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A Comparison Between IMC and Sliding Mode Approaches to Vehicle Yaw Control

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Abstract— **In this paper the problem of vehicle yaw control using a rear active differential is investigated. Due to system uncertainties, time-varying road conditions, and the wide range of operating conditions, which are typical of the automotive context, a robust control technique is required to solve this problem. In this paper, two different robust control schemes, based on enhanced Internal Model Control and Second Order Sliding Mode control, respectively, are designed and their performances are compared in simulation. In order to improve the transient behaviour of the proposed control schemes a feedforward control contribution has been added giving rise to a two degree of freedom structure. Improvements on understeer characteristics and damping properties in impulsive manoeuvres are shown relying on simulations performed on an accurate 14 degree of freedom nonlinear model.**

I. INTRODUCTION

Vehicle yaw dynamics may show unexpected dangerous behaviour in presence of unusual external conditions such as lateral wind force, different left-right side friction coefficients and steering steps needed to avoid obstacles. Moreover, in standard turning manoeuvres understeer phenomena may deteriorate handling performances in manual driving and cause uncomfortable feelings to the human driver. Vehicle active control systems aim to enhance driving comfort characteristics ensuring stability in critical situations. Different approaches to active chassis control have been proposed in the literature during recent years. All the proposed strategies modify the vehicle dynamics by applying suitable yaw moments that can be generated in different ways (see e.g. [1]-[7]). A point which is common to all solutions is the fact that they are able to generate limited values of the yaw moment. As a consequence, the input variable may saturate and this could deteriorate the control performances. Moreover, the active control system has to guarantee safety (i.e. stability) performances robustly in face of the uncertainties arising from the wide range of speed, load, friction, etc., under which the vehicle operates. Robustness of active vehicle systems is a widely studied topic and interesting results have appeared (see e.g. [4], [5]).

In this paper, the problem of yaw control is addressed making reference to a vehicle equipped with a Rear Active Differential (RAD) device [6]. The proposed control structure employs a reference generator, designed to improve vehicle handling, a feedforward contribution, and a feedback controller. The feedforward contribution is used to enhance system performances in the transient phase, while the feedback controller is designed in order to guarantee robust stability. In particular, two robust control design techniques, based on Internal Model Control (IMC) and Second Order Sliding Mode (SOSM) methodologies respectively, have been proposed for the feedback controller. The choice of these two control methodologies is motivated by their robustness properties against the wide range of uncertainties which arises during vehicle operations. In particular, Internal Model Control techniques are well established control methodologies able to handle in an effective way both robustness (see [8]) and saturation (see e.g. [9]) issues. The enhanced IMC structure presented in [10], which guarantees robust stability as well as improved performances during saturation, will be employed, since it proved to give quite good results in the context of vehicle stability control (see [6], [7]). As for sliding mode control technique [11], its well-known robustness properties make this control methodology particularly suitable to deal with uncertain nonlinear time varying systems like the considered automotive system. Yet, conventional sliding mode control laws produce discontinuous control inputs [12] which can generate high frequency chattering, with the consequent excessive mechanical wear and passengers' discomfort. A possible counteraction to eliminate or, at least, reduce the vibrations induced by the controller consists in the approximation of the discontinuous control signals with continuous ones. However, this kind of solution makes the controlled system state evolves in a boundary layer of the ideal sliding subspace, and all the appreciable features, which can induce the controller designer to rely on the sliding mode control methodology, are lost [12]. In order to circumvent the inconvenience of the vibrations induced by sliding mode controllers, a second order sliding mode control scheme [13], based on the so–called sub–optimal control algorithm [14], is designed. Second order sliding mode controllers generate continuous control actions, since the discontinuity is confined to the first time derivative of the control signal. Nevertheless, the generated sliding modes are ideal, in contrast to what happens for solutions which relies on continuous approximations of the discontinuous control laws.

In order to compare and to show in a realistic way the effectiveness of the proposed control approaches, simulations will be performed using a detailed nonlinear 14 degrees of freedom model of the vehicle.

