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Conditions for the Simultaneous Stabilizability of a Segment of Systems

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Abstract— **This paper examines conditions for the simultaneous stabilizability of a segment of linear systems. Some criteria of positive semidefinite matrix and of sums of squares of polynomials are presented to check the Hurwitzness of the corresponding Diophantine equation. These results yield to a tractable method to test the simultaneous stabilizability of uncertain segments of systems in some cases when it may be given a simultaneous compensator for the two endpoints of this segment.**

I. INTRODUCTION

This paper regards the questions of the simultaneous stabilizability of a family of single input single output systems $G_\lambda(s)$ which are assumed to be linear, time invariant represented by a segment of uncertain systems as hereafter

$$G_\lambda(s) = \frac{N_\lambda(s)}{D_\lambda(s)} = \frac{(1-\lambda)N_1(s) + \lambda N_0(s)}{(1-\lambda)D_1(s) + \lambda D_0(s)} \tag{1}$$

where $\lambda \in [0,1]$ and $N_1(s)$, $N_0(s)$, $D_1(s)$, $D_0(s)$, $N_\lambda(s)$, $D_\lambda(s)$ are real polynomials and $N_\lambda(s)$ and $D_\lambda(s)$ are defined as two line segments of constant degree. This family of systems (1) may be viewed as a continuum of transfer functions described by the two distinct transfer functions $G_0(s)$ and $G_1(s)$ given by $\lambda = 1$ and $\lambda = 0$ respectively.

The objective of this paper is to present some solutions to the basic control problem: "*Is there one single linear controller that stabilizes a whole segment of systems described by (1)?*" This problem is firstly formulated and solved in Nyquist plot framework, then verifiable conditions are proposed for checking the simultaneous stabilizability of a segment such that (1) when there exists a simultaneous compensator for the two endpoints of this set of systems.

One can notice that this family of uncertain systems has attracted the attention of many researchers worried by the problem of strong stabilization, see [6], [8]. These authors have stated existence conditions of stable regulators being able to stabilize each members belonging to this family (1). That does not imply existence conditions of a single controller that stabilizes the whole set of systems. This problem is more complex. The question of the simultaneous stabilization of a segment of systems given by (1) was initially tackled by [9], [10] and [1] but no tractable and complete conditions to check the simultaneous stabilizability of such systems have been given. To study in a satisfactory way this question, it is useful to consider the works in the area of the simultaneous control as well as those formulated in the topic of the polynomial control of uncertain systems, see [2], [12], [5], [7], [11].

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II. PROBLEM FORMULATION

First of all, let us give some notations.

1) $\mathbb{R}^+_{-\{0\}}$ denotes the set of strictly positive reals belonging to the interval $]0, \infty]$.
2) $\mathbb{C}_-$ denotes the open left half complex plane $\mathbb{C}$.

The problem of the simultaneous stabilization of a family of systems $G_\lambda(s)$ as described in (1) may be translated into the following way: "*Does it exist a compensator $C(s)$ stabilizing all the systems belonging to the family $G_\lambda(s)$ whatever $\lambda \in [0,1]$?*" or "*does it exist two real polynomials $X(s)$ and $Y(s) \neq 0$ with $C(s) = \frac{X(s)}{Y(s)}$ such that the closed-loop segment of systems $G_\lambda(s)\left(1 + C(s)G_\lambda(s)\right)^{-1}$ has all its poles in $\mathbb{C}_-$, for any $\lambda \in [0,1]$?*" That means examining the existence conditions of two real polynomials $X(s)$ and $Y(s)$ solutions of the following Diophantine equation

$$\lambda \in [0,1], \qquad A_\lambda(s) = X(s)N_\lambda(s) + Y(s)D_\lambda(s) \tag{2}$$

where $A_\lambda(s)$ is an Hurwitz polynomial segment of constant degree for all $\lambda \in [0,1]$, i.e. all the polynomials belonging to this segment (2) are Hurwitz. Let us remark that if $A_\lambda(s)$ is not of same degree for all $\lambda \in [0,1]$ then this loss of degree also implies loss of bounded-input bounded-output stability.

The relationship (2) is rewritten like the following one

$$\lambda \in [0,1],\ A_\lambda(s) = (1-\lambda)(X(s)N_1(s) + Y(s)D_1(s)) + \lambda\,(X(s)N_0(s) + Y(s)D_0(s)) \tag{3}$$

The examination of the existence conditions of the solutions $X(s)$ and $Y(s)$ of (3) is equivalent to check that there exists a compensator $C(s)$ for the system (1) such that the roots of the polynomial $A_\lambda(s)$ are all in $\mathbb{C}_-$ for any $\lambda \in [0,1]$. Hence, the problem of the Hurwitzness of the uncertain polynomial $A_\lambda(s)$ requires the existence of two Hurwitz polynomials $A_0(s)$ and $A_1(s)$ given by (5) that must verify the equality (4) hereafter

$$\lambda \in [0,1], \qquad A_\lambda(s) = (1-\lambda)A_1(s) + \lambda A_0(s) \tag{4}$$

where

$$\begin{cases} A_0(s) = X(s)N_0(s) + Y(s)D_0(s) \\ A_1(s) = X(s)N_1(s) + Y(s)D_1(s) \end{cases} \tag{5}$$

The two Hurwitz polynomials $A_0(s)$ and $A_1(s)$ are assumed to be of same degree and of same signs (i.e. $A_0(0)A_1(0) > 0$), see [4] and [6]. Moreover, we suppose that there exist a simultaneous compensator $C(s)$ for the two systems $G_0(s)$ and $G_1(s)$ verifying the relations (5). In the next sections, algebraic conditions will be given so that the Diophantine equation (2) is Hurwitz when the two endpoints $A_0(s)$ and $A_1(s)$ of the polynomial segment $A_\lambda(s)$ are Hurwitz and of same sign and of same degree.

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III. HURWITZNESS OF THE SEGMENT $A_\lambda(s)$

Before developing our main results, let us recall the following theorem.

