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A Lyapunov Based Multi-level Controller for a Semi-active Suspension System with an MRF Damper

Feng Tyan, Shun-Hsu Tu and Wes S. Jeng

***Abstract*— In this work, we study the implementation of magnetorheological fluid (MRF) to the semi-active suspension. Owing to the nonlinear hysteretic phenomenon, the analysis and synthesis of a controller is not trivial. The kinematic energy and spring potential function of the suspension system plus an integral term of the hysteretic component of an MR damper is chosen as the Lyaupnov function to verify the stability and dissipativity of the system. Then a multi-level controller, which is constructed in virtue of stability analysis, turns out to be effective in vibration suppression. In addition, the controller algorithm is simple and easy to implement, requires only the measurements of relative displacement and velocity between sprung and unsprung masses, and the damping force of the MR damper.**

keywords: **semi-active controller, quarter vehicle suspension system, MR damper, Lyapunov function, dissipativity**

I. INTRODUCTION

For years, vibration attenuation of various dynamic systems has received broad attentions from both academic and industry. In the automobile industry the perceived comfort level and ride stability of a vehicle are two of the most important factors in a subjective evaluation of a vehicle. There are many aspects of a vehicle that influence these two properties, the most important ones of which are the primary suspension components, which isolate the frame of the vehicle from the axle and wheel assemblies.

If a primary suspension is designed to optimize the handling and stability of the vehicle, the operator often perceives the ride to be rough and uncomfortable. On the other hand, if the suspension is designed for ride comfort alone, the vehicle may not be stable during maneuvers. As such, the performance of primary suspensions is always defined by the compromise between ride and handling.

The concept of semi-active suspension and semi-active vibration control in connection with the power consumed was introduced by Karnopp [1]. A semi-active suspension consists of a spring and a damper but, unlike a passive suspension, the value of the damper coefficient "c" can be controlled and updated. Various semi-active devices have been proposed to dissipate vibration energy in a structural or vehicle suspension system (see [2] and the references therein). The magneto-rheological (MR) dampers are new devices that use MR fluid to alter the damping coefficient. These fluids demonstrate dramatic changes in their rheological behaviors in response to a magnetic field.

To control the MR dampers, various control strategies have been proposed, but most of them are either complicated or no direct stability analysis is provided, for example, sliding mode control [3, 4] (needs reference model), H_∞ control [5] (needs inverse model of MR damper), clipped-optimal control [6] (on-off type, needs another desired control force hence no direct stability analysis). While the proposed controller has the feature of simplicity and it is obtained directly from the stability analysis of the closed-loop suspension system. In the mean time, this stability analysis also explains why we need the measurements of relative displacement and velocity between sprung and unsprung masses, and the damping force of MR damper.

This paper is organized as follows. At first we construct the quarter vehicle model with an MR damper utilizing a modified Bouc-Wen model in section II. Then the stability analysis is conducted in section III-A, after that a multi-level dissipative controller is proposed to suppress the vehicle vibration. Finally, a numerical example is used to demonstrate the effectiveness of the controller.

II. QUARTER VEHICLE MODEL WITH MR DAMPER

A. Quarter Vehicle Model

As the vertically oscillating behavior of a vehicle is considered we investigate response of vertical dynamics. The response can be mathematically described with a relatively simple set of dynamic equations known as a quarter-car simulation.The frequency response of the quarter car extends from approximately 0.5 to 20

Feng Tyan is with Faculty of Dept. of Aerospace Engineering, TamKang University, Taipei County 25147, Taiwan, R.O.C. tyanfeng@mail.tku.edu.tw

Shun-Hsu Tu and Wes S. Jeng are with Sky Leading Corporation, Chupei, Hsin Chu County, Taiwan 30204, R. O. C.

Hz with some emphasis on roughness at the body bounce frequency and the axle resonance frequency. The rationale favoring the quarter car is the fact that it covers the appropriate frequency range responsible for exciting vehicle vibrations and emphasizes those that excite modal resonances.

