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Partial Predictors for the Supervision of a Multimodel Direct Generalized Predictive Control of Highly Non Stationary Systems

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Abstract— **The multimodel approach is recently developed in order to resolve the problems of the increasing complexity of many industrial processes. In this paper we propose a multimodel generalized predictive control based on a commutation algorithm. This commutation is controlled by partial predictors associated to the local controllers. These predictors constitute the supervisor. The control law is established to ensure the desired performances in closed loop in the presence of a highly non stationary system. The obtained results are very satisfactory and show a very good precision relatively to the case in which the classical direct adaptive generalized predictive control is adopted.**

I. INTRODUCTION

With the successive developments of the modern automation, we perceived that a 'fixed' regulator could not always provide an acceptable behavior of the system in all situations. The control of dynamic systems in the presence of large uncertainties and constraints is of great interest for several applications [1], [3], [6]. In such cases, the controller has to determine the specific situation that exists at any instant and take the appropriate control action. Accomplishing this rapidly, accurately and in stable fashion is the objective in control design.

The direct adaptive generalized predictive control (*DAGPC*) is one of the few techniques that are able to cope with constraints and modelling errors in an explicit manner [1], [6]. It has proved to be efficient and successful for industrial applications. Moreover, for systems with time-varying parameters, acceptable performances may be maintained with adaptive controllers.

In addition, the presence of complex processes involve modelling and robustness problems, while using the Direct adaptive generalized predictive control.

To overcome this problem, it is necessary to introduce an intelligent modelling and control strategies. Indeed, multimodel and multicontrol approaches are considered to be very suitable and able to identify and control complex systems with high performances.

The multimodel and multicontrol approaches are known as powerful techniques to overcome difficulties encountered in conventional modelling and control techniques. These approaches are useful for the industrial processes which are, often, complex (nonlinear or/and non stationary). The basic idea of the above approaches is the decomposition of the full operation range of the process into a number of operating regimes. In each operating regime, a simple local model or controller is applied. These local models called model's library and controllers are then combined in some way to yield a global model or controller [4], [5], [8], [11], [19], [23].

To overcome the problem of direct adaptive generalized predictive control, we propose, in this work, the multimodel and multicontrol approaches.

For each local model, a generalized predictive control law is elaborated. The effective control law applied to the process is a switching between all these elementary laws. At any given instant during the operation of the system, the main task will be to determine which model approximates at the best the plant in order to apply the corresponding controller. Three inherent parts are necessary in the multicontrol approach: the first one is a set of local controllers; the second one is a 'switching system' whose task is to design the control law from the set of local controllers and the last one is a supervisory system which controls the 'switching system' and more precisely, it indicates the most appropriate controller to the latter.

Firstly, we present in this paper the development of the direct adaptive generalized predictive control. An example of simulation is given thereafter to show the limits of this control law in the presence of a highly non stationary system. In a second part, we propose a solution for these problems through the synthesis of a multimodel generalized predictive control law with a supervisor. A simulation example is proposed, illustrating that the proposed multicontrol approach is more precisely and presents good performances by comparison with the control without supervisor.

II. DIRECT ADAPTATIVE GENERALIZED PREDICTIVE CONTROL LAW (*DAGPC*)

Let's consider the CARIMA model (Controller Auto-Regressive Integrated Moving Average).

$$A(q^{-1})y(k) = q^{-d}B(q^{-1})u(k) + C(q^{-1})\frac{v(k)}{D(q^{-1})}. \quad (1)$$

with:

$$A(q^{-1}) = 1 + a_1 q^{-1} + a_2 q^{-2} + ... + a_{n_A} q^{-nA}.$$
$$B(q^{-1}) = b_0 + b_1 q^{-1} + b_2 q^{-2} + ... + b_{n_B} q^{-nB}.$$
$$C(q^{-1}) = 1 + c_1 q^{-1} + ... + c_{n_C} q^{-nc}.$$
$$D(q^{-1}) = 1 - q^{-1}.$$

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From the previous model (1) we can write:

$$y\left(k+j\right)=q^{-d}\frac{B\left(q^{-1}\right)}{A\left(q^{-1}\right)D\left(q^{-1}\right)}D\left(q^{-1}\right)u\left(k+j\right) + \frac{C\left(q^{-1}\right)}{A\left(q^{-1}\right)D\left(q^{-1}\right)}v(k+j) \tag{2}$$

The j step predictor $\hat{y}(k+j/k)$ is calculated by minimizing a simple criterion function J given by (3):

$$J=E\left\{[y(k+j)-\hat{y}(k+j/k)]^2/M_k\right\}. \tag{3}$$

with M_k is a set of known measures at instant k.