Theorem 1: see [4], *Let us consider the polynomial segment $A_\lambda(s)$ given by (4) and let us assume the two Hurwitz polynomials $A_0(s)$ and $A_1(s)$ of same degree and of same sign. Then, the segment $A_\lambda(s)$ is Hurwitz and of constant degree if and only if the rational function $\frac{A_1(s)}{A_0(s)}$ does not cut the negative real axis of the Nyquist plot.*

This condition is a direct consequence of the "Boundary Crossing Theorem", see [4] which reduces here the Hurwitzness of $A_\lambda(s)$ to the study on the complex plane of the frequency domain image of this segment. For explaining this result, it is necessary to recall that by hypothesis the segment $A_\lambda(s)$ is assumed to be of constant degree. In these conditions, the parametric continuity of the space of polynomial family when λ moves in a continuous way implies also the continuity of the space of the family of zeros. Then the Hurwitzness of the polynomial $A_\lambda(s)$ is guaranteed, see [3], if and only if firstly one polynomial of this family is stable and secondly for any $\lambda \in [0, 1]$, the polynomial family $A_\lambda(s)$ does not have zeros on the imaginary axis. What comes down to say that for any value of $\lambda \in [0,\ 1]$, $A_\lambda(jw)$ cannot contain the origin as root when w varies in the interval $]0, +\infty[$. Hence this property holds in relation with the Zero Exclusion Principle, if for $s = jw$ (the stability boundary) we have

$$\forall\lambda \in [0,\ 1] \text{ and } \forall w \in \mathbb{R}^{+}{}_{-\{0\}},\quad A_\lambda(jw) \neq 0, \tag{6}$$

or

$$\forall\lambda \in [0,\ 1],\ \ \forall w \in \mathbb{R}^{+}{}_{-\{0\}},\ \ \frac{A_1(jw)}{A_0(jw)} \neq -\frac{\lambda}{(1-\lambda)} \tag{7}$$

Consequently, if the condition (7) is checked, the rational function $\frac{A_1(s)}{A_0(s)}$ cannot cut the negative real axis of the Nyquist plot and all the zeros of the polynomial family $A_\lambda(s)$ are in $\mathbb{C}_-$. After these recalls, let us rewrite (7) in another manner.

$$\begin{array}{l} A_0(s) = A_0^{even}(s) + A_0^{odd}(s) \\ A_1(s) = A_1^{even}(s) + A_1^{odd}(s) \end{array} \tag{8}$$

such that

$$\begin{cases} A_0^{even}(s) = a_{0,0} + a_{0,2}\ s^2 + ... +\ a_{0,2i}\ s^{2i} \\ A_0^{odd}\ (s) = a_{0,1}\ s + a_{0,3}\ s^3 + a_{0,5}\ s^5 + ... +\ a_{0,2i+1}\ s^{2i+1} \\ A_1^{even}(s) = a_{1,0} + a_{1,2}\ s^2 + ... +\ a_{1,2i}\ s^{2i} \\ A_1^{odd}\ (s) = a_{1,1}\ s + a_{1,3}\ s^3 + a_{1,5}\ s^5 + ... +\ a_{1,2i+1}\ s^{2i+1} \end{cases}$$

Consider $s = jw$. Then, we have

$$\begin{array}{l} A_0(jw) = A_0^e(jw) + jwA_0^o(jw) \\ A_1(jw) = A_1^e(jw) + jwA_1^o(jw) \end{array} \tag{9}$$

where

$$\begin{cases} A_0^{even}(jw) = \quad A_0^e(jw) \\ A_0^{odd}\ (jw) = jwA_0^o(jw) \\ A_1^{even}(jw) = \quad A_1^e(jw) \\ A_1^{odd}\ (jw) = jwA_1^o(jw) \end{cases} \tag{10}$$

Theorem 2: Let $A_0(s)$ and $A_1(s)$ be two Hurwitz polynomials of same degree and of same sign and let us assume that there are strictly positive real zeros w_i such as

$$A_0^e(jw_i)A_1^o(jw_i)\ -\ A_0^o(jw_i)A_1^e(jw_i) = 0 \tag{11}$$

Then, the polynomial segment $A_\lambda(s)$ defined by (3) is Hurwitz if and only if the scalars w_i verify the constraint (12a) or the constraint (12b).

$$A_0^e(jw_i)A_1^e(jw_i) > 0 \tag{12a}$$

$$A_0^o(jw_i)A_1^o(jw_i) > 0 \tag{12b}$$

In the case where there does not exist any strictly positive real zero w_i such that (11) holds, the segment $A_\lambda(s)$ is Hurwitz.

Proof: Let us assume that Theorem 1 is checked. Thus, we can rewrite the relation (7) by considering (9). It follows that for any $\lambda \in [0,\ 1]$ and for any $w \in \mathbb{R}^{+}{}_{-\{0\}}$, we have

$$\frac{A_1^e(jw) + jw\ A_1^o(jw)}{A_0^e(jw) + jw\ A_0^o(jw)} \neq -\frac{\lambda}{(1-\lambda)} \tag{13}$$

As for any $w \in \mathbb{R}^{+}{}_{-\{0\}}$, $A_0(jw) \neq 0$, relationship (13) is equivalent to

$$\frac{(A_1^e(jw) + jwA_1^o(jw))(A_0^e(jw) - jwA_0^o(jw))}{(A_0^e(jw) + jwA_0^o(jw))(A_0^e(jw) - jwA_0^o(jw))} \neq -\frac{\lambda}{(1-\lambda)} \tag{14}$$

Let us denote by $\widehat{\chi}(jw)$ and $\widehat{\delta}(jw)$ the following expressions

$$\begin{cases} \widehat{\chi}(jw) &= \dfrac{A_1^e(jw)A_0^e(jw)\ + w^2\ A_1^o(jw)A_0^o(jw)}{A_0^e(jw)A_0^e(jw)\ + w^2\ A_0^o(jw)A_0^o(jw)} \\ \widehat{\delta}(jw) &= \dfrac{A_0^e(jw)A_1^o(jw)\ -\ A_0^o(jw)A_1^e(jw)}{A_0^e(jw)A_0^e(jw)\ + w^2\ A_0^o(jw)A_0^o(jw)} \end{cases} \tag{15}$$

Hence, equation (13) becomes

$$\widehat{\chi}(jw) + jw\ \widehat{\delta}(jw) \neq -\frac{\lambda}{(1-\lambda)} \tag{16}$$

The existence conditions given by the inequality (16) may be decomposed as the two following cases

1) If for all $w \in \mathbb{R}^{+}{}_{-\{0\}}$, $\widehat{\delta}(jw) \neq 0$, then (16) holds.
2) If there exists $w \in \mathbb{R}^{+}{}_{-\{0\}}$ such that $\widehat{\delta}(jw) = 0$ and $\widehat{\chi}(jw) > 0$, then (16) holds.

