2008 American Control Conference
Westin Seattle Hotel, Seattle, Washington, USA
June 11-13, 2008

Stochastic Flow Modeling for Control of Manufacturing System Quality

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Abstract— **The importance of quality control and management has long been demonstrated, with quality being shown to be as important as manufacturing productivity. The goal of this paper is to propose an accurate stochastic model that can represent quality data for supervisory control of manufacturing lines. In this paper, we develop a new application of the stochastic-flow modeling (SFM) approach to model the quality behavior of a manufacturing system. This work extends the basic one-product, one-station SFM to that of a multi-product, multi-station manufacturing system. The quality SFM-based model provides aggregation by machine, product, and operational shift. Bernoulli-type rejects, persistent rejects, and quality-loop rejects are addressed in the modeling. Following a presentation of the application physical system, results are given for different examples where the effectiveness of the SFM model is shown.**

I. INTRODUCTION

In manufacturing systems, quality is one of the most important measures of system performance. Quality metrics pervade all manufacturing-related activities beginning with design, development, production, installation, and on to servicing of the product in the field. However, after a product has been designed, the control of final quality becomes the responsibility of manufacturing. Though key design changes to improve product quality are always possible, such decisions require longer feedback times. At the other end of the spectrum, local machine control provides the most immediate feedback and improvement in quality performance. Such quality improvement can be seen as iterative processes described by the Shewhart [1] cycle (Plan-Do-Check-Act) popularized by the Total Quality Management (TQM) approach where the ultimate objective is to obtain perfect quality throughout the system.

Other approaches attempt to effect control at a higher system level where management is more involved in decision making. These approaches include Quality Circles [2], Zero Defects, Statistical Quality Control [3] , Six Sigma [4], or quality management awards and organizations (Malcolm Baldridge National Quality Award, JD Power, etc.). Though these various approaches have been successful, there still remains a gap for controlling quality between the management and local process control levels. This level may be thought of as the multi-machine, or production line. This intermediate system level is particularly important because of the dependencies or interactions present between processes or machines. Local, process-based control is ineffective at improving quality when variations or problems occur in multiple, but related machines.

Different problems can cause a decrease in quality that cannot be solved by decisions at a single machine controller. For example degradation of material handling fixtures, human interactions with assemblies, delays between required maintenance activities, etc. Furthermore, in a manufacturing line, products are inspected and analyzed at different points along the entire line, leading to more control delay in the identification of nonconforming products (usually termed rejects or failures). When quality failures occur, such defective products are isolated from the production line to be repaired if possible.

Having a quality model along with specific knowledge of the performance behavior of a given manufacturing line can aid the resolution of complex quality failures. A system quality model can provide decision-making insight and anticipate expected quality performance. It is difficult to predict quality with deterministic approaches due to the random nature of reject occurrences. In serial manufacturing lines, two main types of rejects exist defined as Bernoulli-type quality failures and as persistent-type quality failures [5], [6]. The first type of failure is random with variations in quality being independent of a station. The second type, as its name implies, are consecutive, persistent rejects linked to a problem at a specific station. A third failure type may be considered if the manufacturing line has quality loops (also known as rework loops). This type may be described as cyclic quality failures associated with rejects occurring various times originating at different stations. The actual failure is identified with a specific product at a station in the quality loop (control and repairing stations).

Manufacturing system operations can be primarily modeled as finite state machines (e.g., Discrete Event Systems (DES), Petri Nets, *etc...*). This class of system representations is widely recognized in queuing systems [7]. The DES model formulation can be complemented to incorporate the real-time behavior of manufacturing systems by use of timed-DES (TDES) models [8]. A timed-DES model is a DES in which the occurrence of each event is marked by a time variable counter. Although the TDES formulation can be derived easily for simple systems, the complex, multi-scale nature of large manufacturing systems limits its applicability in most realistic applications.

In previous research [9], the addition of continuous-time dynamics to DES models using a hybrid control approach was shown to provide a greater flexibility by simplifying the modeling. In this type of dynamical system approach, each event can be characterized by specific dynamical properties such as process delay and process dynamics. This formulation can be used to introduce real-time information such as product demand changes, personnel changes, *etc...* , along with all the dynamically relevant variables that may influence real-time decision making processes.

