

Predictive Control of an Intermittently Aerated Activated Sludge Process

Qian Chai and Bernt Lie

Abstract—This paper presents model-based optimal control and predictive control of a biological wastewater treatment process with intermittent aeration. The objective of the control is to design an aeration strategy which minimizes the energy consumption induced by the aeration system, with adherence to the EU effluent standards and the operating constraints. The developed optimization problem is used with a receding horizon in nonlinear MPC based on the complete ASM3 model. The MPC aeration profile guarantees that the plant fulfills the effluent requirements at any time over long time periods. Significant energy saving is also obtained when comparing MPC to three traditional rule-based control strategies.

I. INTRODUCTION

Due to legislation on wastewater treatment, especially the strict EU Guideline *Urban Wastewater Directive 91/271/EEC*, there are strong incentives to upgrade existing wastewater treatment plants in order to comply with the effluent standards and to reduce operation costs. The activated sludge process (ASP) is the most generally applied biological wastewater treatment method. In the classical biological treatment systems, aerobic nitrification and anoxic denitrification are maintained in separated zones. In the last two decades, intermittently aerated ASPs have been developed in a way through which the aerobic and anoxic zones are periodically exchanged [1], [2].

The intermittently aerated ASP generally consists of a single aeration tank with alternating aerobic-anoxic conditions by switching the supplied air on and off, and a settler where the microbial culture is separated from the liquid being treated. Most of the culture is recycled and mixed with incoming wastewater to maintain convenient sludge age characteristics and high reaction rates. The alternating aerobic-anoxic technique can easily be applied to the existing nitrogen removal plants with nitrogen removal efficiencies of 70–90% [3]. Compared to the classical biological treatment processes, the intermittently aerated ASP offers significant energy savings and easy plant retrofitting. An important feature of the intermittently aerated ASP is its flexible control ability, which makes the process better accommodate variable influent loading conditions and makes it suitable for optimization of operating costs [2]. Generally, control of the aeration system is of great importance since (i) the concentration of dissolved oxygen is directly related to nitrogen removal and (ii) the energy consumption of the aeration system is the main operating cost.

Corresponding author: Bernt.Lie@hit.no; tel: +47 35575169.
Address: Telemark University College, P.O. Box 203, N-3901 Porsgrunn, Norway.

Process control of ASPs is a challenging task since the processes are characterized by large disturbances, significant nonlinearities, and stiff dynamics. To date, in many municipal treatment plants, the lengths of aerobic and anoxic phases are typically fixed or scheduled daily using a plant's supervisory control and data acquisition system. A more advanced control approach is rule-based feedback control, which uses varying phase length by establishing a switch point for each nitrification/denitrification phase. Since the on-line monitoring of ammonia and nitrate is difficult, some indirect measurements of ammonia and nitrate (e.g. oxidation-reduction potential (ORP), pH, dissolved oxygen (DO)) are commonly used to control ASPs [4]. Recently, with the development of mathematical models, particularly the Activated Sludge Models (ASMs) [5], application of optimal control has been the subject of a number of studies [6], [7], [8], [9]. Moreover, the application of MPC in the intermittently aerated ASPs has been studied in [10], [11], and [12] using simplified models, but the application of MPC with the complete ASM models is not studied much in the literature.

In this paper, optimal control of the aeration system is considered for improving the efficiency and reliability of an intermittently aerated ASP, used for removal of nitrogen from domestic wastewater. The objective of the control is to design an aeration strategy (air-on and air-off periods) which minimizes the energy dissipated by the aeration system, with adherence to the limits of the effluent requirements and the operating constraints. In Section II, the configuration and dynamic model of the nitrogen removal plant are described. In Section III, the optimization problem is formulated, and the optimization method and results are discussed. In Section IV, special emphasis is placed on using dynamic optimization in MPC with a receding horizon for a usable online implementation and to show the long-term effects of the optimal aeration strategy. In Section V, the simulation results of MPC are compared to three traditional control strategies for nitrogen removal in the intermittently aerated ASP. Finally, some conclusions are drawn in Section VI.