The first case is obvious. Let us study the second case. For that, we can observe that to check the inequality (16) is equivalent to find the strictly positive real zeros w_i of $\widehat{\delta}(jw)$ and to verify that these zeros yield to $\widehat{\chi}(jw_i) > 0$. Equivalently, the real part of the rational function $\frac{A_1(jw)}{A_0(jw)}$ in the Nyquist plot must be strictly positive when the imaginary part of this unit is null. Let us denote by $\mathfrak{W}$, the set of strictly positive real zeros w_i of $\widehat{\delta}(jw)$. The equality $\widehat{\delta}(jw_i) = 0$ may be written like one of the two following relations (17) or (18).

1) If there is $w_i \in \mathfrak{W}$ such that $A_1^e(jw_i) \neq 0$ and $A_0^e(jw_i) \neq 0$, then the numerator of the rational function $\widehat{\delta}(jw)$ verifies

$$\frac{A_1^o(jw_i)}{A_1^e(jw_i)} = \frac{A_0^o(jw_i)}{A_0^e(jw_i)} \tag{17}$$

2) If there is $w_i \in \mathfrak{W}$ such that $A_1^o(jw_i) \neq 0$ and $A_0^o(jw_i) \neq 0$, then the numerator of the rational function $\widehat{\delta}(jw)$ verifies

$$\frac{A_1^e(jw_i)}{A_1^o(jw_i)} = \frac{A_0^e(jw_i)}{A_0^o(jw_i)} \tag{18}$$

Note that the cases $A_0^e(jw_i) = 0$ and $A_0^o(jw_i) = 0$ or $A_1^e(jw_i) = 0$ and $A_1^o(jw_i) = 0$ are not possible thus if $A_0^e(jw_i) = 0$ and $A_0^o(jw_i) = 0$ then $A_0^e(jw_i)$ and $A_0^o(jw_i)$ have the same root w_i. In the same way, if $A_1^e(jw_i) = 0$ and $A_1^o(jw_i) = 0$ then $A_1^e(jw_i)$ and $A_1^o(jw_i)$ have the same root. In these conditions, the polynomials $A_0(s)$ and $A_1(s)$ do not satisfy the Hermite-Biehler Theorem, see [14]. Consequently these polynomials are not Hurwitz. In this case, we are in contradiction with the starting assumption.

For examining the two cases (17) and (18) in regard to the condition $\widehat{\chi}(jw_i) > 0$, the condition $\widehat{\chi}(jw_i) > 0$ is translated in the two possible factorizations as listed hereafter.

1) Let us consider the case where there is $w_i \in \mathfrak{W}$ such that $A_0^e(jw_i) \neq 0$ and $A_1^e(jw_i) \neq 0$. Then $\widehat{\chi}(jw_i)$ is strictly positive if we verify

$$\widehat{\chi}(jw_i) = A_0^e(jw_i)A_1^e(jw_i)\left(1 + w_i^2\frac{A_0^o(jw_i)A_0^o(jw_i)}{A_0^e(jw_i)A_0^e(jw_i)}\right) \times \left(\frac{1}{A_0^e(jw_i)A_0^e(jw_i) + w_i^2 A_0^o(jw_i)A_0^o(jw_i)}\right) > 0$$

As the two following inequalities are always true

$$\left(1 + w_i^2\frac{A_0^o(jw_i)A_0^o(jw_i)}{A_0^e(jw_i)A_0^e(jw_i)}\right) > 0$$

$$\left(\frac{1}{A_0^e(jw_i)A_0^e(jw_i) + w_i^2 A_0^o(jw_i)A_0^o(jw_i)}\right) > 0$$

then the following relationship must be only checked

$$A_0^e(jw_i)A_1^e(jw_i) > 0 \tag{19}$$

2) Now, we consider the case where there is $w_i \in \mathfrak{W}$ such that $A_0^o(jw_i) \neq 0$ and $A_1^o(jw_i) \neq 0$. The relationship $\widehat{\chi}(jw_i) > 0$ can be rewritten in the equivalent form

$$\widehat{\chi}(jw_i) = A_0^o(jw_i)A_1^o(jw_i)\left(w_i^2 + \frac{A_0^e(jw_i)A_0^e(jw_i)}{A_0^o(jw_i)A_0^o(jw_i)}\right) \left(\frac{1}{A_0^e(jw_i)A_0^e(jw_i) + w_i^2 A_0^o(jw_i)A_0^o(jw_i)}\right) > 0$$

As the two following inequalities are true

$$\left(w_i^2 + \frac{A_0^e(jw_i)A_0^e(jw_i)}{A_0^o(jw_i)A_0^o(jw_i)}\right) > 0$$

$$\left(\frac{1}{A_0^e(jw_i)A_0^e(jw_i) + w_i^2 A_0^o(jw_i)A_0^o(jw_i)}\right) > 0$$

then following relationship must be only checked

$$A_0^o(jw_i)A_1^o(jw_i) > 0 \tag{20}$$

To conclude this proof, let us remark that if we have $A_0^o(jw_i)A_1^o(jw_i) > 0$ or $A_0^e(jw_i)A_1^e(jw_i) > 0$ with w_i given by (17) or (18) then $\widehat{\chi}(jw_i) > 0$ and (16) is verified. Therefore, to test that the no null real zeros w_i verify (16), it suffices to check one of the two relations (19) or (20). ■

Let us consider the two polynomials $A_0(s)$ and $A_1(s)$ defined by (5) and let us compute the polynomials $X(jw)$, $Y(jw)$, $N_0(jw)$, $D_0(jw)$, $N_1(jw)$, $D_1(jw)$ similarly than (9) with the even and odd parts respectively given by $X^e(jw)$, $X^o(jw)$, $Y^e(jw)$, $Y^o(jw)$, $N_0^e(jw)$, $N_0^o(jw)$, $D_0^e(jw)$, $D_0^o(jw)$, $N_1^e(jw)$, $N_1^o(jw)$, $D_1^e(jw)$, $D_1^o(jw)$. Then in accordance with (10), the relationships (9) are rewritten as

$$\begin{cases} A_0(jw) = A_0^e(jw) + jwA_0^o(jw) = \\ (X^e(jw) + jwX^o(jw))\,(N_0^e(jw) + jwN_0^o(jw)) + \\ (Y^e(jw) + jwY^o(jw))\,(D_0^e(jw) + jwD_0^o(jw)) \\ \\ A_1(jw) = A_1^e(jw) + jwA_1^o(jw) = \\ (X^e(jw) + jwX^o(jw))\,(N_1^e(jw) + jwN_1^o(jw)) + \\ (Y^e(jw) + jwY^o(jw))\,(D_1^e(jw) + jwD_1^o(jw)) \end{cases}$$

where

$$\begin{cases} A_0^e(jw) = N_0^e(jw)X^e(jw) - w^2N_0^o(jw)X^o(jw) \\ \qquad + D_0^e(jw)Y^e(jw) - w^2D_0^o(jw)Y^o(jw) \\ A_0^o(jw) = N_0^e(jw)X^o(jw) + N_0^o(jw)X^e(jw) \\ \qquad + D_0^e(jw)Y^o(jw) + D_0^o(jw)Y^e(jw) \\ A_1^e(jw) = N_1^e(jw)X^e(jw) - w^2N_1^o(jw)X^o(jw) \\ \qquad + D_1^e(jw)Y^e(jw) - w^2D_1^o(jw)Y^o(jw) \\ A_1^o(jw) = N_1^e(jw)X^o(jw) + N_1^o(jw)X^e(jw) \\ \qquad + D_1^e(jw)Y^o(jw) + D_1^o(jw)Y^e(jw) \end{cases} \tag{21}$$