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II. PROBLEM FORMULATION AND CONTROL REQUIREMENTS

The first aim of vehicle yaw control is to aid the driver to keep stability in critical maneuvers and in presence of unusual external conditions, such as lateral wind force or different left-right side friction coefficients. Moreover, devices, such as active differentials and rear wheel steering systems, can be employed to change the steady state and dynamic behaviour of the car, enhancing its handling in turning manoeuvres. In order to introduce the control requirements, some basic concepts of lateral vehicle dynamics are now recalled. The steady-state handling and manoeuvrability quality of a vehicle can be characterized by means of *steering diagrams* (see Fig. 1), where the steering angle δ is reported with respect to the lateral acceleration a_y, for a given constant speed v. Such curves are mostly influenced by road friction and depend on the tyre lateral force-slip characteristics. At low acceleration the shape of the steering diagram is linear and its slope is a measure of the readiness of the car: the lower this value, the higher the lateral acceleration reached by the vehicle with the same steering angle, the more the sport feeling and handling quality perceived by the driver (see e.g. [15]). At high acceleration values the behaviour becomes nonlinear showing a saturation value, that is the highest lateral acceleration the vehicle can reach. The intervention of an active stability device can be considered as an external force or moment acting on the car centre of gravity: such a moment is able to vary, under the same steering conditions, the behaviour of a_y, modifying the steering diagram according to some desired requirements. A target steering diagram (as shown in Fig. 1, solid line) can be introduced to take into account the performance improvements to be obtained by the control system. More details about the generation of such target steering diagrams are reported in [6]. Since steady state lateral acceleration is related to vehicle yaw rate $\dot{\psi}$ through the vehicle speed v (i.e. $a_y = v\dot{\psi}$), the choice of yaw rate as the controlled variable is fully justified, also with regard to its reliability and ease of measurement on the car. A reference generator will provide the values for $\dot{\psi}$ needed to achieve the desired performances by means of a suitably designed feedback control law.

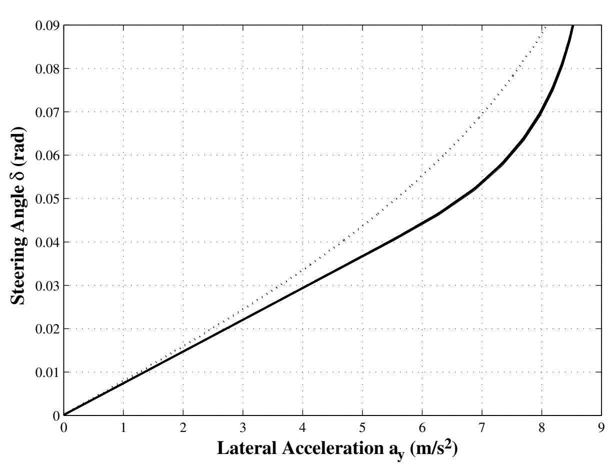

Fig. 1. Uncontrolled vehicle (dotted), and target (solid) steering diagrams. Vehicle speed: 100 km/h

As to the generation of the required yaw moment $M_z(t)$, in this paper a full Rear Active Differential (RAD) is used (see [6], [16] and [17] for details). The main advantage of this system is the capability of generating yaw moment of any value within the actuation system saturation limits, regardless of the input driving torque value and the speed values of the rear wheels. The considered device has a yaw moment saturation value of ± 2500 Nm, due to the physical limits of its electro-hydraulic system.

