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Robustness against Model Uncertainties of Norm Optimal Iterative Learning Control

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Abstract— **In this paper, we study MIMO Iterative Learning Control (ILC) and its robustness against model uncertainty. Although it is argued that, so-called, norm optimal ILC controllers have some inherent robustness, not many results are available that can make quantitative statements about the allowable model uncertainty. In this paper, we derive sufficient conditions for robust convergence of the ILC algorithm in presence of an uncertain system with an additive uncertainty bound. These conditions are applied to norm optimal ILC, resulting in guidelines for robust controller design. Theoretical results are illustrated by simulations.**

I. INTRODUCTION

In many control applications, a system has to perform the same task over and over again. Examples of such systems are robotic manipulators (e.g., a pick and place machine) and chemical batch processes. Iterative Learning Control (ILC) is a control strategy that uses the repetitive nature of the task to improve performance by learning from previous experience. When properly designed, an ILC controller iteratively finds a command signal that yields high system performance. For an introduction to ILC, the reader is referred to [5].

ILC research has always focussed on stability and convergence of the ILC algorithm. The analysis of stability and convergence in presence of model uncertainty, i.e., robust ILC, is less studied. In [10], [12], robustness properties of Linear Quadratic (LQ) ILC controllers (see, e.g., [3], [7], [9], [13]) are studied. Herein, the analysis is performed in the frequency domain and therefore, only approximate (possibly conservative) results are obtained. This is due to the fact that the Fourier transform assumes that signals act on an infinite interval, whereas in ILC, they inherently act on a finite time interval.

Systematic approaches to synthesise robust ILC controllers are studied in [4], [17]. With the control problem posed as an $\mathcal{H}_\infty$ optimal control problem, these results have two drawbacks: First, they are posed in the frequency domain. And second, the solutions are causal, i.e., the command signal in trial $k+1$ at time t only depends on information of trial k at time $[0,\ldots,t-1]$. In [11], [19], it was argued that the real benefit of ILC lies in the noncausality of the command signal. An alternative robust ILC approach [1], [2], represents model uncertainty as interval uncertainty in the system's Markov parameters. Although the resulting controllers are noncausal, synthesis of these controllers is numerically demanding.

Thus, existing tools for analysing and synthesising robust ILC controllers still have a number of deficiencies, caused by frequency domain representations, causality of the ILC controller, and a cumbersome representation of the model uncertainty. In this paper, we develop analysis and synthesis tools that overcome these limitations.

As a main contribution of this paper, we derive robust convergence conditions for first order ILC controllers, using a framework that incorporates the finite time character of ILC. In order to derive these conditions, we present a, for ILC, novel way to represent uncertainty. Moreover, we give a frequency domain interpretation of the robust convergence conditions. As it turns out, the results of [10] form a special case of the work presented here. Finally, we consider design issues for both the aforementioned LQ-ILC solutions, and solutions that incorporate an uncertainty model in the controller [8], [20], for they have a very similar structure.

The remainder of this paper is organised as follows. In Section II, we introduce the necessary ILC notations. Next, in Section III, the robust convergence problem is defined. Subsequently, sufficient conditions for robust convergence, both in finite time and frequency domain, are derived in Section IV. In Section V, we illustrate the robust convergence results by means of simulation examples with LQ-ILC controllers. Finally, some conclusions are drawn in Section VI.

II. NOMENCLATURE

In this paper, we consider discrete time, Linear Time Invariant (LTI) systems, with l outputs and m inputs. Since for these systems the z-transform exists, we can represent a set of perturbed systems Π_z with a bounded additive uncertainty as follows:

$$\Pi_z \; : \; \{J_p(z) = J(z) + W_i(z)\Delta(z)W_o(z) : \|\Delta(z)\|_{i2} \leq 1\}\,. \tag{1}$$

In (1), $J(z)$ represents the nominal model, $W_i(z)$ and $W_o(z)$ form a bound on the additive uncertainty, and $\Delta(z)$ is an arbitrary, stable system. The Frequency Response Function (FRF) is obtained by substituting $z = e^{j\vartheta}$.

