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Transient Responses of Alternative Vehicle Configurations: A Theoretical and Experimental Study on the Effects of Atypical Moments of Inertia

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Abstract— **With the rise of efficient powertrain technologies, future vehicles may exhibit a large variance in mass distribution and inertial parameters. This paper asserts the importance of understanding the effects of atypical inertial properties on vehicle handling and driver experience. Atypically low moments of inertia result in changes to the driver's perception of the lateral acceleration, and the transient development of rear tire force. Both effects can be understood conceptually through an analysis of the center of rotation, and are illustrated experimentally in a comparison of data from two vehicles: a production vehicle with typical mass and inertia properties, and a by-wire electric research vehicle with significantly different properties. The information provided in this paper can be used in the future as a new design tool when considering vehicle mass properties.**

I. INTRODUCTION

The rise in energy conservation efforts has caused the design of efficient vehicles to become progressively necessary. Efficiency may come in forms that modify typical vehicle configurations; for example, the addition of electric motors and battery packs to replace or supplement an internal combustion engine. These redesigned vehicle configurations have the potential to greatly affect the distribution of mass and inertia in the vehicle. This highlights a challenge in modern vehicle dynamics: understanding and predicting the impact of alternative inertial configurations on vehicle handling properties.

Vehicles today fall within a wide range of shapes and sizes, and exhibit a variety of handling characteristics. However, the ratios between mass and inertia properties for most conventional passenger vehicles are remarkably similar. This fact has been noted by several studies, including Brennan's study of non-dimensional vehicle dynamics [3], and Allen's investigation of inertial properties [1], which uses the NHTSA inertial parameter database [5].

Tomorrow's passenger vehicles, however, may be notably different from today's. Already, research and concept vehicles that illustrate new technologies, such as fuel cells and electric drivetrains, have atypical mass and inertia properties. For example, hub-mounted electric motors place the drivetrain mass outboard on the vehicle; center-mounted battery packs place mass near the center of the car; "skateboard" chassis designs, similar to the GM Hywire concept [2], place the entire drivetrain into a low, flat, and wide space.

The purpose of this paper is to determine the effects of atypical moments of inertia on vehicle handling and driver experience. Drivers expect a car that is fun to drive, a car with "on rails" feel – a fast, first order response. This paper shows that vehicles with conventional moments of inertia exhibit a similar dynamic response at all points along the vehicle centerline, while vehicles with atypical inertial parameters may exhibit up to three different dynamic responses at points along the vehicle centerline. These responses significantly impact the driver's perception of the dynamics, as well as the transitive behavior of the rear tire forces.

Fig. 1. Stanford's P1 Steer-by-wire vehicle

This paper clarifies the ways in which different mass and inertia properties impact the transient response of a vehicle to steering commands. The paper opens by presenting a 3DOF vehicle model along with a tire model in Section II. It then provides a method for normalizing mass and inertia properties and discusses the ranges typical of production vehicles in Section III. These concepts are used in Section IV to develop the idea of center of rotation and discuss how it characterizes the effects of mass and inertia properties. Section IV-A details the effects of center of rotation on driver experience, and Section IV-B shows how center of rotation influences rear tire dynamics. Finally, experimental data is provided in Section V from a 1999 Mercedes E320, which has mass and inertia properties representative of many production vehicles, and P1 (Figure 1), an electric by-wire research vehicle built at Stanford with atypical inertia properties.

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II. VEHICLE HANDLING MODEL

A. 3DOF Vehicle Model

The formulation of this paper's 3DOF vehicle model is similar to others in literature (Laws [7], Kim [6], and Cameron [4]) and makes the following assumptions:

- constant longitudinal vehicle speed
- negligible effects from longitudinal tire forces
- negligible effects from track width
- negligible products of inertia
- unsprung body mass center and sprung body roll center are coincident
- sprung and unsprung body mass centers, at zero roll, are coincident in the $x-y$ plane and located a distance h apart in the $z-$direction
- equations of motion are linearized

