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Stabilizing Anti-Windup Controllers for Power Allocation in Cellular Networks

Vishwesh Kulkarni Mayuresh Kothare Michael Safonov

Abstract— **Recently, passivity-based techniques to control the transmit power of mobile nodes have been proposed to ensure the input-output stability of cellular CDMA networks. An inherent assumption in these solutions is that the transmit power synthesized by the controller of choice does not exceed an upper bound p_{max}, which has not been characterized. In practice, an upper bound p_{max} is invariably imposed on the instantaneous transmit power of the mobile user and may lead to windup in the cellular network. We propose an antiwindup mechanism.**

Index Terms— **robust stability, multipliers, sensor networks, noncooperative game theory, power control**

I. Introduction

Recently, [1] has developed passivity-based team-optimized techniques to minimize the transmit power in multi-cell CDMA networks subject to the constraint that the steady state signal-to-interference ratio (SIR) exceeds a pre-designed threshold for each node. An unresolved technicality is that an upper bound on p_{max}, required for the validity of a key result, viz., [1, Lemma 2], has not been characterized. In practice as well, an upper bound p_{max} is invariably set on the instantaneous transmit power either by the cell phone provider or by the cell phone user. As a result, the controller output is subject to a saturation nonlinearity, and is therefore liable to wind-up. In this paper, we describe a method to synthesize an anti-windup controller to allocate the transmit powers across the network so that the input-output stability of the cellular network is maintained in the face of saturation constraints on the mobile transmit power.

For simplicity, we shall consider a single cell network. The system description is as follows (see Fig. 1). The base station receives data from N children over time-varying wireless channels. The uplink transmit power for the i-th mobile node is $p_i \in [0, p_{max}]$. The corresponding signal received at the base station is $x_i = h_i p_i$ where $h_i \in (0, 1)$ is the slowly time-varying channel gain. The SIR of mobile i at the base station is thus given by

$$\gamma_i = \frac{L h_i p_i}{\sum_{j, j \neq i} h_j p_j + \sigma^2},$$

Prof. Vishwesh Kulkarni is with the Department of Electrical Engineering, Indian Institute of Technology Bombay, Mumbai 400 076, India. Email: vishwesh@ee.iitb.ac.in. Prof. Mayuresh Kothare is with the Department of Chemical Engineering, Lehigh University, Bethlehem, PA. Email: mvk2@lehigh.edu. Prof. Michael Safonov is with the Department of Electrical Engineering, University of Southern California, Los Angeles, CA. Email: msafonov@usc.edu. Research supported in parts by Sentina Systems, Inc., Boston, MA.

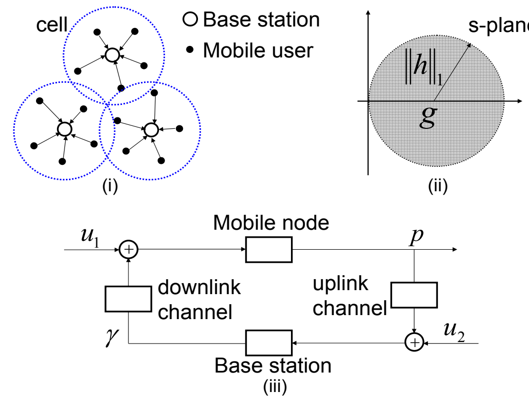

Fig. 1. (i): Cellular network has one base station per cell; a mobile node transmits data to only one base station. (ii) The Nyquist plot of a Zames-Falb multiplier $m(\cdot) = \delta(\cdot) + h(\cdot)$. [2] uses the Zames-Falb multipliers to improve the performance of the system shown in (iii).

