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Fuzzy Model-based Servo Control for Discrete-time Nonlinear Systems

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Abstract— **This paper presents servo control for discrete-time nonlinear systems using the switching fuzzy model-based control approach. We propose the construction method of augmented switching fuzzy servo control system for discrete-time nonlinear systems. Then we introduce the switching fuzzy servo controller which can make outputs of the nonlinear systems converge to target points, and derive the controller design conditions in terms of LMIs. A design example illustrates the utility of this approach.**

I. INTRODUCTION

Recently, fuzzy model-based control has been discussed in a huge number of literatures [2]–[5]. Most of them deal with Takagi-Sugeno (T-S) fuzzy model [1] and LMI-based designs[6]. By employing the T-S fuzzy model, which utilizes local linear system description for each rule, we can devise a control methodology to fully take advantages of linear control theory. However, most of the literatures have mainly dealt with the regulation problem to discuss stability or convergence to the origin. Unfortunately, theoretical controllability of servo control for discrete-time nonlinear systems was not discussed in the literature.

In this paper, we deal with servo control for a class of discrete-time nonlinear systems using the switching fuzzy model-based control approach [11], [12]. We propose the construction method of augmented switching fuzzy servo control system for discrete-time nonlinear systems. Moreover, we introduce the switching fuzzy servo controller which can make outputs of the nonlinear systems converge to target points, and derive the controller design conditions in terms of LMIs. A design example illustrates the utility of this approach.

II. PRELIMINARY RESULTS

In this section, we explain the basic procedures of fuzzy model-based control approach for discrete-time nonlinear systems, servo control for linear discrete systems and the switching fuzzy model construction method.

A. Fuzzy Model-based Control [7]

Consider the following discrete-time nonlinear system.

$$\boldsymbol{x}(t+1) = \boldsymbol{f}_1(\boldsymbol{x}(t)) + \boldsymbol{f}_2(\boldsymbol{x}(t))\boldsymbol{u}(t) \tag{1}$$

$$\boldsymbol{y}(t) = \boldsymbol{g}(\boldsymbol{x}(t)) \tag{2}$$

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where $\boldsymbol{x}(t) = [x_1(t)\ x_2(t)\ \cdots\ x_n(t)]^T$ is the state vector, $\boldsymbol{u}(t) = [u_1(t)\ u_2(t)\ \cdots\ u_m(t)]^T$ is the input vector, $\boldsymbol{y}(t) = [y_1(t)\ y_2(t) \cdots\ y_p(t)]$ is the output vector. For the above nonlinear system, by applying sector nonlinearity concept [7], we can obtain the following T-S fuzzy model.

$$\begin{aligned} &\text{Rule } i: \ \text{IF } z_1(t) \text{ is } M_{i1} \text{ and } \cdots \text{and } z_\ell(t) \text{ is } M_{i\ell} \\ &\qquad \text{THEN} \begin{cases} \boldsymbol{x}(t+1) = \boldsymbol{A}_i\boldsymbol{x}(t) + \boldsymbol{B}_i\boldsymbol{u}(t) \\ \boldsymbol{y}(t) = \boldsymbol{C}_i\boldsymbol{x}(t) \end{cases} \end{aligned} \tag{3}$$

where, $i = 1, 2, \cdots, r$ and r is the number of fuzzy model rules. M_{ij} is the fuzzy set. $z_j(t)$ is the known premise variable. The fuzzy reasoning process is defined as

$$\begin{aligned} \boldsymbol{x}(t+1) &= \frac{\sum_{i=1}^{r} w_i(\boldsymbol{z}(t))\left(\boldsymbol{A}_i\boldsymbol{x}(t) + \boldsymbol{B}_i\boldsymbol{u}(t)\right)}{\sum_{i=1}^{r} w_i(\boldsymbol{z}(t))} \\ &= \sum_{i=1}^{r} h_i(\boldsymbol{z}(t))\left(\boldsymbol{A}_i\boldsymbol{x}(t) + \boldsymbol{B}_i\boldsymbol{u}(t)\right) \end{aligned} \tag{4}$$

$$\begin{aligned} \boldsymbol{y}(t) &= \frac{\sum_{i=1}^{r} w_i(\boldsymbol{z}(t))\boldsymbol{C}_i\boldsymbol{x}(t)}{\sum_{i=1}^{r} w_i(\boldsymbol{z}(t))} \\ &= \sum_{i=1}^{r} h_i(\boldsymbol{z}(t))\boldsymbol{C}_i\boldsymbol{x}(t) \end{aligned} \tag{5}$$

where

$$\boldsymbol{z}(t) = [z_1(t)\ z_2(t)\ \cdots\ z_\ell(t)]$$

$$w_i(\boldsymbol{z}(t)) = \prod_{j=1}^{\ell} M_{ij}(z_j(t)),\ h_i(\boldsymbol{z}(t)) = \frac{w_i(\boldsymbol{z}(t))}{\sum_{i=1}^{r} w_i(\boldsymbol{z}(t))}$$

$M_{ij}(z_j(t))$ is the grade of membership of $z_j(t)$ in M_{ij}. $w_i(\boldsymbol{z}(t))$ and $h_i(\boldsymbol{z}(t))$ have the following properties.

$$\sum_{i=1}^{r} w_i(\boldsymbol{z}(t)) > 0, \quad w_i(\boldsymbol{z}(t)) \geq 0,\ \forall i$$

$$\sum_{i=1}^{r} h_i(\boldsymbol{z}(t)) = 1, \quad h_i(\boldsymbol{z}(t)) \geq 0,\ \forall i$$

To stabilize the T-S fuzzy model (4), we employ the so-called parallel distributed compensation (PDC) control approach [2], [3]. The PDC fuzzy controller is represented

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as

$$\boldsymbol{u}(t)=-\sum_{i=1}^{r}h_i(\boldsymbol{z}(t))\boldsymbol{K}_i\boldsymbol{x}(t) \tag{6}$$

where $\boldsymbol{K}_i$ is a feedback gain. The PDC fuzzy controller design is to determine the feedback gain $\boldsymbol{K}_i$. By substituting (6) into (4), the overall fuzzy control system is represented as follows:

$$\boldsymbol{x}(t+1)=\sum_{i=1}^{r}\sum_{j=1}^{r}h_i(\boldsymbol{z}(t))h_j(\boldsymbol{z}(t))\left(\boldsymbol{A}_i-\boldsymbol{B}_i\boldsymbol{K}_j\right)\boldsymbol{x}(t) \tag{7}$$

The feedback gain $\boldsymbol{K}_i$ is determined by solving Theorem 1.