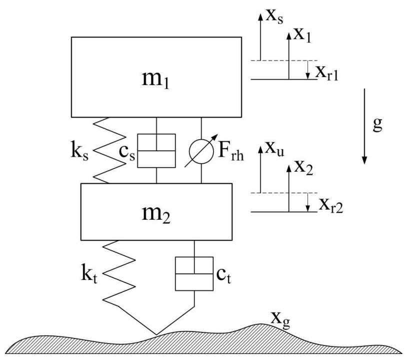

Fig. 1. Quarter Vehicle Suspension model

The equations of motion of the quarter-car model depicted in Fig. 1 can be written as

$$\begin{aligned} &M\begin{bmatrix}\ddot{x}_s\\ \ddot{x}_u\end{bmatrix}+C\begin{bmatrix}\dot{x}_s\\ \dot{x}_u\end{bmatrix}+K\begin{bmatrix}x_s\\ x_u\end{bmatrix} \\ &=\begin{bmatrix}-m_1\\ -m_2\end{bmatrix}g+\begin{bmatrix}0\\ k_t x_g+c_t\dot{x}_g\end{bmatrix}+\begin{bmatrix}-1\\ 1\end{bmatrix}F_{rh}, \end{aligned} \tag{1}$$

where the matrices

$$M\triangleq\begin{bmatrix}m_1 & 0\\ 0 & m_2\end{bmatrix}, C\triangleq\begin{bmatrix}c_s & -c_s\\ -c_s & c_s+c_t\end{bmatrix}, K\triangleq\begin{bmatrix}k_s & -k_s\\ -k_s & k_s+k_t\end{bmatrix},$$

and the quantities

m_1, m_2 are the masses of vehicle body and axle,
x_s, x_u denote vertical displacements of m_1 and m_2,
x_g is the road disturbance,
k_s, c_s represent the stiffness and damping of the uncontrolled suspension,
k_t, c_t denote the stiffness, damping of the tyre,

To remove the gravitational force from the equations of motion, let's define the shifted state variables

$$x=\begin{bmatrix}x_1\\ x_2\end{bmatrix}\triangleq\begin{bmatrix}x_s\\ x_u\end{bmatrix}-x_r, \tag{2}$$

where the the reference point, x_r, is the static equilibrium position

$$x_r=\begin{bmatrix}x_{r1}\\ x_{r2}\end{bmatrix}\triangleq K^{-1}\begin{bmatrix}-m_1\\ -m_2\end{bmatrix}g=\begin{bmatrix}-\frac{m_1+m_2}{k_t}-\frac{m_1}{k_s}\\ -\frac{m_1+m_2}{k_t}\end{bmatrix}g. \tag{3}$$

Then we have the equation of motion for x,

$$M\ddot{x}+C\dot{x}+Kx=\begin{bmatrix}-1\\ 1\end{bmatrix}F_{rh}+\begin{bmatrix}0\\ k_t x_g+c_t\dot{x}_g\end{bmatrix}. \tag{4}$$

For convenience, we further define the state vector

$$x_P\triangleq\begin{bmatrix}x_{P1}\\ x_{P2}\end{bmatrix}\triangleq\begin{bmatrix}x_1-x_2\\ x_2-x_g\end{bmatrix}, \text{ or } x_P=T_P x-\begin{bmatrix}0\\ x_g\end{bmatrix}, \tag{5}$$

where the transformation matrix

$$T_P\triangleq\begin{bmatrix}1 & -1\\ 0 & 1\end{bmatrix}.$$

Note that the inverse matrix $T_P^{-1}=\begin{bmatrix}1 & 1\\ 0 & 1\end{bmatrix}$. Then the equations of motion (4) can be re-formulated as

$$M_P\ddot{x}_P+C_P\dot{x}_P+K_P x_P=\begin{bmatrix}-1\\ 0\end{bmatrix}F_{rh}-\begin{bmatrix}m_1\\ m_1+m_2\end{bmatrix}\ddot{x}_g. \tag{6}$$