As previously described, the improvements on the understeer performances may be obtained using suitable modifications of the vehicle yaw dynamics in steady state conditions. As a matter of fact also in critical manoeuvring situations such as fast path changing at high speed or braking and steering with low and non uniform road friction the vehicle dynamics need to be improved in order to enhance stability and handling performances. Thus, the dynamic vehicle behaviour needs to satisfy good damping and readiness properties, which can be taken into account by a proper design of the feedback controller and the use of a feedforward action based on the driver input (i.e. δ) to increase system readiness. Needless to say that at least safety (i.e. stability) requirements have to be guaranteed in face of the uncertainty arising from the wide range of the vehicle operating conditions of speed, load, tyre, friction, etc. This can be achieved by using a controller whose design procedure takes into account the effects of model uncertainty. In particular, in this paper a SOSM controller and the enhanced IMC scheme introduced in [10] and proposed in [6] for vehicle stability control will be employed and their performance will be compared. Both these controllers are able to handle robust stability issues, as it will be described in Sections IV-A and IV-B.

III. MODEL DESCRIPTION

The control design will be worked out on the basis of a single track vehicle with tyre dynamic force generation description (see [18]). The model dynamic equations are the following

$$\begin{array}{l} mv(t)\dot{\beta}(t) + mv(t)\dot{\psi}(t) = F_{yf,p}(t) + F_{yr,p}(t) \\ J_z\ddot{\psi}(t) = aF_{yf,p}(t) - bF_{yr,p}(t) + M_z(t) \\ F_{yf,p}(t) + l_f/v\dot{F}_{yf,p}(t) = -c_f(\beta(t) + a\dot{\psi}(t)/v(t) - \delta(t)) \\ F_{yr,p}(t) + l_r/v\dot{F}_{yr,p}(t) = -c_r(\beta(t) - b\dot{\psi}(t)/v(t)) \end{array} \quad (1)$$

where m is the vehicle mass, J_z is the moment of inertia around the vertical axis, l is the wheel base, a and b are the distances between the center of gravity and the front and rear axles respectively, l_f and l_r are the front and rear tyre relaxation lengths, c_f and c_r are the front and rear tyre cornering stiffnesses. $F_{yf,p}$ and $F_{yr,p}$ are the front and rear tyre lateral forces, δ is the front steering angle, β is the vehicle sideslip angle, ψ is the vehicle yaw angle and v is the vehicle speed. M_z is the yaw moment applied by the active differential, i.e. the control variable.

As already pointed out, the real vehicle behaviour is influenced by several different factors that introduce model uncertainty. Therefore, in order to perform a robust design, uncertainty intervals are considered for tyre parameters (0%

to -20% front, 0% to +20% rear tyre cornering stiffness and $\pm$ 10% tyre relaxation lengths variations with respect to their nominal values), vehicle speed ($\pm$ 30% of the nominal value) and vehicle mass (0% to +25% of the nominal value with consequent geometrical and inertial parameters changes). Suitable descriptions of system dynamics and related uncertainty will be introduced in Sections IV-B and IV-A, according to the considered control design technique.

IV. Control design

The proposed control structure is depicted in Fig. 2. In such

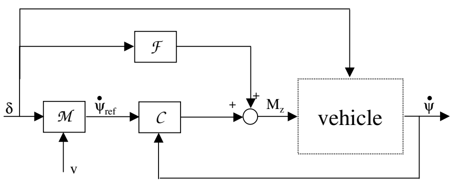

Fig. 2. Considered control structure.

a structure the desired yaw rate behaviour is imposed by the yaw rate reference signal $\dot{\psi}_{\text{ref}}(t)$ which is generated by a static map $\mathcal{M}$ using the values of $\delta(t)$ and $v(t)$. Such a map is computed in order to improve the vehicle maneuverability and increase the lateral acceleration limit. For a detailed description on the criteria followed in the map construction, see [6].

The feedback controller $\mathcal{C}$ computes the yaw torque contribution needed to follow the required yaw rate performances described by $\dot{\psi}_{\text{ref}}(t)$. In order to compare two different approaches to vehicle yaw rate control, the feedback controller $\mathcal{C}$ is designed according to SOSM and IMC methodologies. The same reference map $\mathcal{M}$ is employed with both controllers. Moreover, to improve the yaw rate dynamic properties exploiting the driver input, a feedforward contribution $\mathcal{F}$ from $\delta(t)$ has also been added. Note that, in order to compare the performances of the two proposed feedback controllers, the same filter $\mathcal{F}$ is adopted in the feedforward controller design in the case of SOSM and IMC feedback controllers.