In the next section, some positive real zeros w_i of the rational function $\widehat{\delta}(jw)$ will be determine by using relations (21) in order to compute the signs of the expressions $A_0^e(jw_i)A_1^e(jw_i)$ and $A_0^o(jw_i)A_1^o(jw_i)$.

IV. Study of the positive real zeros of $\widehat{\delta}(jw)$

Let us assume that there exists a simultaneous controller $C(s)$ for the two plants $G_0(s)$ and $G_1(s)$ such that the two Hurwitz polynomials $A_0(s)$ and $A_1(s)$ given by (5) have the same degree and the same sign. In this condition, let us examine if there exists $w \in \mathbb{R}^+{}_{-\{0\}}$ such that

$$A_0^e(jw)A_1^o(jw) - A_0^o(jw)A_1^e(jw) = 0 \tag{22}$$

Relation (22) is an expression resulting of the parametrization of the regulator $C(s)$ as well as of those of the systems $G_0(s)$ and $G_1(s)$. By considering the four terms of this difference (22), it is not possible to isolate on one hand the conditions depending of the systems $G_0(s)$ and $G_1(s)$, and on the other hand the conditions depending of the parametrization of the compensator $C(s)$. It is the reason why in the next paragraphs, this relation will be rewritten of two different manners: either as a function of the compensator $C(s)$ given by the real polynomials $(X^e(jw), X^o(jw), Y^e(jw), Y^o(jw))$ or as a function of the systems $G_0(s)$ and $G_1(s)$ given respectively by the polynomials $(N_0^e(jw), N_0^o(jw), D_0^e(jw), D_0^o(jw))$ and $(N_1^e(jw), N_1^o(jw), D_1^e(jw), D_1^o(jw))$. This second form being the dual form of the first one.

A. *The first writing of the numerator of $\widehat{\delta}(jw)$.*

Let us consider the numerator of $\widehat{\delta}(jw)$, i.e. relation (22) as a function of $C(s)$. Then, we may deduce the following expression

$$A_0^e(jw)A_1^o(jw) - A_0^o(jw)A_1^e(jw) = \varphi^T(jw)Mat_I(jw)\psi(jw)$$

where

$$\varphi^T(jw) = [X^o(jw) \quad X^e(jw) \quad Y^o(jw) \quad Y^e(jw)]$$
$$\psi^T(jw) = [X^e(jw) \quad -w^2X^o(jw) \quad Y^e(jw) \quad -w^2Y^o(jw)]$$

$$Mat_I(jw) =$$

$$\begin{pmatrix} 0 & m_{1,2}(jw) & m_{1,3}(jw) & m_{1,4}(jw) \\ -\,m_{1,2}(jw) & 0 & m_{2,3}(jw) & m_{2,4}(jw) \\ -\,m_{1,3}(jw) & -\,m_{2,3}(jw) & 0 & m_{3,4}(jw) \\ -\,m_{1,4}(jw) & -\,m_{2,4}(jw) & -\,m_{3,4}(jw) & 0 \end{pmatrix}$$

and

$$m_{1,2}(jw) = N_0^o(jw)N_1^e(jw) - N_0^e(jw)N_1^o(jw) \quad (23a)$$
$$m_{1,3}(jw) = D_0^e(jw)N_1^e(jw) - N_0^e(jw)D_1^e(jw) \quad (23b)$$
$$m_{1,4}(jw) = D_0^o(jw)N_1^e(jw) - N_0^e(jw)D_1^o(jw) \quad (23c)$$
$$m_{2,3}(jw) = D_0^e(jw)N_1^o(jw) - N_0^o(jw)D_1^e(jw) \quad (23d)$$
$$m_{2,4}(jw) = D_0^o(jw)N_1^o(jw) - N_0^o(jw)D_1^o(jw) \quad (23e)$$
$$m_{3,4}(jw) = D_0^o(jw)D_1^e(jw) - D_0^e(jw)D_1^o(jw) \quad (23f)$$

This algebraic expression is also noted as

$$B_I(\varphi(jw), \psi(jw)) = \varphi^T(jw)\ Mat_I(jw)\ \psi(jw) \quad (24)$$

Let us notice that the bilinear form (24) is skew-symmetric. Moreover, we observe that this matrix depends only on the parameters of the systems $G_0(jw)$ and $G_1(jw)$ and that the two vectors $\psi(jw)$ and $\varphi(jw)$ depend only of the compensator $C(s)$. At this step, according to (22), we must verify that if there exists $w \in \mathbb{R}^+{}_{-\{0\}}$ such that $B_I(\varphi(jw), \psi(jw))$ is identically zero, then this bilinear form is degenerate or not. To show that, it is useful to recall that the determinant of a skew-symmetric matrix is equal to the square of the pfaffian of that matrix and that a bilinear form is nondegenerate if $det(Mat_I(jw)) \neq 0$. That implies

$$det(Mat_I(jw)) = (pf(Mat_I(jw)))^2$$

where pf denotes the pfaffian of matrix $Mat_I(jw)$ given by the following expression

$$\begin{aligned} pf(Mat_I(jw)) \;\; = \;\; & m_{1,2}(jw)m_{3,4}(jw) - m_{1,3}(jw)m_{2,4}(jw) \\ & + m_{1,4}(jw)m_{2,3}(jw) \end{aligned}$$

