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On simultaneous parameter identification and state estimation for cascade state affine systems

M. GHANES, G. ZHENG and J. DE LEON-MORALES

Abstract— **In this paper, an adaptive observer is proposed to solve the problem of simultaneous parameter identification and state estimation for a class of cascade state affine systems. Sufficient conditions are given in order to guarantee the exponential convergence of the proposed observer. Furthermore, simulation results are given illustrating the performance of the proposed observer when it is applied in the synchronization and identification problem of Rossler's chaotic system.**

Index Terms— **Adaptive observer, synchronization, identification, Rossler's chaotic system**

I. INTRODUCTION

Nonlinear observer design is an important problem in the theory of systems. It is clear that no general procedure exists to construct a nonlinear observer for a general nonlinear system. However, for a class of nonlinear systems, under some extra assumptions, such as Lipchitz, persistent exciting..., it is possible to construct the observer. For the purpose of state estimation, many works have been devoted [14],[13],[9],[11]. However, when dealing with the problem of parameter identification, it becomes more difficult. The simultaneous parameter identification and state estimation problem has been attracted the attention of various research groups, since it is very useful to treat many practical problems, such as fault detection, signal transmission or control, and recently for synchronization of chaotic systems. Motivated by this interest, several approaches have been proposed to simultaneously estimate the state and identify the parameters. In [8], author has proposed a novel adaptive observer with an appropriate adaptation law for the unknown parameters. And in [10], the unknown parameters were treated as the extended state of the system, in such a way the existed 'classical' nonlinear observer design methods can be applied to the augmented system. All these methods can be used to design nonlinear observer for a large class of nonlinear system, such as linear time invariant/variant systems, and a class of state affine systems.

In this paper, our goal is firstly to design an adaptive observer in order to estimate the unmeasurable state variables of the system and to identify the unknown parameters simultaneously for a class of cascade state affine systems. Furthermore, some sufficient conditions in order to guarantee the exponential convergence of the proposed adaptive observer are given.

M. GHANES is with ECS/ENSEA, 6 Avenue du Ponceau, 95014 Cergy-Pontoise, France.

G. Zheng is with INRIA Rhône-Alpes, Inovallée, 655 avenue de l'Europe, Montbonnot Saint Martin, 38334 St. Ismier Cedex, France.

J. DE LEON-MORALES is with Universidad Autónoma de Nuevo Leon, Apdo. Postal 148-F, San Nicolas de Los Garza; N. L 66451, Mexico.

Besides, according to [2], the problem of synchronization of chaotic systems has been related to the concept of observer design from theoretical point of view. Therefore, in order to highlight the feasibility of the proposed observer, we apply it to treat the problem of chaotic synchronization. In fact, the topic of synchronization of chaotic system has attracted many researchers' attention since the work of [3]. After that, many applications of chaotic synchronization have been developed, such as chaotic secure communication based on synchronization of chaotic system [4], [6], [7], [5] and [12].

Inspired by these works, in this paper, secondly, we also try to deal with the synchronization problem for Rossler's chaotic system based on the proposed observer in order to not only estimate the state of system, but also identify its unknown parameters at the same time.

The paper is organized as follows: In section 2, some basic notations about the considered nonlinear systems are introduced. And section 3 is devoted to adaptive observer design for such class of cascade state affine systems. In section 4, an illustrative example dealing with the synchronization and identification parameter problem of Rossler's chaotic systems is given. Simulation results are presented in order to emphasize the performance of the proposed adaptive observer.

II. NOTATIONS

In this paper, we are interested in designing an exponential observer for the following cascade state affine system:

$$\left\{ \begin{array}{l} \dot{z} = A\left(y,u,z,\theta\right)z + \beta\left(y,u,z,\theta\right) + \varphi\left(y,u,z,\theta\right)\theta \\ y = Cz \end{array} \right. \tag{1}$$

where $z \in R^n$, $u \in R^l, y \in R^p$, $\theta \in R^q$ are respectively the state, known input, output of the system and the parameter, function A, β, φ and C are the matrices of appropriate dimensions, and the components of matrix A, and vectors β and φ are continuous functions depending on u, y and $z_1, ..., z_{i-1}, \theta_1, ..., \theta_{i-1}$, for $1 \leq i \leq p$ and uniformly bounded, with

$$z = \begin{pmatrix} z_1 \\ \vdots \\ z_p \end{pmatrix}, \quad y = \begin{pmatrix} y_1 \\ \vdots \\ y_p \end{pmatrix}, \quad \theta = \begin{pmatrix} \theta_1 \\ \vdots \\ \theta_p \end{pmatrix}$$

