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Min-Max Model Predictive Control of a Pilot Plant

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Abstract— **This paper shows the application of a Min-Max Model Predictive Control (MMMPC) strategy to a pilot plant in which the temperature of a reactor is controlled. An approximation of the worst case cost is used to obtain the control action. This approximation can be easily computed yielding a solution of the min-max problem very close to the exact one. The complexity of the algorithm allows the real time implementation for typical prediction and control horizons. The behavior of the system and the controller will be illustrated by means of experimental results.**

I. Introduction

In Min-Max Model Predictive Control (MMMPC) [1], the value of the control signal applied to the controlled process, is computed minimizing the worst possible case of a cost function, usually quadratic. The worst case is calculated maximizing the cost function with respect to all possible cases of disturbances and uncertainties. The solution of these optimization problems requires an enormous amount of calculations, since the problem is NP-hard [2]. Therefore, the implementation of MMMPC is quite difficult, having as a consequence a very limited number of applications reported in the literature, usually to processes with slow dynamics or to complex simulation models [3]. It is not possible to solve the min-max problem in real time for processes with moderately fast dynamics, except when the number of possible cases of the uncertainty is relatively low. When fast dynamics have to be controlled the min-max problem cannot be solved numerically in real time and approximate solutions have to be used [4]. However, these techniques impose a great rigidity in the controller (as well as a certain degree of approximation error) and the controller must be computed again if the process model or the controller parameters change. This is also a problem with the explicit solutions that can be obtained using multiparametric mathematical programming [1], [5]. Another problem is the great amount of memory necessary to store all the regions that compose the explicit description. The number of those regions grows with the prediction horizon in a combinatorial way. Also, searching times can be high and search strategies such as [6] must be used.

The computational burden issue can be circumvented by using an upper bound of the worst case cost instead of calculating it explicitly. In a previous work [7] the authors presented an upper bound of the worst case cost based on simple matrix operations that can be calculated in a fast way and implemented in computers with limited computational capabilities. In addition, the use of the upper bound instead of the worst exact cost does not lead to an excessive error with respect to the exact solution of the original problem [7].

In this work the theoretical results presented in [7] are validated by means of their application to a pilot plant. The pilot plant is used to simulate an exothermic chemical reaction with nonlinear dynamics. This process has been used in previous works, thus the experimental results presented can be compared with other strategies such as nonlinear and linear predictive control [8]. In the experiments, restrictions in the control action and the output have been considered. The results obtained prove the validity of the used control strategy.

The paper is organized in the following way: section II presents the problem description of the predictive control and the notation used in this work. Section III presents the algorithm to calculate the upper bound of the worst case cost and section IV gives a description of the process on which the presented control algorithm has been proven. In section V the experimental results are shown and, finally, in section VI the conclusions.

II. Problem description

Without loss of generality a discrete model in state space with additive uncertainties will be considered [1]:

$$\begin{aligned} x(t+1) &= Ax(t) + Bu(t) + D\theta(t+1) \\ y(t) &= Cx(t) \end{aligned} \quad (1)$$

with $x(t) \in \mathbb{R}^{dimx}$, $u(t) \in \mathbb{R}^{dimu}$, $\theta(t) \in \{\theta \in \mathbb{R}^{dim\theta} : \|\theta\|_\infty \leq \epsilon\}$, $y(t) \in \mathbb{R}^{dimy}$. For simplicity $\epsilon = 1$ is assumed (in opposite case the matrix D can be scaled to ϵD). A control horizon N_u and a prediction horizon N is considered. Furthermore, the cost function $J(\boldsymbol{\theta}, \mathbf{u}, x)$ is a quadratic performance index of the form:

$$\begin{aligned} J(\boldsymbol{\theta}, \mathbf{u}, x) &= \sum_{j=1}^{N} x(t+j|t)^T Q_j x(t+j|t) \\ &\quad + \sum_{j=0}^{N_u-1} u(t+j|t)^T R_j u(t+j|t) \end{aligned} \quad (2)$$

where $x(t+j|t)$ is the prediction of the state for $t+j$ made at t when the future values of the uncertainty are supposed to be given by the sequence $\boldsymbol{\theta} \in \Theta = \{\boldsymbol{\theta} \in \mathbb{R}^{N \cdot dim\theta} : \|\boldsymbol{\theta}\|_\infty \leq 1\}$ and the values of the control action throughout the control horizon are given by $\mathbf{u} \in \mathbb{R}^{N_u \cdot dimu}$. On the other hand, $Q_j \in \mathbb{R}^{dimx \times dimx}$, $R_j \in \mathbb{R}^{dimu \times dimu}$ are symmetrical positive definite matrices used as weighting parameters. The cost function can be rewritten as [1]:

