2008 American Control Conference
Westin Seattle Hotel, Seattle, Washington, USA
June 11-13, 2008

Target Motion Analysis Based on RF Power and Doppler Measurements

Guoxiang Gu[§], Meir Pachter[†], Phil R. Chandler[‡], and Corey J. Schmacher[‡]

Abstract— **This paper investigates target motion analysis (TMA) based on noisy measurements of power and Doppler from a radio frequency (RF) emitter. The research is motivated by the fact that Doppler measurements for RF signals are much easier to obtain and power measurements are always available. More importantly power and Doppler measurements complement each other in terms of the Fisher information content. However both these measurements are highly nonlinear functions of the TMA parameters, specified uniquely by its initial position and velocity. In addition direct use of these measurements in TMA introduces the nuisance parameters leading to poor numerical conditioning because of the very high source frequency, and very small power proportional constant for RF signals. Hence ratios of the neighboring measurements are used to eliminate the nuisance parameters and provide normalization of the measurement equations. An iterative least-squares (LS) algorithm is developed, and is shown to be effective through a numerical example.**

1. Introduction

Target motion analysis (TMA) has been investigated in the past several decades that assumes straight line motion for the target with constant speed because the target is oblivious of its being tracked. The TMA aims at estimation of the initial position and velocity of the target that completely determine the motion of the target. Most of the work in this research area has focused on bearing-only measurements [1], [3], [6], [9], [10]. There are also a few papers on TMA based on both bearing and Doppler [4], [8], termed as Doppler-bearing. In this paper we study the same TMA problem assuming that the target transmits ratio frequency (RF) signals, and both power and Doppler of the RF signal are measured during the time interval of observation. This problem has not been investigated in the past in the best of our knowledge, and is motivated by the fact that the sensing device is considerably simpler than those of Doppler-bearing measurements. A suitably equipped RF receiver suffices, and power measurements are always available with RF signals. More importantly power and Doppler measurements complement each other that can be concluded by analyzing the corresponding Fisher information matrix whose inverse is the much celebrated Cramér-Rao lower bound. Due to the space limit, such an analysis is not included in this conference version.

The research is supported in part by The US Air Force.

[§] Department of Electrical and Computer Engineering, Louisiana State University, Baton Rouge, LA 70803-5901.

[†] Department of Electrical Engineering, Air Force Institute of Technology, Dayton, OH 45433-7531.

[‡] AFRL/VACA, Wright Laboratory, Wright-Patterson Air Force Base, OH 45433-7531.

On the other hand the simpler sensing device also brings in the difficulty for signal processing because both power and Doppler measurements are highly nonlinear functions of the TMA parameters, and it is not possible to obtain quasi-linear models as in the case of bearing-only, or Doppler-bearing. In addition TMA based on power and Doppler measurements gives rise to the problem of nuisance parameters. In the former, the power of an RF signal is inversely proportional to the distance square it travels that introduces the proportional constant, denoted by C. Hence TMA is not possible without knowing C. On the other hand, the Doppler frequency shifts induced by the motion of the target are not directly available. The source frequency, denoted by f_s, is needed in order to obtain the Doppler frequency shifts. One may include both C and f_s as parameters in estimation. However because $C > 0$ is very small, and f_s is very large that is the RF carrier frequency, the nuisance parameters complicate further the nonlinear estimation of the TMA parameters. Indeed they induce divergence of the iterative least-squares (LS) algorithm, widely used in nonlinear estimation, and introduce the numerical conditioning problem in applying the iterative LS algorithm. Moreover C is likely to fluctuate in accordance with environments, and f_s may drift near the nominal carrier frequency. The constant assumptions for C and f_s do not truly hold in practice. These problem pose significant challenges for the TMA problem to be investigated in this paper.

It is the simplicity of the sensing device, complementary nature of the information content, as well as the nonlinearity and numerical difficulty in TMA based on power and Doppler measurements that provide us the impetus for developing a feasible solution. Instead of using the power and Doppler measurements directly, the ratios of the neighboring measurements are employed to eliminate the nuisance parameters. Even though C and f_s may vary considerably during the whole time interval of observation, their variations are very small in a single sampling period that suppress the detrimental effect on TMA by variations of C and f_s. More importantly the numerical conditioning problem induced by C and f_s is weakened significantly. The iterative LS algorithm is convergent for a very large region of the parameter space for a typical application example. The only drawback brought in by ratios of measurements is that the measurement noises become correlated, and the corresponding covariance matrix is tri-diagonal, that is the price to pay with the method of ratios. Our results shed light on using power and Doppler measurements in TMA, and provide an application example for nonlinear estimation.

978-1-4244-2079-7/08/$25.00 ©2008 AACC.

2. PROBLEM FORMULATION AND MOTIVATIONS

Denote positions at time t_k by (T_{x_k}, T_{y_k}) for the target, and by (U_{x_k}, U_{y_k}) for the UAV sensor with $0 \leq k < N$. For TMA, the target is a moving emitter, and assumed to have constant speed and heading. Thus the trajectory of the target is parameterized by the initial position (x_0, y_0) and the velocity (v_x, v_y). Without loss of generality, $t_0 = 0$ is taken. It follows that at time t_k,

$$T_{x_k} = x_0 + v_x t_k, \quad T_{y_k} = y_0 + v_y t_k \tag{1}$$

The positions and velocities of the UAV sensor at different time samples are assumed to be known. The only unknown is the parameter vector of the target:

$$\theta = \begin{bmatrix} x_0 & y_0 & v_x & v_y \end{bmatrix}^T \tag{2}$$

to be estimated based on power and Doppler measurements.