Theorem 1: [7] If there exist positive definite matrix $\boldsymbol{X}$ and $\boldsymbol{M}_i$ satisfying (8), (9) and (10), then the fuzzy model (4) can be stabilized by the fuzzy controller (6).

$$\boldsymbol{X}>\boldsymbol{0}, \tag{8}$$

$$\begin{bmatrix} \boldsymbol{X} & \boldsymbol{X}\boldsymbol{A}_i^T-\boldsymbol{M}_i^T\boldsymbol{B}_i^T \\ \boldsymbol{A}_i\boldsymbol{X}-\boldsymbol{B}_i\boldsymbol{M}_i & \boldsymbol{X} \end{bmatrix}>\boldsymbol{0},\quad \forall i, \tag{9}$$

$$\begin{bmatrix} \boldsymbol{X} & \frac{1}{2}\left\{\boldsymbol{X}\boldsymbol{A}_i^T-\boldsymbol{M}_j^T\boldsymbol{B}_i^T \right. \\ & \left. \boldsymbol{X}\boldsymbol{A}_j^T-\boldsymbol{M}_i^T\boldsymbol{B}_j^T\right\} \\ \frac{1}{2}\left\{\boldsymbol{A}_i\boldsymbol{X}-\boldsymbol{B}_i\boldsymbol{M}_j \right. & \\ \left. \boldsymbol{A}_j\boldsymbol{X}-\boldsymbol{B}_j\boldsymbol{M}_i\right\} & \boldsymbol{X} \end{bmatrix}>\boldsymbol{0}, \tag{10}$$
$$\forall i, i<j,$$

where $\boldsymbol{K}_i=\boldsymbol{M}_i\boldsymbol{X}^{-1}$.

B. Servo Control for Discrete-time Linear Systems

In this section, we explain the servo control for discrete-time linear systems [8]. Consider the following linear system.

$$\boldsymbol{x}(t+1)=\boldsymbol{A}\boldsymbol{x}(t)+\boldsymbol{B}\boldsymbol{u}(t) \tag{11}$$

$$\boldsymbol{y}(t)=\boldsymbol{C}\boldsymbol{x}(t) \tag{12}$$

For the above linear system, we consider the servo control problem, that is, the control problem to make the output $\boldsymbol{y}(t)$ converge to the certain target point $\boldsymbol{r}$, where $\boldsymbol{r}$ is the constant vector. We assume that all states are measurable.

Firstly, we define the difference $\Delta\boldsymbol{x}(t)$ between $\boldsymbol{x}(t)$ and $\boldsymbol{x}(t+1)$ and the error vector $\boldsymbol{e}(t)$ as follows:

$$\Delta\boldsymbol{x}(t)=\boldsymbol{x}(t+1)-\boldsymbol{x}(t) \tag{13}$$

$$\boldsymbol{e}(t)=\boldsymbol{y}(t)-\boldsymbol{r} \tag{14}$$

$$\begin{aligned}\boldsymbol{e}(t+1)&=\boldsymbol{y}(t+1)-r\\&=\boldsymbol{y}(t+1)-\boldsymbol{y}(t)+\boldsymbol{y}(t)-r\\&=\boldsymbol{C}\Delta\boldsymbol{x}(t)+\boldsymbol{e}(t)\end{aligned} \tag{15}$$

Then, the next state of the difference can be obtained as follows:

$$\begin{aligned}\Delta\boldsymbol{x}(t+1)&=\boldsymbol{x}(t+2)-\boldsymbol{x}(t+1)\\&=\boldsymbol{A}\boldsymbol{x}(t+1)+\boldsymbol{B}\boldsymbol{u}(t+1)-\boldsymbol{A}\boldsymbol{x}(t)-\boldsymbol{B}\boldsymbol{u}(t)\\&=\boldsymbol{A}\Delta\boldsymbol{x}(t)+\boldsymbol{B}\Delta\boldsymbol{u}(t)\end{aligned} \tag{16}$$

where $\Delta\boldsymbol{u}(t)=\boldsymbol{u}(t+1)-\boldsymbol{u}(t)$.

Next, we construct the following augmented system by adding (15) and (16).

$$\begin{aligned}\begin{bmatrix}\Delta\boldsymbol{x}(t+1)\\ \boldsymbol{e}(t+1)\end{bmatrix}&=\begin{bmatrix}\boldsymbol{A} & \boldsymbol{0}\\ \boldsymbol{C} & \boldsymbol{I}\end{bmatrix}\begin{bmatrix}\Delta\boldsymbol{x}(t)\\ \boldsymbol{e}(t)\end{bmatrix}+\begin{bmatrix}\boldsymbol{B}\\ \boldsymbol{0}\end{bmatrix}\Delta\boldsymbol{u}(t)\\&=\hat{\boldsymbol{A}}\begin{bmatrix}\Delta\boldsymbol{x}(t)\\ \boldsymbol{e}(t)\end{bmatrix}+\hat{\boldsymbol{B}}\Delta\boldsymbol{u}(t)\end{aligned} \tag{17}$$

Finally, we design the following dynamic controller to stabilize the augmented system (17).

$$\Delta\boldsymbol{u}(t)=-\boldsymbol{K}\begin{bmatrix}\Delta\boldsymbol{x}(t)\\ \boldsymbol{e}(t)\end{bmatrix} \tag{18}$$

where $\boldsymbol{K}$ is a feedback gain. The controller design is to determine the feedback gain $\boldsymbol{K}$. By substituting (18) into (17), we can obtain the following linear control system.

$$\begin{bmatrix}\Delta\boldsymbol{x}(t+1)\\ \boldsymbol{e}(t+1)\end{bmatrix}=\left(\hat{\boldsymbol{A}}-\hat{\boldsymbol{B}}\boldsymbol{K}\right)\begin{bmatrix}\Delta\boldsymbol{x}(t)\\ \boldsymbol{e}(t)\end{bmatrix} \tag{19}$$

The feedback gain $\boldsymbol{K}$ is determined by solving Theorem 2.