where $L > 1$ is the spreading gain of the network and $\sigma^2 > 0$ is the variance of the background noise. The cost function $C_i(p_i)$ of the i-th mobile is strictly convex and continuously differentiable, and can be represented as the difference between a strictly convex pricing function of the transmit power p_i (e.g., the battery consumption) and a strictly concave utility function denoting the willingness of the mobile i to increase γ_i. The optimization problem can now be stated as follows:

$$\min_p \sum_{i=1}^{N} C_i(p_i) \quad \text{s. t.} \quad \gamma_i \geq \overline{\gamma}_i, 0 \leq p_i \leq p_{max} \quad \forall i, \tag{1}$$

where $p \doteq [p_1 \ p_2 \ \ldots \ p_N]^T$, $\overline{\gamma}_i$ is the target SIR for the mobile i, and p_{max} is chosen sufficiently high so that $p_i(t) \leq p_{max}$ $\forall t$. Let

$$A \doteq \begin{bmatrix} h_1 & -h_2\frac{\overline{\gamma}_1}{L} & \ldots & -h_N\frac{\overline{\gamma}_1}{L} \\ -h_1\frac{\overline{\gamma}_2}{L} & h_2 & \ldots & -h_N\frac{\overline{\gamma}_2}{L} \\ \vdots & \vdots & \ldots & \vdots \\ -h_1\frac{\overline{\gamma}_N}{L} & -h_2\frac{\overline{\gamma}_N}{L} & \ldots & h_N \end{bmatrix} \tag{2}$$

$$b \doteq [\overline{\gamma}_1\frac{\sigma^2}{L} \ \ldots \ \overline{\gamma}_N\frac{\sigma^2}{L}]^T \tag{3}$$

$$\Omega \doteq \{p \in \mathbb{R}^N : Ap \geq b, \ p_i \in [0, p^*] \ \forall i\}.$$

Then, the optimization problem given by (1) is recast as

$$\min_p \sum_{i=1}^{N} C_i(p_i) \quad \text{subject to} \quad p \in \Omega. \tag{4}$$

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[1] has derived a decentralized solution p^* to the above optimization problem, and has used well known passivity theory results to prove the global asymptotic stability of the resulting network. Such controllers are liable to windup if an upper bound p_{max} is pre-specified on the transmit power. In this paper, we assume that the mobiles have no access to each other's state. We propose a decentralized scheme in which the mobiles themselves synthesize the required stabilizing anti-windup controller. We also facilitate the largest possible class of dynamic nonlinear controllers that ensures the strict passivity of the base station if it chooses to implement a certain feedback structure, viz., a convolution operator acting on a repeated memoryless monotone nonlinearity.

II. Notation and Background Results

A. Notation and Definitions

Our notation mostly follows [3], [4], [5], and [6]. The causal truncation P_T of a function f is defined as $P_T f(t) = f(t)$ for all $t \le T$ and zero otherwise. The extended space $\mathcal{L}_{2e}$ comprises all functions f having the property that the causal truncation $P_T f \in \mathcal{L}_2$ for all finitely valued T. We say that the projection $(f(x))_x^+$ is active if it is zero-valued, and inactive otherwise. We denote i-th row of a matrix A as $\text{row}_i(A)$. A feedback system $\dot{x} = f(x,u),\ y = h(x)$ is said to be passive if there exists a continuously differentiable positive semi-definite function, i.e. a storage function, $V(x)$ such that

$$\dot{V} = \frac{\partial V}{\partial x} f(x,u) \le u^T h(x) \quad \forall x, u.$$

The system $y = h(t,u)$ is passive if $u^T y \ge 0\ \forall u$. A multiplier is a convolution operator such as, e.g., the Popov multipliers, the Zames-Falb multipliers, and the RC multipliers [7], [8], [9].

Definition 1: `[monotone nonlinearity]`
The class $\mathcal{N}_M$ of monotone nonlinearities consists of all memoryless mappings $N : \mathbb{R}^n \mapsto \mathbb{R}^n$ such that:

1) N is the gradient of a convex real-valued function; and
2) there exists $C \in \mathbb{R}^+$ s.t. $\|N(x)\| \le C\|x\| \quad \forall x \in \mathcal{L}_2$.