The power measurements from the RF emitter or target are assumed to be of the form:

$$\widehat{P}_k = P_k + \eta_k, \quad P_k = \frac{C}{R_k^2} \tag{3}$$

$$R_k = \sqrt{(T_{x_k} - U_{x_k})^2 + (T_{y_k} - U_{y_k})^2} \tag{4}$$

where $0 \leq k < N$. The measurement noise η_k is assumed to be uncorrelated Gauss with mean zero and variance $\sigma_{p_k}^2$. It is assumed that the variance of $\frac{\eta_k}{P_k}$ is a constant that is the inverse of the signal-to-noise (SNR), denoted by SNR_p.

Clearly $R_k = R_k(\theta)$ that is a function of the parameter vector in light of (1). On the other hand, the measurements of the Doppler shifted frequency at time t_k are of the form

$$\widehat{f}_k = f_k + \zeta_k, \quad f_k = f_s\left(1 - \frac{v_k}{\mathrm{c}}\right) \tag{5}$$

where f_s is the carrier frequency from the target source, c is the speed of light, and v_k is the (combined) projected velocity to the radial direction from the UAV sensor to the target. Note that f_k is the Doppler shifted frequency at time k, and ζ_k is the measurement noise, assumed to be uncorrelated Gauss with mean zero and constant variance σ_f^2. The expression of the radial velocity v_k is given by

$$v_k = \frac{(v_x - \dot{U}_{x_k})(T_{x_k} - U_{x_k}) + (v_y - \dot{U}_{y_k})(T_{y_k}k - U_{y_k})}{\sqrt{(T_{x_k} - U_{x_k})^2 + (T_{y_k} - U_{y_k})^2}} \tag{6}$$

where $\dot{U}_{x_k}$ and $\dot{U}_{y_k}$ denote the velocities of the UAV sensor along x and y axis, respectively, that are assumed to be known. It is thus easy to obtain the expression for $\widehat{f}_k$, the Doppler measurements at time t_k in (5) by substituting the expression in (6).

Remark 2.1: Two-dimensional space is assumed for both position and velocity due to simplicity of notations, but all our results apply to the case of three-dimensional space without difficulty in derivation that is contrast to bearing-only, and Doppler-bearing measurements.

As mentioned earlier, TMA has been investigated by many researchers based on bearing-only, and on Doppler-bearing measurements. The use of both bearing and Doppler measurements require an additional sensing device such as antenna arrays. We are motivated to develop simpler and cheaper sensing devices by removing the bearing sensor that requires employment of antenna arrays. On the other hand power measurements are always available when Doppler measurements are taken. Therefore the sensing device requires no more than an RF receiver. The question is whether or not power and Doppler measurements provide similar information content to those of Doppler-bearing. Our analysis on the corresponding FIM yields more than an affirmative answer. However due to the space limit, the analysis is omitted for this conference version.

3. BATCHED ESTIMATION FOR TMA

The use of both bearing and Doppler measurements has helped to improve the TMA performance [4], [8] compared to that of bearing-only. However its primary focus is on sonar or acoustic applications where the signal propagation speed is much slower than the speed of light, and more importantly, the source frequency is rather modest that does not constitute a nuisance in estimation. In contrast the source frequency for a typical RF signal is much higher than the Doppler shifts induced by target and sensor that brings in the numerical conditioning problem, and thus become a nuisance. We begin with TMA based on power and Doppler measurements, and reveal its numerical difficulty in this particular nonlinear estimation problem.

TMA Estimation Based on Power Measurements

Recall the power measurements in (3) and (T_{x_k}, T_{y_k}) in (1). Denote

$$L_k = \begin{bmatrix} U_{x_k} \\ U_{y_k} \\ t_k U_{x_k} \\ t_k U_{y_k} \end{bmatrix}, \quad Q_k = \begin{bmatrix} 1 & 0 & t_k & 0 \\ 0 & 1 & 0 & t_k \\ t_k & 0 & t_k^2 & 0 \\ 0 & t_k & 0 & t_k^2 \end{bmatrix} \tag{7}$$

Then it can be verified that with $\rho_k = \sqrt{U_{x_k}^2 + U_{y_k}^2}$,

$$\begin{aligned} R_k^2 &= (x_0 + v_x t_k - U_{x_k})^2 + (y_0 + v_y t_k - U_{y_k})^2 \\ &= \theta^T Q_k \theta - 2L_k^T \theta + \rho_k^2 \end{aligned} \tag{8}$$

Clearly estimation of θ is not possible without knowing C. A simple way is to take C into one of the parameters in estimation by taking $\theta_p = \begin{bmatrix} \theta^T & C/C_0 \end{bmatrix}^T$ as the estimation parameter vector.