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$$A(y,u,z,\theta) = diag\begin{pmatrix} A_1(y,u), \\ \cdots, \\ A_p(y,u,z_1,...,z_{p-1},\theta_1,...,\theta_{p-1}) \end{pmatrix},$$

$$C = diag\begin{pmatrix} C_1, & \cdots, & C_p \end{pmatrix},$$

$$\beta(y,u) = \begin{pmatrix} \beta_1(y,u) \\ \vdots \\ \beta_p\left(y,u,z_1,...,z_{p-1},\theta_1,...,\theta_{p-1} \right) \end{pmatrix},$$

$$\varphi(y,u,z,\theta) = diag\begin{pmatrix} \varphi_1(y,u), \\ \cdots, \\ \varphi_p(y,u,z_1,...,z_{p-1},\theta_1,...,\theta_{p-1}) \end{pmatrix}$$

where $z_i \in R^{n_i}$, $\theta_i \in R^{q_i}$, $y_i \in R$, A_i, β_i, φ_i, C_i are the matrices of appropriate dimensions, for $1 \le i \le p$, $u \in R^l$. And $\sum_{i=1}^p n_i = n$, $\sum_{i=1}^p q_i = q$.

In the sequel, the dependence of $u, y, z_1, ..., z_{i-1}, \theta_1, ..., \theta_{i-1}$ and $\hat{z}_{i-1}, \hat{\theta}_1, ..., \hat{\theta}_{i-1}$, with A_i, φ_i, β_i and $\hat{A}_i$, $\hat{\varphi}_i$, $\hat{\beta}_i$ will be omitted in order to lighten notations.

III. Adaptive observer for cascade systems

Before introducing our main result, we establish the following assumptions:

Assumption A1. If the input is persistently exciting, in the sense that there exist $\alpha_{2,i} > \alpha_{1,i} > 0$, $T_{1,i} > 0$ and $t_0 \ge 0$ such that for all initial condition x_0, the following condition for all $t \ge t_0$ is satisfied:

$$\alpha_{1,i} I \le \int_t^{t+T_{1,i}} \Psi^T_{(u,x_0,i)}(s,t)\, C_i^T \Sigma_i(s)\, C_i \Psi_{(u,x_0,i)}(s,t)\, ds \le \alpha_{2,i} I$$

where $\Psi_{(u,x_0,i)}$ denotes the transition matrix for the system

$$\begin{cases} \dot{z}_i = A_i(y,u,z_1,...,z_{i-1},\theta_i,...,\theta_{i-1})\, z_i, \\ y_i = C_i z_i \end{cases}$$

and Σ_i is some positive definite bounded matrix, for $1 \le i \le p$.

Assumption A2. Considering matrix

$$\Lambda = diag\{\Lambda_1, ..., \Lambda_p\},$$

where Λ_i is a matrix defined by

$$\dot{\Lambda}_i = \left\{A_i - S_i^{-1} C_i^T \Sigma_i C_i\right\} \Lambda_i + \varphi_i$$

Assume that φ_i is persistency exciting so that there exist $\gamma_{2,i} > \gamma_{1,i} > 0$, $T_{2,i} > 0$ and $t_0 \ge 0$ and some positive definite matrix Σ, such that the following inequality holds

$$\gamma_{1,i} I \le \int_t^{t+T_{2,i}} \Lambda_i^T(s)\, C_i^T \Sigma_i(s)\, C_i \Lambda_i(s)\, ds \le \gamma_{2,i} I$$

for all $t \ge t_0$ and for $1 \le i \le p$.

