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Passivity based Iterative Learning Control for Mechanical Systems subject to Dry Friction

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Abstract— **This paper considers iterative learning control applied to a mass-damper system subject to dry friction. The dry friction nonlinearity is discontinuous, and therefore poses challenges to the conventional learning control methods. We apply the passivity based analysis in learning control and show that it is applicable to the case with velocity output. In the case of combined position and velocity output, the passivity approach is not directly applicable. We derive a modified update law based on the 2D system perspective and adjust the combination coefficient in every iteration step to ensure passivity. Asymptotic convergence is shown under the condition that the combination coefficient does not asymptotically vanish. Simulation results are included to demonstrate the performance of these algorithms.**

I. INTRODUCTION

Iterative learning control (ILC) is a control technique that improves the performance of dynamical systems working in a repetitive mode. In the same way a tennis player improves shots after practicing over and over, an iterative learning controller uses previous trial information to get better performance of a system with respect to some desired performance objective. The ILC concept was first proposed for robot motion tracking in repetitive tasks [1] and later popularized in [2]–[4]. The ILC technique has been shown to be effective in compensating for nonlinear effects such as gravity, Coriolis, and centrifugal forces without the need for a precise dynamical model. Since the initial introduction, there has been a proliferation of ILC applications such as batch chemical processes [5], injection molding machines [6], power electronics [7], aerospace [8], and magnetic bearings [9], among many others. A comprehensive survey of ILC publications has been given in [10].

This paper considers the application of ILC to specific classes of nonsmooth systems: a mass-damper system subject to dry friction. A nonsmooth dynamical system contains one or more nonsmooth functions in its dynamics. Such system occurs in many applications, including mechanical systems (dry friction, intermittent contacts, hysteresis), electrical circuits (MOS transistors, relays, switched power converter), thermal systems (phase changes), and control systems (gain scheduling controller, sliding mode controller). There have been some limited studies on the application of ILC to systems subject to dry friction [11], [12], but no general theory has been developed so far. In this paper, we will limit our consideration to the dry friction nonlinearity, but the underlying theory is applicable to more general nonsmooth systems.

For linear systems, the convergence of ILC depends on a spectral radius condition. For nonlinear systems, there are two main approaches: passivity based [13]–[17] and optimal control [18], [19] (which results in a gradient descent). In this paper, we will consider the application of the passivity approach. For the simple mass-damper example, we consider two different outputs: velocity and a combination of position and velocity. In the velocity output case, the passivity approach may be directly applied to show asymptotic convergence of the velocity to the desired velocity profile. In the combined position and velocity output case, the passivity approach is not directly applicable. We first derive a general class of modified ILC update law, motivated by the two-dimensional (2D) discrete time passivity interpretation of passivity based ILC. This leads to a new approach to ILC where the combination between position and velocity is adjusted in every iteration to ensure the passivity condition holds. Under the assumption that the coefficient of combination does not asymptotically vanish (which holds true in simulation but has not been analytically proven), the ILC would converge asymptotically. Simulation results are included to demonstrate the performance of the proposed methods.

II. ILC APPROACHES: A REVIEW

Consider a dynamic system, G, with input $u(t) \in \mathbb{R}^m$, output $y(t) \in \mathbb{R}^m$, and a desired output $y_d(t)$, $t \in [0,T]$. Assume that a unique input u_d exists, such that $y_d = G(u_d)$. The objective of ILC is to find an iterative form of learning control law $u_{k+1} = F(u_k, e_k)$, $u_k = \{u_k(t)\}_{t=0}^{T}$ etc., to achieve the output tracking objective $y_k(t) \to y_d(t)$ uniformly for $t \in [0,T]$ as $k \to \infty$, where e_k is the tracking error in the kth iteration: $e_k(t) = y_k(t) - y_d(t)$.

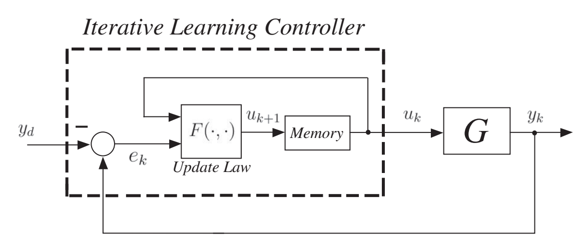

Fig. 1. Iterative learning control schematic.