It is noted that R_k^2 in (8) can be written as

$$\begin{aligned} R_k^2 = & \left(\theta - \widehat{\theta}\right)^T Q_k \left(\theta - \widehat{\theta}\right) - 2\left(L_k - Q_k\widehat{\theta}\right)^T \left(\theta - \widehat{\theta}\right) \\ & + \left(\widehat{\theta}^T Q_k \widehat{\theta} - 2L_k^T \widehat{\theta} + \rho_k^2\right) \end{aligned} \tag{9}$$

for any given $\widehat{\theta}$. Next rewrite the measurement equations in (3) as

$$g_k(\theta) := R_k^2 - \frac{C}{\widehat{P}_k} = R_k^2 \frac{\eta_k}{\widehat{P}_k} \tag{10}$$

By ignoring the second order term and assuming $\eta_k \approx 0$ lead to

$$g_k(\theta) \approx -2(L_k - Q_k\widehat{\theta})^T \theta + \rho_k^2 - \widehat{\theta}^T Q_k \widehat{\theta} - \widehat{P}_k^{-1} C \approx 0 \tag{11}$$

By packing $A_p(\theta)$ and $B_p(\theta)$ as

$$\begin{aligned} A_p(\theta) &= \begin{bmatrix} 2(L_0 - Q_0\theta)^T & C_0\widehat{P}_0^{-1} \\ \vdots & \vdots \\ 2(L_{N'} - Q_{N'}\theta)^T & C_0\widehat{P}_{N'}^{-1} \end{bmatrix} \\ B_p(\theta) &= \begin{bmatrix} \rho_0^2 - \theta^T Q_0 \theta \\ \vdots \\ \rho_{N'}^2 - \theta^T Q_{N'}\theta \end{bmatrix}, \quad N' = N-1 \end{aligned}$$

there holds $A_p(\widehat{\theta})\theta_p \approx B_p(\widehat{\theta})$, yielding the iterative LS algorithm $\widehat{\theta}_p^{(i+1)} = D_i^{-1}N_i$ for $i = 0, 1, \cdots$, where

$$\begin{aligned} D_i &= A_p(\widehat{\theta}^{(i)})^T \Sigma_p(\widehat{\theta}^{(i)})^{-1} A_p(\widehat{\theta}^{(i)}) \\ N_i &= A_p(\widehat{\theta}^{(i)})^T \Sigma_p(\widehat{\theta}^{(i)})^{-1} B_p(\widehat{\theta}^{(i)}) \\ \Sigma_p(\theta) &\approx \mathrm{SNP}_p^{-1}\mathrm{diag}(R_0^4, R_1^4, \cdots, R_{N'}^4) \end{aligned}$$

initialized by some $\widehat{\theta}_p^{(0)}$. Because of the scaling, C/C_0 is not very small any more that helps to improve estimation. However any variation of C may still impact the estimation results negatively.

TMA Estimation Based on Doppler Measurements

Recall the measurements of Doppler shifted frequency in (5). Denote $\varphi_k = \begin{bmatrix} U_{x_k} & U_{y_k} & \dot{U}_{x_k} & \dot{U}_{y_k} \end{bmatrix}^T$, and

$$\begin{aligned} Q_{f_k} &= \begin{bmatrix} 0 & 0 & 0.5 & 0 \\ 0 & 0 & 0 & 0.5 \\ 0.5 & 0 & t_k & 0 \\ 0 & 0.5 & 0 & t_k \end{bmatrix} \\ S_f &= \begin{bmatrix} 0 & 0 & 0.5 & 0 \\ 0 & 0 & 0 & 0.5 \\ 0.5 & 0 & 0 & 0 \\ 0 & 0.5 & 0 & 0 \end{bmatrix} \end{aligned}$$

Then it is straightforward to verify that v_k in (6) can be written as

$$v_k = \frac{\theta^T Q_{f_k}\theta - \varphi_k^T(Q_{f_k} + S_f)\theta + \varphi_k^T S_f \varphi_k}{\sqrt{\theta^T Q_k \theta - 2L_k^T\theta + \rho_k^2}} \tag{12}$$

in light of the TMA assumption or $T_{x_k} = x_0 + v_x t_k$ and $T_{y_k} = y_0 + v_y t_k$. It follows that $\widehat{f}_k = f_k + \zeta_k$ with

$$f_k = f_s\left(1 - \frac{\theta^T Q_{f_k}\theta - \varphi_k^T(Q_{f_k} + S_f)\theta + \varphi_k^T S_f \varphi_k}{\mathrm{c}\sqrt{\theta^T Q_k \theta - 2L_k^T\theta + \rho_k^2}}\right) \tag{13}$$

The measurement equations for Doppler are more complex than those of power. Rewrite (5) as

$$h_k(\theta) := \mathrm{c}\left(\frac{\widehat{f}_k}{f_s} - 1\right) + v_k = \mathrm{c}\frac{\zeta_k}{f_s} \tag{14}$$

where v_k is given as in (12). Define $L_{f_k} := \frac{1}{2}(Q_{f_k} + S_f)\varphi_k$ for $0 \le k < N$. The above can be converted into

$$h_k(\theta) = h_k(\widehat{\theta}) + \left(\frac{\partial h_k(\widehat{\theta})}{\partial\theta}\right)^T\left(\theta - \widehat{\theta}\right) + \mathcal{O}(\|\theta - \widehat{\theta}\|^2) \approx 0 \tag{15}$$

by assuming that $\zeta_k \approx 0$, where

$$\frac{\partial h_k(\theta)}{\partial\theta} = \frac{2\left(Q_{f_k}\theta - L_{f_k}\right)}{R_k} - \frac{\left(Q_k\theta - L_k\right)v_k}{R_k^2} \tag{16}$$

Recall v_k in (12) and R_k in (8).