The class $\mathcal{N} \doteq \{N \in \mathcal{N}_M | N(0) = 0\}$ and the class $\mathcal{N}_{odd} \doteq \{N \in \mathcal{N} | N(x) = -N(-x)\ \forall x\}$. □

Definition 2: `[Zames-Falb multipliers]`
$\mathcal{M}_{ZF}$ of Zames-Falb multipliers denotes the class of convolution operators, either continuous-time or discrete-time, such that the impulse response of an $M \in \mathcal{M}_{ZF}$ is of the form

$$m(\cdot) = g\ \delta(\cdot) + h(\cdot) \quad \text{with} \quad \|h\|_1 < g,$$

where $g, h(\cdot) \in \mathbb{R}$. □

Remark 1: A multiplier preserves positivity of a $\mathcal{N}_M$ nonlinearity if and only if it is in $\mathcal{M}_{ZF}$ [9], [5]. □

B. Background Results

Lemma 1: (Lemma 3.1 of [1])
If Ω is nonempty, the optimization problem given by (3) has a unique global minimum. □

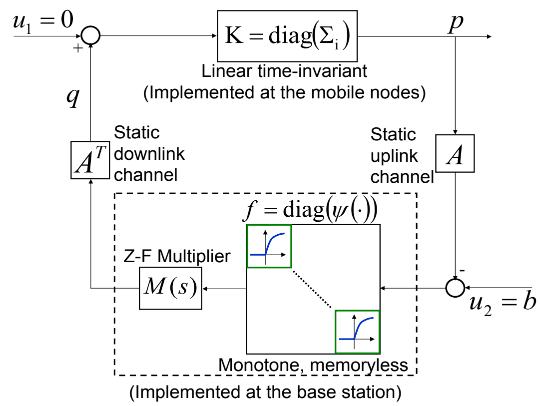

Fig. 2. Block-diagram decomposition of the passivity-based power allocation mechanism presented in [1] and [2]. Exogenous inputs u_1 and u_2 are set to 0 and b respectively, where the choice of b is as per the base station administrator. The feedback nonlinearity $f(\cdot)$ is repeated monotone single-input single-output (SISO). [2] synthesizes a dynamic feedback nonlinearity at the base station by multiplying the output of $f(\cdot)$ by a Zames-Falb multiplier $M(s)$.

Lemma 2: (Lemma 3.2 of [1])
If $\theta \doteq \sum_j \overline{\gamma}_j/(\overline{\gamma}_j + L) < 1$ and if p_{max} is chosen sufficiently large, then Ω is nonempty and every p satisfying $Ap \ge b$ satisfies $p > 0$. Furthermore, Ω is empty if $\theta \ge 1$. □

The above two results have been used by [1] to synthesize the decentralized power control algorithms as follows. If $\theta < 1$, Lemma 2 establishes an optimal solution p^* to the optimization problem given by (3). Furthermore, it establishes that p^* is componentwise positive if p_{max} is large enough. It may be seen that p^* minimizes the Lagrangian

$$L(p, \lambda) \doteq \sum_i C_i(p_i) - \lambda^T(Ap - b). \tag{5}$$

Since A is full-rank, λ is unique. Let $q \doteq A^T\lambda, r \doteq \text{diag}(p_i)q$. Then, (5) is recast as

$$L(p, \lambda) \doteq \sum_i (C_i(p_i) - r_i) - \lambda^T b, \tag{6}$$

with p^* satisfying $\frac{dC(p)}{dp}|_{p=p^*} - q = 0$. Define the convex user and network problems as

$$\text{user } i\text{: } \min_{r_i} C_i(r_i/q_i) - r_i,\ \ r_i \ge 0 \tag{7}$$

$$\text{network: } \min_p \sum_i -r_i \log(p_i), \quad p \in \Omega. \tag{8}$$

[1] shows that there exist p, q, r with $r_i = p_i q_i$ such that r_i solves the user i problem and p solves the network problem so that p is the unique solution to the optimization problem given by (3). As shown in [1], a primal update algorithm is as follows:

$$q = A^T f(Ap), \tag{9}$$

$$\Sigma_i : \ \dot{p_i} = -k_i\left(\frac{dC_i}{dp_i} - q_i\right) \text{ with } k_i > 0, \tag{10}$$