The past approaches to ILC can be classified by the update laws:

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- Proportional update law [1]–[4]

$$u_{k+1} = u_k - Le_k \ , \tag{1}$$

where $L : L_2^m \to L_2^m$, $L_2^m := L_2^m[0,T]$.
- Gradient update law [18]–[23]

$$u_{k+1} = u_k - \alpha \nabla_{u_k} G(e_k) \ , \tag{2}$$

where $\alpha > 0$ and $\nabla_{u_k} G$ is the Fréchet derivative of G with respect to u_k.

A. Proportional ILC: Linear Case

In the linear case, the convergence condition may be easily derived for the proportional update law using a Hilbert space approach [22]. Consider $G : L_2^m \to L_2^m$ as a linear operator. The output is then related to the input by $y = Gu$. The error propagation under proportional update is given by

$$e_{k+1} = Gu_{k+1} - y_d = G(u_k - Le_k) - y_d = (I - GL)e_k. \tag{3}$$

Then $e_k \to 0$ as $k \to 0$ if and only if $\rho(I - GL) < 1$ where ρ is the spectral radius. A modified update law, $u_{k+1} = Q(u_k - Le_k)$, called Q-modification, has also been proposed [24]. The convergence condition becomes less restrictive, but results in a nonzero steady state error. In the case that G is known, L may be chosen based on G^* (the non-causal adjoint operator of G), e.g., $L = \alpha G^*$, provided that G is an onto operator (i.e., G^* is one-to-one, or G^* has a trivial null space).

B. Proportional ILC: Passivity Analysis

Now consider (1) where $L > 0$ is a positive definite operator. Let the input error be defined as $e_{u_k} = u_k - u_d$. Then $e_{u_{k+1}} = e_{u_k} - Le_k$, where $e_k = G(u_k) - G(u_d)$. We have the following propagation:

$$\begin{aligned} \left\langle e_{u_{k+1}}, L^{-1} e_{u_{k+1}} \right\rangle &= \left\langle e_{u_k} - Le_k, L^{-1}(e_{u_k} - Le_k) \right\rangle \\ &= \left\langle e_{u_k}, L^{-1} e_{u_k} \right\rangle - 2\left\langle e_k, e_{u_k} \right\rangle + \left\langle e_k, Le_k \right\rangle. \end{aligned} \tag{4}$$

Assume G is strictly output incremental passive (SOIP), i.e., there exists $\gamma > 0$ such that for any u_1 and u_2

$$\langle G(u_1) - G(u_2), u_1 - u_2 \rangle \geq \gamma \|G(u_1) - G(u_2)\|^2. \tag{5}$$

Note that the inequality above assumes that the system is initially at rest. It follows

$$\left\langle e_k, e_{u_k} \right\rangle \geq \gamma \|e_k\|^2 \geq \gamma \frac{\langle e_k, Le_k \rangle}{\|L\|}.$$

Denote $\left\langle e, L^{-1} e \right\rangle$ by $\|e\|_{L^{-1}}^2$. Then from (4),

$$\left\|e_{u_{k+1}}\right\|_{L^{-1}}^2 \leq \left\|e_{u_k}\right\|_{L^{-1}}^2 - \underbrace{\left(\frac{2\gamma}{\|L\|} - 1\right)}_{\gamma_1} \|e_k\|_L^2. \tag{6}$$

If the update gain is bounded as

$$\|L\| \leq 2\gamma, \tag{7}$$

we have $\gamma_1 > 0$ and

$$\left\|e_{u_{k+1}}\right\|_{L^{-1}}^2 \leq \left\|e_{u_0}\right\|_{L^{-1}}^2 - \gamma_1 \sum_{i=0}^{k} \|e_i\|_L^2.$$

It follows that $\|e_k\|_L$ is an $\ell_2[0,\infty)$ sequence, which implies e_k converges to 0 as $k \to \infty$.

III. Dry Frictional Example

Consider the mechanical system with dry friction shown in Figure 2 and described by

$$m\ddot{x} + \beta\dot{x} + \sigma \mathrm{sgn}(\dot{x}) = u. \tag{8}$$

We will first consider the output to be the velocity $\dot{x}$, and then the combination of position and velocity.

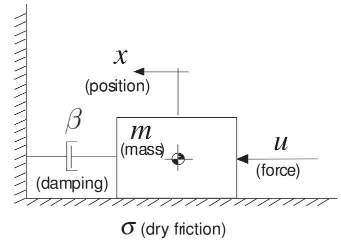

Fig. 2. Mechanical system with dry friction.