Let $\theta_f = \begin{bmatrix} \theta^T & \mathrm{c}f_s^{-1} \end{bmatrix}^T$. As in the power case, packing A_f and B_f as

$$\begin{aligned} A_f(\theta_f) &= \begin{bmatrix} \left(\frac{\partial h_0}{\partial\theta}\right)^T & \hat{f}_0 \\ \vdots & \vdots \\ \left(\frac{\partial h_{N'}}{\partial\theta}\right)^T & \hat{f}_{N'} \end{bmatrix} \\ B_f(\theta_f) &= \begin{bmatrix} \left(\frac{\partial h_0}{\partial\theta}\right)^T\theta + \widehat{f}_0\mathrm{c}f_s^{-1} - h_0(\theta) \\ \vdots \\ \left(\frac{\partial h_{N'}}{\partial\theta}\right)^T\theta + \widehat{f}_{N'}\mathrm{c}f_s^{-1} - h_{N'}(\theta) \end{bmatrix} \end{aligned}$$

yields $A_f(\widehat{\theta}_f)\theta_f \approx B_f(\widehat{\theta}_f)$ by taking $\zeta_k \approx 0$ for each k, leading to the iterative LS algorithm $\widehat{\theta}_f^{(i+1)} = D_i^{-1}N_i$ for $i = 0, 1, \cdots$, where

$$\begin{aligned} D_i &= A_f(\widehat{\theta}_f^{(i)})^T \Sigma_f^{-1} A_f(\widehat{\theta}_f^{(i)}) \\ N_i &= A_f(\widehat{\theta}_f^{(i)})^T \Sigma_f^{-1} B_p(\widehat{\theta}_f^{(i)}) \end{aligned}$$

and $\Sigma_f = \frac{\mathrm{c}^2\sigma_f^2}{f_s^2}I$, initialized by some $\widehat{\theta}_f^{(0)}$. Note that in the expression of $B_f(\theta_f)$,

$$\widehat{f}_k\mathrm{c}f_s^{-1} - h_k(\theta) = c - \frac{\theta^T Q_{f_k}\theta - \varphi_k^T(Q_{f_k} + S_f)\theta + \varphi_k^T S_f\varphi_k}{\sqrt{\theta^T Q_k\theta - 2L_k^T\theta + \rho_k^2}}$$

It is important to point out that $A_f(\theta_f)$ is badly conditioned, even though scaling of f_s is used. Because f_s is very large, and the Doppler frequency shifts induced by motions of the target and sensor are so small, $\widehat{f}_k$ are very large. It follows that $A_f(\theta)$ has only one large singular value with rest much smaller. That is, the discrepancy between the large and small singular values is very large that introduce the conditioning problem in iterative LS algorithm, that persists even if power measurements are jointly used in TMA estimation as described next.

TMA Estimation Based on Both Measurements

If both power and Doppler measurements are available, then the previous derivations can be combined. Denote I_n as identity matrix with size n, and

$$\begin{aligned} A(\theta_{pf}) &= \begin{bmatrix} A_p(\theta_p)\begin{pmatrix} I_4 \\ 0 \end{pmatrix} & A_p(\theta_p)\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \\ A_f(\theta)\begin{pmatrix} I_4 \\ 0 \end{pmatrix} & A_f(\theta)\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \end{bmatrix} \\ B(\theta) &= \begin{bmatrix} B_p(\theta_p) \\ B_f(\theta_f) \end{bmatrix}, \quad \theta_{pf} = \begin{bmatrix} \theta \\ C/C_0 \\ \mathrm{c}f_s^{-1} \end{bmatrix} \end{aligned}$$

Then $A(\widehat{\theta}_{pf})\theta_{pf} \approx B(\widehat{\theta}_{pf})$, leading to the iterative LS algorithm $\widehat{\theta}_{pf}^{(i+1)} = D_i^{-1}N_i$ for $i = 0, 1, \cdots$, where

$$\begin{aligned} D_i &= A(\widehat{\theta}_{pf}^{(i)})^T \Sigma(\widehat{\theta}_{pf}^{(i)})^{-1} A(\widehat{\theta}_{pf}^{(i)}) \\ N_i &= A(\widehat{\theta}_{pf}^{(i)})^T \Sigma(\widehat{\theta}_{pf}^{(i)})^{-1} B(\widehat{\theta}_{pf}^{(i)}) \\ \Sigma(\theta_{pf}) &= \mathrm{diag}(\Sigma_p(\theta), \Sigma_f) \end{aligned}$$

initialized by some $\theta_{pf}^{(0)}$.

Often the estimation performance is assessed by the Cramér-Rao lower bound. Although the FIMs for power and Doppler are presented in the previous section, more convenient forms for MATLAB implementation are given as follows:

$$\mathrm{FIM}_p = 4\mathrm{SNR}_p \sum_{k=0}^{N-1} \frac{(Q_k\theta - L_k)(Q_k\theta - L_k)^T}{(\theta^T Q_k \theta - 2L_k^T\theta + \rho_k^2)^2} \tag{17}$$

On the other hand the FIM for TMA estimation based on N Doppler measurements is given by

$$\mathrm{FIM}_f \approx \frac{f_s^2}{\mathrm{c}^2\sigma_f^2} \sum_{k=0}^{N-1} \left[\frac{\partial h_k(\theta)}{\partial \theta}\right] \left[\frac{\partial h_k(\theta)}{\partial \theta}\right]^T \tag{18}$$

where the partial derivative of $h_k(\theta)$ is given in (16). If both the power and Doppler measurements are employed for estimation, then the corresponding FIM is the sum of FIM_p and FIM_f. The Gauss assumption is crucial in computing the above FIMs.