Since ILC explicitly acts on a finite time interval $t \in [0,1,\ldots,N-1]$, we can use the lifted setting, as first introduced in [16], to express our systems and filters. In this setting, every time signal in trial k is stored in either an lN- or an mN-dimensional column vectors, e.g.:

$$y_k = \begin{bmatrix} y_k^T(0), & y_k^T(T_s), & \dots, & y_k^T\left((N-1)T_s\right) \end{bmatrix}^T, \quad (2)$$

where T_s denotes the sampling time. For brevity of notation, T_s is omitted in the remainder of this paper. In the same setting, systems are represented by their convolution matrix:

$$J = \begin{bmatrix} j(0) & 0 & \dots & 0 \\ j(1) & j(0) & & \vdots \\ \vdots & \ddots & \ddots & 0 \\ j(N-1) & \dots & j(1) & j(0) \end{bmatrix}, \quad (3)$$

where the sequence $\{j(0), j(1), \dots, j(N-1)\}$, with $j(t) \in \mathbb{R}^{l\times m}$, denotes the system's Markov parameters. The Markov parameters result from observing the system's response to a unit pulse. The matrices W_i and W_o are derived from $W_i(z)$ and $W_o(z)$, respectively, similar as J from $J(z)$. Using the lifted notation, a finite time representation of (1) can be written as:

$$\Pi \ : \ \{J_p = J + W_i \Delta W_o : \|\Delta\|_{i2} \leq 1\}. \quad (4)$$

The set Π now maps an input vector $f_k \in \mathbb{R}^{mN}$ to an output vector $y_k \in \mathbb{R}^{lN}$, i.e., $y_k = J_p f_k$. The lifted system Δ of (4) represents an arbitrary norm bounded, lower triangular, block Toeplitz matrix. Although, representing uncertainty as in (4) is a novel idea for ILC, it is a matured concept in the field of robust control theory (see, e.g., [18]).

In this paper, we use both the z-domain and the lifted description. To avoid any confusion, all z-domain signals and systems will have the index z.

Furthermore, in this paper, we make extensive use of norms. Given a lifted description, the induced 2-norm is defined as follows:

$$\|J\|_{i2} = \sup_{f\neq 0} \frac{\|Jf\|_2}{\|f\|_2} = \overline{\sigma}(J), \quad (5)$$

where $\|f\|_2 = \sqrt{\langle f, f\rangle}$ denotes the 2-norm for vectors and $\overline{\sigma}$ denotes the maximum singular value. For a transfer function description, the induced 2-norm is given by:

$$\|J(z)\|_{i2} = \sup_{f(z)\neq 0} \frac{\|J(z)f(z)\|_2}{\|f(z)\|_2} = \sup_{\vartheta\in[-\pi,\pi]} \overline{\sigma}\left(J(e^{j\vartheta})\right). \quad (6)$$

III. The Robust Monotonic Convergence Problem

In this Section, we define the robust monotonic convergence problem, that we will subsequently solve for general norm optimal ILC controllers in Section IV. Yet first, let us consider the ILC control structure used in this paper. This control structure is similar to the one used in [21] and is shown in Fig. 1. The corresponding trial domain dynamics are:

$$\begin{cases} f_{k+1} &= \mathcal{Q} f_k + L e_k \\ e_k &= r - J_p f_k, \end{cases} \quad (7)$$

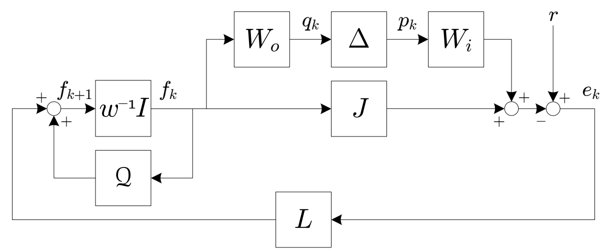

Fig. 1: General ILC control structure.

with corresponding closed loop dynamics:

$$f_{k+1} = \left(\mathcal{Q} - LJ_p\right) f_k + Lr. \quad (8)$$

Eq. (7) is robustly asymptotically stable in the trial domain if and only if all eigenvalues of $\mathcal{Q} - LJ_p$ are within the unit circle for all $J_p \in \Pi$ [5], [14]. More important, however, is the notion of monotonic convergence. The command signal of the ILC algorithm converges monotonically if there exists a $0 \leq \alpha < 1$ such that:

$$\|f_{k+1} - f_\infty\|_2 \leq \alpha \|f_k - f_\infty\|_2, \quad (9)$$

where $f_\infty = \lim_{k\to\infty} f_k$. If we extend the concept of monotonic convergence to include model uncertainty, we can define robust monotonic convergence.