A. *Velocity Output Case*

Consider the velocity as the output of the system, $y = \dot{x}$. Let (u_1, u_2) be two input trajectories with the correspondent outputs (y_1, y_2). Considered the storage function

$$V(t) = \frac{1}{2} m(\dot{x}_1(t) - \dot{x}_2(t))^2 \tag{9}$$

where $t \in [0,T]$. The derivative of V along the solution is

$$\begin{aligned} \dot{V} &= m(\dot{x}_1 - \dot{x}_2)(\ddot{x}_1 - \ddot{x}_2) \\ &= (\dot{x}_1 - \dot{x}_2)\big(-\beta(\dot{x}_1 - \dot{x}_2) + (u_1 - u_2) \\ &\quad -\sigma(sgn(\dot{x}_1) - sgn(\dot{x}_2))\big) \\ &\leq -\beta(y_1 - y_2)^2 + (y_1 - y_2)(u_1 - u_2). \end{aligned}$$

We shall assume that each iteration starts with the same initial condition, so $V(kT) = V(0)$, $k = 1, 2, \ldots$. By integrating both sides, it follows that the system is SOIP, i.e.,

$$\langle y_1 - y_2, u_1 - u_2 \rangle \geq \beta \|y_1 - y_2\|^2. \tag{10}$$

Thus the proportional ILC is asymptotically convergent as before. This result is generalized in the following theorem:

Theorem 3.1: Consider the dynamical system given by

$$\dot{X}(t) = AX(t) + B(u(t) - \eta(y(t))) \ \ ; \ \ y(t) = CX(t). \tag{11}$$

Assume $(A, B, C, 0)$ is strictly positive real (SPR) and η is incrementally sector bounded in $[0, \infty)$, i.e., $(y_1 - y_2)(\eta(y_1) - \eta(y_2)) \geq 0$ for all y_1, y_2. Then the proportional ILC update law (1) is asymptotically convergent if $L > 0$ and $\|L\|$ is sufficiently small.

Proof: Since $(A, B, C, 0)$ is SPR, there exist symmetric positive definite matrices P and Q such that

$$A^T P + PA = -Q \tag{12}$$

$$PB = C^T. \tag{13}$$

Consider two input trajectories, (u_1, u_2). Let the corresponding states and outputs be (X_1, X_2) and (y_1, y_2). Consider the following storage function

$$V = \frac{1}{2}(X_1 - X_2)^T P (X_1 - X_2). \tag{14}$$

The derivative of V along the solution is

$$\begin{aligned}\dot{V} &= \frac{1}{2}(X_1-X_2)^T(A^TP+AP)(X_1-X_2) \\ &\quad +(X_1-X_2)^TPB(u_1-u_2-(\eta(y_1)-\eta(y_2))) \\ &= -\frac{1}{2}(X_1-X_2)^TQ(X_1-X_2)+(y_1-y_2)^T(u_1-u_2) \\ &\quad -(y_1-y_2)^T(\eta(y_1)-\eta(y_2)) \\ &\leq -\frac{1}{2}\lambda_{\min}(Q)\|X_1-X_2\|^2+(y_1-y_2)^T(u_1-u_2) \\ &\leq -\frac{1}{2}\frac{\lambda_{\min}(Q)}{\|C\|^2}\|y_1-y_2\|^2+(y_1-y_2)^T(u_1-u_2).\end{aligned}$$

By integrating both sides, it follows that the system is SOIP. From the passivity analysis for ILC, this implies that $\|e_k\| \to 0$ as $k \to \infty$ if

$$\|L\| \leq \frac{\lambda_{\min}(Q)}{\|C\|^2}.$$

□

For the mass damper case,

$$X = \dot{x},\ A = -d/m,\ B = 1/m,\ C = 1,\ \eta = \text{sgn}.$$

Clearly, the conditions for Theorem 3.1 are all satisfied.

B. Combined Position and Velocity Output Case

In most applications, it is desired to track a position trajectory rather than the velocity. In this section, we consider the output to be $y = \dot{x} + cx,\ c > 0$ (motivated by robot adaptive control [25], [26]). However, for the dry friction example above, the SOIP property no longer holds. We will now develop a modified proportional ILC to address this problem. We will first re-examine the passivity based ILC approach and interpret it from the 2D discrete passive system perspective. The proportional update law is then generalized to contain an output feedback term. The SOIP convergence condition is weakened in this analysis. Finally, this approach is applied to the combined velocity and position feedback case.