$$\begin{aligned} J(\boldsymbol{\theta}, \mathbf{u}, x) &= \mathbf{u}^T M_{uu} \mathbf{u} + \boldsymbol{\theta}^T M_{\theta\theta} \boldsymbol{\theta} + 2\boldsymbol{\theta}^T M_{\theta u} \mathbf{u} \\ &\quad + 2x^T M_{uf}^T \mathbf{u} + 2x^T M_{\theta f}^T \boldsymbol{\theta} + x^T M_{ff} x \end{aligned} \quad (3)$$

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The considered initial scheme of predictive control is the Min-Max Predictive Control [1] in which the optimal sequence $\mathbf{u}^*$ is calculated solving a min-max problem:

$$\begin{aligned} \mathbf{u}^*(x) = \arg \quad & \min_{\mathbf{u}} \quad J^*(\mathbf{u}, x) \\ & \text{s.t.} \quad L\mathbf{u} \leq c + Fx \end{aligned} \tag{4}$$

with

$$J^*(\mathbf{u}, x) = \max_{\boldsymbol{\theta} \in \Theta} \; J(\boldsymbol{\theta}, \mathbf{u}, x) \tag{5}$$

with $L \in \mathbb{R}^{nc \times (N_u \cdot dimu)}$, $F \in \mathbb{R}^{nc \times dimx}$ and $c \in \mathbb{R}^{nc}$ (being nc the number of restrictions). The solution of this problem is applied using receding horizon strategy, habitual in all predictive control schemes. The results presented in this work are valid for MMMPC with open loop predictions or *semi-feedback* strategies [7], [9].

With the model being linear in x, u, θ, and assuming $R_j > 0$, the quadratic cost function is convex in $\boldsymbol{\theta}$ and $\mathbf{u}$ and the solution of the maximization problem is attained at least at one of the vertices of the unitary hypercube θ [1]. Therefore the maximization problem (5) is equivalent to

$$J^*(\mathbf{u}, x) = \max_{\boldsymbol{\theta} \in \text{vert}\{\Theta\}} \; J(\boldsymbol{\theta}, \mathbf{u}, x) \tag{6}$$

The resolution of the maximization problem is a well known NP hard problem. The obligatory evaluation of each one of the $2^{N \cdot dim\theta}$ vertices of θ leads to an exponential complexity. Therefore this problem, and as a consequence the min-max problem (4), can only be solved for small prediction horizons.

The adopted strategy in the control scheme used in this work is directed to reduce the computacional cost of problem (4). The idea is to replace the worst case cost $J^*(\mathbf{u}, x)$ in (4) by an upper bound that can be calculated easily. Section III presents the mentioned bound, which can be computed with a complexity $O(n^3)$ instead of $O(2^n)$.

III. Calculation of the upper bound of the worst case cost

This section presents the algorithm to compute the upper bound of the worst case cost. The computation of this bound is based on simple matrix operations allowing an easy implementation. Only the fundamental results are described, for a complete description see [7].

It can be seen from (3) that:

$$J^*(\mathbf{u}, x) = \max_{\boldsymbol{\theta} \in \text{vert}\{\Theta\}} \boldsymbol{\theta}^T S \boldsymbol{\theta} + 2\boldsymbol{\theta}^T p(\mathbf{u}, x) + r(\mathbf{u}, x) \tag{7}$$

with

$$\begin{aligned} S &= M_{\theta\theta} \\ p(\mathbf{u}, x) &= M_{\theta u}\mathbf{u} + M_{\theta f} x \qquad (8) \\ r(\mathbf{u}, x) &= \mathbf{u}^T M_{uu} \mathbf{u} + 2x^T M_{uf}^T \mathbf{u} + x^T M_{ff} x \qquad (9) \end{aligned}$$

Therefore, the calculation of $J^*(\mathbf{u}, x)$ turns out to be a mathematical problem of the following type:

$$\gamma^* = \max_{\theta \in \text{vert}\{\Theta\}} \theta^T S \theta + 2\theta^T p + r \tag{10}$$

This problem is, as will be shown in the following proposition, equivalent to an augmented quadratic maximization problem. The proof can be found in [7].