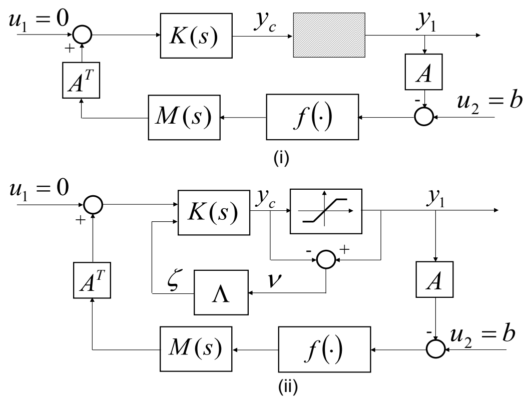

Fig. 3. (i): A prerequisite for the stability results established in [1] is that controller output y_c does not exceed p_{max}, which is left uncharacterized. In practice, an upper bound p_{max} is invariably imposed on y_c. The shaded block represents a saturation nonlinearity that captures this constraint. (ii) A block diagram representation of the single cell after incorporating our proposed two degrees-of-freedom anti-windup controller.

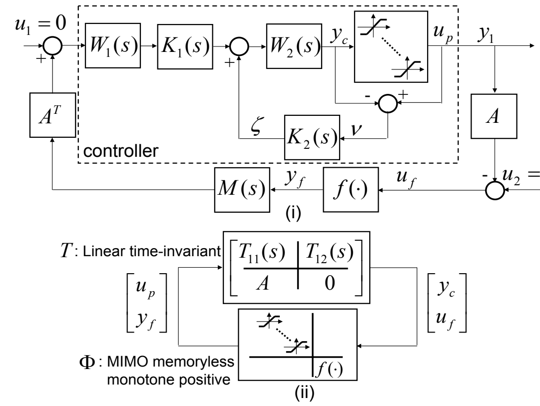

Fig. 4. (i): Block-diagram decomposition of our passivity-based power allocation solution incorporating an anti-windup controller. Exogenous inputs u_1 and u_2 are set to 0 and b respectively, where the choice of b is as per the base station administrator. The feedback nonlinearity $f(\cdot)$ is repeated monotone single-input single-output (SISO). We synthesize a dynamic feedback nonlinearity at the base station by multiplying the output of $f(\cdot)$ with a multiplier from the class $\mathcal{M}^{RS}$. (ii): We build on the class $\mathcal{M}^{RS}$ of multipliers to establish the input-output stability of the shown feedback interconnection, which is a recast form of the system shown in (i).

where $f(Ap) \doteq [\psi_1(\text{row}_1(Ap)) \ldots \ \psi_K(\text{row}_K(Ap))]^T$, and the user-specific $\psi_i(\zeta)$ is memoryless monotone continuous and zero-valued if its argument is negative-valued. Let us refer to this feedback system as $\mathcal{S}_1$. A block diagram representation of $\mathcal{S}_1$ is shown in Fig. 2(i). The question of interest to [2] and [1] is as follows: how should a user choose K and f so that $\mathcal{S}_1$ is input-output stable? In brief, the passivity based solution to the input-output stability problem, as derived by [2], is as follows. Let $\mathcal{S}_2$ denote the system obtained from $\mathcal{S}_1$ by inserting a Zames-Falb multiplier at the output of $f(\cdot)$. Then, the passivity theorem readily establishes the following result.

Theorem 1: Consider $\mathcal{S}_2$ with $M \in \mathcal{M}_{ZF}$. Then, $\mathcal{S}_2$ is input-output stable at the Nash equilibrium p^*. □

An unresolved technicality is that an upper bound on p_{max}, required for the validity of Lemma 2, is not specified in the literature thus far. In practice as well, an upper bound p_{max} is invariably set on the instantaneous transmit power either by the cell phone provider or by the cell phone user. As a result, the output u_p of the plant G is subject to a saturation nonlinearity (see Fig. 3 (i)), and is therefore liable to wind-up. The connection between saturation and wind-up is well discussed in [10], [11], [12], and [13]. We now describe a method to synthesize an anti-windup controller, shown in Fig. 3(ii), to allocate the transmit powers across the network so that the input-output stability of the cellular network is maintained in the face of saturation constraints on the mobile transmit power.