1) 2D Interpretation of ILC and Modified Update Law: The passivity based ILC may be considered from a 2D dynamical system perspective. Consider the iterative control law as a discrete time system, F, mapping input $-e_k$ to the output $e_{u_k} = u_k - u_d$. Suppose that F is a discrete time passive system, i.e., there exists a positive definite storage function S_k such that

$$S_{k+1} \leq S_k + \langle -e_k, e_{u_k} \rangle. \tag{15}$$

If the plant G is SOIP, i.e., $\langle e_k, e_{u_k} \rangle \geq \gamma \|e_k\|^2$, then $S_{k+1} \leq S_k - \gamma\|e_k\|^2$, which implies e_k converges to zero as $k \to \infty$ as before.

Note that not all discrete passive system may be used for F since u_d is not known. However, the proportional ILC update law (1) is admissible:

$$F : \begin{cases} z_{k+1} = z_k + L(-e_k) \\ w_k = e_{u_k} - \frac{L}{2}e_k = z_k + \frac{L}{2}(-e_k) \end{cases}, \tag{16}$$

where z_k represents the discrete time state. The passivity property of F follows from the storage function $S_k = \langle z_k, L^{-1}z_k \rangle$. A continuous time SOIP system may be viewed as a output positive operator:

$$\langle e, e_u \rangle = \langle G(e_u), e_u \rangle \geq \gamma \|G(e_u)\|^2 = \gamma \|e\|^2$$

which is *static* in terms of discrete time iteration. It follows that

$$\left\langle e, e_u - \frac{L}{2}e \right\rangle \geq \left(\gamma - \frac{\|L\|}{2}\right)\|e\|^2.$$

Therefore, the SOIP system is discrete time strictly output passive under condition (7). The asymptotic convergence of the proportional ILC update then follows from the interconnection of the two discrete passive systems. More generally, with input w_k (a linear combination of e_{u_k} and e_k), if the plant is a discrete time strictly output passivity system, i.e., there exists $V_k \geq 0$ such that

$$V_{k+1} \leq V_k + \langle w_k, e_k \rangle - \alpha \|e_k\|^2, \tag{17}$$

then combining with the discrete passivity of F, we get

$$S_{k+1} + V_{k+1} \leq S_{k+1} + V_{k+1} - \alpha \|e_k\|^2. \tag{18}$$

Using the same analysis as before, we can show that $\|e_k\|$ is an ℓ_2 sequence and hence $\|e_k\| \to 0$ as $k \to \infty$.

Now consider a feedforward term $\Gamma > 0$ be added to F as show in Figure 3:

$$F' : \begin{cases} z_{k+1} = z_k + L(-e_k) \\ e_{u_k} = z_k + \Gamma(-e_k) \end{cases} \tag{19}$$

Let $S_k = \langle z_k, L^{-1}z_k \rangle$. Then

$$S_{k+1} - S_k = -2\langle e_k, e_{u_k} \rangle - 2\left\langle e_k, \left(\Gamma - \frac{L}{2}\right)e_k \right\rangle. \tag{20}$$

The condition on the plant G may now be significantly relaxed. Suppose G is bounded below:

$$\langle e_k, e_{u_k} \rangle \geq -\nu \|e_k\|^2 \tag{21}$$

where $\nu \geq 0$ (note that Γ could affect ν). If $\nu = 0$, the plant G is only required to be passive. For $\nu > 0$, the system may not be passive at all, it only needs to be bounded below as above. The evolution of the storage function is then bounded by

$$S_{k+1} - S_k \leq 2\left(\lambda_{\min}(\Gamma - \frac{L}{2}) - \nu\right)\|e_k\|^2. \tag{22}$$

Under the condition that $\Gamma - L/2$ is sufficiently large:

$$\lambda_{\min}(\Gamma - \frac{L}{2}) > \nu \tag{23}$$

$\|e_k\|$ is an ℓ_2 sequence and $\|e_k\|$ converges to zero as $k \to \infty$.

From (19), the update law now is of the form

$$u_{k+1} = u_k - (L - \Gamma)e_k - \Gamma e_{k+1}, \tag{24}$$

where the last term is implemented as a direct output feedback. Note that direct output feedback augmentation has been proposed in [27] for linear ILC systems.