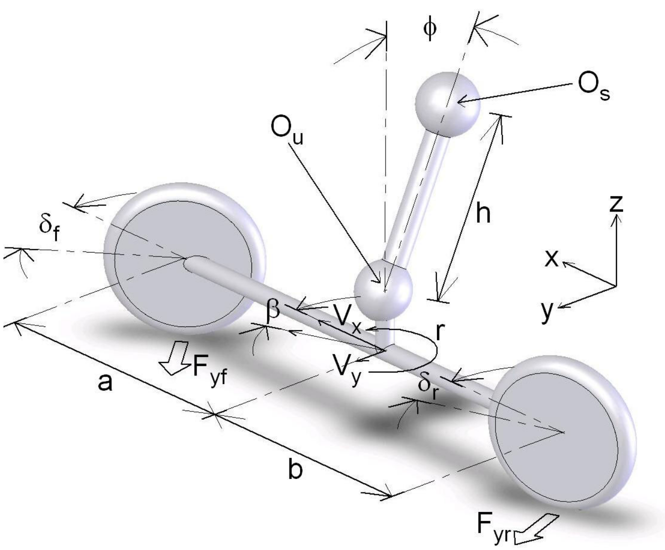

Fig. 2. Diagram of 3DOF Vehicle Model

A diagram of the model is given in Figure 2. The 3DOF model has separate sprung and unsprung bodies whose mass centers are O_S and O_U, respectively. The unsprung body is constrained to $x-y$ planar motion with longitudinal velocity V_x, lateral velocity V_y, sideslip angle β, and yaw rate r. The external lateral tire forces F_{yf} and F_{yr} act directly on the unsprung body. The relative motion of the sprung body to the unsprung body is constrained to rotation about the x-axis by the roll angle ϕ. Roll stiffness (K_ϕ) and damping (b_ϕ) torques act between the two bodies. The front and rear steer angles are δ_f and δ_r. The coordinate system follows the ISO 8855 standard. Lagrange's equations or Kane's method generates a set of three nonlinear equations. The linearization of these equations is given by:

$$\begin{bmatrix} m & 0 & -m_s h \\ 0 & I_z & 0 \\ -m_s h & 0 & I_x + m_s h^2 \end{bmatrix} \begin{bmatrix} V_x\dot{\beta} \\ \dot{r} \\ \ddot{\phi} \end{bmatrix} + \ldots$$

$$\begin{bmatrix} 0 & mV_x & 0 \\ 0 & 0 & 0 \\ 0 & -m_s h V_x & b_\phi \end{bmatrix} \begin{bmatrix} V_x\beta \\ r \\ \dot{\phi} \end{bmatrix} + \ldots$$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & K_\phi - m_s g h \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ \phi \end{bmatrix} =$$

$$\begin{bmatrix} 1 & 1 \\ a & -b \\ 0 & 0 \end{bmatrix} \begin{bmatrix} F_{yf} \\ F_{yr} \end{bmatrix}$$

It is important to note that the parameter h is not the mass center height of the vehicle, but rather the vertical distance between the sprung body mass center and the roll center. Measurements of total vehicle mass center heights, such as those in the NHTSA database [5], require that the roll center heights be subtracted from them before being used as the parameter h. The unsprung and sprung bodies each have their own masses and inertias. During equation derivation and subsequent linearization, the yaw moments of inertia are lumped together into that of the whole vehicle (I_z), and the unsprung roll moment of inertia is neglected. Total vehicle inertial measurements, such as those in [5], need to be altered when used as the sprung body roll moment of inertia (I_x).

The front and rear slip angles (α_f and α_r, respectively) are given by:

$$\alpha_f = \beta + \frac{a}{V_x} r - \delta_f \quad (1)$$

$$\alpha_r = \beta - \frac{b}{V_x} r \quad (2)$$

B. Lateral Tire Force Model

In addition to the vehicle model given in Section II-A, a tire model is necessary to generate dynamic responses. This paper will consider the effects of a linear tire model with and without relaxation lengths.

First consider a simple, linear tire model, given by the following equations:

$$F_{yf} = -C_{\alpha f}\alpha_f \quad (3)$$

$$F_{yr} = -C_{\alpha r}\alpha_r \quad (4)$$

where $C_{\alpha f}$ and $C_{\alpha r}$ are the front and rear axle cornering stiffnesses, respectively.