III. Stabilizing Anti-Windup Controller

We now present a method to synthesize a stabilizing anti-windup controller. The structure of a two degrees-of-freedom anti-windup controller, as considered in [14], [15], and [16], is shown in Fig. 3(ii). We shall consider a special case thereof, as shown in Fig. 4(i). The objective behind including the pre-filter $W_1(s)$ is to ensure that the transfer function from the reference signal u_1 to the controlled output y_1 matches the desired model, if any, exactly at the steady-state. Output of the controller implemented at every mobile node is subject to the saturation nonlinearity sat$(\cdot)$ as follows. Let $(y_c)_i$ and $(u_p)_i$ denote the input and the output, respectively, for the i-th controller saturation nonlinearity. Then,

$$(u_p)_i \doteq \begin{cases} p_{min} & \text{if } (y_c)_i < p_{min}, \\ (y_c)_i & \text{if } p_{min} < (y_c)_i < p_{max}, \\ p_{max} & \text{else.} \end{cases}$$

Pulling out this multi-input multi-output (MIMO) saturation nonlinearity and the base station nonlinearity $f(\cdot)$, the system shown in Fig. 4(i) can be recast as the feedback interconnection, shown in Fig. 4(ii), of the linear time-invariant $T(s)$ and the nonlinearity $\Phi \doteq \text{diag}(\text{sat}(\cdot), f(\cdot))$, where

$$\begin{aligned} T(s) &\doteq \begin{bmatrix} T_{11}(s) & T_{12}(s) \\ A & 0 \end{bmatrix}, \\ T_{11}(s) &\doteq -\Lambda(s) K_2(s), \\ T_{12}(s) &\doteq -\Lambda(s) K_1(s) W_1(s) A^T M(s), \\ \Lambda(s) &\doteq W_2(s) \left[I + K_2(s) W_2(s) \right]^{-1}. \end{aligned}$$

Following [6], stability of such a feedback system can be ascertained using the composite integral quadratic constraint (IQC) for Φ. Let us now introduce the notion of a repeated monotone nonlinearity.

Definition 3: `[repeated SISO monotone]` The class of *repeated SISO monotone* nonlinearities is the subclass $\mathcal{N}^{RS}$ of $\mathcal{N}$ with element $N \in \mathcal{N}^{RS}$ of the form

$$N(\zeta) \doteq [\phi(\zeta_1) \ \phi(\zeta_2) \ \ldots \phi(\zeta_p)]^T \quad \forall \zeta \in \mathbb{R}^p \tag{11}$$

where $\phi \in \mathcal{N}$, ϕ SISO. A shorthand notation for (11) is $N = \text{diag}(\phi)$. The class $\mathcal{N}^{RS}_{odd}$ is defined by replacing $\mathcal{N}$ in the definition of $\mathcal{N}^{RS}$ by $\mathcal{N}_{odd}$. □

Observe that the MIMO saturation nonlinearity belongs to $\mathcal{N}^{RS}$, as does the base station nonlinearity f. We shall now note down background results needed to derive the required composite IQC.

Definition 4: `[sector]`
We say that H is a sector $[k_1, k_2]$ operator if it holds that $\langle Hx - k_1 x, Hx - k_2 x\rangle \geq 0$ for all finite energy x. □

Definition 5: `[similarly ordered, unbiased]`
The sequences $\{x\}$ and $\{y\}$ of real scalars are said to be *similarly ordered* if $x(k) < x(l)$ implies $y(k) \leq y(l)$ for all $k, l \in \mathbb{Z}$. They are said to be *unbiased* if $x(k)y(k) \geq 0\ \forall k$. They are said to be *similarly ordered and symmetric* if they are unbiased and, in addition, the sequences $\{|x|\}$ and $\{|y|\}$ are similarly ordered. □