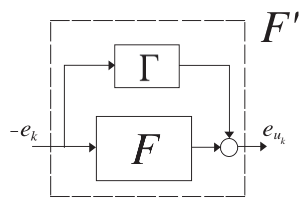

Fig. 3. Modified ILC update law.

2) Modified Update Law for Combined Position/Velocity Output: Motivated by the modified ILC update (24) to the mass-damper example (8), we now re-examine the combined position and velocity output case:

$$y = cx + \dot{x}. \tag{25}$$

We first choose a position feedback loop to ensure the linear portion of the system is stable:

$$u = w - Kx. \tag{26}$$

The system then becomes

$$m\ddot{x} + \beta\dot{x} + Kx + \sigma \text{sgn}\dot{x} = w. \tag{27}$$

Choose the desired trajectory to satisfy the same dynamics:

$$m\ddot{x}_d + \beta\dot{x}_d + Kx_d + \sigma \text{sgn}\dot{x}_d = w_d. \tag{28}$$

We now apply the modified ILC algorithm (24) to w:

$$w_{k+1} = w_k - (L-\Gamma)e_{y_k} - \Gamma e_{y_{k+1}}, \tag{29}$$

where $e_{y_k} = y_k - y_d$. In terms of the original control, we have

$$u_{k+1} = u_k - (L-\Gamma)e_{y_k} - \Gamma e_{y_{k+1}} - Ke_{x_{k+1}}. \tag{30}$$

As shown in (23), if the following system with input e_w and output e_y is passive (i.e., $\nu = 0$ in (21)),

$$\begin{aligned} m\ddot{e}_x + \beta\dot{e}_x + Ke_x + \sigma(\text{sgn}\dot{x} - \text{sgn}\dot{x}_d) &= e_w \\ e_y &= ce_x + \dot{e}_x, \end{aligned} \tag{31}$$

then $\|e_{y_k}\|$ converges to zero if

$$\Gamma > \frac{L}{2}. \tag{32}$$

In the state space form, $X = \begin{bmatrix} e_x & \dot{e}_x \end{bmatrix}^T$, (31) is of the form

$$\begin{aligned} \dot{X} &= \underbrace{\begin{bmatrix} 0 & 1 \\ -\frac{K}{m} & -\frac{\beta}{m} \end{bmatrix}}_{A} X + \underbrace{\begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix}}_{B} (e_w - \sigma(\text{sgn}\dot{x} - \text{sgn}\dot{x}_d)) \\ e_y &= \underbrace{\begin{bmatrix} c & 1 \end{bmatrix}}_{C} X. \end{aligned} \tag{33}$$

For

$$P = \begin{bmatrix} K + c\beta & mc \\ mc & m \end{bmatrix}, \; Q = 2\begin{bmatrix} cK & 0 \\ 0 & \beta - mc \end{bmatrix},$$

$K + c\beta > mc^2$ and $c > 0$, we have $P, Q > 0$ and

$$A^T P + PA = -Q, \; PB = C^T.$$

Hence $(A, B, C, 0)$ is SPR.

Let $V = \frac{1}{2}X^T PX$. Then the derivative of V may be manipulated to satisfy:

$$\dot{V} \le -\frac{1}{2}X^T QX - c\sigma e_x^T(\text{sgn}\dot{x} - \text{sgn}\dot{x}_d) + e_y^T e_w.$$

For a fixed c, in each iteration, we may choose K sufficiently large to ensure that

$$\begin{cases} \frac{1}{2}\langle X, QX\rangle + c\sigma\langle e_x, (\text{sgn}\dot{x} - \text{sgn}\dot{x}_d)\rangle \\ = cK\|e_x\|^2 + (\beta - mc)\|\dot{e}_x\|^2 + c\sigma\langle e_x, (\text{sgn}\dot{x} - \text{sgn}\dot{x}_d)\rangle > 0 \end{cases} \tag{34}$$

which implies that the system in (31) is passive. There are two potential problems with this approach: The value of K for the kth run depends on the result of the kth run, and K may become unbounded. The first issue may require re-running an iteration with a larger K. The second problem has indeed been observed in simulation. It may be addressed by bounding K by a maximum gain $K_{\max}$. A bound on the growth of K may be obtained from (34):

$$K\|e_x\|^2 \ge \sigma\|e_x\| \implies K \ge \frac{\sigma}{\|e_x\|}. \tag{35}$$

Given the upperbound $K_{\max}$, then the worst case $\|e_x\|$ such that (34) is still satisfied is $\frac{\sigma}{K_{\max}}$.

