fig 7. The computed workspace for the cable robot 2 when the end-effector's orientation is fixed by linear algebr; (a) the overall view, and (b) the detail view of the same workspace

Figure 8 display the slices the workspace in the X-Y-Z for better shown of the workspace.

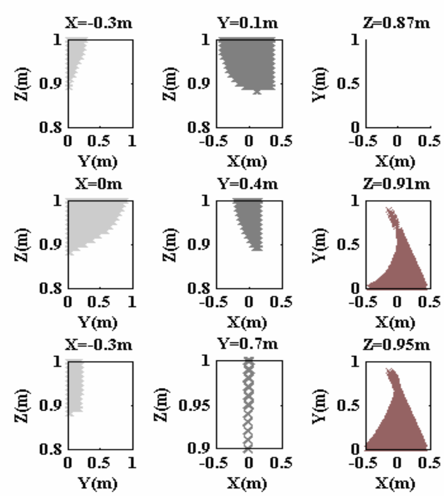


Fig 8. Slices of the workspace along the X, Y and Z axes.

Figure 9-10 display the workspace in terms of the Euler angles when the end-effector's position is fixed at (X,Y,Z)=(0, 0.4,0.95) in the base frame.

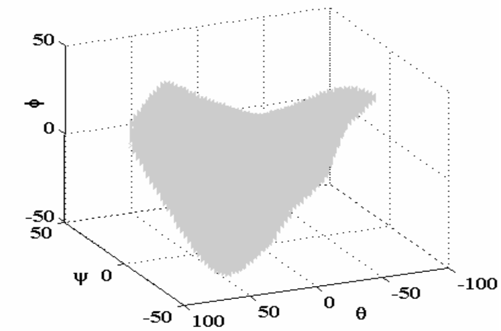


Fig 9. Workspace when the end-effector's position is fixed, cable robot 2.

where ψ rotation about x, Θ rotation about y, φ rotation about z.

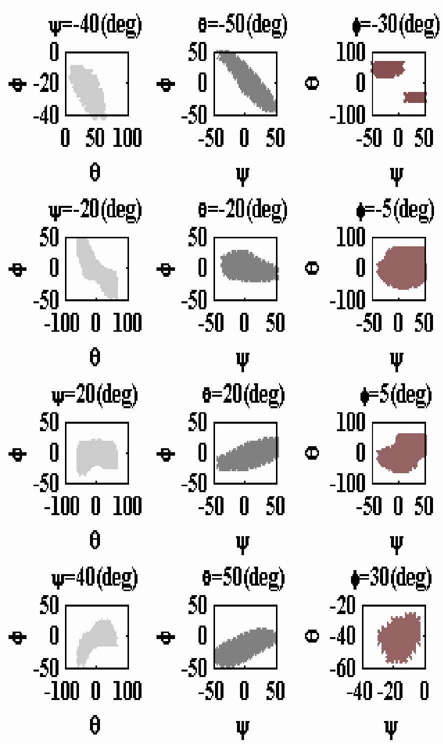


Fig 10. Slices of workspace along the ψ, Θ and φ axes.

The same observation as for planar robots can be made for spatial cable robots.

IV. CONCLUSION

We have approached the problem of find all of poses reaching by the platform. We address two analytical methods for solving this problem. It is worth nothing that the approach presents here finds natural applications in other systems with unilateral point contact model such system include multifingered grasp and include are considered.

V. FUTURE WORK

Cable intertference (cable/cable interference and cable/endeffector interference) can be a serious problem for cable robots. Therefore, if analytical expression were formulated for the condition of interference, these would constitute additional workspace. Our going work address the find collosion-free workspace as the subset of workspace that can be reached without cable interference for all type of cable robots.

REFERENCES

[1] K. Maeda, S. Tadokoro, T. Takamori, M Hiller , and R. Verhoeven , "On design of a redundant wire-driven parallel robot WARP manipulator," *Proceedings of the IEEE International Conference on Robotics and Automation*, Detroit, MI, May 1999, pp. 895–900.

[2] R. L., II Williams and P. Gallina , "Planar cable-direct-driven robots: design for wrench exertion," *Journal of intelligent and robotic systems*. vol. 35, pp. 203–219, 2002.

[3] A. B. Alp and S. K. Agrawal, "Cable suspended robots: design, planning and control," *Proceedings of the IEEE International Conference on Robotics and Automation*, May 2002, pp. 4275–4280.

[4] A. Fattah and S. K. Agrawal, "Design of cable-suspended planar parallel robots for an optimal workspace," in *Proc. Workshop on Fundam. Issues Future Res. Directions for Parallel Mech. Manip.*, Quebec City, QC, Canada, Oct. 2002, pp. 195–202.

[5] A. Fattah and S. K. Agrawal, "Workspace and design analysis of cable-suspended planar parallel robots," in *Proc. ASME DETC*,Montreal, QC, Canada, Oct. 2002, pp. 1–9.

[6] A. Ming and T. Higuchi, "Study on multiple degree-of-freedom positioning mechanism using wires (part 2)—Development of a planar completely restrained positioning mechanism," *Int. J. Jpn. Soc. Precision Eng.*, vol. 28, no. 3, pp. 235–242, Sep. 1994.

[7] J. Albus, R. Bostelman, and N. Dagalakis, "The NIST RoboCrane," *J. Nat. Inst. Standards Technol.*, vol. 97, no. 3, May–Jun. 1992.

[8] P. Bosscher., and I. Ebert-Uphoff, 2004, "Wrench Based Analysis of Cable-Driven Robots," *Proceedings of the IEEE International Conference on Robotics and Automation*, New Orleans, LA, pp. 4950–4955.

[9] E. Stump and V. Kumar, "Workspace of Cable-Actuated Parallel Manipulators" *journal of Mechanical Design,* January 2006, vol. 128, pp. 159-167.

[10] D. Avis and B. Kaluzny, "Solving Inequalities and Proving Farkas's Lemma Made Easy", March 2003.

[11] K. Hoffman and R. Kunze (1971),*'Linear algebra"* chapter one. Prentice-Hall.