It is get $det(Mat_I(jw)) = 0$. That means that $Mat_I(jw)$ is a degenerate matrix and has at least two eigenvalues to zero thus the eigenvalues of an even dimensional skew-matrix come in pairs and are all pure imaginary. Let us calculate precisely these eigenvalues $\zeta_I(jw)$

$$det(Mat_I(jw) - \zeta_I(jw)I_4) = \zeta_I^4(jw) + \zeta_I^2(jw)\varepsilon(jw)$$

where I_4 denotes the 4×4 identity matrix and

$$\begin{aligned} \varepsilon(jw) \;\; = \;\; & m_{1,2}^2(jw) + m_{1,3}^2(jw) + m_{1,4}^2(jw) + m_{2,3}^2(jw) \\ & + m_{2,4}^2(jw) + m_{3,4}^2(jw) \end{aligned}$$

Let us assume $\varepsilon(jw) \neq 0$. It follows that the two non-zero eigenvalues are $\pm j\sqrt{\varepsilon(jw)}$. The following relation is deduced

$$Mat_I(jw) = P_I^T(jw)\Lambda_I(jw)P_I(jw)$$

where

$$\Lambda_I(jw) = \begin{pmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

and

$$P_I(jw) = \begin{pmatrix} N_1^e(jw) & N_1^o(jw) & D_1^e(jw) & D_1^o(jw) \\ N_0^e(jw) & N_0^o(jw) & D_0^e(jw) & D_0^o(jw) \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

B. *The second writing of the numerator of* $\widehat{\delta}(jw)$.

Now develop the numerator of $\widehat{\delta}(jw)$ as skew-symmetric form, dual of the first one. For that, let us express the bilinear form (22) like an expression depending only of the compensator $C(s)$. By construction, we may write

$$A_0^e(jw)A_1^o(jw) - A_0^o(jw)A_1^e(jw) = \widetilde{\varphi}^T(jw)\widetilde{Mat}_I(jw)\widetilde{\psi}(jw)$$

where

$$\begin{aligned} \widetilde{\varphi}^T(jw) &= [N_0^e(jw) \;\; N_0^o(jw) \;\; D_0^e(jw) \;\; D_0^o(jw)] \\ \widetilde{\psi}^T(jw) &= [N_1^e(jw) \;\; N_1^o(jw) \;\; D_1^e(jw) \;\; D_1^o(jw)] \end{aligned}$$

$$\widetilde{Mat}_I(jw) =$$

$$\begin{pmatrix} 0 & \widetilde{m}_{1,2}(jw) & \widetilde{m}_{1,3}(jw) & \widetilde{m}_{1,4}(jw) \\ -\,\widetilde{m}_{1,2}(jw) & 0 & \widetilde{m}_{2,3}(jw) & \widetilde{m}_{2,4}(jw) \\ -\,\widetilde{m}_{1,3}(jw) & -\,\widetilde{m}_{2,3}(jw) & 0 & \widetilde{m}_{3,4}(jw) \\ -\,\widetilde{m}_{1,4}(jw) & -\,\widetilde{m}_{2,4}(jw) & -\,\widetilde{m}_{3,4}(jw) & 0 \end{pmatrix}$$

and with

$$\widetilde{m}_{1,2}(jw) = X^e(jw)X^e(jw) + w^2X^o(jw)X^o(jw) \quad (25a)$$
$$\widetilde{m}_{1,3}(jw) = X^e(jw)\,Y^o(jw) - X^o(jw)Y^e(jw) \quad (25b)$$
$$\widetilde{m}_{1,4}(jw) = X^e(jw)\,Y^e(jw) + w^2X^o(jw)Y^o(jw) \quad (25c)$$
$$\widetilde{m}_{2,3}(jw) = -w^2X^o(jw)\,Y^o(jw) - X^e(jw)Y^e(jw) \quad (25d)$$
$$\widetilde{m}_{2,4}(jw) = -w^2X^o(jw)\,Y^e(jw) + w^2X^e(jw)Y^o(jw) \quad (25e)$$
$$\widetilde{m}_{3,4}(jw) = Y^e(jw)\,Y^e(jw) + w^2Y^o(jw)Y^o(jw) \quad (25f)$$

This algebraic expression is also noted as

$$\widetilde{B}_I(\widetilde{\varphi}(jw), \widetilde{\psi}(jw)) = \widetilde{\varphi}^T(jw)\ \widetilde{Mat}_I(jw)\ \ \widetilde{\psi}(jw) \quad (26)$$

We can observe that the matrix $\widetilde{Mat}_I(jw)$ is a skew-symmetric matrix which depends only on the parameters of the compensator $(X(jw), Y(jw))$ and which is independent of the systems $G_0(jw)$ and $G_1(jw)$. As previously for the bilinear form $B_I(\varphi(jw), \psi(jw))$, it can be showed that this form is degenerate thus we have $det(\widetilde{Mat}_I(jw)) = 0$. That means that $\widetilde{Mat}_I(jw)$ has at least two eigenvalues to zero. Let us calculate precisely these eigenvalues $\widetilde{\zeta}_I(jw)$

$$det(\widetilde{Mat}_I(jw) - \widetilde{\zeta}_I(jw)I_4) = \widetilde{\zeta}_I^4(jw) + \widetilde{\zeta}_I^2(jw)\widetilde{\varepsilon}(jw)$$

where

$$\begin{aligned} \widetilde{\varepsilon}(jw) \;\; = \;\; & \widetilde{m}_{1,2}^2(jw) + \widetilde{m}_{1,3}^2(jw) + \widetilde{m}_{1,4}^2(jw) + \widetilde{m}_{2,3}^2(jw) \\ & + \widetilde{m}_{2,4}^2(jw) + \widetilde{m}_{3,4}^2(jw) \end{aligned}$$

Let us assume $\widetilde{\varepsilon}(jw) \neq 0$. It follows that the two non-zero eigenvalues are $\pm j\sqrt{\widetilde{\varepsilon}(jw)}$. The following relation is deduced

$$\widetilde{Mat}_I(jw) = \widetilde{P}_I^T(jw)\widetilde{\Lambda}_I(jw)\widetilde{P}_I(jw) \quad (27)$$

where

$$\widetilde{\Lambda}_I(jw) = \begin{pmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

and

$$\widetilde{P}_I(jw) = \begin{pmatrix} X^e(jw) & -w^2X^o(jw) & Y^e(jw) & -w^2Y^o(jw) \\ X^o(jw) & X^e(jw) & Y^o(jw) & Y^e(jw) \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

In the next subsection, by considering the two equivalent bilinear forms $B_I(\varphi(jw), \psi(jw))$ and $\widetilde{B}_I(\widetilde{\varphi}(jw), \widetilde{\psi}(jw))$, it will be showed some conditions that assures $\widehat{\delta}(jw)$ identically zero.