Proposition 1: The problem (10) is equivalent to the following augmented problem:

$$\gamma^* = \max_{\left[\begin{smallmatrix} \theta_e \\ \theta \end{smallmatrix}\right] \in \text{vert}\{\Theta_A\}} \left[\begin{smallmatrix} \theta_e \\ \theta \end{smallmatrix}\right]^T \left[\begin{smallmatrix} r & p^T \\ p & S \end{smallmatrix}\right] \left[\begin{smallmatrix} \theta_e \\ \theta \end{smallmatrix}\right] \tag{11}$$

where $\theta_e \in \mathbb{R}$ and Θ_A is the augmented unitary hypercube:

$$\Theta_A = \left\{ \left[\begin{smallmatrix} \theta_e \\ \theta \end{smallmatrix}\right], \; \theta_e \in \mathbb{R}, \; |\theta_e| \leq 1, \; \theta \in \Theta \right\}.$$

□

Therefore the augmented problem can be rewritten like:

$$\gamma^* = \max_{z \in \text{vert}\{\Theta_A\}} z^T H z \tag{12}$$

with $H \in \mathbb{R}^{n \times n}$ a symmetric matrix. Assume now a diagonal matrix T that verifies[1] $T \geq H$, thus:

$$\begin{aligned} z^T H z &\leq z^T T z = \sum_{i=1}^{n} T_{ii} z_i^2 \leq \\ &\leq trace(T) \|z\|_\infty^2 \leq trace(T) \end{aligned}$$

and therefore:

$$\gamma^* \leq \text{trace}(T)$$

That means that the trace of T is an upper bound of γ^*. The strategy used in this paper is to obtain a diagonal matrix $T \geq H$ with a computationally efficient procedure that keeps the upper bound (i.e. $\text{trace}(T)$) close to γ^*. The details of the computation procedure and properties of this upper bound are described in depth in [7]. In the following, the computation procedure is briefly described.

Matrix H will be diagonalized by adding $n-1$ positive semidefinite matrices (with n the dimension of H), all of the form $v_i v_i^T$ such that:

$$H + v_1 v_1^T + v_2 v_2^T + v_3 v_3^T + \cdots + v_{n-1} v_{n-1}^T = T$$

where T is diagonal matrix and v_i vectors of appropriate dimension. Vectors v_i are computed in such a way that: a) T is diagonal and b) the trace of T yields a close upper bound of γ^*. This is accomplished by the following procedure in which the upper bound σ_u is computed:

Procedure 1: Procedure to compute the upper bound of the worst case cost $\sigma_u(H) \geq \max_{z \in \text{vert}\{\Theta_A\}} z^T H z$.

1) Let $T = H \in \mathbb{R}^{n \times n}$.
2) For $k = 1$ to $n-1$
3) Let $H_{sub} = [T_{ij}]$ for $i, j = k \cdots n$.
4) Compute α for $H_{sub} = \left[\begin{smallmatrix} a & b \\ b^T & H_r \end{smallmatrix}\right]$ using the expression $\alpha = \sqrt{\|b\|_1}$.
5) Make $v_k^T = \left[\begin{smallmatrix} \alpha & \frac{-b^T}{\alpha} \end{smallmatrix}\right]$.
6) Make $v_e^T = \left[\begin{smallmatrix} 0 & \cdots & 0 & v_k^T \end{smallmatrix}\right] \in \mathbb{R}^n$.
7) Update T by making $T = T + v_e v_e'$.
8) Endfor.

[1] In this work a matrix inequality of the type $T \geq H$ is fulfilled if and only if $T - H$ is positive semidefinite.

Fig. 1. Pilot plant used to apply the MMMPC.

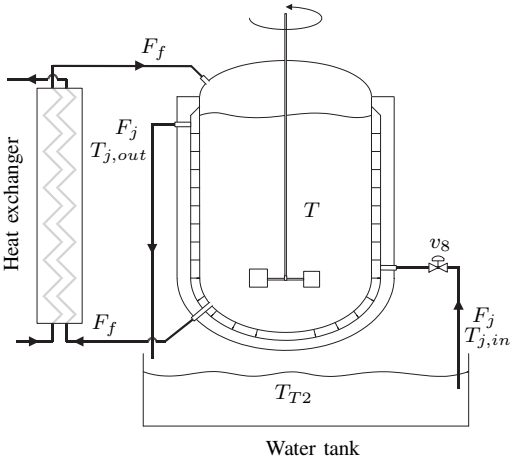

Fig. 2. Diagram of the pilot plant with its four main elements: reactor, heat exchanger, cooling jacket and valve.