IV. Simulation

Numerical simulation results for this system working in a repetitive mode with different ILC update laws are presented in this section. The numerical integration is done using the classical fourth-order Runge-Kutta method with a fixed step size of 0.1ms. The desired position, velocity, and acceleration trajectories (x_d, $\dot{x}_d$, and $\ddot{x}_d$) are shown in Figure 4. The

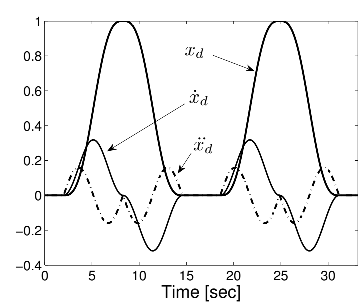

Fig. 4. Desired position x_d, desired velocity $\dot{x}_d$ and desired acceleration $\ddot{x}_d$.

parameters utilized in the simulations are shown in Table I. The value of σ has been chosen so that dry friction significantly affects the behavior of the system. The input at the first iteration is chosen to be zero, i.e., $u_0 = 0$.

Property	Parameter	Value	Units
mass	m	0.01	Kg
damping	β	0.05	Ns/m
dry friction	σ	0.2	N

TABLE I

SIMULATION PARAMETERS.

A. Velocity Output Case

When the output is just the velocity, we use the following proportional update rule:

$$u_{k+1} = u_k - 0.1(\dot{x}_k - \dot{x}_d). \quad (36)$$

The gain is chosen to satisfy (7). Figure 5 shows the monotonic convergence of the velocity tracking error. Figure 6 shows the convergence of the input to a discontinuous u_d. However, the position error in Figure 7 oscillates as the iteration progresses.

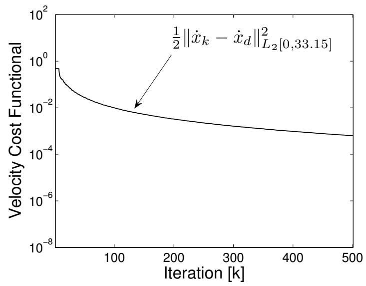

Fig. 5. Velocity convergence: $\frac{1}{2}\|\dot{x}_k - \dot{x}_d\|^2_{L_2[0,33.15]} \to 0$ as $k \to \infty$.

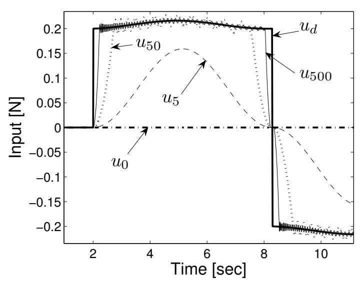

Fig. 6. Input convergence: $u_k \to u_d$ as $k \to \infty$.

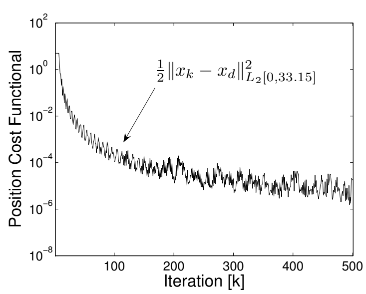

Fig. 7. Position convergence: $\frac{1}{2}\|x_k - x_d\|^2_{L_2[0,33.15]} \to 0$ as $k \to \infty$.

B. Combined Position/Velocity Output Case

For the combined position and velocity feedback case, the output is chosen to be ($c = 1$)

$$y = x + \dot{x}.$$

The modified update law in (30) with $\Gamma = 1$ and $L = 1.95$ (to satisfy (32)) is

$$u_{k+1} = u_k - 0.95 e_{y_k} - e_{y_{k+1}} - K e_{x_{k+1}}. \quad (37)$$

The adjustment of K to ensure the passivity condition (34) for each iteration is shown in Figure 8. The evolution of K is shown in Figure 9, which increases as $\|e_x\|$ decreases. The velocity and position errors both now converge monotonically as shown in Figures 10–11. Figure 12 shows the convergence of the input to a discontinuous u_d.