Definition 3.1 (Robust Monotonic Convergence): Given a $\mathcal{Q}$ and L, the ILC system (8) has the property Robust Monotonic Convergence (RMC) if there exists a $0 \leq \alpha < 1$, for all $J_p \in \Pi$, such that:

$$\|f_{k+1} - f_\infty\|_2 \leq \alpha \|f_k - f_\infty\|_2, \quad (10)$$

with:

$$\alpha = \|\mathcal{Q} - LJ_p\|_{i2}. \quad (11)$$

The difference between monotonic convergence and RMC is that in the former case we only guarantee the command signal to converge for $J_p = J$.

Definition 3.2 (Robust Performance): Robust Performance (RP) of the ILC system (8) is defined as the error for $k \to \infty$, i.e.:

$$e_\infty = \left(I - J_p \left(I - \mathcal{Q} + LJ_p\right)^{-1} L\right) r. \quad (12)$$

Note that (12) can equal zero for all $J_p \in \Pi$ if and only if $\mathcal{Q} = I$ and $r \in \operatorname{Im}(J_p)$.

IV. Robust Monotonic Convergence of Norm Optimal ILC

In this Section, we derive sufficient conditions for RMC for a norm optimal ILC controlled system in both the lifted and frequency domain.

The norm optimal ILC controller is a generalisation of the LQ-ILC controller, as studied in [3], [7], [9], [13]. In norm optimal ILC, the control problem is to minimise the following cost functional:

$$\mathcal{J} = e_{k+1}^T Q e_{k+1} + f_\Delta^T R f_\Delta + f_{k+1}^T S f_{k+1}, \quad (13)$$

where $f_\Delta = f_{k+1} - f_k$, and $Q = Q^T > 0$, $R = R^T \geq 0$, and $S = S^T \geq 0$ denote weighting matrices. Note the difference between $\mathcal{Q}$ and Q: the former is a filter, while the latter is a weighting matrix. Substituting $e_{k+1} = e_k - Jf_\Delta$ and taking $\frac{\partial \mathcal{J}}{\partial f_{k+1}} = 0$, yields the following LQ optimal ILC controller:

$$f_{k+1} = \left(J^TQJ + R + S\right)^{-1}\left(\left(J^TQJ + R\right)f_k + J^TQe_k\right). \tag{14}$$

This can be put in the framework of (7) by defining:

$$\mathcal{Q} = \left(J^TQJ + R + S\right)^{-1}\left(J^TQJ + R\right), \tag{15a}$$

$$L = \left(J^TQJ + R + S\right)^{-1}J^TQ. \tag{15b}$$

While in LQ-ILC, the weighting matrices Q, R, and S are taken in the form of βI, where β is a scalar, in norm optimal ILC we no longer restrict Q and S to have this structure. In [8], for example, a robust ILC control structure is derived that explicitly incorporates an uncertainty model in the controller's weighting matrices.

After combining (4) with (15a) and (15b), condition (11) shows that this ILC algorithm is RMC if:

$$\|\left(J^TQJ + R + S\right)^{-1}\left(R - J^TQW_i\Delta W_o\right)\|_{i2} < 1, \tag{16}$$

for all $\|\Delta\|_{i2} < 1$. Since this expression is still a function of Δ, we cannot use it to guarantee RMC: Any effort to remove this Δ using algebraic manipulations, results in very conservative results.

In the remainder of this Section, we present two ways to circumvent this problem: First, we derive conditions for RMC in the lifted domain using concepts from robust feedback control, to be more specifically, μ-analysis (see, e.g., [15], [18]). Subsequently, we employ a frequency domain interpretation to derive results for Single-Input-Single-Output (SISO) systems.