9) Compute the upper bound from $\sigma_u(H) = \sum_{i=1}^{n} T_{ii}$.

It is evident from the algorithm that $T \geq H$. Therefore

$$\max_{z \in \text{vert}\{\Theta_A\}} z^T H z \leq \max_{z \in \text{vert}\{\Theta_A\}} z^T T z = \sigma_u(H).$$

What implies that,

$$\begin{aligned} J^*(\mathbf{u}, x) &= \max_{z \in \text{vert}\{\Theta_A\}} z^T \begin{bmatrix} r(\mathbf{u}, x) & p^T(\mathbf{u}, x) \\ p(\mathbf{u}, x) & S \end{bmatrix} z \\ &\leq \sigma_u \left(\begin{bmatrix} r(\mathbf{u}, x) & p^T(\mathbf{u}, x) \\ p(\mathbf{u}, x) & S \end{bmatrix} \right). \end{aligned}$$

Then, $\sigma_u(\cdot)$ calculated in procedure 1 is an upper bound of the worst case cost.

Note that procedure 1 can be coded easily as only simple matrix operations are needed to compute the upper bound. This is relevant because a difficult implementation is a drawback when applying complex control strategies in the industry.

IV. Process description

A real process represented by a pilot plant has been chosen for the application of the proposed algorithm. The process has been studied previously by several authors [8], [10].

A. Laboratory process

The used pilot plant (see Fig. 1) serves to simulate exothermic chemical reactions based on temperature changes. It has been used as a benchmark for control purposes by several researchers [4], [8]. The main elements of the pilot plant are the reactor, the heat exchanger, the cooling jacket and the valve to manipulate the flow rate through the cooling jacket (see Fig. 2).

For the temperature reduction in the reactor a cooling jacket is used. The heat dissipation can be regulated by the valve v_8 which manipulates the flow rate F_j through the cooling jacket. The cooling fluid, water, enters the cooling jacket with a constant temperature. In order to maintain the chemical reaction running, the reactive of the chemical reaction is supplied to the reactor by the feed $F_{f,in}$. Before entering the reactor, the feed passes through a heat exchanger in order to adopt the temperature of the reactor content. The outflow $F_{f,out}$ is used to keep the volume of the reactor content constant.

To simulate exothermic reactions, the reactor possesses an electrical resistance in order to supply caloric energy. The energy to be supplied by the electrical resistance is calculated by means of a mathematical model of the simulated reaction. The use of a resistance means that no chemical reaction takes place in the reactor, instead the reaction is emulated on basis of temperature changes, as done by [11].

B. Mathematical model

Although it is not necessary to have a mathematical model for the design of the min-max predictive controller, this section shows the process model to emphasize its nonlinear character. The mathematical model also justifies the way to emulate the heat generated by the chemical reaction with the aid of the resistance.

The emulated chemical reaction, representing a refinement process, was used previously in [8]. With $F_f = F_{f,in} = F_{f,out}$ and a constant volume, the model of the chemical reaction can be defined as:

$$\begin{aligned} \frac{\mathrm{d}T}{\mathrm{d}t} &= -\frac{F_j}{V}(T_{j,in} - T_{j,out}) \\ &\quad + \frac{(-\Delta H) \cdot V}{M C_p} k_0 \, e^{-E/(RT)} C_A^2 \qquad (13) \\ \frac{\mathrm{d}C_A}{\mathrm{d}t} &= \frac{F_f}{V}(C_{A,in} - C_A) - k_0 \, e^{-E/(RT)} C_A^2 \qquad (14) \end{aligned}$$

denoting F_j, $T_{j,in}$ and $T_{j,out}$ the flow rate through the jacket and the temperature of the water entering and leaving the cooling jacket, respectively. C_A and $C_{A,in}$ represent the reactive concentration in the reactor and in the feed, respectively. As the feed passes through the heat exchanger and enters the reactor nearly with the temperature of the reactor content, it was assumed that no heat removal or supply takes place by the feed.

For the heat exchange in the cooling jacket the empirical model:

$$F_j \cdot (T_{j,out} - T_{j,in}) = \frac{T - \alpha}{\beta}(1 - e^{-\gamma F_j}) \qquad (15)$$

with $\alpha = 292.19\,\mathrm{K}$, $\beta = 14.94\,\mathrm{s/l}$ and $\gamma = 13.18\,\mathrm{s/l}$ was used.