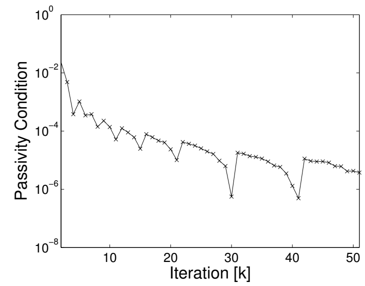

Fig. 8. Passivity condition to ensure convergence: $cK\|e_x\|^2 + (\beta - mc)\|\dot{e}_x\|^2 + c\sigma\langle e_x, (\text{sgn}\dot{x} - \text{sgn}\dot{x}_d)\rangle > 0$.

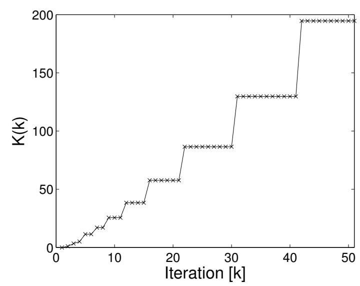

Fig. 9. Adjustment of the feedback gain K.

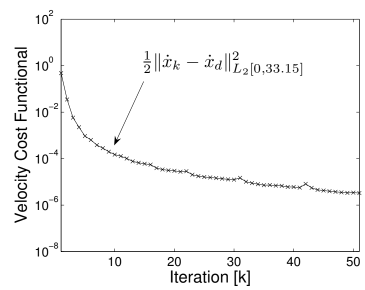

Fig. 10. Velocity convergence: $\frac{1}{2}\|\dot{x}_k - \dot{x}_d\|^2_{L_2[0,33.15]} \to 0$ as $k \to \infty$.

V. Conclusion

This paper presented the application and extension of the passivity based ILC to a mass-damper system under dry friction. The approach is potentially applicable to more general nonsmooth dynamical systems as well. A 2D system theoretic interpretation of passivity based ILC is also provided with an extension that is equivalent to an additional output feedback. In the case that the passivity analysis is not directly applicable, such as a combined velocity and position output, we showed that by adjusting the combination

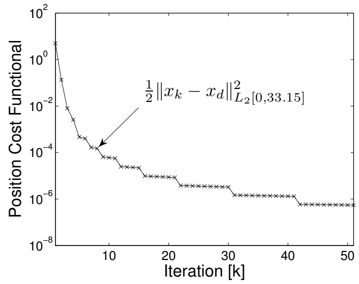

Fig. 11. Position convergence: $\frac{1}{2}\|x_k - x_d\|^2_{L_2[0,33.15]} \to 0$ as $k \to \infty$.

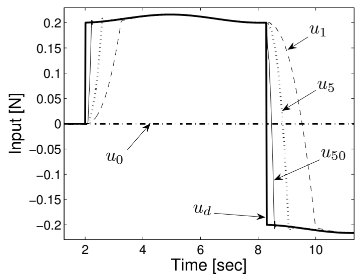

Fig. 12. Input convergence: $u_k \to u_d$ as $k \to \infty$.

between the position and velocity outputs (essentially a two-output system) to ensure passivity, asymptotic convergence may still be obtained under certain conditions. This approach may have broader application to sensor rich systems where there are more outputs than inputs.

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References

[1] M. Uchiyama, "Formulation of high-speed motion pattern of a mechanical arm by trial (in japanese)," *Transactions of the Society for Instrumentation and Control Engineers*, vol. 14, pp. 706–712, 1978.

[2] S. Arimoto, S. Kawamura, and F. Miyazaki, "Bettering operation of robots by learning," *Journal of Robotic Systems*, vol. 1, pp. 123–140, 1984.

[3] G. Casalino and G. Bartolini, "A learning procedure for the control of movements of robotic manipulators," in *IASTED Symposium on Robotics and Automation*, pp. 108–111, 1984.

[4] J. Craig, "Adaptive control of manipulators through repeated trials," in *Proceedings of the American Control Conference*, pp. 1566–1573, 1984.

[5] J. Xu, Q. Hu, T. Lee, and S. Yamamoto, "Iterative learning control with smith time delay compensator for batch processes," *Journal of Process Control*, vol. 11, pp. 321–328, 2001.

[6] H. Havlicsek and A. Alleyne, "Nonlinear control of an electrohydraulic injection molding machine via iterative learning," in *Proceedings of the 1999 American Control Conference*, (San Diego, California), pp. 176–181, June 1999.