A. IMC controller design

The design of the feedback controller in the case of Internal Model Control approach relies on H_∞ methodologies to guarantee robust stability in face of model uncertainty. In order to exploit this design technique, the vehicle model equations (1) in nominal conditions are employed to obtain the following transfer functions in the Laplace domain

$$\dot{\psi}(s) = G_\delta(s)\delta(s) + G_M(s)M_z(s) \tag{2}$$

As the control input variable is the yaw moment M_z and the controlled output is the yaw rate $\dot{\psi}$, transfer function $G_M(s)$ is used in the IMC feedback controller design. In this framework, the considered model uncertainty is described by means of an additive linear model set of the following form:

$$\mathcal{G}_M(G_M, \Gamma(\omega)) = \{G_M(s) + \Delta(s) : |\Delta(\omega)| \leq \Gamma(\omega)\} \tag{3}$$

The enhanced robust IMC structure proposed in [10] and used in [6], [7] for vehicle yaw control, is here employed. The considered control scheme is reported in Fig. 3. It includes an IMC controller with anti-windup structure (made up by filters $Q_1(s)$, $Q_2(s)$) and a feedforward linear filter $F_r(s)$ acting at the reference generation level. H_∞ opti-

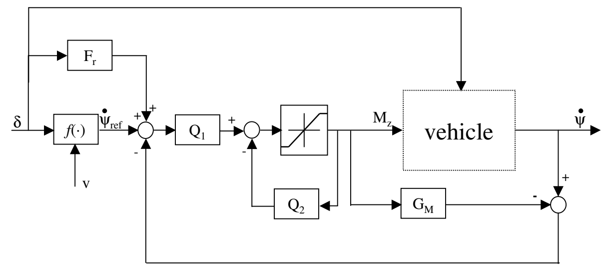

Fig. 3. The proposed control scheme for IMC control system.

mization methodologies have been employed in the design of the feedback controller taking into account the effect of model uncertainty and a desired frequency behaviour of the sensitivity function $1 - Q_1(s)/(1+Q_2(s))G(s)$ described by the weight $W_S(s)$.

The feedforward contribution is computed by means of a linear filter $F^{\text{IMC}}(s)$ to match the open loop yaw rate behaviour given by (2) with the one described by an objective transfer function $T_\delta^{\text{des,IMC}}(s)$

$$F^{\text{IMC}}(s) = \frac{T_\delta^{\text{des,IMC}}(s) - G_\delta(s)}{G_M(s)} \tag{4}$$

In order to deactivate the feedforward action in steady state conditions, the dc-gains of $T_\delta^{\text{des}}(s)$ and $G_\delta(s)$ have to be the same. Moreover, to avoid performance degradation during saturation, the feedforward contribution is injected at the reference level (as shown in Fig. 3) using the linear filter $F_r(s)$, whose expression can be computed by straightforward manipulations as

$$F_r(s) = \left(\frac{1 + Q_2(s)}{Q_1(s)} - G_M(s)\right) F^{\text{IMC}}(s) \tag{5}$$

B. SOSM controller design

A SOSM is a mode of a dynamic system confined to a particular subspace, named sliding manifold, which can be mathematically described in Filippovs' sense [19]. The SOSM is determined by

$$S(x) = \dot{S}(x) = 0 \tag{6}$$

where $S(x)$, the so–called sliding variable, is a smooth function of the state x of the considered dynamical system, and $S(x) = 0$ identifies the sliding manifold. In the case of relative degree one systems, SOSM control generalizes the basic sliding mode control idea, acting on the second order time derivative of the system deviation from the constraint (the sliding manifold), instead that on the first deviation derivative, as it happens in standard (first order) sliding mode control design [12]. The main advantage of SOSM control [13], with respect to the first order case, is that it can generate a continuous control action, while keeping the same robustness with respect to matched uncertainties.