As can be easily seen from the model equations (13) and (14) the chemical reaction possesses nonlinearities in the dynamics of the temperature and the concentration due to the quadratic terms of the concentration. For further details on the model parameters see [8]

V. Experimental results

In this section the strategy of control described in section III is applied to the refinement process. The achieved experimental results will be exposed and discussed. CARIMA type prediction models with bounded additive uncertainties were used in the experiments. This type of model extends the concept of noise in traditional CARIMA models so that an uncertainty is considered:

$$A(z^{-1})y(t) = z^{-d}B(z^{-1})u(t-1) + C(z^{-1})\frac{\theta(t)}{\Delta}$$

with $\Delta = 1 - z^{-1}$, $\theta(t) \in \{\theta \in \mathbb{R}^{dimy} : \|\theta\|_\infty \leq \epsilon\}$, and $dimy$ the dimension of $y(t)$. The use of this type of prediction models results in a control law without error in steady state. The differences between implementing the algorithm of section III for a state space model and a CARIMA model with bounded additive uncertainties are minimal. They reduce to the method used to find the matrices of the prediction equation [1]. The cost function is the same as in (3).

In the following sections the control system in the pilot plant will be described, the necessary steps to obtain a prediction model will be presented and the experimental results will be exposed.

A. Description of the control system

The sensors and actuators in the plant are connected to a PMC-10 control unit. The PMC-10 is connected by ARCnet to a personal computer that runs the control and monitoring system *Simatic-IT*. The control algorithm has been implemented directly in *Matlab* and the communication with *Simatic-IT* is done using the *OPC* protocol (*OLE for Process Control*). Both *Simatic-IT* and the controller run on the same personal computer, based on a Pentium II processor with 300 Mhz. This computer does not have enough computational power to solve exactly the min-max problem of a typical MMMPC, but can compute the control action using the proposed strategy.

B. Identification of the prediction model

A *PRMSS* (*Pseudo-Random Multilevel Step Sequence*) has been applied to the recirculation valve with the objective of collecting data for the parameter identification of the prediction model. The periods of the *PRMSS* have been chosen sufficiently long to observe the reaction of the pilot plant to changes in the input (see Fig. 3). It can be seen that the temperature of the tank reaches steady state in each step in something more than two hours, although the variations in steady state are of several degrees. The reagent concentration also suffers variations in steady state. It can be observed that the input–output gain is negative and clearly variable (greater

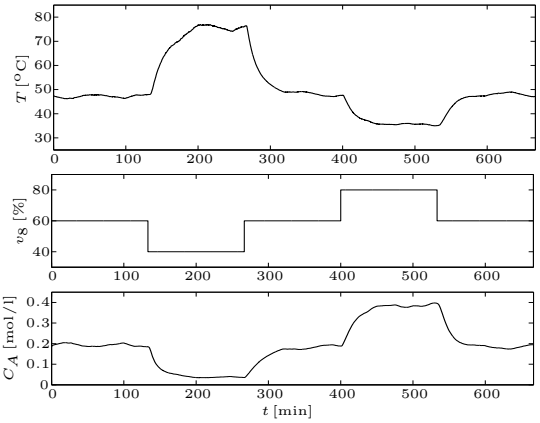

Fig. 3. Experiment for the prediction model identification. From top to bottom: Tank temperature (T), valve opening (v_8) y reagent concentration (C_A).

gain for low openings of v_8). A first order transfer function model with delay is proposed as prediction model. This low order model cannot correctly describe the dynamics of the plant, but it is a good approach to check the robustness of the controller in presence of uncertainties and disturbances.

Using the data of Fig. 3 the following model has been identified:

$$G(s) = \frac{-0.975}{950s + 1}e^{-31.25s} \tag{16}$$

This model was discretized with a sampling time of $T_s = 60$. The delay was rounded to 1 sampling time in order to avoid approximations of the time delay, e.g. Padé approximation. Thereby, the following CARIMA model was obtained:

$$y(t+1) = 0.939\,y(t) - 0.0597\,u(t-1) + \frac{\theta(t)}{\Delta} \tag{17}$$

with the noise polynomial $C(z^{-1}) = 1$.

C. Experimental results of the controller

In the experiments (17) has been used as a the prediction model in the proposed controller. The prediction and control horizons have been chosen equal to $N = 15$ and $N_u = 12$. Note that the prediction horizon includes approximately one time constant of the process, a common value for this parameter. On the other hand, since the prediction model has an additional delay, the prediction horizon would be defined between $\hat{y}(t+2|t)$ and $\hat{y}(t+16|t)$. The control effort will be weighed by the factor $R_j = 2$. The parameter ϵ has been chosen based on the prediction error one step ahead as shown in Fig. 4. The finally chosen value is $\epsilon = 0.25$. In fact, in 97% of the samples the prediction error one step ahead is bounded by the chosen value.