4. Batched Estimation with Ratios of Measurements

While the iterative LS algorithm has been widely used in nonlinear estimation, its direct use as in the previous section fails to give good results because of the nuisance parameters C and f_s of which C is very small easily leading to negative estimate due to the presence of noise, and f_s is very large leading to the poor conditioning number for the corresponding A-matrix. For these reasons, ratios of the measurements are introduced to remove the unknown C and f_s. Another motivation for using ratios is the possible fluctuation of C due to changes of the environment, and existence of drifting in carrier frequency f_s.

TMA Based on Ratios of Power Measurements

As discussed earlier, it is more sensible to work with the ratios of the power measurements taken in the neighboring time samples:

$$\widehat{\mu}_k = \frac{\widehat{P}_k}{\widehat{P}_{k-1}} = \frac{1 + \frac{\eta_k}{P_k}}{1 + \frac{\eta_{k-1}}{P_{k-1}}} \times \frac{R_{k-1}^2}{R_k^2} \tag{19}$$

for $k = 1, 2, \cdots, N-1$. The expression in (19) can be alternatively written as

$$\widehat{R}_{k-1}^2 := R_{k-1}^2 \left(1 + \frac{\eta_k}{P_k}\right) = R_k^2 \widehat{\mu}_k \left(1 + \frac{\eta_{k-1}}{P_{k-1}}\right) \tag{20}$$

Using the notations in the previous section, (19) can be rewritten as

$$\begin{aligned} \widehat{g}_k(\theta) &:= \left(\theta^T Q_{k-1}\theta - 2L_{k-1}^T\theta + \rho_{k-1}^2\right) \\ &\quad - \widehat{\mu}_k \left(\theta^T Q_k \theta - 2L_k^T\theta + \rho_k^2\right) \\ &= \widehat{\mu}_k \left(\theta^T Q_k \theta - 2L_k^T\theta + \rho_k^2\right) \eta_{k-1}/P_{k-1} \\ &\quad - \left(\theta^T Q_{k-1}\theta - 2L_{k-1}^T\theta + \rho_{k-1}^2\right) \eta_k/P_k \end{aligned} \tag{21}$$

Let $\widehat{\theta}$ be an estimate of θ. Then

$$\begin{aligned} \widehat{g}_k(\theta) &= \left(\theta - \widehat{\theta}\right)^T \Delta\widehat{Q}_k \left(\theta - \widehat{\theta}\right) \\ &\quad - 2\left(\Delta\widehat{L}_k - \Delta\widehat{Q}_k\widehat{\theta}\right)^T \left(\theta - \widehat{\theta}\right) \\ &\quad + \widehat{\theta}^T \Delta\widehat{Q}_k \widehat{\theta} - 2\Delta\widehat{L}_k^T\widehat{\theta} + \Delta\widehat{\rho}_k \end{aligned} \tag{22}$$

where $\Delta\widehat{Q}_k$, $\Delta\widehat{L}_k$, and $\Delta\widehat{\rho}_k$ are given respectively by

$$\Delta\widehat{Q}_k = Q_{k-1} - \widehat{\mu}_k Q_k, \quad \Delta\widehat{L}_k = L_{k-1} - \widehat{\mu}_k L_k \tag{23}$$

and $\Delta\widehat{\rho}_k = \rho_{k-1}^2 - \widehat{\mu}_k\rho_k^2$. If SNR_p is adequately large, then $\widehat{g}_k(\theta, \eta) \approx 0$, and thus

$$\begin{aligned} 2\left(\Delta\widehat{L}_k - \Delta\widehat{Q}_k\widehat{\theta}\right)^T \theta &\approx \Delta\widehat{\rho}_k - \widehat{\theta}^T \Delta\widehat{Q}_k \widehat{\theta} \\ &\quad + \left(\theta - \widehat{\theta}\right)^T \Delta\widehat{Q}_k \left(\theta - \widehat{\theta}\right) \end{aligned} \tag{24}$$

Recall $N' = N - 1$. Pack for $k = 1, 2, \cdots, N-1$ with

$$\begin{aligned} L_p(\widehat{\theta}) &= 2\begin{bmatrix} \Delta\widehat{L}_1^T - \widehat{\theta}^T\Delta\widehat{Q}_1 \\ \vdots \\ \Delta\widehat{L}_{N'}^T - \widehat{\theta}^T\Delta\widehat{Q}_{N'} \end{bmatrix} \\ \psi_p(\widehat{\theta}) &= \begin{bmatrix} \Delta\widehat{\rho}_1 - \widehat{\theta}^T\Delta\widehat{Q}_1\widehat{\theta} \\ \vdots \\ \Delta\widehat{\rho}_{N'} - \widehat{\theta}^T\Delta\widehat{Q}_{N'}\widehat{\theta} \end{bmatrix} \\ q(\theta, \widehat{\theta}) &= \begin{bmatrix} \left(\theta - \widehat{\theta}\right)^T \Delta\widehat{Q}_1 \left(\theta - \widehat{\theta}\right) \\ \vdots \\ \left(\theta - \widehat{\theta}\right)^T \Delta\widehat{Q}_{N'} \left(\theta - \widehat{\theta}\right) \end{bmatrix} \end{aligned}$$