where the transformed matrices

$$M_P\triangleq T_P^{-\mathrm{T}}MT_P^{-1}=\begin{bmatrix}m_1 & m_1\\ m_1 & m_1+m_2\end{bmatrix},$$

$$C_P\triangleq T_P^{-\mathrm{T}}CT_P^{-1}=\begin{bmatrix}c_s & 0\\ 0 & c_t\end{bmatrix},$$

$$K_P\triangleq T_P^{-\mathrm{T}}KT_P^{-1}=\begin{bmatrix}k_s & 0\\ 0 & k_t\end{bmatrix}.$$

B. Modified Bouc-Wen MR Damper Model

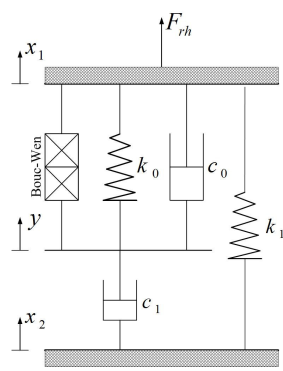

Fig. 2. Modified Bouc-Wen model

A modified Bouc-Wen model (see Fig. 2) for better predicting the response of the MR damper in the region of the yield point was proposed by Spencer [2] and is adoppted in this work. The equations for the force in this model are given by

$$\dot{x}_1-\dot{y}=\frac{1}{c_0+c_1}\left[-k_0(x_1-y)-\alpha z+c_1\dot{x}_{P1}\right], \tag{7}$$

$$\dot{z}=(\dot{x}_1-\dot{y})\left\{\delta-|z|^n\left[\beta+\gamma\mathrm{sgn}(\dot{x}_1-\dot{y})\mathrm{sgn}(z)\right]\right\}, \tag{8}$$

and the equation governing the force exerted by the MRF damper, F_{rh}, is

$$\begin{aligned} F_{rh}&=-c_1(\dot{x}_1-\dot{y})+c_1\dot{x}_{P1}+k_1(x_{P1}-\bar{x}_0), \\ &=\frac{c_1}{c_0+c_1}\left[k_0(x_1-y)+\alpha z\right]+\frac{c_0c_1}{c_0+c_1}\dot{x}_{P1} \\ &\quad+k_1(x_{P1}-\bar{x}_0), \end{aligned} \tag{9}$$

where

$x_1 - y$ and z are the internal relative displacement and hysteretic component of the MR damper, respectively,

δ, β, γ and n are positive constant parameters, and α is a scaling value for Bouc-Wen model,

k_0, k_1 are spring constants,

$\bar{x}_0$ corresponds to the initial displacement.

The voltage dependent parameters are modeled by

$$\alpha = \alpha_a + \alpha_b u,\ c_0 = c_{0a} + c_{0b} u,\ c_1 = c_{1a} + c_{1b} u,$$

where $\alpha_a, \alpha_b, c_{0a}, c_{0b}$ and c_{1a}, c_{1b} are positive constants. Furthermore, the command voltage is accounted for through the first-order filter

$$\dot{u} = -\eta(u - v), u(0) = 0, \tag{10}$$

where v is the command voltage sent to the current driver, and η is a positive number that reflects the time lag of the MR damper. An efficient identification method for the MR fluid damper can be found in, for example [7].

To reflect the real situation, the command input v is confined to be finite positive. As a result, u is also limited to be positive finite, that is,

$$0 \leq v \leq V_{\max} \text{ and } 0 \leq u \leq V_{\max},$$

where $V_{\max}$ is the maximum voltage to the current driver associated with the saturation of the magnetic field in the MR fluid damper. It follows that all the related parameters α, c_0 and c_1 are all finite positive as well.