Theorem 2: If there exist positive definite matrix $\boldsymbol{X}$ and $\boldsymbol{M}$ satisfying (20) and (21), then the augmented system (17) can be stabilized by the dynamic controller (18).

$$\boldsymbol{X}>\boldsymbol{0}, \tag{20}$$

$$\begin{bmatrix}\boldsymbol{X} & \boldsymbol{X}\hat{\boldsymbol{A}}^T-\boldsymbol{M}^T\hat{\boldsymbol{B}}^T\\ \hat{\boldsymbol{A}}\boldsymbol{X}-\hat{\boldsymbol{B}}\boldsymbol{M} & \boldsymbol{X}\end{bmatrix}>\boldsymbol{0}, \tag{21}$$

where $\boldsymbol{K}=\boldsymbol{M}\boldsymbol{X}^{-1}$ and $\boldsymbol{u}(t+1)=\boldsymbol{u}(t)+\Delta\boldsymbol{u}(t)$.

By using the designed controller, we can make the output $\boldsymbol{y}(t)$ of the linear system (11) converge to the target point $\boldsymbol{r}$.

C. Switching Fuzzy Model Construction

In this section, we show the switching fuzzy model construction method [11], [12].

1) Switching Planes and Continuity Matrix: To begin with, determine the dividing planes. We assume that the dividing planes contain the origin. Dividing planes are represented by the following linear equations.

$$\lambda_{\gamma 1}x_1+\lambda_{\gamma 2}x_2+\cdots+\lambda_{\gamma n}x_n=\boldsymbol{\Lambda}_\gamma\boldsymbol{x}=0$$

where $\gamma=1,2,\cdots,\Gamma$ and Γ is the number of dividing planes. We define Region R_q constructed by dividing planes as follows.

Definition 1: One dividing plane divides the input space into the following two regions.

$$\overline{S}_\gamma=\{\boldsymbol{x}|\boldsymbol{\Lambda}_\gamma\boldsymbol{x}\geq 0\},\quad \underline{S}_\gamma=\{\boldsymbol{x}|\boldsymbol{\Lambda}_\gamma\boldsymbol{x}\leq 0\}$$

The input space is divided into Q regions by Γ dividing planes. Note that $Q=2^\Gamma$ is not necessarily satisfied. One region constructed by dividing planes is defined as follows:

$$\begin{aligned}R_q=\{\boldsymbol{x}|&\boldsymbol{\Lambda}_1\boldsymbol{x}\geq 0,\boldsymbol{\Lambda}_2\boldsymbol{x}\leq 0,\boldsymbol{\Lambda}_3\boldsymbol{x}\leq 0,\\&\boldsymbol{\Lambda}_4\boldsymbol{x}\geq 0,\cdots,\boldsymbol{\Lambda}_\Gamma\boldsymbol{x}\geq 0\}\end{aligned}$$

We represent the region as follows:

$$\begin{aligned}&R_q(s_1,s_2,s_3,s_4,\cdots,s_\Gamma)\\&s_1=1,s_4,\cdots,s_\Gamma=1,s_2,s_3=0\end{aligned}$$

or

$$R_q(1,0,0,1,\cdots,1)$$

where

$$s_\gamma = \begin{cases} 1 & \boldsymbol{\Lambda}_\gamma \boldsymbol{x} \geq 0 \\ 0 & \boldsymbol{\Lambda}_\gamma \boldsymbol{x} \leq 0 \end{cases}$$

Next we define the continuity matrix $\boldsymbol{K}_q \in \mathbb{R}^{(2\Gamma+n)\times n}$.

Definition 2:

$$\boldsymbol{K}_q = \Big[\eta_{q11}\boldsymbol{\Lambda}_1^T \ \eta_{q12}\boldsymbol{\Lambda}_2^T \ \cdots \ \eta_{q1\Gamma}\boldsymbol{\Lambda}_\Gamma^T \\ -\eta_{q21}\boldsymbol{\Lambda}_1^T \ -\eta_{q22}\boldsymbol{\Lambda}_2^T \ \cdots \ -\eta_{q2\Gamma}\boldsymbol{\Lambda}_\Gamma^T \ \boldsymbol{I}_n\Big]^T$$

where

$$\eta_{q1\gamma} = \begin{cases} 1 & R_q \in \overline{S}_\gamma \\ 0 & R_q \notin \overline{S}_\gamma \end{cases} \quad \eta_{q2\gamma} = \begin{cases} 1 & R_q \in \underline{S}_\gamma \\ 0 & R_q \notin \underline{S}_\gamma \end{cases}$$

$$\overline{S}_\gamma = \{\boldsymbol{x}|\boldsymbol{\Lambda}_\gamma\boldsymbol{x} \geq 0\}, \quad \underline{S}_\gamma = \{\boldsymbol{x}|\boldsymbol{\Lambda}_\gamma\boldsymbol{x} \leq 0\}.$$

The continuity matrix $\boldsymbol{K}_q$ satisfies the following condition on region boundaries.

$$\boldsymbol{K}_{q_1}\boldsymbol{x} = \boldsymbol{K}_{q_2}\boldsymbol{x}, \quad \boldsymbol{x} \in R_{q_1} \cap R_{q_2}.$$

2) Switching Fuzzy Model Construction: Consider the following nonlinear function.

$$y = f(\boldsymbol{x}) = f(x_1, x_2, \cdots, x_n) \tag{22}$$

where $f(\boldsymbol{0}) = 0$. By solving the following conditions, we can construct the tight sector which can cover the nonlinear function (22) and satisfies the continuity on region boundaries.