This simple tire model can be refined with the addition of relaxation length:

$$
\begin{aligned}
F_{yf} &= -C_{\alpha f}\bar{\alpha}_f && (5)\\
F_{yr} &= -C_{\alpha r}\bar{\alpha}_r && (6)\\
\dot{\bar{\alpha}}_f &= \frac{V_x}{\sigma_f}\left(\alpha_f - \bar{\alpha}_f\right) && (7)\\
\dot{\bar{\alpha}}_r &= \frac{V_x}{\sigma_r}\left(\alpha_r - \bar{\alpha}_r\right) && (8)
\end{aligned}
$$

where the effective front and rear slip angles $\bar{\alpha}_f$ and $\bar{\alpha}_r$ have a velocity dependent, first-order lag determined by the front and rear relaxation lengths σ_f and σ_r.

III. INERTIA EFFECTS

Passenger vehicles today range from sedans to trucks and vans, and all of these cars are designed to exhibit very different handling characteristics. However, most conventional passenger vehicles, as denoted in the NHTSA inertial database [5] have very similar ratios between their mass and inertial properties. A comparison of these ratios is possible through inertia normalization. The roll (I_x) and yaw (I_z) moments of inertia can be normalized as follows:

$$
\begin{aligned}
\bar{I}_x &= \frac{I_x}{m_s h^2} && (9)\\
\bar{I}_z &= \frac{I_z}{mab} && (10)
\end{aligned}
$$

where $\bar{I}_x$ and $\bar{I}_z$ are the normalized moments of inertia in roll and yaw, respectively. These normalized inertias are remarkably similar for most current passenger vehicles. Consider the inertial parameters for cars and trucks given in the NHTSA database [5] as an example. Since the database does not give h or m_s, $\bar{I}_x$ can be approximated as follows:

$$\bar{I}_{x,alt} = \frac{I_x}{mH^2} \tag{11}$$

where m is the complete vehicle mass and H is the CG height. Using $\bar{I}_{x,alt}$ and $\bar{I}_z$, over 90% of the normalized inertias from the database fall within the following ranges:

$$
\begin{aligned}
0.70 < &\ \bar{I}_{x,alt} < 1.33 && (12)\\
0.86 < &\ \bar{I}_z < 1.18 && (13)
\end{aligned}
$$

Because $m > m_s$ and $H > h$ (with the rare exception of vehicles exhibiting a roll center below the ground), $\bar{I}_{x,alt} < \bar{I}_x$. For typical passenger cars, $\bar{I}_x$ is approximately 1.5 to 2 times $\bar{I}_{x,alt}$.

A. Direct Effect of Inertia

There are a growing number of technologies that may broaden the narrow parameter space of conventional vehicles. For example, a vehicle that uses hub-mounted electric motors may have a notably higher $\bar{I}_x$ and $\bar{I}_z$, while a vehicle that places a heavy battery pack near the center of the car may have a notably lower $\bar{I}_z$.

One can imagine that the most direct effect of changing the moments of inertia of the vehicle will be related to response time. For example, a vehicle with a very high yaw moment of inertia will have a slower yaw rate response than a vehicle with a very low moment of inertia. This suggests that the effect of lowering the moments of inertia will result in a vehicle that is more responsive in handling, and potentially more enjoyable for the driver. While this simple analysis suggests that a lower moment of inertia is preferable, there is more to consider when taking a closer look at the dynamics of the vehicle.

IV. CENTER OF ROTATION

One metric that quantifies the effects of varying inertia on the transient response is the location of the vehicle's center of rotation at the instant a front lateral tire force F_{yf} is applied. This is the point on the vehicle at which the initial change in lateral velocity is zero for a step steer input. Using this relationship, one can derive the location of the center of rotation at a distance c behind the CG along the vehicle centerline.