Remark 2.1: The original form for $\dot{y}$ given by Spencer is motivated by the force balance as given in the following (with moving x_2),

$$c_1(\dot{y} - \dot{x}_2) = \alpha z + k_0(x_1 - y) + c_0(\dot{x}_1 - \dot{y}),$$

or

$$\dot{y} = \frac{1}{c_0 + c_1}\left[k_0(x_1 - y) + \alpha z + c_1\dot{x}_2 + c_0\dot{x}_1\right],$$

equivalently. While equations (7)-(9) make it clear that the inputs of the modified Bouc-Wen model are $x_{P1}, \dot{x}_{P1}$ and the state variables are $x_1 - y$ and z.

Remark 2.2: On the $(x_1 - y, z)$-plane there are two sets of trajectories, which are determined by either of the following two differential equations,

$$d(x_1 - y) = \frac{dz}{\delta - |z|^n(\beta + \gamma)},$$
$$d(x_1 - y) = \frac{dz}{\delta - |z|^n(\beta - \gamma)}.$$

The "+" or "−" sign between β and γ is determined by the sign of the product $(\dot{x}_1 - \dot{y})z$, while the sign of $\dot{x}_1 - \dot{y}$ is directly controlled by the input to the MR damper, $\dot{x}_{P1}$. For $n = 2$, the two corresponding sets of trajectories are

$$x_1 - y = \frac{1}{\sqrt{\delta(\beta + \gamma)}}\tanh^{-1}\frac{(\beta + \gamma)z}{\sqrt{\delta(\beta + \gamma)}} + C_1,$$
$$x_1 - y = \frac{1}{\sqrt{\delta(\beta - \gamma)}}\tanh^{-1}\frac{(\beta - \gamma)z}{\sqrt{\delta(\beta - \gamma)}} + C_2,$$

where C_1 and C_2 are constants that depend on initial conditions.

III. Stability Analysis and Controller Synthesis

A. System Stability

Although it is well known that an MR damper is a dissipative device, while to the best of authors' knowledge, a direct proof of this property has not been provided. Hence, in this section, we prove this fundamental property of the MR damper when implemented on a vehicle suspension system.

Theorem 3.1: The quarter-car system (6) with the passive MR damper force F_{rh} defined by (9) and (7), (8) is dissipative if

1) $\delta > 0, \beta > 0, \gamma \geq 0$, the voltage dependent parameters $\alpha \geq 0, c_0 > 0, c_1 > 0$,
2) the initial condition of z satisfies $|z(0)| \leq z_M$, $z_M \triangleq \sqrt[n]{\frac{\delta}{\beta - \gamma}}$,

Proof: Define the Lyapunov candidate function for the quarter car model

$$V_{qc} \triangleq \frac{1}{2}\dot{x}_P^{\mathrm{T}} M_P \dot{x}_P + \frac{1}{2}x_P^{\mathrm{T}} K_P x_P, \tag{11}$$

it follows immediately that the time derivative of V_{qc} along the system trajectories is

$$\begin{aligned}\dot{V}_{qc} &= -\dot{x}_P^{\mathrm{T}} C_P \dot{x}_P + \dot{x}_P^{\mathrm{T}}\begin{bmatrix}-1\\0\end{bmatrix}F_{rh} - \dot{x}_P^{\mathrm{T}}\begin{bmatrix}m_1\\m_1 + m_2\end{bmatrix}\ddot{x}_g,\\ &= -\dot{x}_P^{\mathrm{T}} C_P \dot{x}_P - \dot{x}_{P1}F_{rh} - [m_1\dot{x}_{P1} + (m_1 + m_2)\dot{x}_{P2}]\ddot{x}_g,\end{aligned}$$

where the power consumed by the MR damper can be written in detail as

$$\begin{aligned}-\dot{x}_{P1}F_{rh} = &-\frac{c_0 c_1}{c_0 + c_1}\dot{x}_{P1}^2 - k_1\dot{x}_{P1}(x_{P1} - \bar{x}_0)\\ &- \frac{c_1\dot{x}_{P1}}{c_0 + c_1}\left[k_0(x_1 - y) + \alpha z\right].\end{aligned}$$