A. Lifted Domain RMC

For deriving RMC conditions in the lifted domain, we consider the $N\Delta$ structure, see Fig 2a. This structure can be obtained by reorganising Fig. 1, i.e.:

$$\begin{bmatrix} q_k^T & f_{k+1}^T \end{bmatrix}^T = N\begin{bmatrix} p_k^T & f_k^T \end{bmatrix}^T, \tag{17}$$

with:

$$N = \begin{bmatrix} 0 & W_o \\ -LW_i & \mathcal{Q} - LJ \end{bmatrix}. \tag{18}$$

This formulation enables us to use generally well known ideas from robust control theory: With $\|\Delta\|_{i2} \leq 1$, the upper linear fractional transformation:

$$\|\mathcal{F}_u(N, \Delta)\|_{i2} = \|\mathcal{Q} - L\underbrace{(J + W_i\Delta W_o)}_{J_p}\|_{i2} < 1, \tag{19}$$

is exactly equal to (11). Because we require $\|\mathcal{F}_u(N,\Delta)\|_{i2} < 1 \ \forall \ \|\Delta\|_{i2} \leq 1$, we can include this requirement in the Δ-block, as shown in Fig. 2b.

A standard result from μ-analysis, [15], [18], yields that $\|\mathcal{F}_u(N, \Delta_N)\|_{i2} < 1$ if:

$$\inf_{D \in \mathcal{D}} \|DND^{-1}\|_{i2} < 1, \tag{20}$$

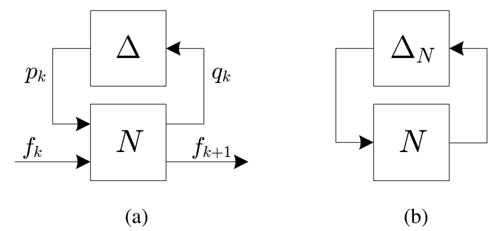

Fig. 2: NΔ-structure.

where $\mathcal{D} = \{D \ : \ D\Delta_N = \Delta_N D \ : \ \Delta_N \in \mathbf{\Delta}\}$, and $\mathbf{\Delta} = \{\text{diag}(\Delta, \Delta_f) \ : \ \|\Delta\|_{i2}, \|\Delta_f\|_{i2} \leq 1\}$.

We now present our main results using the following two propositions.

Proposition 4.1: Given system (8) and ILC controller (14), with $R = 0$. Then, for MIMO systems (4), the ILC algorithm is RMC if:

$$\|W_o\|_{i2} \cdot \|\left(J^TQJ + S\right)^{-1}J^TQW_i\|_{i2} < 1. \tag{21}$$

Furthermore, for SISO systems (4) and W_o square, the ILC algorithm is RMC if:

$$\|\left(J^TQJ + S\right)^{-1}J^TQW_iW_o\|_{i2} < 1. \tag{22}$$

Proof: Consider the $N\Delta$ structure of Fig. 2a. Substitution of (15a) and (15b) into (20) and taking $R = 0$ yields:

$$\inf_D \left\|\begin{bmatrix} D & 0 \\ 0 & I \end{bmatrix}\begin{bmatrix} 0 & W_o \\ -LW_i & 0 \end{bmatrix}\begin{bmatrix} D^{-1} & 0 \\ 0 & I \end{bmatrix}\right\|_{i2} < 1, \tag{23}$$

with D an arbitrary lower triangular block Toeplitz matrix of the form:

$$D = \begin{bmatrix} d_oI_m & 0 & \cdots & 0 \\ d_1I_m & d_oI_m & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ d_{N-1}I_m & \cdots & d_1I_m & d_0I_m \end{bmatrix}, \tag{24}$$

where $I_m \in \mathbb{R}^{m\times m}$ identity matrix. Since (23) has an antidiagonal structure, the inequality is satisfied if and only if:

$$\inf_D \max\left\{\|DW_o\|_{i2}, \|L_iW_iD^{-1}\|_{i2}\right\} < 1. \tag{25}$$

If we take $D = dI$, then there exists a $d \in \mathbb{R}^+$, for which (25) is equal to (21). If we take $D = W_o^{-1}$, then (25) is equal to (22). ■

Proposition 4.2: Assume that (8) with ILC controller (14) is RMC for $R = 0$. Then (8) is RMC for $R = \rho I \geq 0$, where $\rho \in \mathbb{R}$.