II. PROCESS CONFIGURATION AND MODELING

A. Process configuration

In this study, a model of a laboratory-scale nitrogen removal plant is considered. The process consists of a unique aeration tank ($V_a = 401$) and a cylindrical settler ($V_{set} = 2.51$). More details of the process are given in [13]. The sludge retention time (SRT) for the overall process is maintained at 15 d. The influent wastewater to the plant is

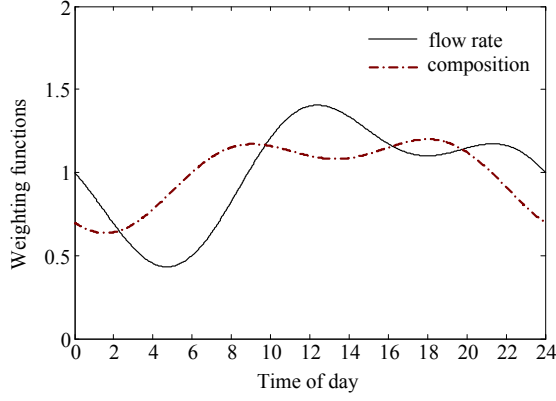


Fig. 1. Typical daily variations of inlet wastewater flow rate and composition, taken from [14].

primarily domestic wastewater. The average influent flow rate (Q_{in}) is 961/d which gives a hydraulic retention time (HRT) of 10 h, and the average total chemical oxygen demand (COD_{in}) and total nitrogen (TN_{in}) concentrations are 260 g m^{-3} and 25 g m^{-3} , respectively. Typical daily variations of dry weather conditions are simulated using weighting functions defined in [14] for both inlet wastewater flow rate and composition, see Fig. 1.

B. Process model

The plant model consists of an aeration tank model and a settler model. The complete Activated Sludge Model No. 3 (ASM3) describes the biological processes involved in the aeration tank. The states of the model are grouped into the concentration of soluble components S_j and particulate components X_j . Assuming perfect mixing in the reactor, the mass balance in the aeration tank results in:

$$\frac{dx_a}{dt} = \frac{Q_{in}x_{in} + Q_{rs}x_{rs} - (Q_{in} + Q_{rs})x_a}{V_a} + r + e_1 \mathcal{A}_{O_2}, \quad (1)$$

where x_{in} , x_{rs} , $x_a \in \mathbb{R}^{13}$ contain the concentrations in the influent, in the recycled sludge, and in the reactor, respectively; their components are

$$x_l = [S_{O_2,l} \ S_{I,l} \ S_{S,l} \ S_{NH_4,l} \ S_{N_2,l} \ S_{NOX,l} \ S_{ALK,l} \ X_{I,l} \ X_{S,l} \ X_{H,l} \ X_{STO,l} \ X_{A,l} \ X_{SS,l}]^T,$$

$l \in \{in, rs, a\}$; $r \in \mathbb{R}^{13}$ is the vector formed by the reaction rates of each component (defined in ASM3); e_1 is the standard basis for the first coordinate in \mathbb{R}^{13} ; and \mathcal{A}_{O_2} describes the oxygen transfer:

$$\mathcal{A}_{O_2} = u_a \cdot K_L a (S_{O_2}^{sat} - S_{O_2,a}), \quad (2)$$

where $K_L a$ is the oxygen transfer coefficient which is a function of the supplied air flow rate, and $S_{O_2}^{sat}$ is the saturated dissolved oxygen concentration ($S_{O_2}^{sat} \approx 10 \text{ g m}^{-3}$). Variable u_a is a binary sequence switching between 0 and 1 and represents the state of the blower (on/off) aerating the plant. It is assumed that the blower is on at time $t = 0$.