Maurice Olley first used the concept of center of rotation in the 1930's [8]. Assuming a 2DOF bicycle model, Olley estimated the center of rotation to be a distance $c^\star$ behind the CG:

$$c^\star = \bar{I}_z b \tag{14}$$

As this paper is interested in the effects of both $\bar{I}_x$ and $\bar{I}_z$, the effects of roll must be included. The inclusion of roll effects allows for a more precise estimate of the center of rotation than does Olley's 2DOF equation. Using the 3DOF vehicle model given in Section II-A, the location of the center of rotation is found at a distance c behind the CG:

$$c = \bar{I}_z \left(1 + \frac{1}{\bar{I}_x}\right) b \tag{15}$$

Referring again to the inertial parameters of passenger vehicles given in the NHTSA database [5], the following ranges for $c^\star$ and c hold for over 90% of the tested vehicles (approximating $\bar{I}_x$ as $1.67\bar{I}_{x,alt}$):

$$
\begin{aligned}
0.86b < &\ c^\star < 1.18b && (16)\\
1.39b < &\ c < 1.96b && (17)
\end{aligned}
$$

The value of c determines the step response of the vehicle at various points along its centerline, divided into three sections: at the center of rotation, before the center of rotation, and behind the center of rotation. At the center of rotation, there is initially zero change in lateral velocity at the start of a step steer, so the lateral velocity and sideslip response at the center of rotation will look similar to a first order response. Ahead of the center of rotation, the change in lateral velocity will initially be positive for a positive step steer, generating an initially positive sideslip angle for any points ahead of the center of rotation. However, the steady state tire forces must be positive, and the steady state sideslip angles must be negative; therefore, the sideslip response for points ahead of the center of rotation will be non-minimum phase. Lastly, for points on the car behind the center of rotation, the change in lateral velocity will initially be negative for a positive step steer, generating initially negative sideslip angles for points behind the center of rotation. This will lead to a minimum-phase, possibly oscillatory response. This can be derived equivalently using the transfer function from steer angle to yaw rate; however, the center of rotation is a more intuitive description.

Figure 3 and Figure 4 show the $\dot{V}_y$ gradient at the start of a step steer for vehicles with centers of rotation behind and ahead of the rear axle. Note that for vehicles with $c > b$, there is only one dynamic response for all points along the vehicle centerline, whereas for vehicles with $c < b$, there are three different dynamic responses for points along the vehicle centerline. It is important to note that for any point on the vehicle centerline, the steady state lateral velocity will always be the same for a given maneuver; therefore, the placement of the center of rotation affects only the transients.

All vehicles in the NHTSA inertial database fall within the first case ($c > b$) when the center of rotation calculation includes roll. This illustrates a commonality in conventional automotive design that is not evident using Olley's simplifications.

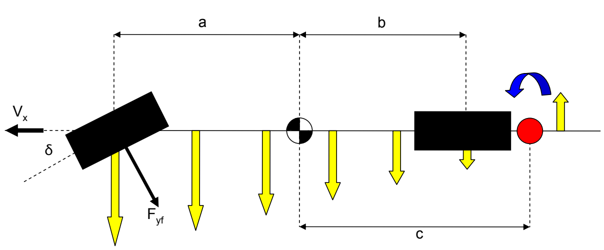

Fig. 3. $\dot{V}_y$ gradient at the start of a step steer for $c > b$

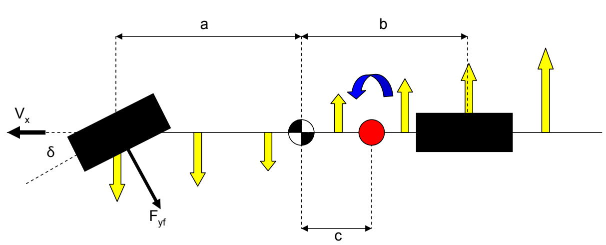

Fig. 4. $\dot{V}_y$ gradient at the start of a step steer for $c < b$

A. *Effect of Center of Rotation on Driver Perception*

Because the center of rotation marks the point at which the dynamics along the centerline of the vehicle change, it is important to examine how the position of the center of rotation will effect the driver's perception of the vehicle's handling. Figure 5 shows how center of rotation effects the lateral acceleration experienced by the driver in a step steer maneuver.