A. Matrix formulation of the predictor

The minimization of equation (3) written in a matrix form provides the future control sequence, for $j\in[HI,HP]$; (HI and HP are, respectively, the horizons of initialisation and prediction):

$$\underline{\hat{Y}}=R^{**}\underline{DU}+G^{**}\underline{Y}+Q^{**}\underline{DU}_p \tag{4}$$

with :

$\underline{\hat{Y}}=[\hat{y}(k+HI/k)\ldots\hat{y}(k+HP/k)]^T$

$\underline{DU}=[Du(k-1)\ldots Du(k-n_B)]^T$

$\underline{Y}=[y(k)\ldots y(k-n_A)]^T$

$\underline{DUp}=[Du(k)\ldots Du(k+HP-1)]^T$;

$\dim[\underline{\hat{Y}}]=(HP-HI+1,1)$; $\dim[\underline{DU}]=(n_B,1)$;

$\dim[\underline{Y}]=(n_A+1,1)$; $dim[\underline{DUp}]=(HP,1)$;

$\dim[R^{**}]=(HP-HI+1,n_B)$.

$\dim[G^{**}]=(HP-HI+1,n_A+1)$.

$\dim[Q^{**}]=(HP-HI+1,HP)$.

The matrix R^{**}, G^{**} et Q^{**} are derived by solving diophantine equations, with unique solutions [1]. Let's define a vector of prediction which depends on present and past measurements:

$$\underline{E\hat{Y}}_a=\underline{\hat{Y}}_a-\underline{YC}=R^{**}\underline{DU}+G^{**}\underline{Y}-\underline{YC} \tag{5}$$

with:
$\underline{\hat{Y}}_a=R^{**}\underline{DU}+G^{**}\underline{Y}$: is a vector of prediction relative to present and past measurements.

$\underline{YC}=[y_c(k+HI)\ldots y_c(k+HP)]^T$: is a set point vector.

$$dim[\underline{YC}]=dim[\underline{\hat{Y}}_a]=(HP-HI+1,1).$$

The prediction $\underline{E\hat{Y}}_p$ depending on the future control sequence is defined by:

$$\underline{E\hat{Y}}_p=Q^{**}\underline{DU}_p \tag{6}$$

B. Calculation of control law

The objective of the direct adaptive generalized predictive control is to calculate the optimal control sequence $[Du(k),Du(k+1),\ldots,Du(k+HC-1)]$ by minimizing the criterion given by relation (7).

$$J=\frac{1}{2}\left\{\sum_{j=HI}^{HP}[\hat{y}(k+j/k)-y_c(k+j)]^2+\lambda\sum_{j=0}^{HC-1}Du(k+j)^2\right\} \tag{7}$$

HC is the horizon of control and λ is a weight factor.

The dimension of the vector $\underline{DU}_p$ is reduced to $(HC,1)$ and the $(HP-HC)$ columns of the matrix Q^{**} are not taken into account. We obtain the new matrix Q_*.
Taking into account this last definition, the equations (4) and (6) can be written as follows:

$$\begin{aligned}\underline{\hat{Y}}&=R^{**}\underline{DU}+G^{**}\underline{Y}+Q_*\underline{DU}_p\\ \underline{E\hat{Y}}_p&=Q_*\underline{DU}_p\end{aligned} \tag{8}$$