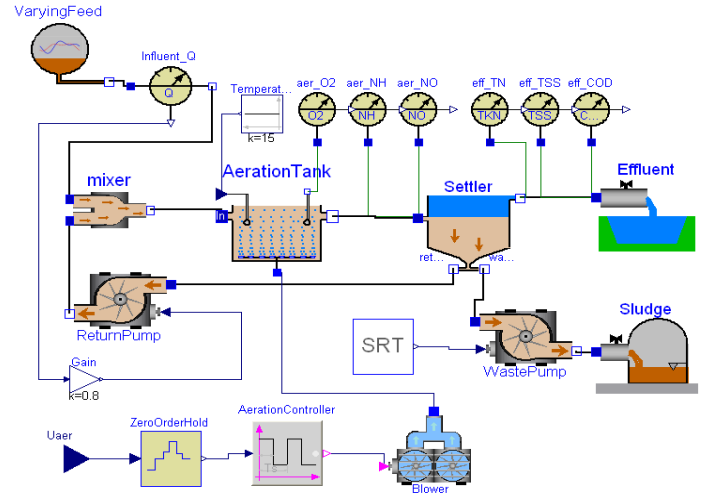


Fig. 2. Model diagram of the intermittently aerated ASP implemented in Modelica/Dymola.

Here, the control variable that influences the operation of the plant is the sequence of switching times, i.e., times when the blower switches on/off.

For simplicity, the settler is often considered a perfect splitter, thus the settler model can be expressed as follows:

Effluent concentrations are

$$S_{j,eff} = S_{j,a}, \quad X_{j,eff} = 0 \quad (3)$$

Recycled sludge concentrations are

$$S_{j,rs} = S_{j,a}, \quad X_{j,rs} = \frac{Q_{in} + Q_{rs}}{Q_{rs} + Q_w} X_{j,a}. \quad (4)$$

In summary, the general model of the dynamic system can be simply described as a set of ODEs:

$$\frac{dx_a}{dt} = \mathcal{F}(x_a, u_a, t). \quad (5)$$

The dynamic model is implemented in the object-oriented modeling language Modelica [15] using the Dymola simulation environment [16], see Fig. 2, based on the modified Modelica library WasteWater [17]. Dymola generates a convenient interface to Matlab such that Modelica models can be executed within Matlab. An application executing a Modelica model in Matlab is described in [18].

III. OPTIMIZATION PROBLEM

A. Definition of optimization problem

The aeration process can be seen as a succession of cycles where each cycle consists of an air-on period followed by an air-off period, i.e. the period between two consecutive starts of the blower, see Fig. 3. For a given optimization horizon T_h , let there be N_c aeration cycles. In the k th aeration cycle, a_k is the duration of the cycle and d_k is the length of the duty cycle, i.e. the air-on period. The aeration fraction f_k , which is defined as $f_k = d_k/a_k$, is often introduced instead of d_k . Hence, the aeration time can be optimized

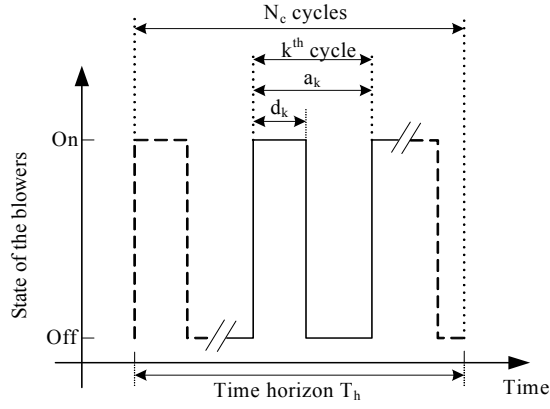


Fig. 3. Parameters of the aeration cycles, based on [19].

by manipulating a_k and f_k for $k = 1, \dots, N_c$. In order to avoid a *mixed integer programming* problem that may complicate the solution, let the number of cycles N_c be fixed. In addition, the length of the aeration cycles are made constant, $a = T_h/N_c$. The set of optimized parameters is then reduced from $2 \times N_c$ to N_c parameters. Also, the optimization horizon is chosen to be 24 h.