Assumption A3. We assume that the components of z and θ are bounded i.e. there exist positive constants such that

$$\|z_i\| \le \delta_i^z, \quad \|\theta_i\| \le \delta_i^\theta$$

and the following inequalities hold

$$\|\hat{A}_i - A_i\| \le \sum_{j=1}^{i-1} \delta_j^A \|e_j\| + \sum_{j=1}^{i-1} \varrho_j^A \|e_{\theta_j}\|$$

$$\|\hat{\beta}_i - \beta_i\| \le \sum_{j=1}^{i-1} \delta_j^\beta \|e_j\| + \sum_{j=1}^{i-1} \varrho_j^\beta \|e_{\theta_j}\|$$

$$\|\hat{\varphi}_i - \varphi_i\| \le \sum_{j=1}^{i-1} \delta_j^\varphi \|e_j\| + \sum_{j=1}^{i-1} \varrho_j^\varphi \|e_{\theta_j}\|$$

where $e_i = z_i - \hat{z}_i$ and $e_{\theta_i} = \theta_i - \hat{\theta}_i$, for $1 \le i \le p$.

Theorem 1: Consider system (1), if assumptions A1, A2 and A3 are satisfied, then the following system

$$\begin{aligned} \dot{\hat{z}} &= A\left(y,u,\hat{z},\hat{\theta}\right)\hat{z} + \beta\left(y,u,\hat{z},\hat{\theta}\right) + \varphi\left(y,u,\hat{z},\hat{\theta}\right)\hat{\theta} \\ &\quad + \left\{S^{-1}C^T + \Lambda\Gamma^{-1}\Lambda^T C^T\right\}\Sigma(y - C\hat{z}) \end{aligned}$$

where

$$\begin{cases} \dot{\hat{z}}_i = \hat{A}_i \hat{z}_i + \hat{\beta}_i + \hat{\varphi}_i \hat{\theta}_i \\ \quad + \left\{S_i^{-1}C_i^T + \Lambda_i \Gamma_i^{-1} \Lambda_i^T C_i^T\right\}\Sigma_i(y_i - C_i\hat{z}_i) \\ \dot{S}_i = -\rho_i S_i - \hat{A}_i^T S_i - S_i \hat{A}_i + C_i^T \Sigma_i C_i \\ \dot{\Lambda}_i = \left\{\hat{A}_i - S_i^{-1} C_i^T \Sigma_i C_i\right\}\Lambda_i + \hat{\varphi}_i \\ \dot{\Gamma}_i = -\lambda_i \Gamma_i + \Lambda_i^T C_i^T \Sigma_i C_i \Lambda_i \\ \dot{\hat{\theta}}_i = \Gamma_i^{-1} \Lambda_i^T C_i^T \Sigma_i (y_i - C_i \hat{z}_i) \end{cases} \tag{2}$$

is an exponential observer for system (1), where ρ_i and λ_i are sufficiently large positive constants and Σ_i are some positive definite matrices for $1 \le i \le p$.

Proof: Set $e = (e_1, ..., e_p)^T$ and $e_\theta = \left(e_{\theta_1}, ..., e_{\theta_p}\right)^T$, the state estimation error and the parameter estimation error respectively, where $e_i = z_i - \hat{z}_i$ and $e_{\theta_i} = \theta_i - \hat{\theta}_i$, and θ_i is a constant, for $1 \le i \le p$.

For a general case of i, for $1 \le i \le p$, the estimation error dynamics for $e_i = z_i - \hat{z}_i$ and $e_{\theta_i} = \theta_i - \hat{\theta}_i$ can be obtained as follows

$$\begin{aligned} \dot{e}_i &= \left(A_i - S_i^{-1} C_i^T \Sigma_i C_i - \Lambda_i \Gamma_i^{-1} \Lambda_i^T C_i^T \Sigma_i C_i\right) e_i \\ &\quad + \varphi_i e_{\theta_i} + \left(\hat{A}_i - A_i\right) z_i + \left(\hat{\beta}_i - \beta_i\right) \\ &\quad + (\hat{\varphi}_i - \varphi_i)\, \theta_i \end{aligned}$$

and

$$\dot{e}_{\theta_i} = -\Gamma_i^{-1}\Lambda_i^T C_i^T \Sigma_i C_i e_i$$

Introducing the following change of variable

$$\epsilon_i = e_i - \Lambda_i e_{\theta_i} \tag{3}$$

we get

$$\begin{aligned}\dot{\epsilon}_i &= \left(A_i - S_i^{-1}C_i^T\Sigma_i C_i\right)\epsilon_i + \left(\hat{A}_i - A_i\right) z_i \\ &\quad + \left(\hat{\beta}_i - \beta_i\right) + \left(\hat{\varphi}_i - \varphi_i\right)\theta_i\end{aligned}$$