The determination of the vector $\underline{DU}_p$ requires to put the criterion (7) under a matrix form.The optimal vector $\underline{DU}_P$ will be:

$$\underline{DU}_P=-M\left[R^{**}\underline{DU}+G^{**}\underline{Y}-\underline{YC}\right] \tag{9}$$

with:

$$M=[Q_*^TQ_*+\lambda I_{HC}]^{-1}Q_*^T=\begin{bmatrix}\underline{m_1}^T\\ \underline{m_2}^T\\ \vdots\\ \underline{m_{HC}}^T\end{bmatrix}=ZQ_*^T \tag{10}$$

with: $Z=[Q_*^TQ_*+\lambda I_{HC}]^{-1}$.
We can write the equation (9) under the following matrix form:

$$M\underline{YC}=MG^{**}\underline{Y}+\underline{DU}_P+M\ R^{**}\underline{DU}=\theta^T\phi(k) \tag{11}$$

with:

$$\begin{aligned}\theta^T&=[MG^{**}\ I_{HC}\ MR^{**}]\\ \phi\left(k\right)&=\left[y\left(k\right)...y\left(k-n_A\right)\quad DU_p^T\right.\\ &\qquad\left.Du\left(k-1\right)\ ...\ Du\left(k-n_B\right)^T\right]^T\end{aligned}$$

θ, $\phi(k)$ are, respectively, the matrix of parameters with dimension (n_A+n_B+1+HC,HC) and the input-output measurements vector with dimension $(n_A+HC+n_B+1,1)$.
In a classic way, for the direct adaptive generalized predictive control, only the first element of vector $\underline{DU}_P$ is considered. The control law applied to the process at each iteration k will be deducted from this last vector as follows:

$$Du(k)=-\underline{m_1}^T\left(R^{**}\underline{DU}+G^{**}\underline{Y}-\underline{YC}\right) \tag{12}$$

C. RST structure of the DAGPC control

This particular structure of the generalized predictive control law synthesizes an equivalent polynomial control law form with the three polynomials R, S and T [1], [6].

From the equation (12), we can write:

$$\begin{aligned} Du(k) = & -\underline{m_1}^T q^{-1} R^{**}[1 \dots q^{-n_B+1}]^T Du(k) \\ & -\underline{m_1}^T G^{**}[1 \dots q^{-n_A}]^T y(k) \\ & +\underline{m_1}^T [y_c(k+HI) \dots y_c(k+HP)]^T \end{aligned} \quad (13)$$

and also if we make:

$$R^{**}(q^{-1}) = R^{**}[1 \dots q^{-n_B+1}]^T;$$
$$G^{**}(q^{-1}) = G^{**}[1 \dots q^{-n_A}]^T;$$

we get:

$$\begin{aligned} & D(q^{-1})u(k)\left[1 + \underline{m_1}^T R^{**}(q^{-1})\, q^{-1}\right] = \\ & \quad -\underline{m_1}^T G^{**}(q^{-1})y(k) + \underline{m_1}^T \left[q^{HI} \dots q^{HP}\right]^T y_c(k) \end{aligned} \quad (14)$$

This relation must correspond to the following equation:

$$S(q^{-1})D(q^{-1})u(k) = -R(q^{-1})y(k) + T(q)y_c(k) \quad (15)$$

By identification, the three polynomials $R(q^{-1})$, $S(q^{-1})$ and $T(q^{-1})$ are:
$S(q^{-1}) = 1 + q^{-1}\underline{m_1}^T R^{**}(q^{-1}) = 1 + q^{-1}S^*(q^{-1})$,
$R(q^{-1}) = \underline{m_1}^T \underline{G^{**}(q^{-1})}$;
$T(q) = \underline{m_1}^T \underline{[q^{HI} \dots q^{HP}]^T}$;
$\deg(S(\underline{q^{-1}})) = n_B; \deg(R(q^{-1})) = n_A; \deg(T(q)) = HP$.