Proof: Observe that (16) is equivalent to:

$$\|\left(J^TQJ + R + S\right)^{-1}\left(R + \left(J^TQJ + S\right)Y\right)\|_{i2} < 1, \tag{26}$$

with $Y = -\left(J^TQJ+S\right)^{-1}J^TQW_i\Delta W_o$ and $\|Y\|_{i2} < 1$. For $R = \rho I \geq 0$, (26) has an upper bound given by:

$$\|\left(J^TQJ+\rho I+S\right)^{-1}(\rho I + (1-\epsilon)\left(J^TQJ+S\right))\|_{i2} < 1, \quad (27)$$

with $0 < \epsilon \leq 1$. With $J^TQJ + S$ square, symmetric and positive definite, this inequality is satisfied for all $\rho \geq 0$. ■

Remark: From Proposition 4.2 we conclude that the parameter R does not influence the robustness of norm optimal ILC against model uncertainty. A similar statement has also been made in [12]. □

B. Frequency Domain RMC

Although we argued that the frequency domain is not the preferred domain to analyse RMC, it can give us valuable insight in the RMC properties of norm optimal ILC controllers. In [14], a sufficient condition for monotonic convergence is given by:

$$|\mathcal{Q}(e^{j\vartheta})|\ |I - L^\star(e^{j\vartheta})J(e^{j\vartheta})| < 1, \quad \vartheta \in [-\pi,\pi], \quad (28)$$

with $L^\star(e^{j\vartheta}) = \mathcal{Q}^{-1}(e^{j\vartheta})L(e^{j\vartheta})$. Using this result, we can derive a sufficient condition for RMC, given by:

$$|\mathcal{Q}(e^{j\vartheta})|\ |I - L^\star(e^{j\vartheta})J_p(e^{j\vartheta})| < 1 \quad \forall J_p(e^{j\vartheta}) \in \Pi_z. \quad (29)$$

In the following result, we restrict ourselves to SISO systems, since the derivation requires the matrix multiplications to commute. In other words, we require: $W_i(z)\Delta(z)W_o(z) = W_i(z)W_o(z)\Delta(z) = W_A(z)\Delta(z)$. Furthermore, we require Q and S to correspond to an LTI system, so that their z-transform exists, and R to be a constant. The following result is a generalisation of [10].

Proposition 4.3: Given the set of SISO systems (1), ILC controller:

$$\mathcal{Q}(z) = \left(J(z^{-1})Q(z)J(z)+R+S(z)\right)^{-1}\left(J(z^{-1})Q(z)J(z)+R\right), \quad (30a)$$

$$L(z) = \left(J(z^{-1})Q(z)J(z)+R+S(z)\right)^{-1}J(z^{-1})Q(z), \quad (30b)$$

and $R \geq 0$. Then the ILC controlled system is RMC if:

$$|\left(J(e^{-j\vartheta})Q(e^{j\vartheta})J(e^{j\vartheta})+S(e^{j\vartheta})\right)^{-1} J(e^{-j\vartheta})Q(e^{j\vartheta})W_A(e^{j\vartheta})| < 1. \quad (31)$$

Proof: Substituting (30b) into (29) yields:

$$|\mathcal{Q}(e^{j\vartheta})|\ |\left(J(e^{-j\vartheta})Q(e^{j\vartheta})J(e^{j\vartheta})+R\right)^{-1}\left(R - J(e^{-j\vartheta})Q(e^{j\vartheta})W_A(e^{j\vartheta})\Delta(e^{j\vartheta})\right)| < 1. \quad (32)$$

With $\left(J(e^{-j\vartheta})Q(e^{j\vartheta})J(e^{j\vartheta})+R\right)^{-1}$ a zero-phase filter, we can, after substituting (30a), rewrite this expression as:

$$\begin{aligned}&|\left(J(e^{-j\vartheta})Q(e^{j\vartheta})J(e^{j\vartheta})+R+S(e^{j\vartheta})\right)^{-1}|\\ &\qquad |R - J(e^{-j\vartheta})Q(e^{j\vartheta})W_A(e^{j\vartheta})\Delta(e^{j\vartheta})| < 1 \quad \Leftrightarrow\\ &\left|R - J(e^{-j\vartheta})Q(e^{j\vartheta})W_A(e^{j\vartheta})\Delta(e^{j\vartheta})\right| <\\ &\qquad \left|J(e^{-j\vartheta})Q(e^{j\vartheta})J(e^{j\vartheta})+R+S(e^{j\vartheta})\right|, \quad (33)\end{aligned}$$