$$\underset{\boldsymbol{a}_1, \boldsymbol{a}_2}{\text{minimize}} \sum_{q=1}^{Q} |Y_{q1}(\boldsymbol{a}_1) - Y_{q2}(\boldsymbol{a}_2)| \tag{23}$$

$$\text{subject to}$$

$$y_{q1}(\boldsymbol{a}_1, \boldsymbol{x}) - f(\boldsymbol{x}) \geq 0, \quad \boldsymbol{x} \in R_q, \ \forall q$$

$$f(\boldsymbol{x}) - y_{q2}(\boldsymbol{a}_2, \boldsymbol{x}) \geq 0, \quad \boldsymbol{x} \in R_q, \ \forall q$$

where

$$\boldsymbol{a}_i = [\, a_{i1} \ a_{i2} \ \cdots \ a_{i(2\Gamma+n)} \,]$$

$$Y_{qi}(\boldsymbol{a}_i) = \boldsymbol{a}_i \boldsymbol{K}_q \boldsymbol{D}_q$$

$$\boldsymbol{D}_q = \int \cdots \int \int_{R_q} \boldsymbol{x} \ dx_1 dx_2 \cdots dx_n$$

$$y_{qi}(\boldsymbol{a}_i, \boldsymbol{x}) = \boldsymbol{a}_i \boldsymbol{K}_q \boldsymbol{x}$$

$$|x_1| \leq d_1, |x_2| \leq d_2, \cdots, |x_n| \leq d_n$$

The sector in qth region is represented by the following two linear models.

$$y_{q1}(\boldsymbol{x}) = \boldsymbol{a}_1 \boldsymbol{K}_q \boldsymbol{x} \tag{24}$$

$$y_{q2}(\boldsymbol{x}) = \boldsymbol{a}_2 \boldsymbol{K}_q \boldsymbol{x} \tag{25}$$

By using (24) and (25), the switching fuzzy model can be constructed as follows:

Region q :

Rule 1 : IF $(x_1, \cdots, x_n)$ is h_{q1} THEN $y_{q1} = \boldsymbol{a}_1 \boldsymbol{K}_q \boldsymbol{x}$

Rule 2 : IF $(x_1, \cdots, x_n)$ is h_{q2} THEN $y_{q2} = \boldsymbol{a}_2 \boldsymbol{K}_q \boldsymbol{x}$

The membership function can be represented by the following equations.

$$h_{q1}(\boldsymbol{x}) = \frac{f(\boldsymbol{x}) - y_{q2}(\boldsymbol{x})}{y_{q1}(\boldsymbol{x}) - y_{q2}(\boldsymbol{x})}$$

$$h_{q2}(\boldsymbol{x}) = \frac{y_{q1}(\boldsymbol{x}) - f(\boldsymbol{x})}{y_{q1}(\boldsymbol{x}) - y_{q2}(\boldsymbol{x})}$$

where $h_{q1}(\boldsymbol{x}) \geq 0$, $h_{q2}(\boldsymbol{x}) \geq 0$ and $h_{q1}(\boldsymbol{x}) + h_{q2}(\boldsymbol{x}) = 1$. The fuzzy reasoning process is defined as

$$y = \sum_{q=1}^{Q} \sum_{i=1}^{2} v_q(\boldsymbol{x}) h_{qi}(\boldsymbol{x}) \boldsymbol{a}_i \boldsymbol{K}_q \boldsymbol{x}$$

where

$$v_q(\boldsymbol{x}) = \begin{cases} 1, & \boldsymbol{x} \in R_q, \\ 0, & \boldsymbol{x} \notin R_q. \end{cases}$$

Remark 1: We show the important property of the switching fuzzy model construction. Consider the following nonlinear functions.

$$\psi_1 = f(\boldsymbol{\phi}_1)$$

$$\psi_2 = f(\boldsymbol{\phi}_2)$$

where $\boldsymbol{\phi}_1 = [\phi_{11} \ \cdots \ \phi_{1n}]$ and $\boldsymbol{\phi}_2 = [\phi_{21} \ \cdots \ \phi_{2n}]$. ψ_1 and ψ_2 are scalar outputs. We construct difference function which consists of above two functions.

$$y = \mathcal{F}(\boldsymbol{\phi}_1, \boldsymbol{\phi}_2) = f(\boldsymbol{\phi}_1) - f(\boldsymbol{\phi}_2)$$

By using the switching fuzzy model construction method, the difference function can be converted into the switching fuzzy model which is linear with respect to $\boldsymbol{\phi}_1 - \boldsymbol{\phi}_2$.

We define the following n dividing planes.

$$\phi_{11} - \phi_{21} = 0$$

$$\phi_{12} - \phi_{22} = 0$$

$$\vdots$$

$$\phi_{1n} - \phi_{2n} = 0$$

By solving the switching fuzzy model construction condition (23) with $\boldsymbol{a}_i = [a_{i1} \ a_{i2} \ \cdots \ a_{in} \ 0 \ \cdots \ 0]$, then we can obtain the following sector.

$$y_{q1}(\boldsymbol{\phi}_1, \boldsymbol{\phi}_2) = \boldsymbol{a}_1 \boldsymbol{K}_q [\boldsymbol{\phi}_1^T \ \boldsymbol{\phi}_2^T]^T = \hat{\boldsymbol{a}}_{q1}(\boldsymbol{\phi}_1 - \boldsymbol{\phi}_2)$$

$$y_{q2}(\boldsymbol{\phi}_1, \boldsymbol{\phi}_2) = \boldsymbol{a}_2 \boldsymbol{K}_q [\boldsymbol{\phi}_1^T \ \boldsymbol{\phi}_2^T]^T = \hat{\boldsymbol{a}}_{q2}(\boldsymbol{\phi}_1 - \boldsymbol{\phi}_2)$$

where $\hat{\boldsymbol{a}}_{q1} = [a_{q11} \ a_{q12} \ \cdots \ a_{q1n}]$ and $\hat{\boldsymbol{a}}_{q2} = [a_{q21} \ a_{q22} \ \cdots \ a_{q2n}]$. This means that we can obtain the following switching fuzzy model.

$$\begin{aligned} y &= f(\boldsymbol{\phi}_1) - f(\boldsymbol{\phi}_2) \\ &= \sum_{q=1}^{Q} \sum_{i=1}^{2} v_q(\boldsymbol{\phi}_1 - \boldsymbol{\phi}_2) h_{qi}(\boldsymbol{\phi}_1, \boldsymbol{\phi}_2) \\ &\qquad \times \hat{\boldsymbol{a}}_{qi}(\boldsymbol{\phi}_1 - \boldsymbol{\phi}_2) \end{aligned}$$

The switching fuzzy model is linear with respect to $\boldsymbol{\phi}_1 - \boldsymbol{\phi}_2$. We utilize the above switching fuzzy model in Section III-A.