1) *Objective function*: The objective of this study is to determine the aeration profile that minimizes the energy consumption. The energy consumption is mainly decided by the aeration time of the process, and the extra power consumption induced by starting the aeration system is neglected. Energy optimization is therefore achieved by minimizing the aeration time. Thus, the objective function J is defined as the total aeration time divided by the total optimization time:

$$J = \frac{\sum_{k=1}^{N_c} a \cdot f_k}{T_h}. \quad (6)$$

2) *Inequality constraints*: The inequality constraints are defined in order to cope with the EU effluent standards on chemical oxygen demand COD, suspended solids SS, and total nitrogen TN:

$$\text{COD}_{\text{eff}} \leq \text{COD}_{\text{max}} = 125 \text{ g m}^{-3} \quad (7)$$

$$\text{SS}_{\text{eff}} \leq \text{SS}_{\text{max}} = 35 \text{ g m}^{-3} \quad (8)$$

$$\text{TN}_{\text{eff}} \leq \text{TN}_{\text{max}} = 10 \text{ g m}^{-3}. \quad (9)$$

In this work, we assume that there are no particulate components in the effluent ($\text{SS}_{\text{eff}} \equiv 0$). Also, the COD constraint is generally easily satisfied as a large part of the biodegradable organic carbon is consumed during denitrification stages. Based on these considerations, only the constraint on TN_{eff} is therefore considered in the optimization problem.

To ensure the feasibility of the computed aeration strategies and to prevent the blower from damage, constraints on the aeration and non-aeration sequences are introduced:

$$t_{\text{on},\text{min}} \leq a \cdot f_k \leq t_{\text{on},\text{max}} \quad (10)$$

$$t_{\text{off},\text{min}} \leq a \cdot (1 - f_k) \leq t_{\text{off},\text{max}}, \quad (11)$$

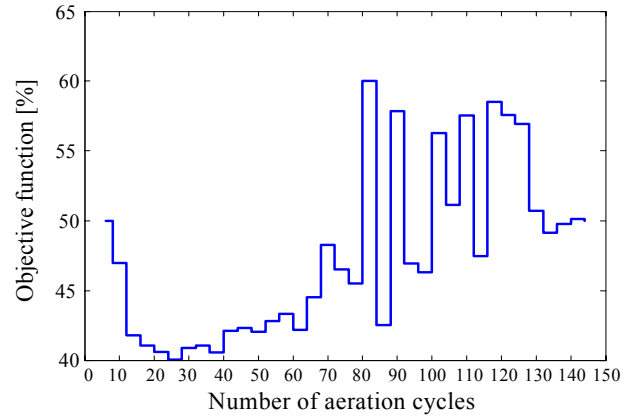


Fig. 4. Objective function J vs. number of cycles N_c .

where we define $t_{\text{on},\text{min}} = t_{\text{off},\text{min}} = 5$ min and $t_{\text{on},\text{max}} = t_{\text{off},\text{max}} = 2$ h with reference to [19].

To decide the constraints for the number of cycles N_c , we first get the constraints of the aeration cycle duration a by adding inequalities 10 and 11:

$$t_{\text{on},\text{min}} + t_{\text{off},\text{min}} \leq a \leq t_{\text{on},\text{max}} + t_{\text{off},\text{max}}. \quad (12)$$

Then, N_c must be constrained to

$$\frac{T_h}{a^u} \leq N_c \leq \frac{T_h}{a^l} \quad (13)$$

where a^u and a^l are the upper and lower boundaries of a in inequality 12, respectively.