Since $\Delta(e^{j\vartheta})$ is an arbitrary, stable system, there exists a $\Delta(e^{j\vartheta})$ for which the upper bound of the triangular inequality is achieved. Therefore, without introducing extra conservatism, (33) can be rewritten to:

$$\begin{aligned}&R + \left|J(e^{-j\vartheta})Q(e^{j\vartheta})W_A(e^{j\vartheta})\right| <\\ &\qquad \left|J(e^{-j\vartheta})Q(e^{j\vartheta})J(e^{j\vartheta})+S(e^{j\vartheta})\right| + R \quad \Leftrightarrow\\ &\left|J(e^{-j\vartheta})Q(e^{j\vartheta})W_A(e^{j\vartheta})\right| <\\ &\qquad \left|J(e^{-j\vartheta})Q(e^{j\vartheta})J(e^{j\vartheta})+S(e^{j\vartheta})\right|.\end{aligned}$$

The latter expression can be rewritten to (31). ■

The results of Proposition 4.3 confirm that the parameter R does not influence the robustness properties of the ILC controller.

Corollary 4.4: Proposition 4.3 shows that some uncertainty can be allowed while still retaining maximum performance. If the multiplicative uncertainty $|W_M(e^{j\vartheta})| = |J^{-1}(e^{j\vartheta})W_A(e^{j\vartheta})| < 1$ for all $\vartheta \in [-\pi,\pi]$, the norm optimal ILC solution with $R = S = 0$ is RMC. This result is very similar to that of [6], in which model based feedforward is discussed.

V. Simulation Example

In this Section, we illustrate the results of Section IV using an LQ-ILC controller. Hence, we restrict Q, R, and S to have a diagonal form.

A. System Description

For this example, we consider a model of the two-mass system used in [21]. The continuous time dynamics of this system are governed by the following transfer function:

$$G(s) = \frac{ds+k}{m_1m_2s^4 + (m_1+m_2)ds^3 + (m_1+m_2)ks^2}, \quad (34)$$

where $m_1 = 2\cdot 10^{-4}$, $m_2 = 1.6\cdot 10^{-4}$, $d = 5.66\cdot 10^{-4}$, and $k = 9.8$. Uncertainty is introduced by perturbing the values d and k between 95% and 105% of their nominal values. A discrete time equivalent of this model is obtained by using a 'zero-order-hold' approximation with a sampling frequency of 1kHz. Since this system is marginally stable, it is controlled using feedback with the following controller:

$$K(s) = 0.2\frac{\left(\frac{1}{2\pi\cdot 3}s+1\right)\left(\frac{1}{(2\pi\cdot 52)^2}s^2 + \frac{0.02}{2\pi\cdot 52}s+1\right)}{\left(\frac{1}{2\pi\cdot 20}s+1\right)\left(\frac{1}{(2\pi\cdot 52)^2}s^2 + \frac{2}{2\pi\cdot 52}s+1\right)}, \quad (35)$$

which is implemented in discrete time using a Tustin approximation with a prewarp frequency of 52Hz. In case we use feedback control in conjunction with ILC, the process sensitivity is the relevant transfer function for ILC:

$$J(z) = \left(I + G(z)K(z)\right)^{-1}G(z). \quad (36)$$

The nominal system model is obtained by taking the nominal values for k and d. The additive uncertainty bound of the process sensitivity is obtained by taking a Tustin approximation of the following continuous time bound:

$$W_A(s) = 5 \cdot 10^{-6} \cdot \frac{\left(\frac{1}{(2\pi\cdot 0.2)^2}s^2 + \frac{2}{2\pi\cdot 0.2}s + 1\right)}{\left(\frac{1}{(2\pi\cdot 5.2)^2}s^2 + \frac{0.6}{2\pi\cdot 5.2}s + 1\right)} \cdot \frac{\left(\frac{1}{(2\pi\cdot 51)^2}s^2 + \frac{1.1}{2\pi\cdot 51}s + 1\right)\left(\frac{1}{(2\pi\cdot 54.5)^2}s^2 + \frac{1.1}{(2\pi\cdot 54.5)}s + 1\right)}{\left(\frac{1}{(2\pi\cdot 51)^2}s^2 + \frac{0.04}{2\pi\cdot 51}s + 1\right)\left(\frac{1}{(2\pi\cdot 54.5)^2}s^2 + \frac{0.042}{(2\pi\cdot 54.5)}s + 1\right)}. \tag{37}$$

Since this example is a SISO system, all matrix multiplications commute, and we can take either $W_i(z) = W_A(z)$ and $W_o(z) = 1$ or $W_i(z) = 1$ and $W_o(z) = W_A(z)$.