Because S_i and Γ_i are some positive definite matrices according to Assumptions A1 and A2, a candidate Lyapunov function can be chosen as follows

$$v_i = \epsilon_i^T S_i \epsilon_i + e_{\theta_i}^T \Gamma_i e_{\theta_i}$$

Hence its time derivative is given by

$$\begin{aligned}\dot{v}_i &= -\rho_i\epsilon_i^T S_i\epsilon_i - \lambda_i e_{\theta_i}^T\Gamma_i e_{\theta_i} \\ &\quad - \left(\epsilon_i + \Lambda_i e_{\theta_i}\right)^T C_i^T\Sigma_i C_i\left(\epsilon_i + \Lambda_i e_{\theta_i}\right) \\ &\quad + 2\epsilon_i^T S_i \left\{ \begin{array}{c} \left(\hat{A}_i - A_i\right) z_i + \left(\hat{\beta}_i - \beta_i\right) \\ + \left(\hat{\varphi}_i - \varphi_i\right)\theta_i \end{array} \right\}\end{aligned}$$

According to Assumptions A1, A2 and A3, we obtain

$$\begin{aligned}\dot{v}_i &\leq -\rho_i\epsilon_i^T S_i\epsilon_i - \lambda_i e_{\theta_i}^T\Gamma_i e_{\theta_i} \\ &\quad + 2\|\epsilon_i\|\|S_i\|\left\{\delta_i^z\left(\sum_{j=1}^{i-1}\delta_j^A\|e_j\| + \sum_{j=1}^{i-1}\varrho_j^A\|e_{\theta_j}\|\right)\right\} \\ &\quad + 2\|\epsilon_i\|\|S_i\|\left\{\sum_{j=1}^{i-1}\delta_j^\beta\|e_j\| + \sum_{j=1}^{i-1}\varrho_j^\beta\|e_{\theta_j}\|\right\} \\ &\quad + 2\|\epsilon_i\|\|S_i\|\left\{\delta_i^\theta\left(\sum_{j=1}^{i-1}\delta_j^\varphi\|e_j\| + \sum_{j=1}^{i-1}\varrho_j^\varphi\|e_{\theta_j}\|\right)\right\}\end{aligned}$$

which can be rearranged as follows

$$\begin{aligned}\dot{v}_i &\leq -\rho_i\epsilon_i^T S_i\epsilon_i - \lambda_i e_{\theta_i}^T\Gamma_i e_{\theta_i} \\ &\quad + 2\|\epsilon_i\|\|S_i\|\sum_{j=1}^{i-1}\left\{\delta_i^z\delta_j^A + \delta_j^\beta + \delta_i^\theta\delta_j^\varphi\right\}\|e_j\| \\ &\quad + 2\|\epsilon_i\|\|S_i\|\sum_{j=1}^{i-1}\left\{\delta_i^z\varrho_j^A + \varrho_j^\beta + \delta_i^\theta\varrho_j^\varphi\right\}\|e_{\theta_j}\|\end{aligned}$$

Hence, using the following inequality

$$\|x\|\|y\| \leq \frac{1}{2}\left(\|x\|^2 + \|y\|^2\right)$$

we get

$$\begin{aligned}\dot{v}_i &\leq -\rho_i\epsilon_i^T S_i\epsilon_i - \lambda_i e_{\theta_i}^T\Gamma_i e_{\theta_i} \\ &\quad + \sum_{j=1}^{i-1}\left\{L_{j,1}^i + L_{j,2}^i\right\}\|\epsilon_i\|^2 + \sum_{j=1}^{i-1}L_{j,1}^i\|e_j\|^2 \\ &\quad + \sum_{j=1}^{i-1}L_{j,2}^i\|e_{\theta_j}\|^2\end{aligned}$$

where

$$\begin{aligned}L_{j,1}^i &= \|S_i\|\left\{\delta_i^z\delta_j^A + \delta_j^\beta + \delta_i^\theta\delta_j^\varphi\right\} \\ L_{j,2}^i &= \|S_i\|\left\{\delta_i^z\varrho_j^A + \varrho_j^\beta + \delta_i^\theta\varrho_j^\varphi\right\}\end{aligned}$$

for $1 \leq i \leq p$, $1 \leq j \leq i-1$, with $L_{j,1}^1 = L_{j,1}^1 = 0$.