C. *Sufficient conditions that assure $\widehat{\delta}(jw)$ identically zero.*

Let us remark that relation (22) can be rewritten as

$$\begin{aligned} & m_{1,2}(jw_i)\widetilde{m}_{1,2}(jw_i) + m_{1,3}(jw_i)\widetilde{m}_{1,3}(jw_i) + \\ & m_{1,4}(jw_i)\widetilde{m}_{1,4}(jw_i) + m_{2,3}(jw_i)\widetilde{m}_{2,3}(jw_i) + \\ & m_{2,4}(jw_i)\widetilde{m}_{2,4}(jw_i) + m_{3,4}(jw_i)\widetilde{m}_{3,4}(jw_i) \quad = 0 \end{aligned} \tag{28}$$

Theorem 3: If there exists w in $\mathbb{R}^{+}{}_{-\{0\}}$ such that $Mat_I(jw)$ or $\widetilde{Mat}_I(jw)$ are zero matrices then relation (22) is verified.

Proof: Obvious, see the previous equation (28). ■

It follows that to study $Mat_I(jw) = 0$, we have to consider only 3 equations instead of 6 equations resulting from $Mat_I(jw) = 0$. That is if there exists common roots w_i such that $m_{1,2}(jw_i) = m_{1,3}(jw_i) = m_{1,4}(jw_i) = m_{2,3}(jw_i) = m_{2,4}(jw_i) = m_{3,4}(jw_i) = 0$, then the solutions of these equations are equivalent to the common roots w_i of the following expressions

$$\left\{ \begin{array}{lcl} N_0^o(jw_i)\; N_1^e(jw_i) - N_0^e(jw_i)\; N_1^o(jw_i) & = & 0 \\ D_0^e(jw_i)\; N_1^o(jw_i) - N_0^o(jw_i)\; D_1^e(jw_i) & = & 0 \\ D_0^o(jw_i)\; N_1^o(jw_i) - N_0^o(jw_i)\; D_1^o(jw_i) & = & 0 \end{array} \right. \tag{29}$$

Similarly, to study $\widetilde{Mat}_I(jw_i) = 0$, we have to consider only 2 equations instead of 6 equations resulting from $\widetilde{Mat}_I(jw) = 0$. That is if there exists common roots w_i such that $\widetilde{m}_{1,2}(jw_i) = \widetilde{m}_{1,3}(jw_i) = \widetilde{m}_{1,4}(jw_i) = \widetilde{m}_{2,3}(jw_i) = \widetilde{m}_{2,4}(jw_i) = \widetilde{m}_{3,4}(jw_i) = 0$, then the solutions of these equations are equivalent to the common roots w_i of the expressions below

$$\left\{ \begin{array}{lcl} X^e(jw_i)Y^o(jw_i) - X^o(jw_i)Y^e(jw_i) & = & 0 \\ X^e(jw_i)Y^e(jw_i) + w_i^2 X^o(jw_i)Y^o(jw_i) & = & 0 \end{array} \right. \tag{30}$$

V. Test of strict positivity of $A_0^e(jw_i)A_1^e(jw_i)$ and $A_0^o(jw_i)A_1^o(jw_i)$

In this section, by considering the relations (29) satisfying $Mat_I(jw_i) = 0$, positivity conditions are given in order to evaluate the signs of the expressions $A_0^e(jw_i)A_1^e(jw_i)$ and $A_0^o(jw_i)A_1^o(jw_i)$. These symmetric bilinear forms may be transformed as below

$$A_0^e(jw_i)A_1^e(jw_i) = \psi^T(jw_i)Mat_R(jw_i)\psi(jw_i) \tag{31a}$$

$$A_0^o(jw_i)A_1^o(jw_i) = \varphi^T(jw_i)Mat_R(jw_i)\varphi(jw_i) \tag{31b}$$

where $Mat_R(jw_i) =$

$$\begin{pmatrix} n_{1,1}(jw_i) & n_{1,2}(jw_i) & n_{1,3}(jw_i) & n_{1,4}(jw_i) \\ n_{1,2}(jw_i) & n_{2,2}(jw_i) & n_{2,3}(jw_i) & n_{2,4}(jw_i) \\ n_{1,3}(jw_i) & n_{2,3}(jw_i) & n_{3,3}(jw_i) & n_{3,4}(jw_i) \\ n_{1,4}(jw_i) & n_{2,4}(jw_i) & n_{3,4}(jw_i) & n_{4,4}(jw_i) \end{pmatrix}$$

and

$$n_{1,2}(jw_i) = N_0^e(jw_i)N_1^o(jw_i) = N_0^o(jw_i)N_1^e(jw_i) \tag{32a}$$

$$n_{1,3}(jw_i) = N_0^e(jw_i)D_1^e(jw_i) = D_0^e(jw_i)N_1^e(jw_i) \tag{32b}$$

$$n_{1,4}(jw_i) = N_0^e(jw_i)D_1^o(jw_i) = D_0^o(jw_i)N_1^e(jw_i) \tag{32c}$$

$$n_{2,3}(jw_i) = N_0^o(jw_i)D_1^e(jw_i) = D_0^e(jw_i)N_1^o(jw_i) \tag{32d}$$

$$n_{2,4}(jw_i) = N_0^o(jw_i)D_1^o(jw_i) = D_0^o(jw_i)N_1^o(jw_i) \tag{32e}$$

$$n_{3,4}(jw_i) = D_0^e(jw_i)D_1^o(jw_i) = D_0^o(jw_i)D_1^e(jw_i) \tag{32f}$$

$$n_{1,1}(jw_i) = N_0^e(jw_i)N_1^e(jw_i) \tag{32g}$$

$$n_{2,2}(jw_i) = N_0^o(jw_i)N_1^o(jw_i) \tag{32h}$$

$$n_{3,3}(jw_i) = D_0^e(jw_i)D_1^e(jw_i) \tag{32i}$$

$$n_{4,4}(jw_i) = D_0^o(jw_i)D_1^o(jw_i) \tag{32j}$$

Hence, $Mat_R(jw_i)$ is a symmetric matrix. In the next theorem, a condition will be presented to check that $Mat_R(jw_i)$ is semidefinite positive and no null matrix when the positive real zeros w_i are given by $Mat_I(jw_i) = 0$.