A set of quadratic equations are obtained as follows:

$$L_p(\widehat{\theta})\theta \approx \phi(\widehat{\theta}), \quad \phi(\widehat{\theta}) = \psi_p(\widehat{\theta}) + q(\theta, \widehat{\theta}) \tag{25}$$

By ignoring the quadratic term, iterative LS can be used to compute iteratively the estimate as $\widehat{\theta}_{i+1} = D_i^{-1}N_i$ for $i = 0, 1, \cdots$, where

$$\begin{aligned} D_i &= L_p(\widehat{\theta}_i)^T \widehat{\Sigma}_p(\widehat{\theta}_i)^{-1} L_p(\widehat{\theta}_i) \\ N_i &= L_p(\widehat{\theta}_i)^T \widehat{\Sigma}_p(\widehat{\theta}_i)^{-1} \psi_p(\widehat{\theta}_i) \end{aligned}$$

initialized by some θ_0 where $\Sigma_p(\theta)$ is the covariance matrix for the noise term ignored in (24) that is tri-diagonal with diagonal element $\widehat{\mu}_k^2 R_k^4 + R_{k-1}^4$, and off diagonal element $\widehat{\mu}_{k+1} R_{k-1}^2 R_{k+1}^2$ on right for the kth row.

TMA Based on Ratios of Doppler Measurements

It is also sensible to bypass f_s in estimation of the parameter vector θ, leading to the following ratios of the

Doppler measurements in the neighboring time samples:

$$\widehat{\nu}_k = \frac{\widehat{f}_k}{\widehat{f}_{k-1}} = \frac{1+\frac{\zeta_k}{f_k}}{1+\frac{\zeta_{k-1}}{f_{k-1}}} \times \frac{\left(1-\frac{v_k}{\mathrm{c}}\right)}{\left(1-\frac{v_{k-1}}{\mathrm{c}}\right)} \tag{26}$$

where $1 \le k < N$ and v_k is the same as in (6). The above can be converted into

$$\widehat{h}_k(\theta) = \frac{\zeta_k}{f_k}\left(1-\frac{v_k}{\mathrm{c}}\right) - \frac{\widehat{\nu}_k\zeta_{k-1}}{f_{k-1}}\left(1-\frac{v_{k-1}}{\mathrm{c}}\right) \tag{27}$$

The noise term $\frac{\zeta_k}{f_k}$ has variance $\frac{\sigma_f^2}{f_s^2}$ approximately. If σ_f is adequately small, then $\widehat{h}_k(\theta,\zeta) \approx 0$.

Clearly $\widehat{h}_k(\cdot)$ is also a nonlinear function of θ and involves higher order terms than those of $\widehat{g}(\cdot)$ as in the case of power measurements. If the iterative LS algorithm is employed to estimate θ, the Jacobian matrix needs to be evaluated. It can be obtained through some calculation that

$$\mathrm{c}\frac{\partial \widehat{h}_k}{\partial \theta} = \frac{\partial h_k(\theta)}{\partial \theta} - \nu_k \frac{\partial h_{k-1}(\theta)}{\partial \theta} \tag{28}$$

where the partial derivative of $h_k(\theta)$ is given as in (16). Pack $\{\widehat{h}_k(\cdot,\cdot)\}$ into a column vector $\underline{\widehat{h}}(\theta,\zeta)$. Let $L_f(\theta)$ be the Jacobian of $\underline{\widehat{h}}(\theta,\zeta)$, and

$$L_f(\theta) = \mathrm{c}\begin{bmatrix} \frac{\partial \widehat{h}_1}{\partial \theta} & \frac{\partial \widehat{h}_2}{\partial \theta} & \cdots & \frac{\partial \widehat{h}_{N-1}}{\partial \theta} \end{bmatrix}^T \tag{29}$$

In this case the column vector of the functions $\underline{h}(\theta,\zeta)$ has the expansion

$$\mathrm{c}\underline{\widehat{h}}(\theta,\zeta) = \mathrm{c}\underline{\widehat{h}}(\widehat{\theta},\zeta) + L_f(\widehat{\theta})(\theta-\widehat{\theta}) + \mathcal{O}(\mathrm{c}\|\theta-\widehat{\theta}\|^2) \approx 0 \tag{30}$$

The scaling by c for RF signal is necessary that is the speed of light. Indeed there holds now

$$\mathrm{c}\widehat{h}_k(\widehat{\theta}) = (1-\widehat{\nu}_k)\mathrm{c} + \widehat{\nu}_k v_{k-1} - v_k \tag{31}$$

By ignoring the high order terms, there holds approximately the linear equations

$$L_f(\widehat{\theta})\theta \approx \psi_f(\widehat{\theta}), \qquad \psi_f(\widehat{\theta}) = L_f(\widehat{\theta})\widehat{\theta} - \mathrm{c}\underline{\widehat{h}}(\widehat{\theta},\zeta) \tag{32}$$

Hence the iterative LS algorithm can be obtained as $\widehat{\theta}_{i+1} = D_i^{-1}N_i$ for $i = 0, 1, \cdots$, where

$$\begin{aligned} D_i &= L_f(\widehat{\theta}_i)^T \widehat{\Sigma}_f(\widehat{\theta}_i)^{-1} L_f(\widehat{\theta}_i) \\ N_i &= L_f(\widehat{\theta}_i)^T \widehat{\Sigma}_f(\widehat{\theta}_i)^{-1} \psi_f(\widehat{\theta}_i) \end{aligned}$$

that is initialized by some $\widehat{\theta}_0$. The covariance matrix $\frac{f_s^2}{\mathrm{c}^2\sigma_f^2}\widehat{\Sigma}_f(\theta)$ is tri-diagonal as well with diagonal element $\left(1-\frac{v_k}{\mathrm{c}}\right)^2 + \widehat{\nu}_k^2\left(1-\frac{v_{k-1}}{\mathrm{c}}\right)^2$, and off diagonal element $-\widehat{\nu}_{k+1}\left(1-\frac{v_{k+1}}{\mathrm{c}}\right)^2$ on right for the kth row.