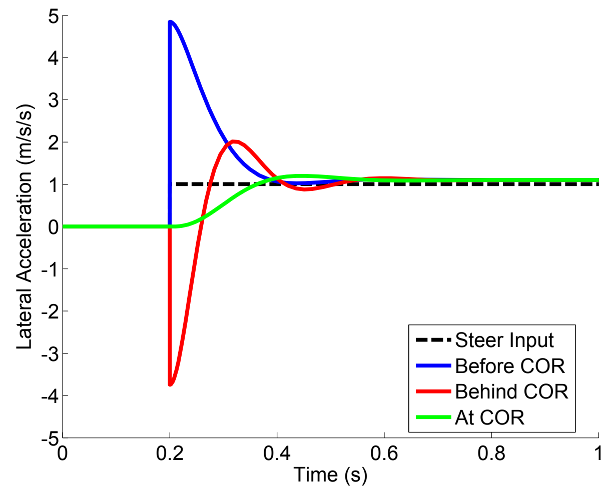

Fig. 5. Lateral acceleration before/at/behind the center of rotation

While sitting ahead of the center of rotation, as is the case in a typical passenger vehicle, where $c > b$, the driver will feel like he is being pushed hard into a turn. If the driver is sitting at the center of rotation, as in a low-inertia vehicle where $c < b$, he will feel as though he is easing into the turn due to a first-order type of response. Lastly, if he is sitting behind the center of rotation, the high initial negative lateral acceleration will cause him to feel as though he is initially pushed out of the turn, and the response can even become oscillatory. Ideally, the driver (and passengers) would sit near the center of rotation so as to experience a smoother lateral acceleration response, which is only possible in atypical, low-inertia vehicles.

B. *Effect of Center of Rotation on Rear Tire Dynamics*

Because center of rotation affects the sideslip dynamics of points along the centerline of the vehicle, it will also affect the kinematically-related tire slip angles. As with sideslip, the slip angle response for tires ahead the center of rotation is non-minimum phase, whereas the slip angle response for tires behind the center of rotation is minimum-phase and oscillatory.

In order to see the effects of the center of rotation as inertia is varied, this section shows the time-domain response of rear tire slip angle α_r to a step steer input δ. Figure 6 provides simulated responses of α_r to a step steer maneuver, using the 3DOF model presented in Section II-A and typical vehicle parameters. The response is plotted for various values of c, generated by changing $\bar{I}_z$, at a vehicle speed of 15 m/s.

For values of c typical of most passenger cars, where $c > b$, the overshoot is minimal or even zero, and the non-minimum phase characteristic is present. However, as c decreases, the overshoot and oscillation increase notably, and the non-minimum phase characteristic disappears (these results are similar to those for acceleration in Section IV-A). This suggests that as moments of inertia decrease to low values, the rear tire dynamics will be drastically different