D. performances error and performances index

We define the prediction vector $X_p(k+HP)$, which contains the predicted output and the future control sequence [6]:

$$X_p(k+HP) = [\underline{\hat{Y}}^T \; \underline{DU}_p^T]^T \quad (16)$$

$$\dim[X_p] = (HP - HI + HC + 1, 1).$$

We also define the target vector $X_c(k+HP)$ from $X_p(k+HP)$, with the same dimension, composed of reference vector $\underline{YC}$ and zero vector Φ. Considering the fact that the output vector $\underline{\hat{Y}}$ has to converge to the reference vector while the control signal $\underline{DU}_P$ has to tend to zero, we have:

$$X_c(k+HP) = [\underline{YC}^T \quad \Phi^T]^T \quad (17)$$

Therefore, the error of " reaching target " can be defined as follows:

$$e(k+HP) = [X_p(k+HP) - Xc(k+HP)] \quad (18)$$

The introduction of a weighting matrix L with dimension $(HP - HI + HC + 1, HC)$ to create a dynamic cancellation of the error defined previously, yields to a new error called "performances error " defined as follows:

$$\begin{aligned} e_f(k+HP) &= L^T e(k+HP) \\ &= L^T \left[X_p(k+HP) - X_c(k+HP)\right] \\ &= L^T X_p(k+HP) - L^T X_c(k+HP) \\ &= Ip_r(k+HP) - Ip_d(k+HP) \end{aligned} \quad (19)$$

with: $L^T = [M \quad \lambda Z]$.
This L matrix imposes the tracking dynamic for the output and a weighting factor on the control values.
The relation (19) introducing the notion of the performances error is presented by the difference of two entities $Ip_r(k+HP)$ and $Ip_d(k+HP)$. The first one is an indication of the measured performances and the second one is an evaluation of the desired performances [6].
These two indicators serve to measure the changing of the process behavior at the instant $(k+HP)$. Indeed, in the case when the system does not evolve, the error of "reaching target" $e(k+HP)$ is equal to zero, that results in the convergence of the measured performances indicator toward the desired performances.
However, the parametric variation generates a difference between the measured performances indicator of the present model and the initial model, controlled by the same control law. Consequently, that the desired performances indicator of the process must be equal to the initial at every instant.

$$Ip_r(k+HP) = M\underline{YC} = \theta^T \phi(k) \quad (20)$$

The performances index to be minimized is a quadratic cost function $\Im$ defined by [6] :

$$\begin{aligned} \Im(k+HP) &= \mathrm{e}_f(k+Hp)^T \mathrm{e}_f(k+Hp) \\ &= \left[Ip_r(k+HP) - \hat{\theta}^T(k+HP-1)\,\phi(k)\right]^T \times \\ & \quad \left[Ip_r(k+HP) - \hat{\theta}^T(k+HP-1)\,\phi(k)\right] \end{aligned} \quad (21)$$

Including the *RST* structure and the performances error, the direct adaptive generalized predictive control (*DAGPC*) diagram is represented in figure 1.

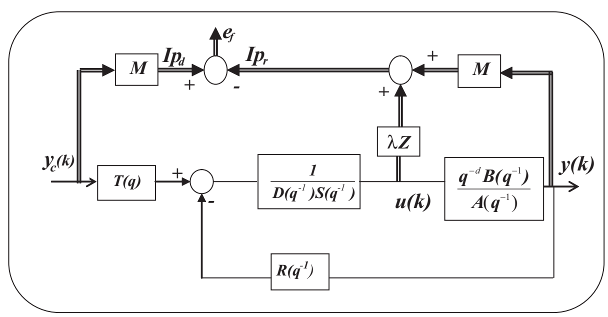

Fig. 1. The Direct Adaptive Generalized Predictive Control structure.

E. Updating of control law

The objective of the direct adaptive generalized predictive control *DAGPC* is to identify directly the parameters of the regulator *RST* using an adaptive algorithm which minimizes the performances index ($\Im(k+HP)$) at each sampling time. The control parameters are updating through the gradient algorithm given by:

$$\hat{\theta}(k+HP) = \hat{\theta}(k+HP-1) - \frac{\Gamma}{2}\frac{\partial\Im(k+HP)}{\partial\hat{\theta}(k+HP-1)} \quad (22)$$

with: Γ is a matrix of adaptation gain.
According to the equation (21), we can write the equation (22) as follow:

$$\hat{\theta}(k+HP) = \hat{\theta}(k+HP-1) + \Gamma\ \phi(k)\ e_f(k+HP)^T \quad (23)$$

where $e_f(k+HP)$ is the priori performances error.