By applying

$$\|x+y\|^2 \leq 2\left(\|x\|^2 + \|y\|^2\right)$$

and after straightforward computations, we obtain

$$\begin{aligned}\dot{v}_i &\leq -\left\{\rho_i - \frac{\sum_{j=1}^{i-1}\left(L_{j,1}^i + L_{j,2}^i\right)}{\eta_i}\right\}\epsilon_i^T S_i\epsilon_i \\ &\quad - \lambda_i e_{\theta_i}^T\Gamma_i e_{\theta_i} \\ &\quad + \sum_{j=1}^{i-1}\left\{\frac{2L_{j,1}^i}{\eta_j}\epsilon_j^T S_j\epsilon_j\right\} \\ &\quad + \sum_{j=1}^{i-1}\left\{\frac{2L_{j,1}^i\|\Lambda_j\|^2 + L_{j,2}^i}{\chi_j}e_{\theta_j}^T\Gamma_j e_{\theta_j}\right\}\end{aligned}$$

which follows

$$\dot{v}_i \leq -\mu_i v_i + \sum_{j=1}^{i-1}\kappa_{i,j}v_j$$

where $\mu_i = \min\left\{\rho_i - \frac{\sum_{j=1}^{i-1}\left(L_{j,1}^i + L_{j,2}^i\right)}{\eta_i}, \lambda_i\right\}$ and $\kappa_{i,j} = \max\left\{\frac{2L_{j,1}^i}{\eta_j}, \frac{2L_{j,1}^i\|\Lambda_j\|^2 + L_{j,2}^i}{\chi_j}\right\}$, with $\kappa_{1,j} = 0$.

As a result, the whole Lyapunov function can be chosen as $v = \sum_{i=1}^p v_i$, whose time derivative is $\dot{v} = \sum_{i=1}^p \dot{v}_i$, and we have

$$\begin{aligned}\dot{v} &\leq \sum_{i=1}^p\left\{-\mu_i v_i + \sum_{j=1}^{i-1}\kappa_{i,j}v_j\right\} \\ &= \sum_{i=1}^p\left(-\mu_i + \sum_{j=i+1}^p\kappa_{j,i}\right)v_i\end{aligned}$$

Consequently, if $\mu = \min\left\{\mu_i | \mu_i > \sum_{j=i+1}^p\kappa_{j,i},\ 1 \leq i \leq p\right\}$, then we obtain

$$\dot{v} \leq -\mu v < 0$$

and this ends the proof. ■

IV. Application into chaotic synchronization and parameter identification

As an illustrative example of adaptive state affine observation, let us consider the synchronization and parameter identification problem of a chaotic system. It is well-known that a chaotic system is a nonlinear deterministic system having a complex and unpredictable behavior. The sensitive dependence on initial conditions and the parameter variations is a prominent feature of chaotic behavior. In the sequel, we apply the proposed observer to simultaneously estimate states and identify parameters for a given chaotic system.

A. Chaotic system

Considering the following 3-dimensional autonomous Rossler's chaotic system described by [15]

$$\begin{cases} \dot{x}_1 = -x_2 - x_3 \\ \dot{x}_2 = x_1 + a x_2 \\ \dot{x}_3 = b + x_3(x_1 - c) \end{cases} \tag{4}$$

where $x_i (1 \leq i \leq 3)$ are the state variables, and a, b, c are all positive real constant parameters. The actual system parameters of (4) are set to $a = 0.2$, $b = 0.2$, and $c = 5.7$ for exhibiting the chaos phenomenon.

B. Change of coordinate and transformation into cascade system

By considering the following change of coordinate

$$\begin{pmatrix} z_1 \\ z_2 \end{pmatrix} = \begin{pmatrix} z_{1,1} \\ z_{1,2} \\ z_{2,1} \end{pmatrix} = \begin{pmatrix} x_1 \\ x_2 \\ x_2 + x_3 \end{pmatrix}$$

where $z_1 \in R^2$, $z_2 \in R$, the chaotic system (4) can be rewritten into cascade system (1) as follows:

$$\begin{cases} \dot{z}_1 = A_1(y,u) z_1 + \beta_1(y,u) + \varphi_1(y,u)\theta_1 \\ \dot{z}_2 = A_2(y,u,z_1,\theta_1) z_2 + \beta_2(y,u,z_1,\theta_1) \\ \quad +\varphi_2(y,u,z_1,\theta_1)\theta_2 \\ y_1 = z_{1,2} \\ y_2 = z_2 \end{cases} \tag{5}$$

where $y_1 \in R$ and $y_2 \in R$, with

$$\begin{cases} A_1(y,u) = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, \\ \beta_1(y,u) = \begin{pmatrix} -y_2 \\ 0 \end{pmatrix}, \\ \varphi_1(y,u) = \begin{pmatrix} 0 \\ y_1 \end{pmatrix}, \\ \theta_1 = a, \end{cases}$$

and

$$\begin{cases} A_2(y,u,z_1,\theta_1) = z_{1,1}, \\ \beta_2(y,u,z_1,\theta_1) = z_{1,1} + y_1(a - z_{1,1}), \\ \varphi_2(y,u,z_1,\theta_1) = \begin{pmatrix} 1 & y_2 - y_1 \end{pmatrix}, \\ \theta_2 = \begin{pmatrix} b & c \end{pmatrix}. \end{cases}$$

From figure 4, the chaotic system (5) with the parameters $a = 0.2$, $b = 0.2$, $c = 5.7$ and initial conditions $z_{1,1} = 2$, $z_{1,2} = 3$, $z_2 = 2$, exhibits the chaotic dynamics.

Now, for this cascade system (5), the proposed adaptive observer (2) is applied and designed as follows:

$$\begin{cases} \dot{\hat{z}}_1 = \hat{A}_1(y,u)\hat{z}_1 + \hat{\beta}_1(y,u) + \hat{\varphi}_1(y,u)\hat{\theta}_1 \\ \quad + \left\{ S_1^{-1} C_1^T + \Lambda_1 \Gamma^{-1} \Lambda_1^T C_1^T \right\} \Sigma_1 (y_1 - C_1 \hat{z}_1) \\ \dot{S}_1 = -\rho_1 S_1 - \hat{A}_1^T S_1 - S_1 \hat{A}_1 + C_1^T \Sigma_1 C_1 \\ \dot{\Lambda}_1 = \left\{ \hat{A}_1 - S_1^{-1} C_1^T \Sigma_1 C_1 \right\} \Lambda_1 + \hat{\varphi}_1 \\ \dot{\Gamma}_1 = -\lambda_1 \Gamma_1 + \Lambda_1^T C_1^T \Sigma_1 C_1 \Lambda_1 \\ \dot{\hat{\theta}}_1 = \Gamma_1^{-1} \Lambda_1^T C_1^T \Sigma_1 (y_1 - C_1 \hat{z}_1) \\ \\ \dot{\hat{z}}_2 = A_2\left(y,u,\hat{z}_1,\hat{\theta}_1\right)\hat{z}_2 + \hat{\beta}_2\left(y,u,\hat{z}_1,\hat{\theta}_1\right) \\ \quad +\hat{\varphi}_2\left(y,u,\hat{z}_1,\hat{\theta}_1\right)\hat{\theta}_2 \\ \quad + \left\{ S_2^{-1} C_2^T + \Lambda_2 \Gamma^{-1} \Lambda_2^T C_2^T \right\} \Sigma_2 (y_2 - C_2 \hat{z}_2) \\ \dot{S}_2 = -\rho_2 S_2 - \hat{A}_2^T S_2 - S_2 \hat{A}_2 + C_2^T \Sigma_2 C_2 \\ \dot{\Lambda}_2 = \left\{ \hat{A}_2 - S_2^{-1} C_2^T \Sigma_2 C_2 \right\} \Lambda_2 + \hat{\varphi}_2 \\ \dot{\Gamma}_2 = -\lambda_2 \Gamma_2 + \Lambda_2^T C_2^T \Sigma_2 C_2 \Lambda_2 \\ \dot{\hat{\theta}}_2 = \Gamma_2^{-1} \Lambda_2^T C_2^T \Sigma_2 (y_2 - C_2 \hat{z}_2) \end{cases} \tag{6}$$

where

$$\begin{cases} \hat{z}_1 = [\hat{z}_{1,1}, \hat{z}_{1,2}]^T, \hat{A}_1 = A_1(y,u), \hat{\beta}_1 = \beta_1(y,u), \\ \hat{\varphi}_1 = \varphi_1(y,u), \hat{\theta}_1 = \hat{a}, C_1 = \begin{pmatrix} 0 & 1 \end{pmatrix} \end{cases}$$