3) *Optimization problem*: Finally, the dynamic optimization problem on a given optimization horizon T_h can be stated as:

$$\min_{f_1, \dots, f_{N_c}} J = \frac{\sum_{k=1}^{N_c} a \cdot f_k}{T_h} = \frac{1}{N_c} \sum_{k=1}^{N_c} f_k \quad (14)$$

$$\text{s.t. } \frac{dx_a}{dt} = \mathcal{F}(x_a, u_a, t)$$

$$0 \leq \text{TN}_{\text{max}} - \text{TN}_{\text{eff}}$$

$$\frac{t_{\text{on},\text{min}}}{a} \leq f_k \leq \frac{t_{\text{on},\text{max}}}{a}$$

$$1 - \frac{t_{\text{off},\text{max}}}{a} \leq f_k \leq 1 - \frac{t_{\text{off},\text{min}}}{a}.$$

B. Optimization methods and results

The resulting optimization problem is solved by the SNOPT solver in the TOMLAB¹ optimization toolbox for Matlab. SNOPT is a fast and robust solver for solving large-scale nonlinear optimization problems with a sequential quadratic programming (SQP) algorithm [20].

First, simulation of the start-up period of the plant is done using an intermittent air supply with $K_L a = 72 \text{ d}^{-1}$, $N_c = 15 \text{ d}^{-1}$ (i.e. $a = 1.6$ h), and

$$f_k = 0.6 \text{ for } k = 1, \dots, N_c. \quad (15)$$

¹For more information, see the TOMLAB home page <http://www.tomlab.biz>.

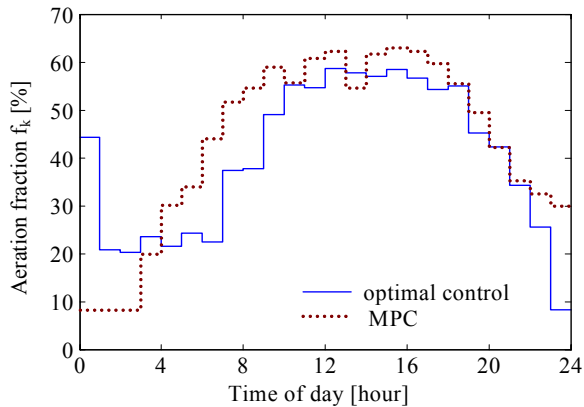


Fig. 5. Evolution of aeration fraction f_k for open-loop optimal control (solid) and for MPC (dotted) over 24 h.

The system approaches steady state after about 30 days according to simulation, see also [21]. In this work, the steady state concentrations are used as the initial conditions, and are used for all control strategies here so that the obtained results can be compared.

The optimal number of aeration cycles N_c is sought by computing optimal aeration profiles for the values of $N_c \in [6, 144] \text{ d}^{-1}$ given in inequality 13 over a 24 h optimization horizon. The corresponding values of the objective function J are given in Fig. 4. The optimal value of N_c is found to be 24 with the lowest value of $J = 40.05\%$, i.e. the length for each aeration cycle is 1 h. The values of objective function have large variations when $N_c > 60 \text{ d}^{-1}$, because the SQP algorithm may easily be captured in local minima with the increasing number of unknown optimization variables.

The optimal aeration strategy for 24 cycles a day is represented in Fig. 5 in solid line. The corresponding effluent concentrations of TN and COD are shown in Fig. 6 (solid lines). The effluent constraint on COD always remains inactive, so this constraint has been removed from the optimization problem. The optimization results demonstrate that the total aeration fraction of 40.05%, i.e. 9 h 37 min of aeration time a day, is sufficient to ensure the effluent constraints over 24 h.

IV. MODEL PREDICTIVE CONTROL

The nonlinear optimal control strategy described above is an open-loop operation which is very sensitive to model errors and unknown disturbances. For a usable online implementation, a nonlinear MPC is applied to change the open-loop optimal control into a closed-loop solution. In order to apply MPC to the model of the pilot plant, we need state estimation to get the current unmeasured states from measurements. In this simulation study, it is assumed that all states are available, therefore the state estimation is not considered here. The study of state estimation of a similar nitrogen removal process is described in [22].