The chosen sliding variable is the error between the actual yaw rate and the reference yaw rate, i.e.,

$$S(t) = \dot{\psi}(t) - \dot{\psi}_{ref}(t) \tag{7}$$

since the control objective is to make this error vanish in finite time. The first and second time derivative of the sliding variable are, respectively

$$\begin{cases} \dot{S}(t) &= 1/J_z(aF_{yf,p}(t) - bF_{yr,p}(t) + M_z(t)) - \ddot{\psi}_{ref}(t) \\ \ddot{S}(t) &= 1/J_z(a\dot{F}_{yf,p}(t) - b\dot{F}_{yr,p}(t) + \dot{M}_z(t)) - \psi^{(3)}_{ref}(t) \end{cases} \tag{8}$$

Introducing the auxiliary variables $y_1(t) = S(t)$ and $y_2(t) = \dot{S}(t)$, system (8) can be rewritten as

$$\begin{cases} \dot{y}_1(t) &= y_2(t) \\ \dot{y}_2(t) &= \gamma(t) + \tau(t) \end{cases} \tag{9}$$

where $\tau(t) = \dot{M}_z(t)/J_z$ is the auxiliary control and $\gamma(t) = (a\dot{F}_{yf,p}(t) - b\dot{F}_{yr,p}(t) - J_z\psi^{(3)}_{ref}(t))/J_z$. On the basis of physical consideration, the quantity $\gamma(t)$ is bounded. To apply the SOSM algorithm is not necessary that a precise evaluation of $\gamma(t)$, is available. In the sequel, it will be only assumed that a suitable bound of $\gamma(t)$, i.e.,

$$|\gamma(t)| \leq \Gamma \tag{10}$$

is known. A conservative estimation for Γ can be determined on the basis of (1), and the tyre characteristic. According to the SOSM sub–optimal control algorithm [14], the auxiliary control variable τ can be defined by the following law

$$\tau(t) = \dot{M}_z(t)/J_z = -K_{SL}\ sign\Big\{S(t) - \frac{1}{2}S_M(t)\Big\} \tag{11}$$

with the constraint

$$K_{SL} > 2\Gamma \tag{12}$$

where $S_M(t)$ is a piece–wise constant function representing the value of the last singular point of $S(t)$ (i.e., the most recent value $S_M(t_{Mi})$ such that $\dot{S}(t_{Mi}) = 0$). As proved in [14], the trajectories on the $SO\dot{S}$ plane obtained by applying control law (11) are confined within limit parabolic arcs which include the origin. Moreover, the absolute value of the coordinates of the trajectory intersections with the S, and $\dot{S}$ axis decrease in time, and the origin of the plane, i.e., $S = \dot{S} = 0$, is reached in a finite time. As regards the feedforward contribution, in this case such action is computed as:

$$F^{\text{SL}}(s) = \frac{T_\delta^{\text{des,SL}}(s) - G_\delta(s)}{G_M(s)} \tag{13}$$

Equation (13) is obtained like equation (4), using the same objective transfer function. Filter F^{SL} is implemented as shown in Fig. 2. In order to take into account the saturation of the control input, in accordance to [20], the actual control law $M_z(t)$ is given by

$$\dot{M}_z(t) = \begin{cases} -M_z(t) & \text{if } |M_z(t)| \geq M_{z,sat} \\ J_z\tau(t) & \text{otherwise} \end{cases} \tag{14}$$

where $\tau(t)$ is given by (11) and $M_{z,sat}$ is the saturation value of the RAD, i.e., $2500\,\text{Nm}$.