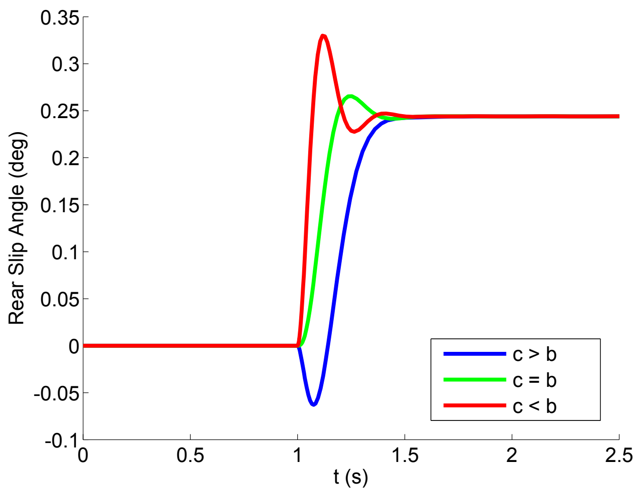

Fig. 6. Simulated α_r response to a front step steer maneuver for various c

TABLE I
VEHICLE PARAMETERS

Parameter		*P1*	*E320*	
Mass	m	1725	1850	kg
Sprung mass	m_s	1600	1570	kg
Normalized roll inertia	$\bar{I}_x$	0.78	1.57	
Modified normalized roll inertia	$\bar{I}_{x,alt}$	0.47	1.03	
Normalized yaw inertia	$\bar{I}_z$	0.41	1.13	
Center of rotation	$^c/_b$	0.93	1.85	
Roll stiffness	K_ϕ	100	90	kNm/rad
Roll damping	b_ϕ	3.50	3.50	kNms/rad
Wheelbase	L	2.50	2.83	m
Distance from CG to roll center	h	0.40	0.45	m
Distance from CG to front axle	a	1.35	1.37	m
Distance from CG to rear axle	b	1.15	1.46	m
Front cornering stiffness	$C_{\alpha f}$	99	162	kN/rad
Rear cornering stiffness	$C_{\alpha r}$	170	244	kN/rad
Front relaxation length	σ_f	0.35	0.40	m
Rear relaxation length	σ_r	0.50	0.70	m

from those of a typical vehicle. Because the front tires are always ahead of the center of rotation for a front-steering vehicle, the dynamics of the front slip angle do not change as drastically with varying inertias.

V. EXPERIMENTAL RESULTS

In order to validate the simulated results, this paper presents transient responses from two vehicles. One is a Mercedes E320, which has a parameter set typical of many production cars. The other is P1, which has an electric drivetrain and heavy lead-acid batteries positioned near the CG of the car. The parameters for both cars are given in Table I, which shows that P1 has a center of rotation $^c/_b$ of 0.93 (ahead of the rear axle), whereas the Mercedes has a center of rotation of 1.85 (behind the rear axle).

The vehicles' responses to a fast ramp steer input show the effects of differing moments of inertia. These experimental data are compared with simulated responses, using the 3DOF model given in Section II. The experimental response of the E320 to a fast ramp steer is shown in Figure 7, and that of P1 is shown in Figure 8. Both tests were completed at a vehicle speed of approximately 10 m/s in the linear tire region. The yaw rate r, sideslip angle β, and rear slip angle α_r are shown for each vehicle. Because these maneuvers exhibit sharp transients at low speeds, it is necessary to include relaxation length to capture the response [7], as is evident in both figures where the vehicle response is plotted with and without relaxation length.

To directly show the effect of inertia, the response of each vehicle is also plotted as if it had the $\bar{I}_z$ of the opposite vehicle. Therefore, the Mercedes data is simulated again with a very low $\bar{I}_z$, and the P1 data is simulated again with a very high $\bar{I}_z$. This allows for the direct comparison of the dynamics of each maneuver as $\bar{I}_z$ is varied.

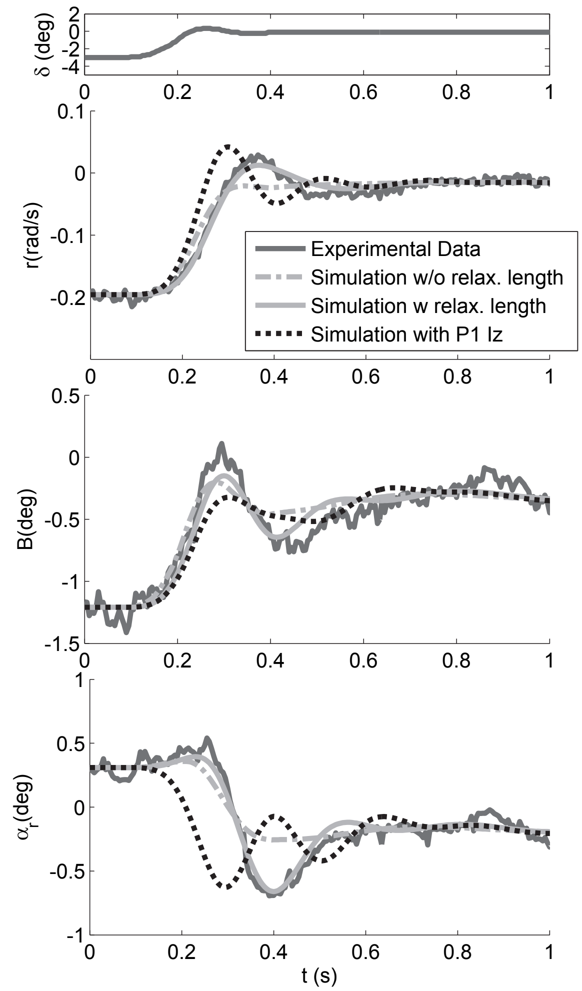

Fig. 7. Experimental response of the E320 to a fast ramp steer maneuver: a) steering input, b) yaw rate, c) sideslip, d) rear slip angle