III. Simulation Example

Let us consider a second order non stationary system described by the following equation:

$$\begin{aligned} y(k) = & -a_1(k)\,y(k-1) - a_2(k)\,y(k-2) + b_1(k)\,u(k-1) \\ & + b_2(k)\,u(k-2) \end{aligned} \quad (24)$$

The parameters $a_i(k)$ and $b_i(k)$ are a time varying parameters. with:

$$a_i(k) = a_{i0} + \Delta a_i\, sin\,(wk)$$
$$b_i(k) = b_{i0} + \Delta b_i\, sin\,(wk)$$

w : is a pulsation, chosen for the parameters variation of the system. (a_{i0} and b_{i0} are the mean values of the parameters): $a_{10} = -0.5$, $a_{20} = 0.1$, $b_{10} = 0.1$ and $b_{20} = 0.2$.
The degrees of parameters variation (Δa_i, Δb_i) are:

$$\Delta a_i = \frac{da_i}{100}\,|a_{i0}| \ \ ; \Delta b_i = \frac{db_i}{100}\,|b_{i0}| \ \ ; \ i = 1,\, 2$$

A. Case of faintly non stationary system

In a first case, we consider a faintly variation of the parameters $(a_i(k), b_i(k))$. A direct adaptive generalized predictive control, with a given synthesis parameters ($HP = 8$, $HC = 2$, $HI = 1$, λ=1) is synthesized. The application of this control law to the considered system gave the results represented by the following figure (figure 2). This figure illustrates the evolutions of the faintly non stationary system output $y(k)$ and of the reference trajectory $y_c(k)$. We remark that the system output follows with precision the desired reference trajectory.

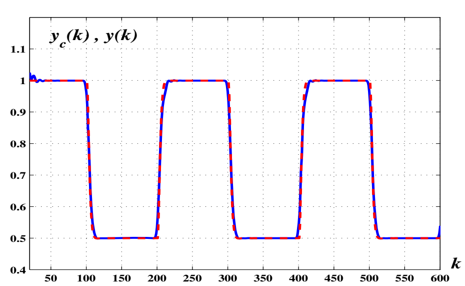

Fig. 2. The evolutions of the desired and the real outputs.

B. Case of highly non stationary system

In presence of a highly non stationary system the performances of direct adaptive generalized predictive control *DAGPC* is considerably deteriorated. It is due to the inability of this control law to act according to parametric variations and to the modelling errors. This is illustrated by figure 3 which lets appear a big error between the process output and the desired reference trajectory.

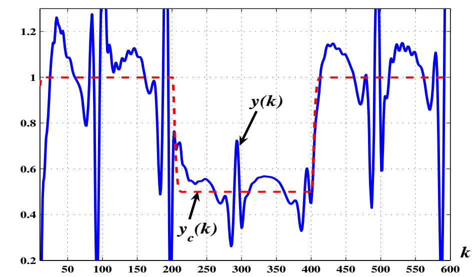

Fig. 3. The evolutions of the desired and the real outputs.

In order to overcome these problems, we propose to apply the multimodel approach. In this way, the highly non stationary system can be represented by a set of N local models called models'base. We associate to each model a local Generalized Predictive Control.
The effective control $u(k)$, applied to the process, can be a result of a switching strategy between all these control laws $u_i(k)$.

IV. The proposed concept of the multimodel generalized predictive control with supervisor

The idea of this proposed strategy is to associate a predictor and a controller to each operating regimes of the system. Therefore a set of predictors constituting the supervisor will be used to describe its full operating range.
Three inherent parts are necessary in the multicontrol approach: the first one is a set of local controllers; the second one is a 'switching system' and the last one is a supervision system which controls the switching system and more precisely, it indicates the most appropriate controller [19]. The general diagram of the proposed concept is given by figure 4.