[7] X. Zha and Y. Chen, "The iterative learning control strategy for hybrid active filter to dampen harmonic resonance in industrial power systems," in *Proceedings of the IEEE International Symposium on Industrial Electronics*, pp. 848–853, 2003.

[8] Y. Chen, C. Wen, J. Xu, and M. Sun, "High-order iterative learning identification of projectile's aerodynamic drag coefficient curve from radar measured velocity data," *IEEE Transactions on Control Systems Technology*, vol. 6, no. 4, pp. 563–570, 1998.

[9] B. Costic, M. de Queiroz, and D. Dawson, "A new learning control approach to the active magnetic bearing benchmark system," in *Proceedings of the 2000 American Control Conference*, pp. 2639–2643, 2000.

[10] K. Moore, "Iterative learning control - an expository overview," *Applied and computational control, signals, and circuits*, vol. 1, pp. 151–214, 1999.

[11] Y. Wang and R. Longman, "Limit cycle behavior and convergence to zero error in learning control with stick-slip friction," in *International Conference on Systems, Man, and Cybernetics, 1994. 'Humans, Information and Technology'*, vol. 3, pp. 2774–2779, 1994.

[12] D. Zheng and A. Alleyne, "Adaptive iterative learning control for systems with non-smooth nonlinearities," in *Proceedings of 2001 ASME International Mechanical Engineering Congress and Exposition*, vol. 70, pp. 609–617, 2001.

[13] S. Kawamura, F. Miyazaki, and S. Arimoto, "Iterative learning control for robotic systems," in *Proceedings of IECON '84*, pp. 393–398, 1984.

[14] S. Arimoto, "Learning control theory for robotic motion," *International Journal of Adaptive Control and Signal Processing*, vol. 4, pp. 543–564, 1990.

[15] S. Arimoto, "Robustness of learning control for robot manipulators," in *Proceedings of 1990 IEEE International Conference on Robotics and Automation*, pp. 1528–1533, 1990.

[16] S. Arimoto, T.Naniwa, and H.Suzuki, "Robustness of p-type learning control with a forgetting factor for robotics," in *Proceedings of the 29th IEEE Conference on Decision and Control*, pp. 2640–2645, 1990.

[17] S. Arimoto, T.Naniwa, and H.Suzuki, "Selective learning with a forgetting factor for robotic motion control," in *Proceedings of the 1991 IEEE International Conference on Robotics and Automation*, pp. 728–733, 1991.

[18] M. Yamakita and K. Furuta, "Iterative generation of virtual reference for a manipulator," *Robotica*, vol. 9, pp. 71–80, 1991.

[19] B. Potsaid and J. Wen, "High performance motion tracking control," in *Proceedings of the 2004 IEEE International Conference on Control Applications*, vol. 1, pp. 718–723, 2004.

[20] M. Togai and O. Yamano, "Analysis and design of an optimal learning control scheme for industrial robots: a discrete system approach," in *Proceedings of the 24th IEEE Conference on Decision and Control*, 1985.

[21] K. Furuta and M. Yamakita, "Iterative generation of optimal input of a manipulator," in *Proceedings of the 1986 IEEE International Conference on Robotics and Automation*, vol. 3, pp. 579–584, 1986.

[22] K. Avrachenkov, "Iterative learning control based on quasi-newton methods," in *Proceedings of the 37th IEEE Conference on Decision and Control*, vol. 1, pp. 170–174, 1998.

[23] J. Ghosh and B. Paden, "A pseudoinverse-based iterative learning control," *IEEE Transactions on Automatic Control*, vol. 47, no. 5, pp. 831–837, 2002.

[24] D. Bristow, M. Tharayil, and A. Alleyne, "A survey of iterative learning control: A learning-based method for high-performance tracking control," *IEEE Control Systems Magazine*, vol. 26, no. 3, pp. 96–114, 2006.

[25] J. Slotine and W. Li, *Applied Nonlinear Control*. Prentice Hall, 1991.

[26] D. Bayard and J. Wen, "A new class of stabilizing control laws for robotic manipulators, part ii: Adaptive case," *International Journal of Control*, vol. 47, no. 5, pp. 1387–1406, 1988.

[27] N. Amann, D. H. Owens, and E. Rogers, "Iterative learning control using optimal feedback and feedforward actions," *International Journal of Control*, vol. 65, no. 2, pp. 277–293, 1996.