Definition 6: `[associated matrix, kernel]`
Given a bounded possibly time varying linear operator $M : \ell_2^p \to \ell_2^p$, $z = My$ is given as

$$z(k) \doteq \sum_{l=-\infty}^{\infty} \overline{m}_{k,l}\ y(l) \quad \forall k \in \mathbb{Z},$$

where $\overline{m}_{k,l} \in \mathbb{R}^{p\times p} \quad \forall k, l$; the *associated matrix* $\widetilde{M}$ of M is defined as

$$\widetilde{M} \doteq \begin{bmatrix} \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots \\ \ddots & \overline{m}_{-1,-1} & \overline{m}_{-1,0} & \overline{m}_{-1,1} & \overline{m}_{-1,2} & \ddots & \ddots \\ \ddots & \overline{m}_{0,-1} & \overline{m}_{0,0} & \overline{m}_{0,1} & \overline{m}_{0,2} & \ddots & \ddots \\ \ddots & \overline{m}_{1,-1} & \overline{m}_{1,0} & \overline{m}_{1,1} & \overline{m}_{1,2} & \overline{m}_{1,3} & \ddots \\ \ddots & \overline{m}_{2,-1} & \overline{m}_{2,0} & \overline{m}_{2,1} & \overline{m}_{2,2} & \overline{m}_{2,3} & \ddots \\ \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots \end{bmatrix}$$

The symbol m_{ij}, $i, j \in \mathbb{Z}$ denotes the (i, j)-th scalar element of the matrix $\widetilde{M}$; for example, m_{00} denotes the upper left entry in the $p \times p$ matrix $\overline{m}_{0,0}$ and $m_{-p,0}$ denotes the upper left entry in the $p \times p$ matrix $\overline{m}_{-1,0}$. If $\overline{m}_{k,l} = \overline{m}_{k+n,l+n}\ \forall k, l, n \in \mathbb{Z}$ then $\widetilde{M}$ is said to be *block Toeplitz* and M is said to be a *time invariant operator* or, alternatively, a *convolution operator*. For a bounded possibly time varying continuous time linear operator $M : \mathcal{L}_2 \to \mathcal{L}_2$

$$z(t) = \int_{-\infty}^{\infty} \overline{m}(t,\tau) y(\tau)\ d\tau \quad \forall t \in \mathbb{R}.$$

the *kernel* $\overline{m}(t, \tau) \in \mathbb{R}^{p\times p}$ is the counterpart of $\overline{m}_{k,l}$. In the continuous time case, M is called a *time invariant operator* or, alternatively, a *convolution operator* if $\overline{m}(t, \tau) = \overline{m}(t + \nu, \tau + \nu)\ \forall t, \tau, \nu \in \mathbb{R}$. For a convolution operator M, a shorthand notation for $\overline{m}(t, \tau)$ and $\overline{m}_{i,j}$ is $\overline{m}(t - \tau)$ and $\overline{m}(i - j)$, respectively with $\overline{m}(t)$ and $\overline{m}(k)$ denoting the respective *impulse response*. □

Definition 7: `[hyperdominance, dominance]`
An operator $M : \ell_2 \to \ell_2$ is said to be *doubly dominant* if the elements m_{ij} of its associated matrix have the following properties.

$$m_{ii} \geq \sum_{j=-\infty, j\neq i}^{\infty} |m_{ij}|, \quad m_{ii} \geq \sum_{j=-\infty, j\neq i}^{\infty} |m_{ji}| \quad \forall i$$

If, in addition, it also holds that

$$m_{ij} \leq 0,\ \forall i \neq j$$

then M said to be *doubly hyperdominant*. For an operator $M : \mathcal{L}_2 \to \mathcal{L}_2$, these notions are defined in terms of its kernel in an analogous manner with integrals suitably replacing sums. □