TMA Based on Ratios of Both Measurements

If both power and Doppler measurements are available, then there holds $L(\widehat{\theta})\theta \approx \psi(\widehat{\theta})$ where

$$L(\widehat{\theta}) = \begin{bmatrix} L_p(\widehat{\theta}) \\ L_f(\widehat{\theta}) \end{bmatrix}, \qquad \psi(\widehat{\theta}) = \begin{bmatrix} \psi_p(\widehat{\theta}) \\ \psi_f(\widehat{\theta}) \end{bmatrix} \tag{33}$$

and $\widehat{\theta}$ an estimate close to the true parameter vector. The iterative LS algorithm leads to

$$\widehat{\theta}_{i+1} = \left[L(\widehat{\theta}_i)^T \widehat{\Sigma}(\widehat{\theta}_i)^{-1} L(\widehat{\theta}_i)\right]^{-1} L(\widehat{\theta}_i)^T \widehat{\Sigma}(\widehat{\theta}_i)^{-1} \psi(\widehat{\theta}_i) \tag{34}$$

initialized by some $\widehat{\theta}_0$ where $\widehat{\Sigma}(\theta) = \mathrm{diag}\left(\widehat{\Sigma}_p(\theta), \widehat{\Sigma}_f(\theta)\right)$.

5. SIMULATION STUDIES

In our simulation studies, the target transmits RF signals at 1.9 GHz over a time interval of two minutes. The target is oblivious of its being tracked and thus moves along a straight line at constant speed of 90 km per hour or 25 meters per second. The small aerial vehicle (SAV) sensor flies at 180 km per hour or 50 meters per second. The following figure shows the trajectories of the target and the sensor.

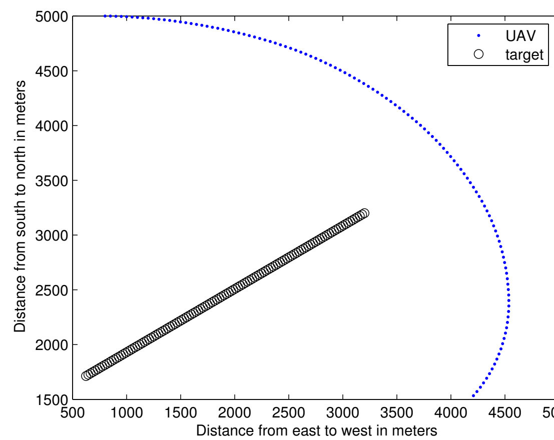

Fig. 1 Trajectories of the target and the SAV

The true parameter vector for the target is given by $\theta = \begin{bmatrix} 3200 & 3200 & -21.65 & -12.5 \end{bmatrix}$. The average distance between the target and the sensor is 2.72 km with maximum 3.63 km and minimum 1.94 km. The power and Doppler measurements are taken every second, and thus a total of 120 samples are obtained. The Cramér-Rao lower bounds (CRLBs) are computed for the above trajectories that lead to the following root mean-squared (RMS) errors from the lower bounds for unbiased estimators where $\mathrm{SNR}_p = 40$ dB and $\sigma_f^2 = 1.9^2$:

Table 1: RMS values for CRLBs in meters

Parameters	Power only	Doppler only	Power & Doppler
x_0	15.7821	15.3020	3.9263
y_0	15.6618	15.7205	3.8172
v_x	0.1252	0.3150	0.0361
v_y	0.4294	0.2005	0.1025

For the case of $\mathrm{SNR}_p = 20$ dB and $\sigma_f^2 = 1.9^2$, the following table is obtained:

Table 2: RMS values for CRLBs in meters

Parameters	Power only	Doppler only	Power & Doppler
x_0	157.8205	15.3020	8.2231
y_0	156.6182	15.7205	8.3520
v_x	1.2518	0.3150	0.1559
v_y	4.2940	0.2005	0.1340

For the case of $\mathrm{SNR}_p = 40$ dB and $\sigma_f^2 = 19^2$, the following table is obtained:

Table 3: RMS values for CRLBs in meters

Parameters	Power only	Doppler only	Power & Doppler
x_0	15.7821	153.0203	14.6164
y_0	15.6618	157.2053	14.4891
v_x	0.1252	3.1496	0.1166
v_y	0.4294	2.0049	0.3973

Based on the CRLBs, it is reasonable to investigate the case of $\mathrm{SNR}_p = 40$ dB and $\sigma_f^2 = 1.9^2$. It is noted that small σ_f^2 for the Doppler measurements holds because of the advanced RF technology. It is also noted that the maximum Doppler shift induced by the moving target is only 158.4 Hz, compared to the standard deviation for the Doppler measurement error that is 1.9 Hz. In addition there is a drifting error in the carrier frequency close to 2.5 Hz over the 2 minutes time interval to be explained later.