and

$$\begin{cases} \hat{z}_2 \in R, \hat{A}_2 = \hat{z}_{1,1}, \hat{\beta}_1 = \hat{z}_{1,1} + y_1(a - \hat{z}_{1,1}), \\ \hat{\varphi}_1 = \begin{pmatrix} 1 & y_2 - y_1 \end{pmatrix}, \hat{\theta}_2 = \begin{pmatrix} \hat{b} & \hat{c} \end{pmatrix}, C_2 = 1. \end{cases}$$

C. Simulation results

The aim here is to illustrate the simulation results obtained by the proposed cascade observer when it is applied in the synchronization and parameter identification problem of Rossler's chaotic system. The chosen numerical values (initial conditions and parameters) for Rossler's chaotic cascade system (5) are given in section (IV-B) while the values of its observer (6) are as follows.
The initial conditions are
$\hat{z}_{1,1} = \hat{z}_{1,2} = \hat{z}_2 = 0$; $\hat{\theta}_1 = \hat{a} = 0$, $\hat{\theta}_2 = \begin{pmatrix} \hat{b} & \hat{c} \end{pmatrix} = 0_{1\times 2}$.
$S_1(0) = 2I_{2\times 2}$, $\Lambda_1(0) = 0_{2\times 2}$, $\Gamma_1(0) = 10I_{2\times 2}$; $S_2(0) = 1$, $\Lambda_2(0) = 0_{1\times 2}$, $\Gamma_2(0) = 10I_{2\times 2}$.
The gain were chosen as $\rho_1 = 20$, $\lambda_1 = 15$; $\rho_2 = 20$, $\lambda_2 = 15$.

The simulation results obtained with the proposed observer are illustrated in Figs. 1 to 7.