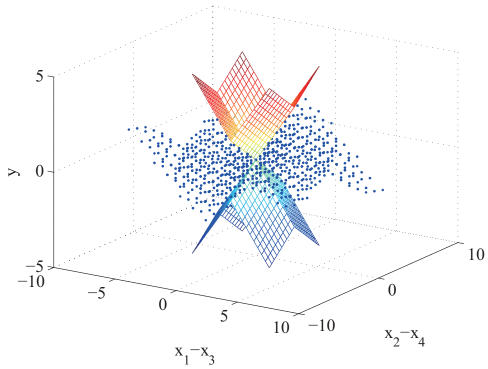

Fig. 1. Sectors which cover the nonlinear function (27).

[Example]

We consider the following difference function.

$$\begin{aligned} y &= f(\boldsymbol{\phi}_1) - f(\boldsymbol{\phi}_2) \\ &= \sin(\phi_{11} - 0.5\phi_{12}) - \sin(\phi_{21} - 0.5\phi_{22}) \end{aligned} \tag{26}$$

where $\boldsymbol{\phi}_1 = [\phi_{11}\ \phi_{12}]^T$ and $\boldsymbol{\phi}_2 = [\phi_{21}\ \phi_{22}]^T$. By solving the condition (23) with the following dividing planes

$$\phi_{11} - \phi_{21} = 0,\ \phi_{12} - \phi_{22} = 0,$$

we can obtain the following switching fuzzy model.

$$\begin{aligned} y &= \sum_{q=1}^{4}\sum_{i=1}^{2} v_q(\boldsymbol{\phi}_1 - \boldsymbol{\phi}_2) h_{qi}(\boldsymbol{\phi}_1, \boldsymbol{\phi}_2) \\ &\quad \times \hat{\boldsymbol{a}}_{qi}(\boldsymbol{\phi}_1 - \boldsymbol{\phi}_2) \end{aligned}$$

where

$$\begin{aligned} &\hat{\boldsymbol{a}}_{11} = [1\ \ 0.5],\ \hat{\boldsymbol{a}}_{12} = [-1\ \ -0.5], \\ &\hat{\boldsymbol{a}}_{21} = [-1\ \ 0.5],\ \hat{\boldsymbol{a}}_{22} = [1\ \ -0.5], \\ &\hat{\boldsymbol{a}}_{31} = [-1\ \ -0.5],\ \hat{\boldsymbol{a}}_{32} = [1\ \ 0.5], \\ &\hat{\boldsymbol{a}}_{41} = [1\ \ -0.5],\ \hat{\boldsymbol{a}}_{42} = [-1\ \ 0.5], \\ &h_{q1}(\boldsymbol{\phi}_1, \boldsymbol{\phi}_2) = \frac{y - \hat{\boldsymbol{a}}_{q2}(\boldsymbol{\phi}_1 - \boldsymbol{\phi}_2)}{\hat{\boldsymbol{a}}_{q1}(\boldsymbol{\phi}_1 - \boldsymbol{\phi}_2) - \hat{\boldsymbol{a}}_{q2}(\boldsymbol{\phi}_1 - \boldsymbol{\phi}_2)} \\ &h_{q2}(\boldsymbol{\phi}_1, \boldsymbol{\phi}_2) = \frac{\hat{\boldsymbol{a}}_{q1}(\boldsymbol{\phi}_1 - \boldsymbol{\phi}_2) - y}{\hat{\boldsymbol{a}}_{q1}(\boldsymbol{\phi}_1 - \boldsymbol{\phi}_2) - \hat{\boldsymbol{a}}_{q2}(\boldsymbol{\phi}_1 - \boldsymbol{\phi}_2)}. \end{aligned}$$

Fig. 1 shows sectors (linear planes) and values of the nonlinear function (dots). X-axis represents $x_1 - x_3$. Y-axis represents $x_2 - x_4$. All dots are covered by linear planes.

III. Fuzzy Model-based Servo Control for Nonlinear Systems

In this section, we propose the servo control to make the output $\boldsymbol{y}(t)$ converge to the target point $\boldsymbol{r}$ for discrete-time nonlinear systems using the switching fuzzy model-based nonlinear control approach.

Consider the following discrete-time nonlinear system.

$$\boldsymbol{x}(t+1) = \boldsymbol{f}(\boldsymbol{x}(t), \boldsymbol{u}(t)) \tag{27}$$

$$\boldsymbol{y}(t) = \boldsymbol{g}(\boldsymbol{x}(t)) \tag{28}$$

We assume that $\boldsymbol{f}$ and $\boldsymbol{g}$ are known. Note that (27) is a more general form than (1). We define the error vector $\boldsymbol{e}(t)$ as follows:

$$\boldsymbol{e}(t) = \boldsymbol{y}(t) - \boldsymbol{r} \tag{29}$$

A. Construction of Augmented Fuzzy System

Firstly, we construct the following time-difference system.

$$\begin{aligned} \Delta\boldsymbol{x}(t+1) &= \boldsymbol{\mathcal{F}}(\boldsymbol{x}(t), \boldsymbol{u}(t), \boldsymbol{x}(t+1), \boldsymbol{u}(t+1)) \\ &= \boldsymbol{f}(\boldsymbol{x}(t+1), \boldsymbol{u}(t+1)) - \boldsymbol{f}(\boldsymbol{x}(t), \boldsymbol{u}(t)) \end{aligned} \tag{30}$$

Then, by time-shifting error vector (29), we can obtain the following equation.

$$\begin{aligned} \boldsymbol{e}(t+1) &= \boldsymbol{y}(t+1) - \boldsymbol{r} \\ &= \boldsymbol{g}(\boldsymbol{x}(t+1)) - \boldsymbol{g}(\boldsymbol{x}(t)) + \boldsymbol{g}(\boldsymbol{x}(t)) - \boldsymbol{r} \\ &= \boldsymbol{g}(\boldsymbol{x}(t+1)) - \boldsymbol{g}(\boldsymbol{x}(t)) + \boldsymbol{e}(t) \end{aligned} \tag{31}$$