A naive choice of the Lyapunov candidate function for the MR damper is

$$V_{rh} \triangleq \frac{1}{2}k_1(x_{P1} - \bar{x}_0)^2 + \frac{1}{2}k_0(x_1 - y)^2 + \int_0^z \frac{\alpha}{\delta}z\,dz. \tag{12}$$

It is easy to see that

$$\dot{V}_{rh} = k_1\dot{x}_{P1}(x_{P1}-\bar{x}_0) + [k_0(x_1-y)+\alpha z](\dot{x}_1-\dot{y}) - \frac{\alpha}{\delta}z(\dot{x}_1-\dot{y})|z|^n[\beta+\gamma\mathrm{sgn}(\dot{x}_1-\dot{y})\mathrm{sgn}(z)].$$

Henceforth, if we let the Lyapunov candidate function for the quarter car and suspension system be

$$V = V_{qc}+V_{rh},$$

it follows that

$$\begin{aligned}\dot{V} = &-\dot{x}_P^{\mathrm{T}}C_P\dot{x}_P - \frac{c_0c_1}{c_0+c_1}\dot{x}_{P1}^2 - \frac{1}{c_0+c_1}[k_0(x_1-y)+\alpha z]^2\\ &-\frac{\alpha}{\delta}z(\dot{x}_1-\dot{y})|z|^n[\beta+\gamma\mathrm{sgn}(\dot{x}_1-\dot{y})\mathrm{sgn}(z)]\\ &+[m_1\dot{x}_{P1}+(m_1+m_2)\dot{x}_{P2}]\ddot{x}_g.\end{aligned}$$

Two cases will be considered in the following.

Case I: $\beta>0,\gamma\geq 0,\beta-\gamma\leq 0$: In this case, we have

$$\begin{aligned}&\dot{V}-[m_1\dot{x}_{P1}+(m_1+m_2)\dot{x}_{P2}]\ddot{x}_g\\ &= -\dot{x}_P^{\mathrm{T}}C_P\dot{x}_P - \frac{c_0c_1}{c_0+c_1}\dot{x}_{P1}^2 - \frac{1}{c_0+c_1}[k_0(x_1-y)+\alpha z]^2\\ &\quad -\frac{\alpha}{\delta}z(\dot{x}_1-\dot{y})|z|^n[\beta+\gamma\mathrm{sgn}(\dot{x}_1-\dot{y})\mathrm{sgn}(z)] \leq 0,\end{aligned}$$

since the last term in the above equation is positive for $(\dot{x}_1-\dot{y})z$ is either positive or negative.

Case II: $\beta>0,\gamma\geq 0,\beta-\gamma>0$: In this case, if $(\dot{x}_1-\dot{y})z$ is positive, similar to Case I, we have $\dot{V}\leq 0$. While if $(\dot{x}_1-\dot{y})z$ is negative, we rewrite the above inequality for $\dot{V}$ as

$$\begin{aligned}&\dot{V}+\frac{\alpha}{\delta}z\dot{z}\frac{|z|^n}{z_M^n-|z|^n}-[m_1\dot{x}_{P1}+(m_1+m_2)\dot{x}_{P2}]\ddot{x}_g\\ &= -\dot{x}_P^{\mathrm{T}}C_P\dot{x}_P - \frac{c_0c_1}{c_0+c_1}\dot{x}_{P1}^2 - \frac{1}{c_0+c_1}[k_0(x_1-y)+\alpha z]^2\\ &\leq 0, \qquad (13)\end{aligned}$$

where the second term on the left-hand side of the equation is obtained by utilizing (8). It has been shown that for bounded input $\dot{x}_{P1}$ (hence bounded $\dot{x}_1-\dot{y}$) if the initial condition of z satisfies $|z(0)|\leq z_M$, then $|z(t)|\leq\max\{|z(0)|,z_m\},\forall t$, where $z_m \triangleq \sqrt[n]{\frac{\delta}{\beta+\gamma}}$ [8]. This further implies that $\dot{z}$ is bounded, hence the system (6) is dissipative with respective to supplied rate $[m_1\dot{x}_{P1}+(m_1+m_2)\dot{x}_{P2}]\ddot{x}_g-\frac{\alpha}{\delta}z\dot{z}\frac{|z|^n}{z_M^n-|z|^n}$ [9].