Theorem 4: Let us assume that the positive reals w_i verifying (22) are given by $Mat_I(jw_i) = 0$. Then the matrix $Mat_R(jw_i)$ is a positive semidefinite no null matrix if and only if the following inequality is verified.

$$\begin{aligned} & D_0^o(jw_i)D_1^o(jw_i) + D_0^e(jw_i)D_1^e(jw_i) + \\ & N_0^o(jw_i)N_1^o(jw_i) + N_0^e(jw_i)N_1^e(jw_i) \quad > \quad 0 \end{aligned} \tag{33}$$

Proof: Obvious thus by taking the relationships (29) into account, we may deduce that $det(Mat_R(jw_i) - \zeta_R(jw_i)I)$ has three null eigenvalues $\zeta_R(jw_i)$ and one no null. This no null eigenvalue $\zeta_R(jw_i)$ is given by (33). ■

In these conditions, if relation (33) is verified, then the polynomial forms $A_0^e(jw_i)A_1^e(jw_i)$ and $A_0^o(jw_i)A_1^o(jw_i)$ are positive semidefinite. Consequently, condition (33) is not a sufficient constraint to guarantee that the quadratic forms (31a) and (31b) are positive definite. Therefore, the question which must be now set is the following: One of the two vectors $\psi(jw_i)$ or $\varphi(jw_i)$ as expressed in (31a) or (31b), does it satisfy one of the two strict positivity constraints established in (12a) and (12b)? That is, by assuming there exists $w_i \in \mathbb{R}^{+}{}_{-\{0\}}$ verifying $Mat_I(jw_i) = 0$ and (33), one of the two conditions a) or b) below must be check.

a) There exists $\psi(jw_i)$ such that

$$\psi^T(jw_i)\; Mat_R(jw_i)\; \psi(jw_i) > 0 \tag{34}$$

b) There exists $\varphi(jw_i)$ such that

$$\varphi^T(jw_i)\; Mat_R(jw_i)\; \varphi(jw_i) > 0 \tag{35}$$

Now, we may show conditions to satisfy the inequalities (34) and (35). Firstly, give a technical lemma.

Lemma 1: [13]. *Let $P(s)$ be an univariate polynomial in s, then $P(s)$ can be written in the form of a sum of squares of terms if and only if $P(s)$ is a positive semidefinite form.*

The proof is immediate: see [13]. ■

In the following theorem, an explicit creterion is provided to determine the Hurwitzness of the polynomial segment $A_\lambda(s)$ according to the simultaneous compensator $C(s)$.

Theorem 5: Let us assume that there exist two stable polynomials $A_0(s)$ and $A_1(s)$ of same degree and of same sign given by (5) and let us consider the case where the strictly positive real zeros w_i are given by $Mat_I(jw_i) = 0$ such that $\psi(jw_i) \neq 0$ or $\varphi(jw_i) \neq 0$. In these conditions, $A_\lambda(s)$ given by (3) is Hurwitz if and only if

a) *the two systems $G_0(jw)$ and $G_1(jw)$ verify (36)*

$$\left\{ \begin{array}{l} \dfrac{N_0^e(jw_i)}{N_1^e(jw_i)} > 0, \\ \\ D_0^o(jw_i)\; D_1^o(jw_i) + \; D_0^e(jw_i)\; D_1^e(jw_i) + \\ \;N_0^o(jw_i)\; N_1^o(jw_i) + \; N_0^e(jw_i)\; N_1^e(jw_i) \; > \; 0 \end{array} \right. \tag{36}$$

b) *the simultaneous controller $C(s)$ stabilizing the two systems $G_0(jw)$ and $G_1(jw)$ verifies one of the four inequalities (37)*

$$\begin{cases} (N_1^e(jw_i)X^e(jw_i))^2 + w_i^4\,(N_1^o(jw_i)X^o(jw_i))^2 + \\ (D_1^e(jw_i)Y^e(jw_i))^2 + w_i^4\,(D_1^o(jw_i)Y^o(jw_i))^2 \neq 0 \\ \\ (N_1^e(jw_i)X^o(jw_i))^2 + (N_1^o(jw_i)X^e(jw_i))^2 + \\ (D_1^e(jw_i)Y^o(jw_i))^2 + (D_1^o(jw_i)Y^e(jw_i))^2 \neq 0 \\ \\ (N_0^e(jw_i)X^o(jw_i))^2 + (N_0^o(jw_i)X^e(jw_i))^2 + \\ (D_0^e(jw_i)Y^o(jw_i))^2 + (D_0^o(jw_i)Y^e(jw_i))^2 \neq 0 \\ \\ (N_0^e(jw_i)X^e(jw_i))^2 + w_i^4\,(N_0^o(jw_i)X^o(jw_i))^2 + \\ (D_0^e(jw_i)Y^e(jw_i))^2 + w_i^4\,(D_0^o(jw_i)Y^o(jw_i))^2 \neq 0 \end{cases} \tag{37}$$

Proof: Let us assume that there is a positive semidefinite no null matrix $Mat_R(jw_i)$ such that relation (33) holds and let us suppose that for any w_i, there exist $\psi(jw_i) \neq 0$, and $\varphi(jw_i) \neq 0$. By considering the two writings of $Mat_R(jw_i)$ given by (32a-32f) and (32g-32j), this symmetric positive semidefinite matrix can be written in the forms (38) where $L(jw_i)$ and $\widetilde{L}(jw_i)$ are given by (39) by taking into account the relations (29)

$$Mat_R(jw_i) = L(jw_i)\;L^T(jw_i) = \widetilde{L}(jw_i)\;\widetilde{L}^T(jw_i) \tag{38}$$

$$\begin{array}{l} L^T(jw_i) = \sqrt{\frac{N_0^e(jw_i)}{N_1^e(jw_i)}}[N_1^e(jw_i)\;N_1^o(jw_i)\;D_1^e(jw_i)\;D_1^o(jw_i)] \\ \widetilde{L}^T(jw_i) = \sqrt{\frac{N_1^e(jw_i)}{N_0^e(jw_i)}}[N_0^e(jw_i)\;N_0^o(jw_i)\;D_0^e(jw_i)\;D_0^o(jw_i)] \end{array} \tag{39}$$

Thus, the polynomial segment $A_\lambda(s)$ may be written in a strictly positive form. For that, it is necessary and sufficient to check (34) or (35), i.e. one of the four expressions below

$$\begin{array}{l} A_0^e(jw_i)A_1^e(jw_i) = \psi^T(jw_i)L(jw_i)L^T(jw_i)\psi(jw_i) > 0 \\ A_0^o(jw_i)A_1^o(jw_i) = \varphi^T(jw_i)L(jw_i)L^T(jw_i)\varphi(jw_i) > 0 \\ A_0^e(jw_i)A_1^e(jw_i) = \psi^T(jw_i)\widetilde{L}(jw_i)\widetilde{L}^T(jw_i)\psi(jw_i) > 0 \\ A_0^o(jw_i)A_1^o(jw_i) = \varphi^T(jw_i)\widetilde{L}(jw_i)\widetilde{L}^T(jw_i)\varphi(jw_i) > 0 \end{array}$$