Next, by adding (30) and (31), the augmented system is constructed as follows:

$$\begin{aligned} &\begin{bmatrix} \Delta\boldsymbol{x}(t+1) \\ \boldsymbol{e}(t+1) \end{bmatrix} \\ &= \begin{bmatrix} \boldsymbol{f}(\boldsymbol{x}(t+1), \boldsymbol{u}(t+1)) - \boldsymbol{f}(\boldsymbol{x}(t), \boldsymbol{u}(t)) \\ \boldsymbol{g}(\boldsymbol{x}(t+1)) - \boldsymbol{g}(\boldsymbol{x}(t)) + \boldsymbol{e}(t) \end{bmatrix} \end{aligned} \tag{32}$$

By applying switching fuzzy model construction method, which is described in Remark 1, to each nonlinear function in the augmented system (32), we can obtain the following augmented switching T-S fuzzy model.

$$\begin{aligned} \begin{bmatrix} \Delta\boldsymbol{x}(t+1) \\ \boldsymbol{e}(t+1) \end{bmatrix} &= \sum_{q=1}^{Q}\sum_{i=1}^{r} v_q(\Delta\boldsymbol{x}(t), \Delta\boldsymbol{u}(t)) \\ &\quad \times h_{qi}(\boldsymbol{x}(t), \boldsymbol{x}(t+1), \boldsymbol{u}(t), \boldsymbol{u}(t+1)) \\ &\quad \times \left(\begin{bmatrix} \boldsymbol{A}_{qi} & \boldsymbol{0} \\ \boldsymbol{C}_{qi} & \boldsymbol{I} \end{bmatrix} \begin{bmatrix} \Delta\boldsymbol{x}(t) \\ \boldsymbol{e}(t) \end{bmatrix} + \begin{bmatrix} \boldsymbol{B}_{qi} \\ \boldsymbol{0} \end{bmatrix} \Delta\boldsymbol{u}(t) \right) \end{aligned} \tag{33}$$

where dividing planes for the switching fuzzy model construction are defined as follows:

$$\begin{aligned} \boldsymbol{x}(t) - \boldsymbol{x}(t+1) &= \boldsymbol{0} \\ \boldsymbol{u}(t) - \boldsymbol{u}(t+1) &= \boldsymbol{0} \end{aligned}$$

The number of dividing planes are $\Gamma = n + m$.

B. Dynamic Fuzzy Servo Controller Design

In this section, we design the dynamic switching fuzzy controller for the augmented switching T-S fuzzy model (33). Consider the following stable linear system.

$$\Delta\hat{\boldsymbol{x}}(t+1) = \alpha \boldsymbol{I}_{2\Gamma}\hat{\boldsymbol{x}}(t) \tag{34}$$

where $|\alpha| < 1$, $\boldsymbol{I}_{2\Gamma}$ is an identity matrix, $\hat{\boldsymbol{x}}(t) = [\hat{x}_1(t)\ \hat{x}_2(t)\ \cdots\ \hat{x}_{2\Gamma}(t)]^T$ is a state vector for the linear system (34).

To stabilize the augmented switching T-S fuzzy model (33), we propose the following dynamic switching fuzzy servo controller.

$$\begin{aligned} \Delta\boldsymbol{u}(t+1) &= \sum_{q=1}^{Q}\sum_{i=1}^{r} v_q(\Delta\boldsymbol{x}(t), \Delta\boldsymbol{u}(t)) \\ &\quad \times h_i(\boldsymbol{x}(t), \boldsymbol{x}(t+1), \boldsymbol{u}(t), \boldsymbol{u}(t+1)) \\ &\quad \times (\boldsymbol{F}_{1qi}\Delta\boldsymbol{x}(t) + \boldsymbol{F}_{2qi}\boldsymbol{e}(t) \\ &\quad + \boldsymbol{F}_{3qi}\Delta\boldsymbol{u}(t) + \boldsymbol{F}_{4qi}\hat{\boldsymbol{x}}(t)) \end{aligned} \tag{35}$$

where $\boldsymbol{F}_{1qi}$, $\boldsymbol{F}_{2qi}$, $\boldsymbol{F}_{3qi}$, $\boldsymbol{F}_{4qi}$ are feedback gains. By utilizing the dynamic controller, note that the membership function $h_i(\boldsymbol{x}(t), \boldsymbol{x}(t+1), \boldsymbol{u}(t), \boldsymbol{u}(t+1))$ can be calculated although the membership function includes the time-shifted control input $\boldsymbol{u}(t+1)$. By adding the dynamic state feedback controller (35) and stable linear system (34) to the augmented switching T-S fuzzy model (33), we define the following augmented control system.

$$\tilde{\boldsymbol{x}}(t+1) = \sum_{q=1}^{Q}\sum_{i=1}^{r} v_q(\tilde{\boldsymbol{x}}(t)) \times h_{qi}(\boldsymbol{x}(t), \boldsymbol{x}(t+1), \boldsymbol{u}(t), \boldsymbol{u}(t+1))\hat{\boldsymbol{A}}_{qi}\tilde{\boldsymbol{x}}(t) \quad (36)$$

where $\tilde{\boldsymbol{x}}(t) = [\Delta\boldsymbol{x}^T(t) \;\; \boldsymbol{e}^T(t) \;\; \Delta\boldsymbol{u}^T(t) \;\; \hat{\boldsymbol{x}}^T(t)]^T$

$$\hat{\boldsymbol{A}}_{qi} = \begin{bmatrix} \boldsymbol{A}_{qi} & \boldsymbol{0} & \boldsymbol{B}_{qi} & \boldsymbol{0} \\ \boldsymbol{C}_{qi} & \boldsymbol{I} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{F}_{1qi} & \boldsymbol{F}_{2qi} & \boldsymbol{F}_{3qi} & \boldsymbol{F}_{4qi} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \alpha\boldsymbol{I}_{2\Gamma} \end{bmatrix}$$