From the above two cases, we conclude that the system is dissipative under the given conditions. ■

Remark 3.1: Note that the minimum value of the Lyapunov function is zero only if $\bar{x}_0=0$.

Remark 3.2: $\dot{V}=0$ implies that $\dot{x}_P=0, k_0(x_1-y)+\alpha z=0$ and $z(\dot{x}_1-\dot{y})=0$, respectively, which in turn imply that $\dot{x}_1-y=0,\dot{z}=0$ and x_P satisfies $K_Px_P=\begin{bmatrix}-1\\0\end{bmatrix}k_1(x_{P1}-\bar{x}_0)$.

B. A Lyapunov Function Based Multi-level Controller

The implementation of (9) to the third term of (13) renders

$$\begin{aligned}&\dot{V}+\frac{\alpha}{\delta}z\dot{z}\frac{|z|^n}{z_M^n-|z|^n}-[m_1\dot{x}_{P1}+(m_1+m_2)\dot{x}_{P2}]\ddot{x}_g\\ &= -\dot{x}_P^{\mathrm{T}}C_P\dot{x}_P - \frac{c_0c_1}{c_0+c_1}\dot{x}_{P1}^2\\ &\quad -\frac{c_0+c_1}{c_1^2}\left[F_{rh}-\frac{c_0c_1}{c_0+c_1}\dot{x}_{P1}-k_1(x_{P1}-\bar{x}_0)\right]^2\\ &= -\dot{x}_P^{\mathrm{T}}C_P\dot{x}_P - c_0\dot{x}_{P1}^2-\frac{c_0+c_1}{c_1^2}[F_{rh}-k_1(x_{P1}-\bar{x}_0)]^2\\ &\quad +\frac{2c_0}{c_1}[F_{rh}-k_1(x_{P1}-\bar{x}_0)]\dot{x}_{P1},\\ &\triangleq -\dot{x}_P^{\mathrm{T}}C_P\dot{x}_P+\dot{V}_a(x_{P1},\dot{x}_{P1},F_{rh},u).\end{aligned}$$

Since the hysteretic component z of an MR damper is fictitious, it can not be measured. Hence the corresponding term is treated as a supply rate to the system. The above expression suggests that the input function v takes the form

$$v = v(x_{P1},\dot{x}_{P1},F_{rh},u),$$

which indicates that the measurements needed to construct the feedback controller are the relative displacement, x_{P1}, and velocity, $\dot{x}_{P1}$, between the sprung and unsprung masses of vehicle, and the damping force, F_{rh}, of MR damper. While the input voltage u is determined by minimizing the augmented function $\dot{V}_a$. Owing to the fast dynamics of the command voltage (10), it is reasonable to assume that $v=u$. Hence, a simple multi-level controller is proposed as follows:

$$\begin{aligned}v=\frac{i}{N}V_{\max},\ \text{if } &\dot{V}_a(x_{P1},\dot{x}_{P1},F_{rh},V_{\max}i/N)\\ &\leq\dot{V}_a(x_{P1},\dot{x}_{P1},F_{rh},V_{\max}j/N),\\ &\forall j\neq i, i=0,\ldots,N.\end{aligned} \qquad (14)$$

where N is the number of levels that the input voltage is divided. To have a smooth input voltage, a first order filter, $\frac{1}{\tau_v s+1}$, may be augmented to the damper controller.

IV. Numerical Example

Consider the quarter car and an MR damper model with the parameters defined by Table I and II, respectively [4]. Assume that the maximum input voltage of the MR damper is $V_{\max}=2.0$V.

Two kinds of road profile inputs are considered in this example.