The constrained nonlinear optimization defined in Section III is used in MPC with a receding horizon. The MPC

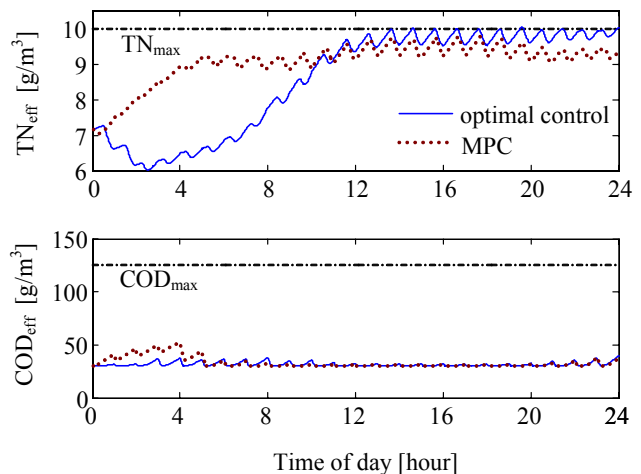


Fig. 6. Results of optimal control (solid) and MPC (dotted) over 24 hours: the effluent concentrations of TN and COD.

horizon is an important tuning parameter for optimization results. In this study, the MPC horizon is tuned to be 4 aeration cycles. The MPC aeration fraction and simulation results over 24 h are shown in Figs. 5 and 6 with dotted lines. We note that the optimal aeration strategy is closely related to the variation of the feed concentration: the high TN load (see Fig. 1) induces the high aeration fraction (e.g. between 8:00 and 20:00), whereas the low TN load results in the low aeration fraction. Compared with optimal control, the TN concentration with MPC increases faster at the beginning due to the shorter MPC horizon. The application of MPC leads to a total aeration fraction of 43.46% over a 24 h period. This value is larger than the one obtained in optimal control ($J = 40.05\%$). The higher aeration fraction results in that the effluent concentration of TN is held at lower values ($8.5\text{--}9.5 \text{ g m}^{-3}$) than the effluent constraint 10 g m^{-3} , see Figs. 6 and 7. This seemingly disappointing result for MPC can be explained as follows: due to the periodic operation, the stationary optimal future sequence of f_k will look similar to the solid line in Fig. 5, and will never reach some “steady state” — there will always be some initial “transient” in f_k .

Also, MPC is applied over a long simulation horizon to guarantee that the effluent requirements are fulfilled and the energy saving is lasting, see Fig. 7. The computation of the optimal f_k over 10 days (for 24 cycles a day) yields a total aeration fraction of 46.42% and more than fulfills the strict effluent requirements at any time.

V. COMPARISON BETWEEN MPC AND RULE-BASED CONTROL

In this section, the MPC controller developed above is compared with the traditional rule-based control strategies for intermittently aerated ASPs. The rule-based control strategies used in practice and in the literature can be categorized as (1) open-loop control using fixed lengths of aerobic and anoxic phases, or (2) feedback control using varying phase lengths by establishing a switch point for each phase. The switch point is typically the concentration of dissolved