The lifted system description of (4) is obtained by defining J, W_i and W_o as given in (3). The perturbed system's impulse response and the defined trajectory for ILC (which is in fact the reference trajectory filtered by the sensitivity function $(I + G(z)K(z))^{-1}$) are depicted in Fig. 3 and Fig. 4, respectively.

B. RMC for $R, S \rightarrow 0$ and Retaining Uncompromised Performance

In Corollary 4.4, we stated that if we require uncompromised performance in combination with RMC, we have to satisfy $|W_M(z)| < 1$. To illustrate this, we consider the following multiplicative uncertainty:

$$W_M(s) = 5.8 \cdot 10^{-7} \cdot \frac{\left(\frac{1}{(2\pi)^2}s^2 + \frac{1.2}{2\pi}s + 1\right)^2}{\left(\frac{1}{(2\pi\cdot 4.5)^2}s^2 + \frac{1}{2\pi\cdot 4.5}s + 1\right)\left(\frac{1}{(2\pi\cdot 52)^2}s^2 + \frac{0.032}{2\pi\cdot 52}s + 1\right)}, \tag{38}$$

which is discretised using a Tustin transformation. The additive uncertainty description is obtained as follows:

$$W_{A,1}(z) = J(z)W_M(z). \tag{39}$$

It can verified that this additive uncertainty bound satisfies the condition of Corollary 4.4. An uncertain system that does not satisfy these conditions is given by:

$$W_{A,2}(z) = 2.5 \cdot J(z)W_M(z). \tag{40}$$

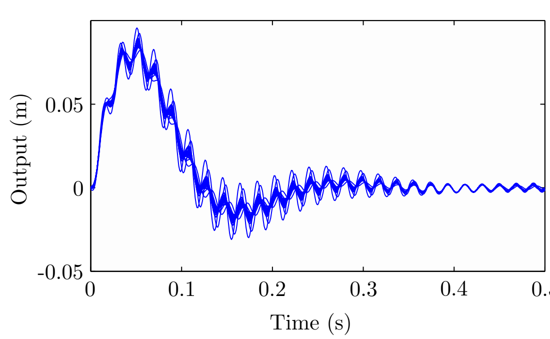

Fig. 3: The impulse response of the uncertain system J_p.

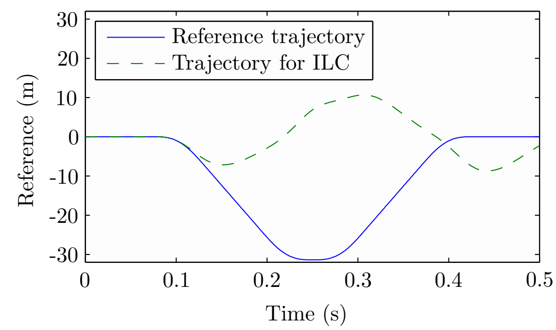

Fig. 4: The applied reference trajectory r.

Fig. 5 and Fig. 6 show the convergence behaviour for several systems in the two sets by choosing $Q = I$, $R = S = 0$. It can be seen that the set using $W_{A,2}$, although it is stable, does not achieve RMC.

C. RMC Cannot be Guaranteed for $R \neq 0$ and $S = 0$

Here, we show that by tuning R, we cannot achieve RMC. For that, we use (37) as our uncertainty bound and choose $Q = I$ and $S = 0$. In Fig. 7, convergence, or lack thereof, of the error signal is shown for $R = 0$, $R = 0.1 \cdot I$ and $R = I$. Increasing R will only postpone the appearance of the instable behaviour. Since for unstable systems f_∞ does not exist, it is not possible to show the convergence of the command signal $\|f_k - f_\infty\|_2$, as we did in Fig. 5. Instead, the lack of convergence of the error is used to illustrate that the system is indeed not RMC.