Definition 8: `[multipliers]`
$\mathcal{M}^{RS}_{odd}$ denotes the class of MIMO convolution operators, either continuous or discrete, such that the impulse response of an $M \in \mathcal{M}^{RS}_{odd}$ is of the form

$$m = g\ \delta - h \tag{12}$$

where $g, h(\cdot) \in \mathbb{R}^{p\times p}$ satisfy

$$g_{ii} \geq \sum_{i=1, i\neq j}^{n} |g_{ij}| + \sum_{i=1}^{n} \|h_{ij}\|_1 \qquad \forall i = 1, 2, \ldots, n \tag{13}$$

$$g_{ii} \geq \sum_{j=1, j\neq i}^{n} |g_{ij}| + \sum_{j=1}^{n} \|h_{ij}\|_1 \qquad \forall i = 1, 2, \ldots, n. \tag{14}$$

The subclass $\mathcal{M}^{RS}$ is obtained by further stipulating

$$g_{ij} \leq 0\ \forall i \neq j, \quad h_{ij}(\cdot) \geq 0\ \forall i, j. \tag{15}$$

Under the restriction

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$$g,\ h \text{ are Hermitian matrices,} \tag{16}$$

the subclass $\mathcal{M}^{D}$ ($\mathcal{M}^{D}_{odd}$) is derived from $\mathcal{M}^{RS}$ ($\mathcal{M}^{RS}_{odd}$). □

Lemma 3: `[5, Willems]`
Let $M : \ell_2 \to \ell_2$ be a bounded linear operator. Then, $\langle x, My\rangle$ is nonnegative for all similarly ordered unbiased (similarly ordered symmetric unbiased) sequences $\{x\}, \{y\} \in \ell_2$ if and only if M is doubly hyperdominant (doubly dominant). □

Theorem 2: `[17, Kulkarni-Safonov]`
A bounded linear operator M mapping ℓ_2^p into ℓ_2^p [or $\mathcal{L}_2$ into $\mathcal{L}_2$] preserves positivity of every $N \in \mathcal{N}^{RS}$ ($N \in \mathcal{N}^{RS}_{odd}$) if and only if its associated matrix [kernel] is doubly hyperdominant (doubly dominant). Furthermore, a bounded convolution operator M mapping $\mathcal{L}_2$ into $\mathcal{L}_2$, or mapping ℓ_2^p into ℓ_2^p, preserves positivity of every $N \in \mathcal{N}^{RS}$ ($N \in \mathcal{N}^{RS}_{odd}$) if and only if $M \in \mathcal{M}^{RS}$ ($M \in \mathcal{M}^{RS}_{odd}$). □

Now, note that the feedback nonlinearity $f(\cdot)$ in the system $\mathcal{S}_2$ is a $\mathcal{N}^{RS}$ nonlinearity. Then, it follows that Theorem 1 and Theorem 2 together prove the following result.

Theorem 3: Consider $\mathcal{S}_2$ with $M \in \mathcal{M}^{RS}$. Then, $\mathcal{S}_2$ is input-output stable at the Nash equilibrium p^*. □

Theorem 3 shows that the network can be input-output stable at the Nash equilibrium p^* even when we allow the inputs and the outputs of the dynamic nonlinearity at the

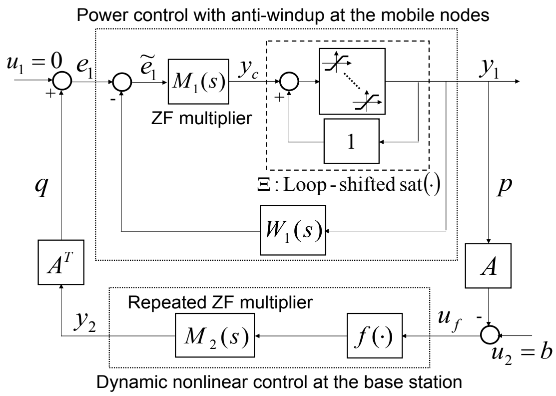

Fig. 5. Our proposed transmit power control scheme to ensure the input-output stability of a single cell network. A dynamic nonlinear controller is implemented at the base station. A dynamic nonlinear controller is implemented at each mobile. The nonlinearity at each mobile stems from the saturation limits encountered in practice. We assume that the mobiles have no access to each other's states and facilitate the largest possible class of the triplet (M_1, M_2, W_1) that ensures the input-output stability of the closed-loop system at the Nash equilibrium.

base station to be correlated provided that the multiplier M operating on the output of the repeated scalar nonlinearity $f(\cdot)$ belongs to the class $\mathcal{M}^{RS}$. Note that the multipliers in $\mathcal{M}^{RS}$ are full-block while the multipliers considered in Theorem 1 are repeated scalars. Theorem 3 furnishes the largest possible class of nonlinear dynamic controllers for the base station that ensures the input-output stability of the network at the Nash equilibrium.