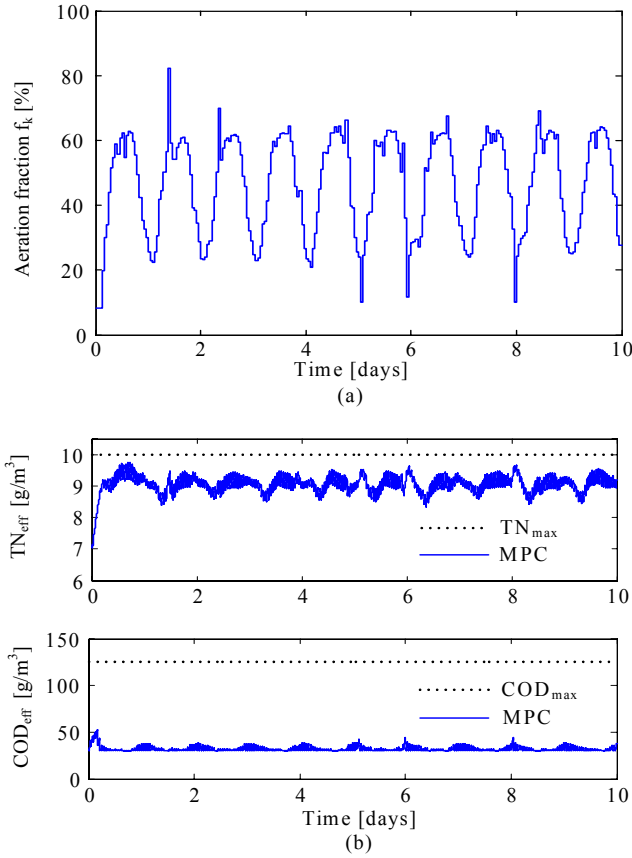


Fig. 7. MPC simulation results over 10 days: (a) MPC aeration fraction and (b) the corresponding effluent concentrations of TN and COD.

oxygen, nitrate, or ammonia [23], [24]. In this work, three rule-based control strategies are compared with MPC: fixed phase length, oxygen based control, and ammonia based control. The comparison is carried out by simulation using the sequence of aeration/non-aeration times as the control input and the TN concentration as the primary output. The objective of control strategies is to minimize the aeration time of the process while satisfying the effluent constraints. Evaluation criterion J is described in Equation 6. The applied rule-based controllers are described as follows:

- *Fixed phase length control*: In this controller, the lengths of aerobic and anoxic phases are fixed. We assume that there are N_c aeration cycles for a given optimization horizon 1 d; then the length of aeration cycles a is $\frac{1\text{d}}{N_c}$. The controller thus has 2 tuning parameters: the number of cycles N_c each day and the aeration fraction f , which are found to be $N_c = 60\text{ d}^{-1}$ and $f = 48.61\%$ by minimizing J over 1 d.
- *Oxygen (DO) based control*: In practice, a well-accepted technique is to turn the aeration system on and off based on the online measurement of DO concentration. In [19], an oxygen based controller is designed as: each aeration cycle has a constant duration of $a = \frac{1\text{d}}{N_c}$ and the aeration system is stopped when the DO concentration reaches DO_{\max} . In this study, to

TABLE I

COMPARISON BETWEEN RULE BASED CONTROL STRATEGIES AND MPC.

Controller	Criterion	Tuning parameters	CPU time for simulating 24 h
Fixed phase	48.61%	$N_c = 60\text{ d}^{-1}$ $f = 48.61\%$	39 s
DO based	53.12%	$N_c = 50\text{ d}^{-1}$ $\text{DO}_{\max} = 1.64\text{ g m}^{-3}$	20 s
S_{NH_4} based	47.59%	$S_{\text{NH}_4,\max} = 7.9\text{ g m}^{-3}$ $S_{\text{NH}_4,\min} = 7.4\text{ g m}^{-3}$	30 s
MPC	43.46%	$N_c = 24\text{ d}^{-1}$ $f_k, k = 1, \dots, N_c$	22 min

fulfill the effluent constraints, N_c and DO_{\max} are tuned to be 50 d^{-1} and 1.64 g m^{-3} , respectively.

- *Ammonia (S_{NH_4}) based control*: The aeration system can also be controlled by the upper and lower bounds of S_{NH_4} based on the measurement of S_{NH_4} (see [25]): the aeration is turned on when $S_{\text{NH}_4} \geq S_{\text{NH}_4,\max}$ and is turned off when $S_{\text{NH}_4} \leq S_{\text{NH}_4,\min}$. In fact, large values of $S_{\text{NH}_4,\max}$ and $S_{\text{NH}_4,\min}$ will result in lower aeration consumption, since ammonia is oxidized to nitrate under the aerobic step. To make the control results comparable, $S_{\text{NH}_4,\max}$ and $S_{\text{NH}_4,\min}$ are tuned to be 7.9 g m^{-3} and 7.4 g m^{-3} because S_{NH_4} varies within this region when using the MPC solution.