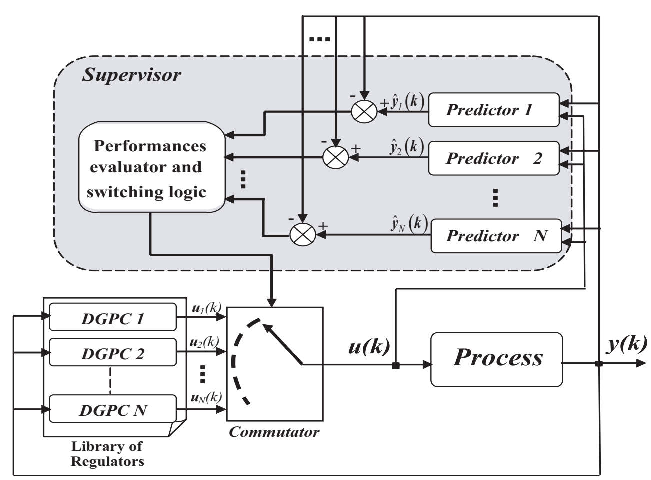

Fig. 4. A general architecture for multimodel switching control with supervisor.

For each local model, we are able to calculate a local controller satisfying the closed loop local objectives.
The supervisor structure is given in figure 4. It consists of a bank of predictors and a block denoted by *Performances evaluator and switching logic*. The supervision mission consists in selecting the predictor i and then applying the corresponding controller (*DGPC*i).
The supervisor task is achieved by comparing a performance criterion $J_i(k)$, based on the error $e_i(k)$, of each predictor, and choosing the controller that corresponds to the minimum at each step [9], [17], [19].
The switching algorithm is based on the minimization of a criterion $J_i(k)$:

$$J_i(k) = \alpha e_i^2(k) + \beta \sum_{j=1}^{k} e^{-\lambda(k-j)} e_i^2(k). \qquad i = 1..N \quad (25)$$

with:

$e_i(k) = y(k) - \hat{y}_i(k)$.
N is the number of local controllers.
$\hat{y}_i(k)$ is the output of the i^{th} predictor.
α ,β and λ are positive tuning parameters.

Where α and β are weighting factors of the terms that incorporate instantaneous and long-term measures of accuracy. The forgetting factor λ determines the memory of the index.
In this work a multimodel generalized predictive control is synthesized. The local predictors are given by the inversion of the correspondent local controllers (local generalized predictive control). The supervisor is constituted of the set of local predictors. $e_i(k)$ is the identification error calculated between the output of the i^{th} predictor and the plant output.

A. Synthesis of supervisor

For the direct adaptive generalized predictive control, only the first element of vector $\underline{DU}_P$ is considered. The control at the instant k applied to the process will be deduced from the vector given by equation (13):

$$Du(k) = -\underline{m_1}^T \left(R^{**}\underline{DU} + G^{**}\underline{Y} - \underline{YC}\right) \quad (26)$$

At each iteration, from $u(k)$ and $y(k)$, it is possible to rebuild the reference signal $y_c(k)$. Taking into account the last equation, we can write:

$$\underline{YC} = (m_1 m_1^T)^{-1} m_1 Du(k) + R^{**}\underline{DU} + G^{**}\underline{Y} \quad (27)$$

Therefore, to find the parameters of the local predictor $\underline{\hat{Y}_i}$ from the controller's structure and while applying the principle of certain equivalence [19], it is sufficient to express $\underline{\hat{Y}_i}$ according to the control law and the output of the process. The output of the predictor can be written by the following equation:

$$\underline{\hat{Y}_i} = (m_{1i} m_{1i}^T)^{-1} m_{1i} Du(k) + R_i^{**}\underline{DU} + G_i^{**}\underline{Y} \qquad i = 1..N \quad (28)$$

with:

$$\underline{Y} = \begin{bmatrix} y(k) \\ y(k-1) \\ \\ y(k-n_A) \end{bmatrix} \; ; \; \underline{DU} = \begin{bmatrix} Du(k-1) \\ Du(k-2) \\ \\ Du(k-n_B-d+1) \end{bmatrix}$$

We only have needed the first element:

$$\hat{y}_i(k) = q^{-HI}\underline{\hat{Y}_i}(1) \qquad i = 1..N \quad (29)$$

The proposed local predictor structure is illustrated by figure 5.