V. Simulation results

The IMC control design has been performed using transfer functions $G_\delta(s)$ and $G_M(s)$ defined in (2) computed at a nominal speed $v = 100\,\text{km/h} = 27.77\,\text{m/s}$ and with the following values of the other involved parameters:
$m = 1715\,\text{kg} \quad J_z = 2700\,\text{kgm}^2$
$a = 1.07\,\text{m} \quad b = 1.47\,\text{m}$
$l_f = 1\,\text{m} \quad l_r = 1\,\text{m}$
$c_f = 95117\,\text{Nm/rad} \quad c_r = 97556\,\text{Nm/rad}$
The following weighting function $W_S(s)$ has been used in the IMC design (see [6])

$$W_S(s) = \frac{s}{s+20} \tag{15}$$

As regards the SOSM controller design, the value of the gain K_{SL} in (11) has been chosen as $K_{\text{SL}} = 5000$.
The objective transfer functions for the feedforward design is chosen as

$$T_\delta^{\text{des,SL}}(s) = T_\delta^{\text{des,IMC}}(s) = \frac{5.67}{1 + \frac{s}{6}} \tag{16}$$

In order to show in a realistic way the performances obtained by the proposed yaw control approach, simulations have been performed using a detailed nonlinear 14 degrees of freedom Simulink model. The model degrees of freedom correspond to the standard three chassis translations and yaw, pitch and roll angles, the four wheel angular speeds and the four wheel vertical movements with respect to the chassis. Nonlinear characteristics obtained on the basis of measurements on the real vehicle have been employed to model the tyre, steer and suspension behaviour.
The following open loop (i.e. without driver's feedback) manoeuvres have been chosen:
- constant speed steering pad performed at 90 km/h: to evaluate steady state vehicle performances, steering angle is slowly increased (i.e. 1°/s) while the vehicle is moving at constant speed, until the vehicle lateral acceleration limit is reached;
- steer reversal test with handwheel angle of 50° at 100 km/h, with a steering wheel speed of 400°/s. This test aims to evaluate the controlled car transient response performances: in Fig. 4 the employed steering angle behaviour is shown. In order to test the control system robustness in front of parameter changes, this manoeuvre has been performed both in nominal conditions and with vehicle mass increased by 300 kg (with consequent geometrical and inertial parameters changes);
- steering wheel frequency sweep performed at 100 km/h in the frequency range 0-4 Hz with steering wheel angle amplitude of 20°.

In Fig. 5 the understeer performance improvement is shown for the considered steering pad manoeuvre. The reference steering diagram and the ones obtained with the Sliding Mode and the IMC controllers are practically superimposed: thus the target vehicle behaviour, characterized by a lower understeer gradient, is reached by both control systems, which show good tracking performances also in the nonlinear

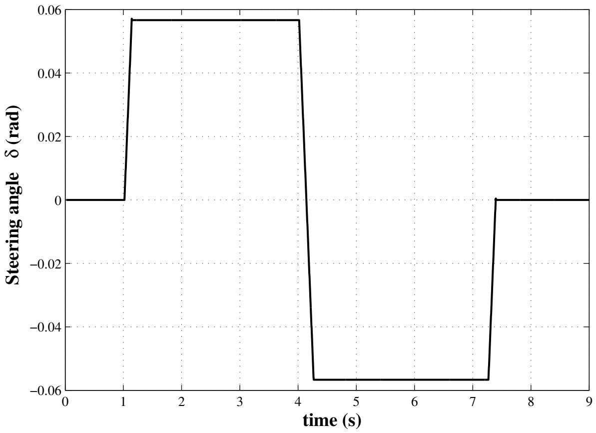

Fig. 4. Steering angle reversal test input corresponding to 50° handwheel angle

tract of the diagram.