TABLE I
QUARTER CAR MODEL PARAMETERS

Parameter	value
m_1	372 kg
m_2	45 kg
k_s	40 kN/m
k_t	190 kN/m
c_s	0 N s/m
c_t	0 N s/m

TABLE II
PARAMETERS FOR THE MR DAMPER RD-1005-1 [4]

Coeff.		Coeff.	
α_a	12441 N/m	c_{0_a}	784 N · s/m
α_b	38430 N/m · V	c_{0_b}	1803 N · s/m · V
β	2059020 m^{-2}	c_{1_a}	14649 N · s/m
γ	136320 m^{-2}	c_{1_b}	34622 N · s/m · V
δ	58	n	2
η	190 s^{-1}	$\bar{x}_0$	0 m
k_0	3610 N/m	k_1	840 N/m

A. Case I: 0.03m height bump disturbance

Let the vehicle be subject to a bump excitation with amplitude 0.03m as shown in Fig. 3. Then we implement the above controller (14) along with a first-order filter to the system, and let $N = 4, \tau_v = 0.02$. The corresponding responses of sprung mass and unsprung mass, force and input voltage history are shown in Fig. 4 and 5, respectively. Note that the "ride" of a motor vehicle is most commonly measured by the acceleration on the body [10].

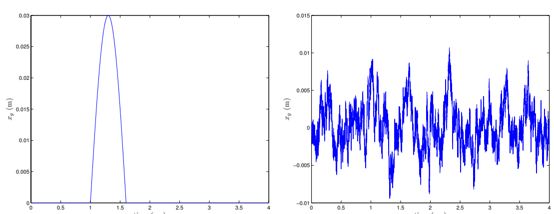

Fig. 3. (Left) Bump Excitation (Right) Grade C random road profile

For comparison purpose, the responses of the cases with constant input voltage $v = 0$V and $v = 2$V are given in Fig. 6 and 7, respectively. As we can tell from the given results, the proposed controller for the MR damper improves the performance for vibration suppression of the sprung mass under bump excitation. The maximum voltage required is only slightly larger than 1V. Also larger damper forces do not always produce better results. In addition, compared with the results shown in [4], the performance of the controller (14) outperforms that of the paper, in the sense of less settling time, smaller peak values of response and acceleration of sprung mass. Besides the proposed algorithm is simple and no reference model is required.