In addition to the previously mentioned parameters, a correction in the prediction of $y(t+1)$, similar to the Smith predictor, has been considered. This is due to the varying delay of real process. Therefore the prediction of the output at time $t+1$ using the nominal model, $\hat{y}_n(t+1|t)$, is corrected in this way:

$$\hat{y}(t+1|t) = \hat{y}_n(t+1|t) + (\hat{y}_n(t|t) - y(t)) \tag{18}$$

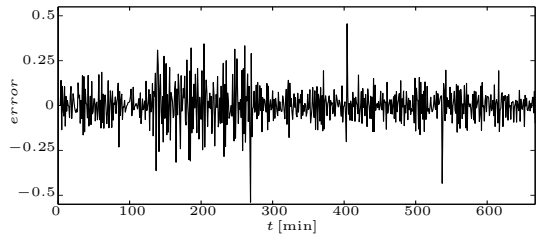

Fig. 4. One step ahead prediction error during the experiment for the model identification.

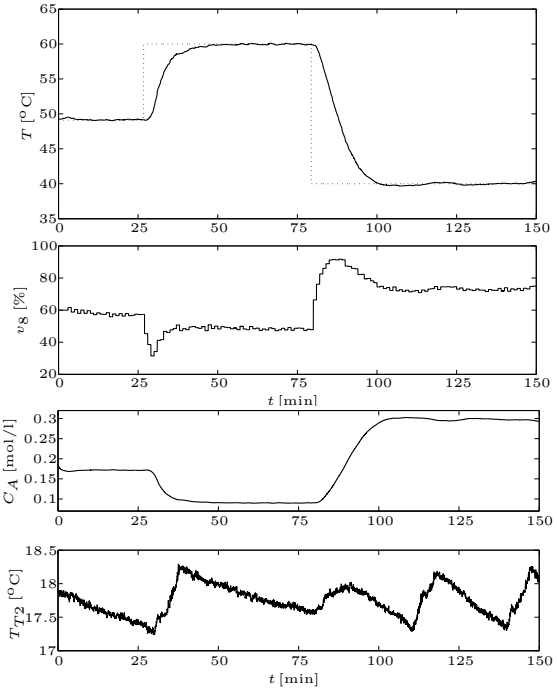

Fig. 5. Reference tracking experiment. From top to bottom: Tank temperature (T), valve opening (v_8), reagent concentration (C_A) and cold water temperature (T_{T2}).

where $y(t)$ is the real process output at time t. This simple correction improves the performance of predictive controllers in the case of time delay systems [12].

Finally, input and output restrictions have been considered in the experiments. The process output, the control signal and the control moves are limited to:

$$\begin{aligned} 30 \leq \quad & \hat{y}(t+j|t) \quad \leq 70 \;\; j = 2, \ldots, 16 \\ 5 \leq \quad & u(t+j|t) \quad \leq 100 \;\; j = 0, \ldots, 11 \\ -20 \leq \quad & \Delta u(t+j|t) \quad \leq 20 \;\; j = 0, \ldots, 11 \end{aligned}$$

Note that in the output restrictions the effect of the uncertainty has to be considered.

With the proposed strategy several experiments with setpoint changes and disturbance rejection have been made. In the tracking experiment (see Fig. 5) the values used for the setpoint are different enough to result in control actions nearly throughout the entire valid interval. It is noteworthy that in the first setpoint change no overshoot appears, although the controller reaction is quite fast. After the second setpoint change a small overshoot (of approximately -0.35°C), totally justified by the nonlinearity of the process, can be observed. In steady state, the output practically remains on the reference, whereas the control signal does not present significant oscillations. The small changes in the control action (smaller than 2%) are necessary to hold the output on the reference due to the variations in the generated heat and the cold water temperature.