These above expressions are equivalent to the following ones

$$\begin{array}{l} A_0^e(jw_i)A_1^e(jw_i) = \|L^T(jw_i)\psi(jw_i)\|^2 > 0 \\ A_0^o(jw_i)A_1^o(jw_i) = \|L^T(jw_i)\varphi(jw_i)\|^2 > 0 \\ A_0^e(jw_i)A_1^e(jw_i) = \|\widetilde{L}^T(jw_i)\psi(jw_i)\|^2 > 0 \\ A_0^o(jw_i)A_1^o(jw_i) = \|\widetilde{L}^T(jw_i)\varphi(jw_i)\|^2 > 0 \end{array} \tag{40}$$

Let us assume that the components of the column matrix $L(jw_i)$ and $\widetilde{L}(jw_i)$ are denoted $l_j(jw_i)$ and $\widetilde{l}_j(jw_i)$ respectively and the components of the vector $\psi(jw_i)$ and $\varphi(jw_i)$ are denoted $\psi_j(jw_i)$ and $\varphi_j(jw_i)$ respectively. Consequently, the forms (40) may be written as sum of squares of terms, see Theorem 1 by substituting s by w_i. Therefore, the expressions (40) become

$$\begin{array}{l} A_0^e(jw_i)A_1^e(jw_i) = \sum_{j=1}^{j=4}\left(l_j^T(jw_i)\psi_j(jw_i)\right)^2 > 0 \\ A_0^o(jw_i)A_1^o(jw_i) = \sum_{j=1}^{j=4}\left(l_j^T(jw_i)\varphi_j(jw_i)\right)^2 > 0 \\ A_0^e(jw_i)A_1^e(jw_i) = \sum_{j=1}^{j=4}\left(\widetilde{l}_j^T(jw_i)\psi_j(jw_i)\right)^2 > 0 \\ A_0^o(jw_i)A_1^o(jw_i) = \sum_{j=1}^{j=4}\left(\widetilde{l}_j^T(jw_i)\varphi_j(jw_i)\right)^2 > 0 \end{array} \tag{41}$$

By considering these relations (41) and the terms $l_j(jw_i)$, $\widetilde{l}_j(jw_i)$, $\psi_j(jw_i)$, $\varphi_j(jw_i)$, the following inequalities are obtained

$$\begin{cases} \dfrac{N_0^e(jw_i)}{N_1^e(jw_i)} > 0 \\ \\ (N_1^e(jw_i)X^e(jw_i))^2 + w_i^4\,(N_1^o(jw_i)X^o(jw_i))^2 + \\ (D_1^e(jw_i)Y^e(jw_i))^2 + w_i^4\,(D_1^o(jw_i)Y^o(jw_i))^2 \neq 0 \\ \\ (N_1^e(jw_i)X^o(jw_i))^2 + (N_1^o(jw_i)X^e(jw_i))^2 + \\ (D_1^e(jw_i)Y^o(jw_i))^2 + (D_1^o(jw_i)Y^e(jw_i))^2 \neq 0 \end{cases} \tag{42}$$

and

$$\begin{cases} \dfrac{N_1^e(jw_i)}{N_0^e(jw_i)} > 0 \\ \\ (N_0^e(jw_i)X^o(jw_i))^2 + (N_0^o(jw_i)X^e(jw_i))^2 + \\ (D_0^e(jw_i)Y^o(jw_i))^2 + (D_0^o(jw_i)Y^e(jw_i))^2 \neq 0 \\ \\ (N_0^e(jw_i)X^e(jw_i))^2 + w_i^4\,(N_0^o(jw_i)X^o(jw_i))^2 + \\ (D_0^e(jw_i)Y^e(jw_i))^2 + w_i^4\,(D_0^o(jw_i)Y^o(jw_i))^2 \neq 0 \blacksquare \end{cases} \tag{43}$$

VI. CONCLUSION

In this paper have been given some algebraic conditions to test the simultaneous stabilizability of a segment of systems defined by (1) by assuming that there exists a controller that stabilizes the two endpoints of this segment.

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REFERENCES

[1] C. Abdallah, P. Dorato, F. Prez, and D. Docampo. Controller synthesis for a class of interval plants. *Automatica*, 31:pp. 341 – 343, 1995.

[2] C. Barlett, V. Hollot, and H. Lin. Root location of an entire polytope of polynomials: it suffices to check the edges. *Mathematics of Controls, Signals and Systems*, 1:pp 61–71, 1988.

[3] B. R. Barmish. *New tools for robustness of linear systems*. Macmillan Publishing Company, 1994. New-York.

[4] S. P. Bhattacharyya, H. Chappellat, and L. H. Keel. *Robust Control, The parametric Approach*. Prentice Hall, 1995.

[5] S. Bialas. A necessary and sufficient condition for the stability of convex combinations of stable polynomials and matrices. *Bulletin of Polish Academy*, 33:473–480, 1985.

[6] V. Blondel. A note on convex combinations of polynomials. *IEEE TAC*, 41(11):pp. 1690–1691, 1996.

[7] N. K. Bose. Test of hurwitz and shur properties of convex combination of complex polynomials. *IEEE Transactions on Automatic Control*, 36:1245–1247, 1989.

[8] G. Chockalingam and S. Dasgupta. Strong stabilizability of systems with multiaffine uncertainties and numerator denominator coupling. *IEEE TAC*, 39(9):pp. 1955–1958, 1994.

[9] K. Ghosh. Some new results on the simultaneous stabilizability of a family of single input, single output systems. *Systems and Control Letters*, 6(1):pp. 39–45, 1985.

[10] K. Ghosh. Simultaneous partial pole placement: A new approach to multimode system design. *IEEE TAC*, 31(5):pp. 440–443, 1986.

[11] S. P. Bhattacharyya H. Chapellat. A generalization of kharitonov's theorem: robust stability of interval plants. *IEEE Transactions on Automatic Control*, 34:306–311, 1989.

[12] V. Hollot and R. Tempo. On the nyquist envelope of an interval plant family. *IEEE-TAC*, 39(2):pp. 391–396, 1994.

[13] P.A. Parrilo. Structured semidefinite programs and semialgebraic geometry methods in robustness and optimization. *California Institute of Technology, Pasadena, CA, USA*, 2000.

[14] C. B. Soh. Generalization of the hermite-biehler theorem and applications. *IEEE Transactions on Automatic Control*, 35:222–225, 1990.