The 50° steer reversal tests at 100 km/h allow to study

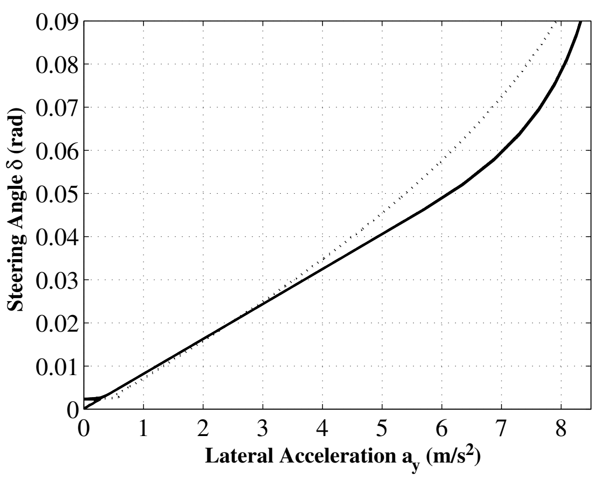

Fig. 5. Steering pad test at 90 km/h: comparison between the reference (thin solid line) steering diagram and the ones obtained with the uncontrolled vehicle (dotted) and with IMC (dashed) and the Sliding Mode (solid) controlled vehicles.

the results obtained when the vehicle reaches the lateral acceleration limit of about 8 m/s^2. In particular, the obtained yaw rate course show that the controlled vehicle dynamic response in nominal conditions is well damped with both the IMC and the Sliding Mode controllers (Fig. 6), while a slight performance degradation occurs with increased mass (Fig. 7). The courses of yaw moment M_z are reported in Fig. 8: it can be noted that chattering of the control variable occurs with the Sliding Mode controller, while a smooth behaviour is obtained with the IMC controller. On the other hand, the control input issued by the IMC controller saturates in all the transients during the test, while the Sliding Mode controller is less aggressive. Both control systems are able to handle saturation well, without worsening of the performances.

In the steering wheel frequency sweep manoeuvre the aim

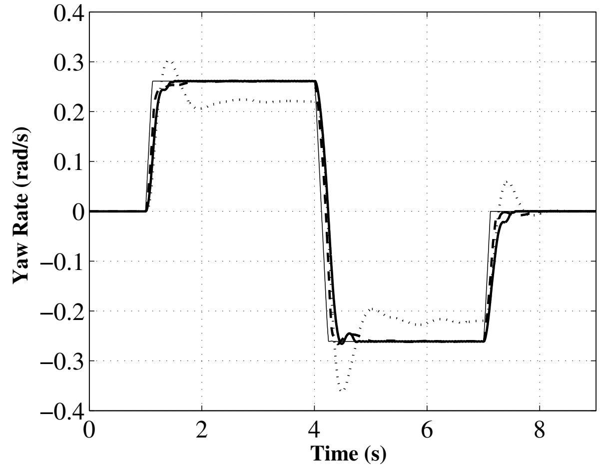

Fig. 6. 50° steer reversal test at 100 km/h, nominal conditions. Comparison between the reference yaw rate course (thin solid line) and the ones obtained with the uncontrolled (dotted) vehicle and the IMC (dashed) and Sliding Mode (solid) controlled vehicles.

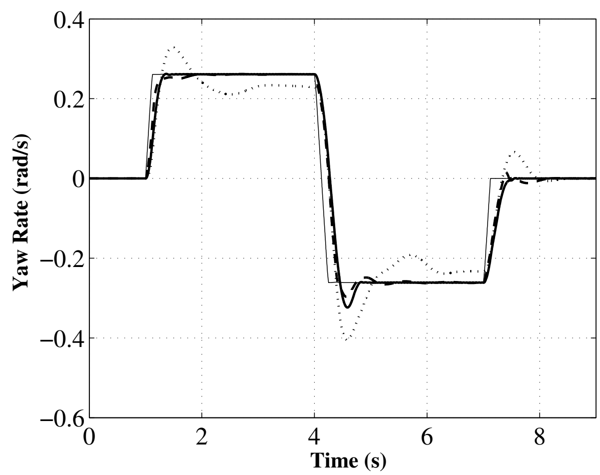

Fig. 7. 50° steer reversal test at 100 km/h with increased vehicle mass. Comparison between the reference yaw rate course (thin solid line) and the ones obtained with the uncontrolled (dotted) vehicle and the IMC (dashed) and Sliding Mode (solid) controlled vehicles.

is to evaluate the bandwidth and resonance peak obtained with the considered control systems. In Fig. 9 the simulated behaviour of the transfer ratio

$$T_m(\omega) = \left| \frac{\dot{\psi}(\omega)}{\dot{\psi}_{\text{ref}}(\omega)} \right|$$

is shown putting into evidence the significant reduction of the resonance peak provided by the Sliding Mode controller. A slightly higher resonance peak but also a higher system bandwidth are obtained with the IMC controller. The controlled vehicle performs better than the uncontrolled one with both the considered control systems.