Using these results, we shall now synthesize the required anti-windup control. A block diagram of the closed-loop system comprising the base station and the mobile users implementing our proposed solution is shown in Fig. 5 wherein we have applied the loop-shift transformation (see [4] and [7]) to transform a repeated sector $(0, 1]$ saturation nonlinearity into a repeated sector $(0, \infty]$ nonlinearity Ξ. Let us denote the class of such systems as $\mathcal{S}_3$. The task on-hand is to choose (M_1, M_2, W_1) so that the closed-loop system is input-output stable. We assume that the mobile users do not have access to each other's states and yet need to implement a decentralized transmit power controller. Therefore, M_1 and W_1 must be diagonal. Then, the passivity theorem yields the required result as follows.

Theorem 4: Consider the class $\mathcal{S}_4$ derived from $\mathcal{S}_3$ by constraining M_1, M_2, and W_1 as follows. M_1 is a diagonal Zames-Falb multiplier, W_1 is a diagonal passive linear time-invariant operator, and $M_2 \in \mathcal{M}^{RS}$. Then, $\mathcal{S}_4$ is input-output stable. $\square$

Proof: We will prove that the mapping from y_2 to Ap is strictly passive. Then, the passivity theorem (see [4, Chapter 6]) can readily be used to prove the input-output stability after observing that Theorem 3 has established that the mapping from u_f to y_2 is strictly passive. Note that $\Xi \in \mathcal{N}^{RS}$. By Theorem 2, a diagonal Zames-Falb multiplier preserves its positivity. Therefore, $M_1\Xi$ is strictly passive. By assumption, W_1 is passive. Therefore, by the passivity theorem, the mapping from $\widetilde{e}_1$ to y_1 is strictly passive. Therefore, since A is a constant matrix, the mapping from y_2 to Ap is passive. Note that $f \in \mathcal{N}^{RS}$ with the graph of the repeating scalar nonlinearity confined to sector $(0, \infty]$. Hence, by Theorem 3, the mapping from u_f to y_2 is strictly passive. Hence, by the passivity theorem, $\mathcal{S}_4$ is input-output stable. QED. $\square$

IV. Discussion

We have followed the Zames-Falb multiplier based approach to synthesize the anti-windup mechanism as opposed to, say, the piecewise quadratic Lyapunov function based approach adopted in [14] because the regions over which the underlying system is affine grows exponentially with the number of mobile users. It is known that the Zames-Falb multipliers do not preserve the incremental positivity of memoryless monotone nonlinearities (see [18]). This technicality implies that if the set points (such as b, p_{min}, and p_{max}) are varied arbitrarily, the deviations in y_1 and y_2 are no longer guaranteed to be small in the systems belonging to the set $\{\mathcal{S}_2, \mathcal{S}_3, \mathcal{S}_4\}$. However, if the set-point varies only by a constant term, the Zames-Falb multipliers do indeed preserve the incremental positivity of memoryless monotone nonlinearities (see [18]). As a result, the condition on p_{max} in Lemma 2 does not have serious consequences on the input-output stability provided that p_{max} is a constant. A number of interesting problems arise if the matrix A is time-varying, and are the focus of our current research.

V. Conclusion

We have built on the passivity-based techniques proposed by [1] and [2] to control the transmit power of mobile nodes in single cell CDMA networks. We have proposed a solution to ensure the input-output stability of the network in the face of transmit power constraints. Our solution uses the Zames-Falb multipliers (see [8], [9], [17]) to ensure passivity of the subsystems at the base station and the mobiles. Then, a straightforward application of the passivity theorem realizes a class of dynamic nonlinear controllers that guarantees the required input-output stability.

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