As can be seen, the general dynamics of each vehicle are closely captured by the 3DOF model from Section II-A. Because the Mercedes' center of rotation is behind the rear axle ($c > b$), both the β and α_r responses are non-minimum phase, with little overshoot. From the plot of the Mercedes with P1's $\bar{I}_z$, it is easily seen that a Mercedes with low moments of inertia ($c < b$) would be more oscillatory with a faster, minimum phase response in r and α_r.

Conversely, P1's response confirms the theoretical results for vehicles with small moments of inertia. As expected, because the center of rotation is just ahead of the rear axle ($c < b$), the r and α_r responses are minimum phase, fast, and slightly oscillatory. Although not obvious from the given maneuver, the β response is still non-minimum phase because the center of gravity is ahead of c. Looking at the simulation of P1 with the Mercedes' $\bar{I}_z$, a P1 with high moments of inertia would exhibit a slower r response and a non-minimum phase α_r response.

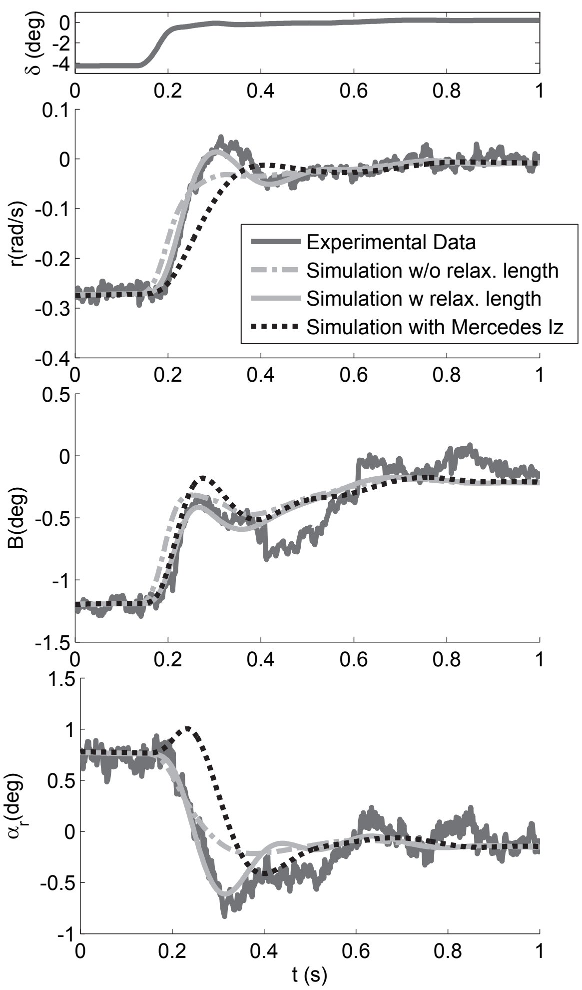

Fig. 8. Experimental response of P1 to a fast ramp steer maneuver: a) steering input, b) yaw rate, c) sideslip, d) rear slip angle

VI. CONCLUSION

As more cars with atypical values of mass and moment of inertia are designed, the importance of understanding the effects of these parameters on the dynamic response of the vehicle will increase. The mass distribution of the car can become a design parameter to identify the speed of response as well as the character of the response. By placing the center of rotation ahead of the rear axle, the vehicle can have a quick response, due to low inertias. As the center of rotation is brought closer to the driver, the lateral acceleration response may become more intuitive. One downside to bringing the center of rotation ahead of the rear wheels is the challenge of an oscillatory rear slip angle response. This challenge must be met - through automatic control or through careful design of the tire and vehicle dynamics - to realize the benefits of reduced inertia.

VII. ACKNOWLEDGMENTS

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