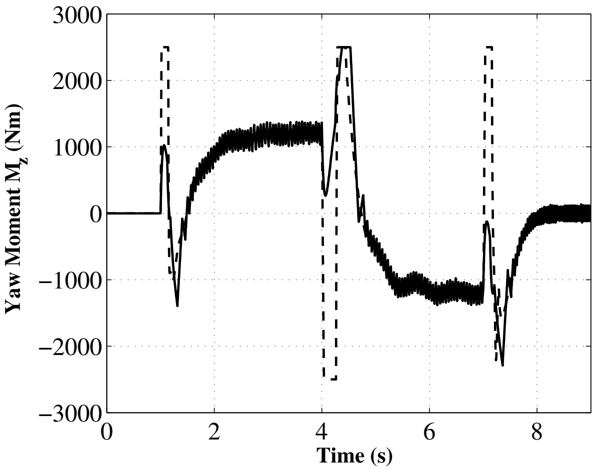

Fig. 8. 50° steer reversal test at 100 km/h, nominal conditions. Comparison between the yaw moment courses obtained with the IMC (dashed) and Sliding Mode (solid) controllers.

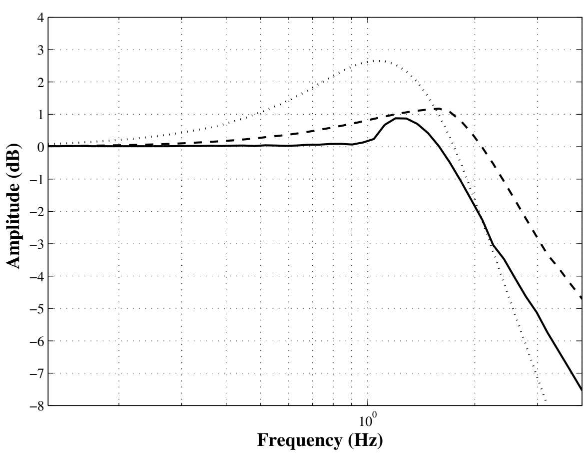

Fig. 9. Steering wheel frequency sweep at 100 km/h. Frequency response: uncontrolled vehicle (dotted), controlled vehicle with IMC (dashed) and Sliding Mode (solid)

VI. Conclusions

The problem of vehicle yaw control using yaw rate feedback and a Rear Active Differential has been investigated. The proposed control structure employs a reference generator, designed to improve vehicle handling, a feedforward contribution which enhances the transient system response, and a feedback controller. In particular, two robust control design techniques, based on SOSM and Internal Model Control methodologies respectively, have been considered for the feedback controller. A comparison of the results obtained with the considered controllers has been carried out through simulation with an accurate 14 d.o.f. vehicle model. The obtained results demonstrate the effectiveness of the proposed control structure with both controllers. Very good tracking performances have been obtained with both control systems during steering pad manoeuvres and good transient performances have been achieved in steer reversal tests, with a less aggressive control variable behaviour in the case of Sliding Mode controller, which shown a little chattering of the input. Finally, a slightly higher system bandwidth, but also a higher resonance peak, has been obtained by the IMC controller in the handwheel sweep test. The robustness of the employed controllers has been also tested, since the considered manoeuvres have been performed with varying vehicle speed and mass. Future works will be devoted to compare the performances of the designed control scheme with a realistic model of the RAD. The possibility of coupling SOSM and IMC control methodologies in order to exploit the benefits of both control approaches will also be investigated.

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