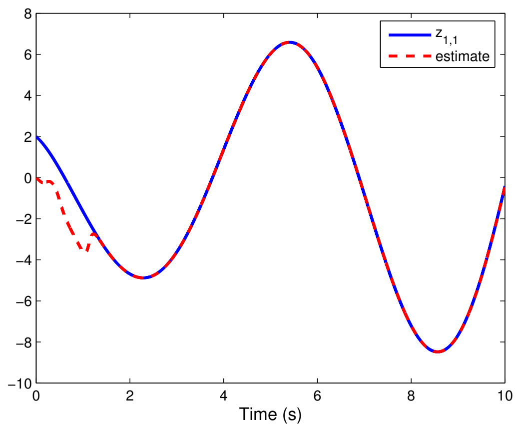

Fig. 1. $z_{1,1}$ and its estimate

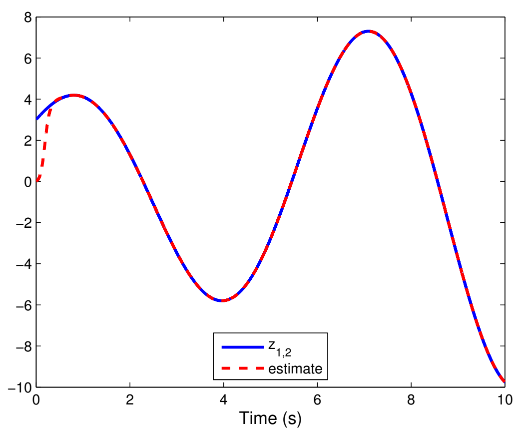

Fig. 2. $z_{1,2}$ and its estimate

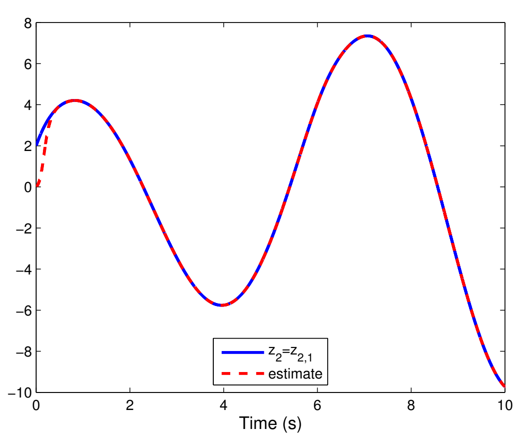

Fig. 3. z_2 and its estimate

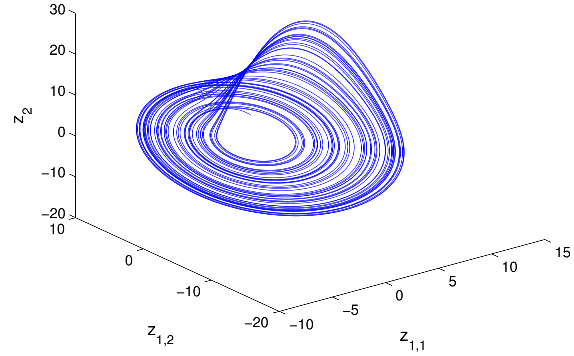

Fig. 4. Three-dimensional phase portrait of $z_{1,1}$, $z_{1,2}$ and z_2 with initial conditions $z_{1,1} = 2$, $z_{1,2} = 3$, and $z_2 = 2$.

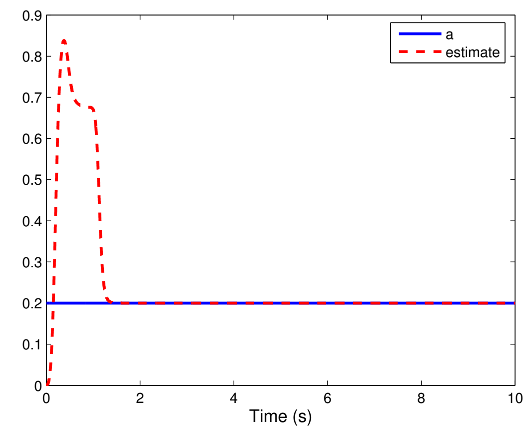

Fig. 5. Parameter a and its estimate

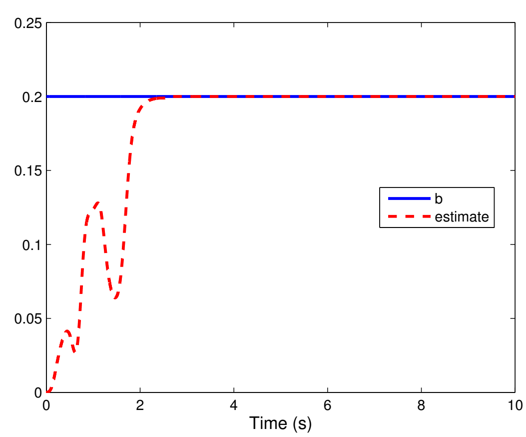

Fig. 6. Parameter b and its estimate

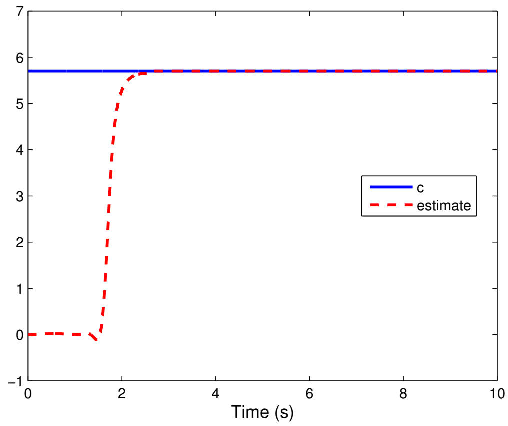

Fig. 7. Parameter c and its estimate

Figs 1, 2 and 3 illustrate the simulation results of the estimation of state variables, i.e. the problem of synchronization. The parameter identification results are depicted from Figs. 5, 6 and 7. Simulation results show that the proposed cascade state affine observer performs well. In all cases, state variables $z_{1,1}$, $z_{1,2}$, z_2 and parameters a, b, c indeed appear to be well estimated.

V. Conclusion

In this paper, the problem of adaptive observer for a class of cascade state affine systems has been discussed. Sufficient conditions have been given in order to guarantee the exponential convergence of the adaptive observer. Furthermore, it has been shown that the adaptive observer has an arbitrarily tunable rate. One practical interest of such observer is to study the synchronization and parameter identification problem of chaotic systems. An example of synchronization and parameter identification of Rossler's chaotic system has been studied in order to illustrate the feasibility of the proposed observer.

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