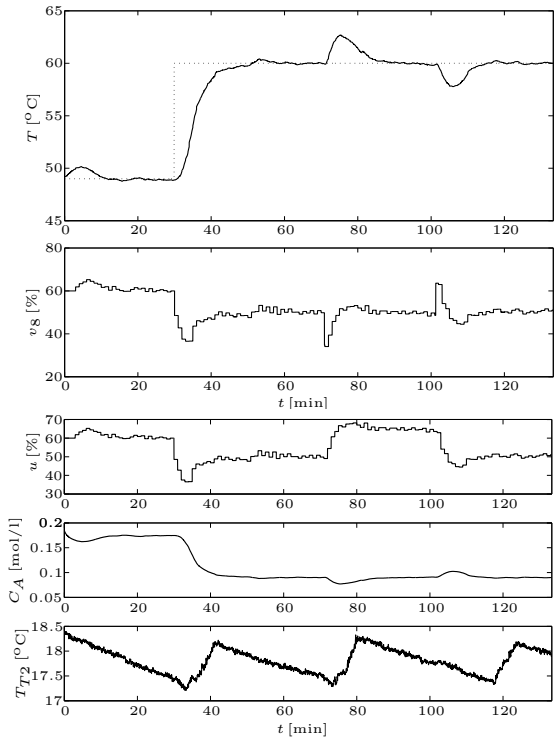

Fig. 6. Experiment with input disturbance rejection. From top to bottom: Tank temperature (T), valve opening(v_8), controller output (u), reagent concentration (C_A) and cold water temperature (T_{T2}).

In second place, two experiments with disturbance rejection are presented. As disturbances the feeding flow F_f, see equation (14), and the opening of the valve v_8 have been chosen. Figure 6 shows the results of the experiment with disturbance rejection in the system input. In this experiment, after $t = 71\,\text{min}$ a constant disturbance in the entrance $\Delta v_8 = 15\,\%$ is applied. As can be observed in the figure the controller totally rejects the disturbance in less than 20 minutes, without significant oscillations neither in the output nor in the control action. The same behavior can be seen after the disappearance of the disturbance in $t = 101\,\text{min}$. At the beginning of this experiment the pilot plant was far from its equilibrium point which explains that the output does not follow immediately the reference in the first sampling periods.

The third experiment is shown in Fig. 7. In this experiment an additive disturbance in the feeding of $\Delta F_f = 0.0125\,\text{l/s}$, which corresponds to an error of $25\,\%$, has been applied. As in the previous case, the disturbance is rejected completely in less than 20 minutes, although in this case a small and slow overshoot with an error inferior to 0.2°C can be observed.

Finally, it is important to mention that the calculation of the control signal took place without problems within the

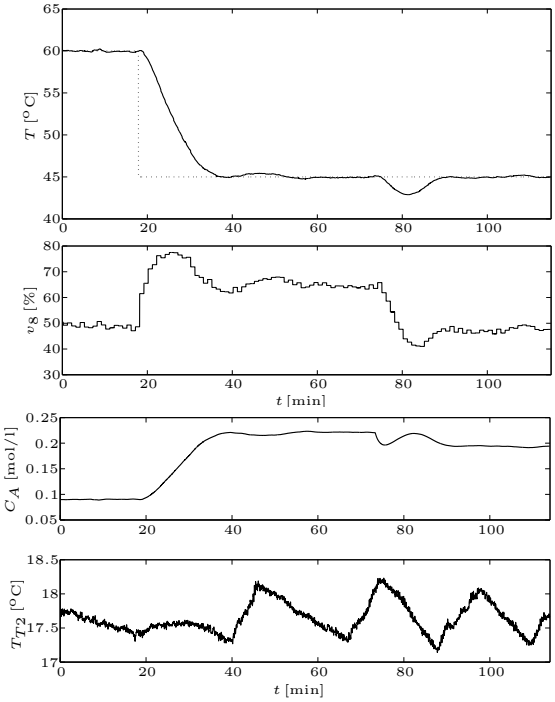

Fig. 7. Experiment with disturbance rejection in the feed flow. From top to bottom: Tank temperature (T), valve opening(v_8), reagent concentration (C_A) and cold water temperature (T_{T2}).

chosen sampling time (60 seconds). During the experiments the average computation time was 9.54 seconds, with a maximum of 16.3 seconds and a minimum of 5.32 seconds. The required computacional effort is quite low considering the used horizons and the low computational power of the computer. For a more exhaustive analysis of the computational cost of the upper bound or the calculation of the control signal see [7].

To compare the obtained results a reference tracking experiment with a linear predictive controller was carried out. The used controller was a GPC based on the model (17). For the horizons and the weighting function the same values as in the case of the MMMPC have been used ($N = 15$, $N_u = 12$, $Q_j = 1$, $R_j = 2$). With the two controllers based on the same discrete model and using the same parameters, the obtained results can be compared directly (see Fig. 8). As can be seen in the figure, the GPC exhibits significant temperature oscillations after setpoint changes. The direct comparison of the results show that the MMMPC stabilises the tank temperature in a more efficient way with less oscillations in the control action.