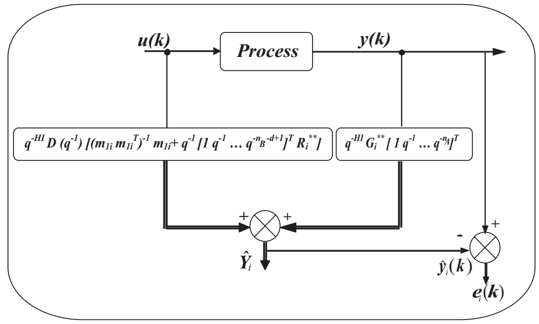

Fig. 5. The proposed local predictor structure.

B. Simulation Example

In order to show the considerable contribution in performances of the multimodel generalized predictive control with a supervisor; we consider the same highly non stationary process described by the equation (24), while preserving the same parametric variations and the same simulation conditions.

1) Determination of a models'base: The determination of a models'library is confided to the method based on the Kohonen networks [22], [23]. The application of this approach requires firstly to determine the number of clusters. The classification of an identification data set is the second stage. Then, there is a stage of structural and parametric estimation in order to determine the local models that can reproduce the behavior of the process [13], [22], [23].
The application of this method to an identification data set, picked out on the highly non stationary system, yields to three second-order systems described by the following transfer functions:

$$H_1(q^{-1}) = \frac{0.1003q^{-1} + 0.1992q^{-2}}{1 - 0.5108q^{-1} + 0.1111q^{-2}} \quad (30)$$

$$H_2(q^{-1}) = \frac{0.0995q^{-1} + 0.1856q^{-2}}{1 - 0.5425q^{-1} + 0.1140q^{-2}} \quad (31)$$

$$H_3(q^{-1}) = \frac{0.1009q^{-1} + 0.1739q^{-2}}{1 - 0.60148q^{-1} + 0.1442q^{-2}} \quad (32)$$

A local generalized predictive control ($DGPC_i$) is synthesized for each model. The multimodel generalized predictive control law with supervisor synthesized as previously is applied to the considered system.
The figure 6 presents the evolutions of the desired reference trajectory $y_c(k)$ and the system output $y(k)$. It shows the results of the application of the multimodel generalized predictive control with supervision to the highly non stationary system. This figure shows a very satisfactory tracking performance.

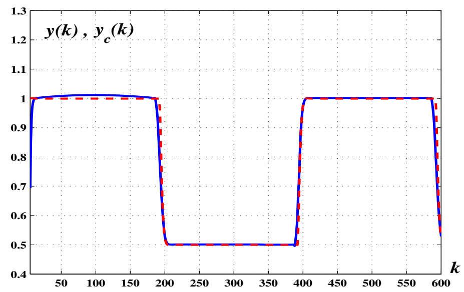

Fig. 6. The evolutions of the desired and the real outputs.

The evolution of the control law is given in figure 7. The state of the commutation of controllers is given in figure 8. The obtained results are very satisfactory and show a very good performances relatively to the case in which the classical direct adaptive generalized predictive control is adopted.

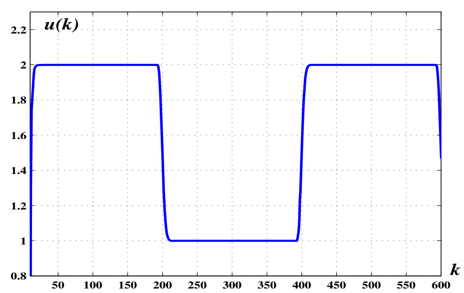

Fig. 7. The control signal.

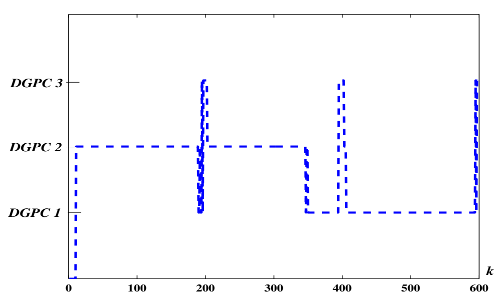

Fig. 8. Illustration of the controllers commutation.

V. Conclusion

In this paper, we have elaborated a multimodel generalized predictive control with supervisor for highly non stationary system. This control law is obtained from a switching between elementary control signals generated by the base's local controllers. We also compared the performances of this proposed strategy to the classical adaptive generalized predictive control. The simulation results show clearly that the proposed strategy leads to a good closed loop performances in the presence of a high parametric variation.

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