D. RMC for $R = 0$ and $S \neq 0$

Propositions 4.1 and 4.3 are both sufficient conditions for RMC. This is due to the fact that even monotonic convergence for the nominal system is stated as a sufficient condition. However, if we violate the aforementioned conditions, the system can become unstable. Using (37), the system is RMC for $Q = I$, $R = 0$, and $S = 0.7 \cdot I$, while for $S < 0.7 \cdot I$, the conditions are not satisfied. Fig. 8 shows that for $S = 0.5 \cdot I$, the system becomes unstable, although one might wrongfully conclude that the system is stable after observing the first few trials.

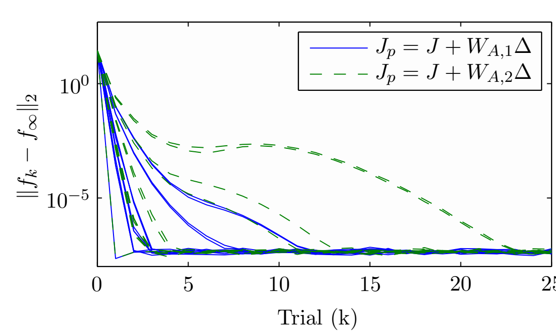

Fig. 5: Convergence of the command signal with $R, S \rightarrow 0$.

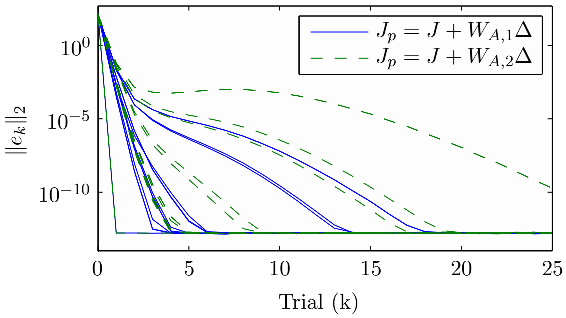

Fig. 6: Convergence of the error with $R, S \to 0$.

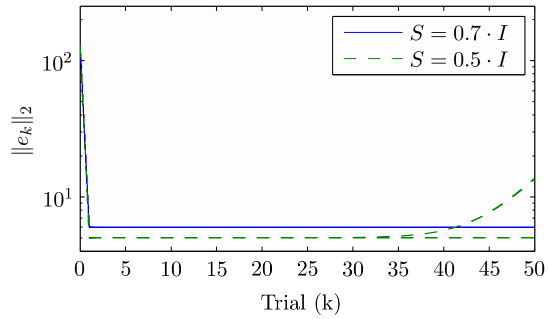

Fig. 8: Convergence of the error with $R = 0$ and $S \neq 0$.

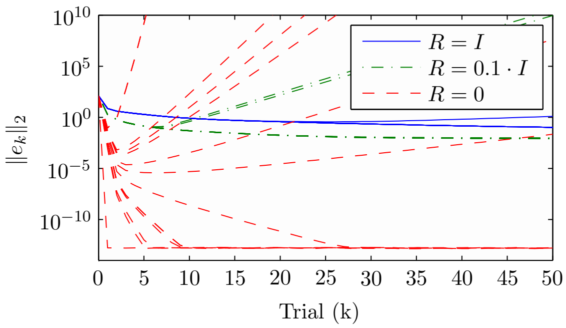

Fig. 7: Convergence of the error with $R \neq 0$ and $S = 0$.

VI. Conclusions

In this paper, we studied robustness against model uncertainty of norm optimal ILC controllers in the finite time domain. For that, we introduced the notion of robust monotonic convergence (RMC) and presented sufficient conditions to guarantee RMC, for a given norm optimal ILC controller and an additive uncertainty bound. It turns out that RMC can be examined by evaluating a single expression. Using this expression, we obtained guidelines for synthesis of robust ILC controllers. Finally, we illustrated these design issues with simulation examples using an LQ-ILC controller.

Using the obtained insight in RMC, in [8], we focus on synthesis of robust ILC controllers that are equipped with an uncertainty model inside the controller. For these controllers, it can be guaranteed that the ILC algorithm is RMC.

Future research could focus on extending the theory to include analysis of higher order ILC, as well as trial varying model uncertainty.

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