VI. Conclusions

In this paper the application of a MMMPC based on the upper bound of the worst case cost to a laboratory process has been presented. The process is a pilot plant where an exothermic chemical reaction is simulated generating the reaction heat by means of an electrical resistance.

The results showed a good system behavior and the stabilization of the plant temperature around the operation point. After changes in the desired reference the controller

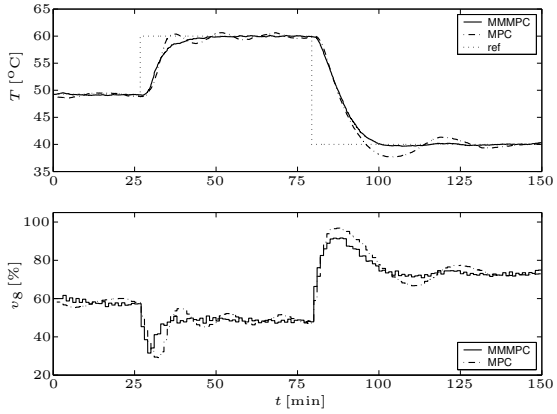

Fig. 8. Reference tracking results of the MMMPC and the GPC. From top to bottom: Tank temperature (T), valve opening (v_8).

quickly compensates the error between output and reference. Furthermore, experiments with disturbances have been made, showing the MMMPC its capacity to compensate errors caused by the disturbances.

The application shown in this work joins the small number of MMMPC applications reported in specialized literature. The low computational requirements of the proposed control strategy allowed the use of appropriate sampling times (approximately 15 times smaller than the typical time constant of the system) and realistic prediction and control horizons (15 and 12, respectively).

References

[1] E.F. Camacho and C. Bordóns, *Model Predictive Control*, Second edition, Springer-Verlag, 2004.

[2] J.H. Lee and Zhenghong Yu, "Worst-case formulations of model predictive control for systems with bounded parameters", *Automatica*, vol. 33, no. 5, pp 763-781, 1997.

[3] Y.H. Kim and W.H. Kwon, "An application of min-max generalized predictive control to sintering processes", *Control Engineering Practice*, vol. 6, pp 999-1007, 1998.

[4] D.R. Ramírez, M.R. Arahal and E.F. Camacho, "Min-Max Predictive Control of a Heat Exchanger using a Neural Network Solver", *IEEE Trans. on Control Systems Technology*, vol. 12, no. 5, pp 776-786, 2004.

[5] A. Bemporad, F. Borrelli and M. Morari, "Min-max Control of Constrained Uncertain Discrete-Time Linear Systems", *IEEE Transactions on Automatic Control*, vol. 48, no. 9, pp 1600-1606, 2003.

[6] P. Tøndel, T.A. Johansen and A. Bemporad, "Evaluation of piecewise affine control via binary search tree", *Automatica*, vol. 39, no. 5, pp 945-950, 2003.

[7] D.R. Ramirez, T. Alamo, E.F. Camacho and D. Muñoz de la Peña, "Min-Max MPC based on a computationally efficient upper-bound of the worst case cost", *Journal of Process Control*, vol. 16, pp 511-519, 2006.

[8] J.K. Gruber and C. Bordons, "Control Predictivo no Lineal Basado en Modelos de Volterra. Aplicación a una Planta Piloto", *Revista Iberoamericana de Automática e Informática Industrial (in spanish)*, vol. 4, no. 3, pp 34-45, 2007.

[9] D.Q. Mayne, "Control of Constrained Dynamic Systems", *European Journal of Control*, vol. 7, pp 87-99, 2001.

[10] F. Szeifert, T. Chovan and L. Nagy, "Process dynamics and temperature control of fed-batch reactors", *Computers & Chemical Engineering*, vol. 19, no. 1, pp 447-452, 1995.

[11] L.O. Santos, P.A.F.N.A. Afonso, J.A.A.M. Castro, N.M.C. Oliveira and L.T. Biegler, "On-line implementation of nonlinear MPC: an experimental case study", *Control Engineering Practice*, vol. 9, no. 8, pp 847-857, 2001.

[12] J.E. Normey-Rico and E.F. Camacho, *Control of Dead-Time Processes*, Springer-Verlag, 2007.