Fuzzy dynamic system (36) can be rewritten as follows:

$$\tilde{\boldsymbol{x}}(t+1) = \sum_{q=1}^{Q}\sum_{i=1}^{r} v_q(\tilde{\boldsymbol{x}}(t)) \times h_{qi}(\boldsymbol{x}(t), \boldsymbol{x}(t+1), \boldsymbol{u}(t), \boldsymbol{u}(t+1)) \times \left(\tilde{\boldsymbol{A}}_{qi} + \tilde{\boldsymbol{B}}\tilde{\boldsymbol{F}}_{qi}\boldsymbol{E}_q\right)\tilde{\boldsymbol{x}}(t) \quad (37)$$

where

$$\tilde{\boldsymbol{A}}_{qi} = \begin{bmatrix} \boldsymbol{A}_{qi} & \boldsymbol{0} & \boldsymbol{B}_{qi} & \boldsymbol{0} \\ \boldsymbol{C}_{qi} & \boldsymbol{I} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \alpha\boldsymbol{I}_{2\Gamma} \end{bmatrix}$$

$$\tilde{\boldsymbol{B}} = \begin{bmatrix} \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{W}_1 & \boldsymbol{W}_2 & \boldsymbol{W}_3 & \boldsymbol{W}_4 \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \end{bmatrix}$$

where $\tilde{\boldsymbol{F}}_{qi} \in R^{(2\Gamma+n+m+p)\times(2\Gamma+n+m+p)}$ is a feedback gain and

$$\boldsymbol{E}_q = [\boldsymbol{K}_q \; \boldsymbol{K}_q^{\perp}] \quad (38)$$

$\boldsymbol{K}_q^{\perp}$ is the orthogonal complement of $\boldsymbol{K}_q$. Note that $\boldsymbol{E}_q$ becomes a nonsingular matrix because of the property of the orthogonal complement. $\boldsymbol{W}_1 \in R^{m\times n}$, $\boldsymbol{W}_2 \in R^{m\times p}$, $\boldsymbol{W}_3 \in R^{m\times m}$ and $\boldsymbol{W}_4 \in R^{m\times(2\Gamma)}$ are arbitrary full row rank matrices, that is,

$$rank(\boldsymbol{W}_1) = rank(\boldsymbol{W}_2) = rank(\boldsymbol{W}_3) = rank(\boldsymbol{W}_4) = m$$

$$[\boldsymbol{W}_1 \; \boldsymbol{W}_2 \; \boldsymbol{W}_3 \; \boldsymbol{W}_4]\tilde{\boldsymbol{F}}_{qi}\boldsymbol{E}_q = [\boldsymbol{F}_{1qi} \; \boldsymbol{F}_{2qi} \; \boldsymbol{F}_{3qi} \; \boldsymbol{F}_{4qi}]$$

The feedback gain $\tilde{\boldsymbol{F}}_{qi}$ can be determined by solving controller design conditions (39) and (40) in Theorem 3. Note that (39) and (40) are represented in terms of LMIs. Hence we can effectively determine the feedback gains by computer software like MATLAB.

Theorem 3: If there exist positive definite matrix $\boldsymbol{X} \in R^{(2\Gamma+n+m+p)\times(2\Gamma+n+m+p)}$ and $\boldsymbol{M}_{qi} \in R^{(2\Gamma+n+m+p)\times(2\Gamma+n+m+p)}$ satisfying (39) and (40) and the initial state is $\tilde{\boldsymbol{x}}(0) = [\Delta\boldsymbol{x}^T(0) \; \boldsymbol{e}^T(0) \; \Delta\boldsymbol{u}^T(0) \; \boldsymbol{0}^T]^T$, then the augmented system (33) can be stabilized by the switching fuzzy dynamic controller (35).

$$\boldsymbol{X} > \boldsymbol{0}, \quad (39)$$

$$\begin{bmatrix} \boldsymbol{X} & \boldsymbol{X}\boldsymbol{E}_q^{-T}\boldsymbol{A}_{qi}^T - \boldsymbol{M}_{qi}^T\boldsymbol{B}^T \\ \boldsymbol{A}_{qi}\boldsymbol{E}_q^{-1}\boldsymbol{X} - \boldsymbol{B}\boldsymbol{M}_{qi} & \boldsymbol{E}_q^{-1}\boldsymbol{X}\boldsymbol{E}_q^{-T} \end{bmatrix} > \boldsymbol{0}, \quad \forall i, q, \quad (40)$$

where $\tilde{\boldsymbol{F}}_{qi} = \boldsymbol{M}_{qi}\boldsymbol{X}^{-1}$.

(Proof) It can be proven from the combination of the proofs of the continuous-time switching fuzzy controller design [11], [12] and discrete-time PDC fuzzy controller design [7].

By using the designed controller, we can make the output $\boldsymbol{y}(t)$ of the nonlinear system (27) converge to the target point $\boldsymbol{r}$.

Remark 2: Note that we discuss the system trajectory with the initial state $\tilde{\boldsymbol{x}}(0) = [\Delta\boldsymbol{x}^T(0) \; \boldsymbol{e}^T(0) \; \Delta\boldsymbol{u}^T(0) \; \boldsymbol{0}^T]^T$. Therefore, the controller (35) reduces to (41), which do not include the augmented state $\tilde{\boldsymbol{x}}(t)$, although we employ (35) in the derivation of the LMIs.

$$\Delta\boldsymbol{u}(t+1) = -\sum_{q=1}^{Q}\sum_{i=1}^{r} v_q(\Delta\boldsymbol{x}(t), \Delta\boldsymbol{u}(t)) \times h_i(\boldsymbol{x}(t), \boldsymbol{x}(t+1), \boldsymbol{u}(t), \boldsymbol{u}(t+1)) \times (\boldsymbol{F}_{1qi}\Delta\boldsymbol{x}(t) + \boldsymbol{F}_{2qi}\boldsymbol{e}(t) + \boldsymbol{F}_{3qi}\Delta\boldsymbol{u}(t)) \quad (41)$$

Remark 3: To guarantee the stability of (27), we suppose that the trajectory of a system can not stay on the region boundaries. Hence, most kind of sliding modes are excluded [13].

The switching fuzzy model construction based on sector nonlinearity requires to consider constraints on control inputs and outputs. Theorem 4 gives an LMI constraint on the control inputs and outputs.