For the power measurements in (3), we use the model of power loss from [5] in the case of 1.9 GHz carrier frequency that yields $C = C_0 := (\lambda/4\pi)^2 = 1.5766 \times 10^{-4}$ using $f_s = 1.9$ GHz.

For the simulation studies, we first carried out estimation based on power only. It turns out that the iterative LS algorithm fails to compute meaningful estimates with $\mathrm{SNR}_p = 40$ dB even if the initial guess of the parameter vector is close to the true parameter vector. If $\mathrm{SNR}_p = 60$ dB is used, then it gives more accurate estimates, though still far from what predicted by the Cramér-Rao lower bounds, but only under the condition that the initial parameter vector is close to the true parameter.

On the other hand, if estimation based on Doppler only is carried out with $\sigma_f^2 = 1.9^2$ and initial parameter vector $(2200, 2700, -16, -8)$, the RMS estimation errors (averaged over 100 runs) are: $(36.2257, 16.1438, 0.4168, 0.5940)$.

For estimation based on both power and Doppler measurements where $\mathrm{SNR}_p = 40$ dB and $\sigma_f^2 = 1.9^2$ are used, the RMS errors (200 runs) are: $(6.2987, 7.6909, 0.1192, 0.1304)$, which are greater than though, but close to the corresponding CRLB RMS values. The interesting observation is that the iterative LS algorithm converges for most of the initial parameter vectors simulated unless the initial parameter vector is too far away from the true one. The above RMS errors use the initial parameter vector $(4900, 1200, 0, 0)$ that is close to the reality. The simulation studies suggest that the use of both power and Doppler measurements complement each other and help to overcome divergence problem even when initial parameter vectors are relatively far away from the true parameter vector.

6. Conclusion

This report investigated TMA problem based on measurements of power and Doppler. The associated FIMs are analyzed and compared with those based on Doppler-bearing measurements to make the case in using power and Doppler measurements for localization and tracking of the ground moving target (GMT). It is a fact that power and Doppler are complementary to each other in estimation of the position and velocity of the GMT. The research work conducted in this paper is also motivated by the simplicity and low cost of the sensing device for power and Doppler measurements. However the development of the TMA tools based on power and Doppler measurements also encounters the difficulties. One of them is the nuisance parameters that may change with respect to time; The other is the numerical difficulty due to much higher source frequency than the Doppler frequency induced by the motions of the target and the sensor. To overcome these difficulties, normalization via ratios of the measurements over neighboring time samples is employed based on which the iterative LS algorithm is developed. It is interesting to observe from the simulation example that the iterative LS algorithm has a much larger region of convergence than the case without using the ratios. In fact our results show that the RMS error is close to what predicted by the Cramér-Rao lower bound. Due to the space limit, our simulation results for the case where variation of constant C and drifting of f_s are skipped. The interested readers can request for the complete report [7] [1]. It needs to be mentioned, though, that the simulation results are much worse when variations of power constant C and source frequency f_s are involved that is currently under our investigation.

References

[1] V.J. Aidala and S.E. Hammel, "Utilization of modified polar coordinates for bearing-only tracking," *IEEE Trans. Automat. Contr.*, vol. 28, pp. 283-294, March 1983.

[2] B.D.O. Anderson and J.B. Moore, *Optimal Filtering*, Prentice-Hall, Englewood Cliffs, N.J., 1979.

[3] J.P. Le Cadre and C. Jauffret, "On the convergence of iterative methods for bearing-only tracking," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 34, pp. 179-193, Jan. 1998.

[4] Y.T. Chan and S.W. Rudnicki, "Bearing-only and Doppler-bearing tracking using instrumental variables," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 28, pp. 1076-1083, Oct. 1992.

[5] V. Erceg, L.J. Greensteein, S.Y. Tjandra, S.R. Parkoff, A. Gupta, B. Kulic, A.A. Julius, and R. Bianchi, "An empirically based path loss model for wireless channels in suburban environments," *IEEE J. Select. Areas Commun.*, vol. 17, pp. 1205-1211, July 1999.

[6] M. Gavish and A.J. Weiss, "Performance analysis of bearing-only target location algorithm," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 26, pp. 22-26, Oct. 1992.

[7] G. Gu, M. Pachter, P.R. Chandler, and C.J. Schmacher, "Target motion analysis based on RF power and Doppler measurements," AFRL, WPAFB, Dayton, Ohio, Aug. 2007.

[8] K.C. Ho and Y.T. Chan, "An asymptotically unbiased estimator for bearing-only and Doppler-bearing target motion analysis," *IEEE Trans. Signal Processing*, vol. 54, pp. 809-822, March 2006.

[9] A.G. Lindgren and K.F. Gong, "Position and velocity estimation via bearing-only observations," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 14, pp. 564-577, July 1978.

[10] S.C. Nardone, A.G. Lindgren, and K.F. Gong, "Fundamental properties and performance of conventional bearing-only target motion analysis," *IEEE Trans. Automat. Contr.*, vol. 29, pp. 775-787, Sept. 1984.

Disclaimer

The views expressed in this article are those of the authors and do not reflect the official policy or position of the United States Air Force, Department of Defence, or U.S. Government.

[1] Email address of the corresponding author: ggu@ece.lsu.edu