A comparison of MPC with the rule-based controllers is given in Table I. All computations are performed on a 1.7GHz Pentium M computer with 1Gbyte RAM. The values of criterion J show that MPC provides the shortest aeration time, whereas the DO based controller is least satisfactory. We find that MPC achieves 18.2% (i.e., 2.3h) aeration reduction each day in comparison to the DO based control. Compared to the fixed phase length and S_{NH_4} based control, which have similar J values, MPC reduces the average aeration time by up to 10.6% and 8.7%. Although the computation time of MPC is much longer than the simpler controllers, it is still feasible with on-line implementation of an MPC controller. Fig. 8 presents the TN concentrations obtained with all these control strategies. It is seen that the performances of the optimally tuned fixed phase length controller and DO based controller are similar, e.g., the aeration switching frequencies are high for both controllers, and the S_{NH_4} based controller is designed to mimic the performance of MPC. It should be noticed that the fixed phase length strategy is limited in that predetermined fixed aerobic and anoxic phases cannot compensate for unanticipated loading variations (e.g. rain/storm events). Although feedback is utilized for the DO and S_{NH_4} based control strategies, the tuning of the parameters are rather sensitive and must be performed carefully. In summary, the MPC presents the following advantages:

- 1) MPC provides the most reduction of the aeration time.
- 2) It is easier to deal with the inequality constraints in both effluent requirements and operating conditions.
- 3) MPC handles disturbances (e.g. load changes) in a more natural way.

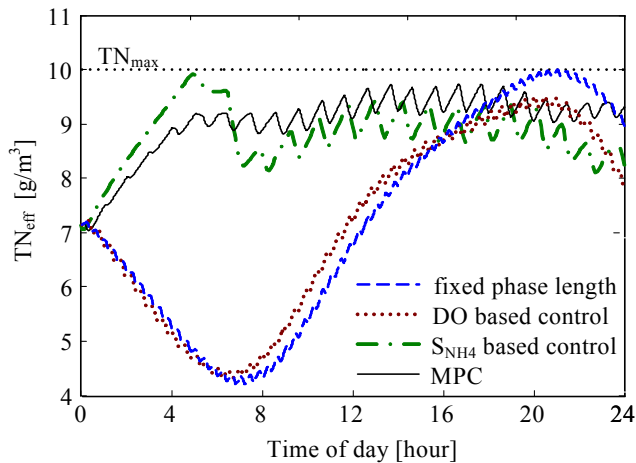


Fig. 8. Comparison of effluent TN concentrations obtained with rule based control strategies and with MPC.

VI. CONCLUSIONS AND FUTURE WORK

In this paper, an optimal control strategy for a small-sized nitrogen removal ASP is developed to reduce the energy consumption induced by the aeration system, with adherence to the limits of the effluent standards. The optimization problem is formulated as a constrained nonlinear optimization problem, which is solved by the SNOPT solver of the TOMLAB optimization toolbox. Then, the optimization method is used with a receding horizon in nonlinear MPC based on the complete ASM3 model. The application of MPC over a long simulation horizon (10 d) is discussed, and the MPC aeration profile guarantees that the plant fulfills the tight EU effluent requirements at any time. Based on the same control objective as MPC, traditional rule-based control strategies, such as fixed phase length, DO based, and S_{NH_4} based control are discussed using well-tuned controller parameters. The comparison between the rule-based controllers and MPC shows that better aeration profiles, with reductions of energy consumption of up to 18.2%, could be obtained by applying MPC.

In the present study, the predictive control strategy has been applied to a simulation model. The plan is to test the strategy on the laboratory plant in the future. It is also of interest to study the applications of MPC to full-scale industrial plants.

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