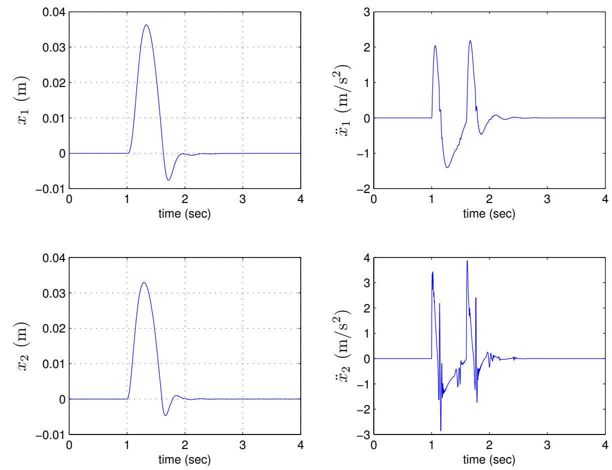

Fig. 4. Responses of sprung and unsprung masses under bump excitation

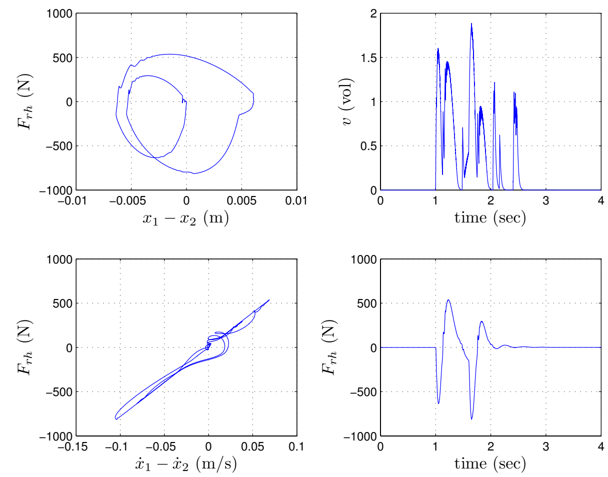

Fig. 5. Damping force and input voltage of MR damper

B. Case II: grade C random road profile

A grade C road profile shown in Fig. 3 is given as the random excitation to the suspension system. Some typical values of the RMS of the acceleration of the sprung, the suspension deflection $(x_1 - x_2)$, and the tyre deflection $(x_2 - x_g)$ for the listed four different cases are given in Table III. These results indicate that the proposed controller provides an acceptable performance compared with that of the passive suspension systems (input voltage $v = 0$V and $v = 2$V).

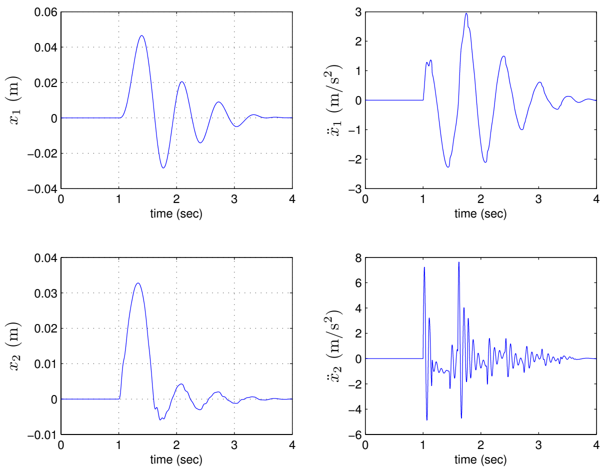

Fig. 6. Responses of sprung and unsprung masses under bump excitation for $v = 0$

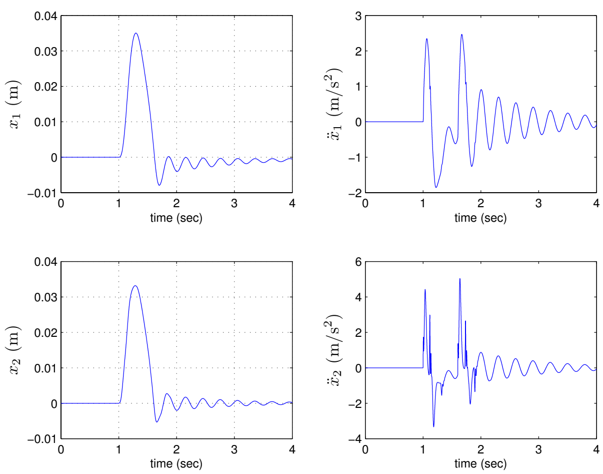

Fig. 7. Responses of sprung and unsprung masses under bump excitation for $v = 2$

TABLE III

RMS ANALYSIS FOR GRADE C ROAD EXCITATION TESTS

Controller	$\ddot{x}_1$(m/sec^2)	$x_1 - x_2$(m)	$x_2 - x_g$(m)
(14)	1.1426	0.0021	0.0026
$v = 0$	0.7035	0.0039	0.0029
$v = 2$	1.3462	0.0019	0.0029

V. CONCLUSIONS AND FUTURE WORKS

In this work, a Lyapunov function consisted of the kinematic energy and spring potential function of a suspension system plus an integral term of the hysteretic component of an MR damper is chosen to examine the stability and dissipativity of the system. To suppress the vibration, an multi-level controller based on the derivative of the Lyapunov function is proposed. Through numerical examples, the proposed controller turns out to be effective and the algorithm is simple to implement. It requires only the measurements of relative displacement and velocity between sprung and unsprung masses, the damping force, and the voltage dependent coefficients c_0, c_1 and k_1, which can be identified in the beginning. While the parameters of quarter-vehicle are not required. In the future, the effects on the parameter uncertainties will be further investigated.

VI. ACKNOWLEDGMENTS

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