Theorem 4: [9] Assume that initial condition $\tilde{\boldsymbol{x}}(0)$ is known. The constraints $\|\Delta u_i(t)\| \le \mu_i$ and $\|\Delta x_i(t)\| \le \mu_i$ is enforced at all times if the LMIs

$$\begin{bmatrix} 1 & \tilde{\boldsymbol{x}}^T(0) \\ \tilde{\boldsymbol{y}}(0) & \boldsymbol{E}_q^{-1}\boldsymbol{X}\boldsymbol{E}_q^{-T} \end{bmatrix} \ge \boldsymbol{0}, \quad (42)$$

$$\begin{bmatrix} \boldsymbol{E}_q^{-1}\boldsymbol{X}\boldsymbol{E}_q^{-T} & \boldsymbol{E}_q^{-1}\boldsymbol{X}\boldsymbol{E}_q^{-T}\tilde{\boldsymbol{C}}_i^T \\ \tilde{\boldsymbol{C}}_i\boldsymbol{E}_q^{-1}\boldsymbol{X}\boldsymbol{E}_q^{-T} & \mu_i^2\boldsymbol{I} \end{bmatrix} \ge \boldsymbol{0}, \quad (43)$$

hold, where $\tilde{\boldsymbol{C}}_i$ is the vector to determine which input or output is constrained, that is, $\Delta u_i(t) = \tilde{\boldsymbol{C}}_i\tilde{\boldsymbol{x}}(t)$ or $\Delta x_i(t) = \tilde{\boldsymbol{C}}_i\tilde{\boldsymbol{x}}(t)$ where,

$$\tilde{\boldsymbol{C}}_i = [\overbrace{\boldsymbol{0}}^{n} \; \overbrace{\boldsymbol{0}}^{p} \; \overbrace{\boldsymbol{e}_{1i}}^{m} \; \overbrace{\boldsymbol{0}}^{2\Gamma}] \quad \text{for } \Delta u_i(t)$$

$$\tilde{\boldsymbol{C}}_i = [\overbrace{\boldsymbol{e}_{2i}}^{n} \; \overbrace{\boldsymbol{0}}^{p} \; \overbrace{\boldsymbol{0}}^{m} \; \overbrace{\boldsymbol{0}}^{2\Gamma}] \quad \text{for } \Delta x_i(t)$$

$\boldsymbol{e}_{1i} \in \boldsymbol{R}^{1\times m}$ and $\boldsymbol{e}_{2i} \in \boldsymbol{R}^{1\times n}$ are vectors whose ith element is 1 and all the other elements are 0.

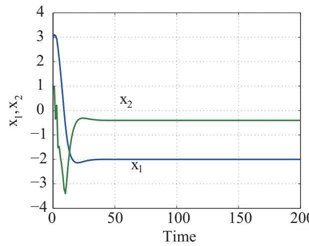

Fig. 2. Control result.

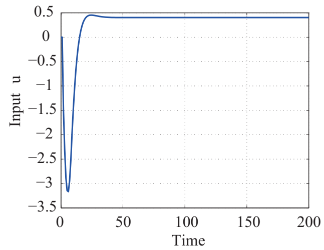

Fig. 3. Control input.

IV. Design Example

To illustrate the utility of this servo control approach, we show a simulation example.

Consider the following nonlinear system.

$$\begin{aligned}\boldsymbol{x}(t+1) &= \begin{bmatrix} x_1(t+1) \\ x_2(t+1) \end{bmatrix} \\ &= \begin{bmatrix} x_1(t) + 0.1x_2(t) + 0.1u(t) \\ \sin(x_1(t) + 0.5x_2(t)) + u(t) \end{bmatrix} \quad (44) \\ y(t) &= x_1(t) \quad (45)\end{aligned}$$

For the above nonlinear system, firstly, we construct the augmented switching fuzzy model (33) by applying the switching fuzzy model construction method which are described in Section III-A and Remark 1. We select the following two dividing planes.

$$x_1(t+1) - x_1(t) = 0$$
$$x_2(t+1) - x_2(t) = 0$$

The state space is divided into 4 regions by the dividing planes. Next, we construct the augmented switching control system (37) with $\alpha = 0.5$, $\boldsymbol{W}_1 = [1\ 1]$, $\boldsymbol{W}_2 = \boldsymbol{W}_3 = 1$, $\boldsymbol{W}_4 = [1\ 1\ 1\ 1]$. Finally, by solving Theorem 3, we can design the switching fuzzy servo controller (35).

Figures 2 and 3 show the control result and the control input, where initial state is $\boldsymbol{x}(0) = [3\ 1]^T$, $u(0) = 0$, $\Delta u(0) = 0$ and target point is $r = -2$. By using the designed controller, the output of the nonlinear system (44) converges to the target point. Figs. 4 and 5 show other simulation results using the same controller. Both of initial states are $[3\ 1]$, $[-7\ 2]$, $[-4\ \ -2]$, and $[10\ \ -4]$. Target points are $r = -2$ for Fig. 4 and $r = 7$ for Fig. 5. All outputs converge to each target point.

V. Conclusions

This paper has presented servo control for discrete-time nonlinear systems using the switching fuzzy model-based control approach. We have proposed the construction method of augmented switching fuzzy servo control system for discrete-time nonlinear systems. Moreover, we have introduced the switching fuzzy servo controller which can make outputs of the nonlinear systems converge to target points, and derive the controller design conditions in terms of LMIs. A design example has illustrated the utility of this approach.

Our future work is to apply this approach to complicated discrete-time nonlinear systems.

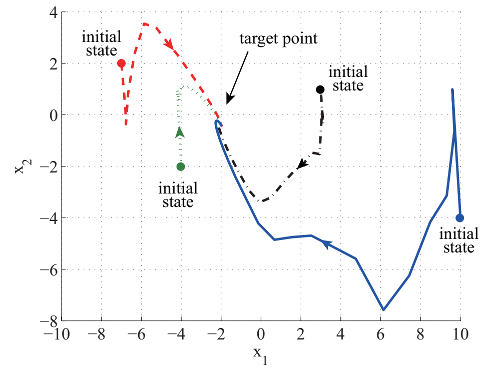

Fig. 4. Simulation results. $r = -2$.

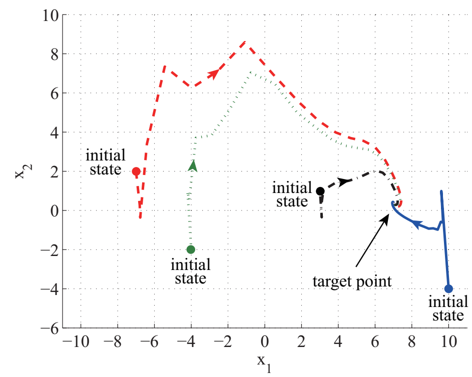

Fig. 5. Simulation results. $r = 7$.

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