Recently, a class of models called Continuous Flow Models (CFM) [10] has been proposed for the analysis of manufacturing systems. For each work station, the continuous flow models monitor the ability of a station to do work, its capacity and its product outflow. CFM's provide an elegant way to develop event-based, continuous-time models that are suitable for decision-making in the control of manufacturing systems. Successful applications of the CFM paradigm have led to new approaches for throughput optimization in manufacturing systems [11], [12], real-time scheduling [13] and optimal resource allocation [14], among others. Continuous-flow models provide the modeling capabilities of the Finite State Machines (FSM) representation of DES. Moreover, the DES formulation requires intensive search algorithms to find suboptimal solutions, whereas it is relatively easy to use CFM in the development of optimal algorithms [15]. For non-deterministic analysis, Stochastic Flow Models (SFM) have also been proposed to take into account the random nature of process operations [16], [17], [18].

In this paper, the flexibility of hybrid systems modeled by SFM is described and applied to the estimation of quality for multiple stations in manufacturing lines. In section II, a complete modeling of quality for an entire manufacturing line is provided where Bernoulli type rejects, persistent rejects and quality-loop rejects are modeled. Section III is devoted to a simulation case study along with a discussion of model computation. In section IV, results of the quality simulation are given and compared to the quality expected value.

II. SYSTEM MODEL

A. Single-Product Type SFM Model

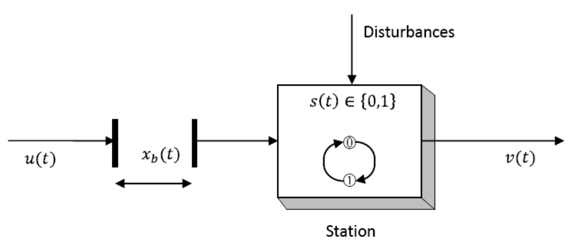

Fig. 1. Single-station Stochastic Flow Model.

In this work, the development of a dynamic model of manufacturing system quality is based on a stochastic flow model for each station of a product assembly line. The model uses information about the probability of the occurrences of reject codes for a defined product type and a production shift to compute the number of rejects on the line for any given period of time. A representation is given in Figure 1 of a single-station SFM.

In Figure 1, the rate of parts entering the station, $u(t)$, is a function of production planning and scheduling, the rate at which the station produces parts, $v(t)$, is a more complex function of processing conditions. Note that the station switches between two states, 0 (OFF) or 1 (ON), such that:

$$s(t) = \begin{cases} 1 & \text{machine working} \\ 0 & \text{otherwise} \end{cases} \quad (1)$$

where $s(t)$ is the operational state of the system. The working and non-working states are triggered by random events that cause the station to change its current state. More elaborate logical statements may also be considered in practice. A simple rule-based operational constraint for the output rate could be easily formulated as follows:

$$v(t) = \begin{cases} 0 & \text{if } s(t)=0 \text{ or } x_b(t)=0 \\ \rho(t) & \text{otherwise} \end{cases} \quad (2)$$

where the buffer content $x_b(t)$ is determined by the following ordinary differential equation:

$$\dot{x}_b(t) = u(t) - v(t) \quad (3)$$

When a product exits the station, it may either be conforming (acceptable quality) or nonconforming (defective quality). If there exists a problem with the product, the product will be given a reject status thus identifying it for repair (as explained in the next section). The total number of rejects is calculated using the following ordinary differential equation:

$$\dot{x}_r(t) = \Delta(t)\delta(t) \quad (4)$$

where x_r is the number of rejects (with initial conditions $x_r(0)=0$), $\Delta(t)$ a decision function, $\delta(t)$ a unit impulse.

Before defining the function $\Delta(t)$, some definitions must be introduced. First, a parameter called $\lambda > 1$ is introduced to represent the average rate of produced parts between two rejects. Historical data in this work fit an exponential distribution $\mathscr{E}(\lambda)$ which has a density function $f_{\mathscr{E}(\lambda)}(t) = \lambda e^{-\lambda t}$. Based on this assumption, a maximum likelihood estimate of λ is obtained for various plant conditions.

From the property of density function, the integral of $f_{\mathscr{E}(\lambda)}(t)$ from 0 to ∞ is equal to 1 (100%) but there is also $\frac{100}{\lambda}\%$ that corresponds to the reject rate and thus $\left(100 - \frac{100}{\lambda}\right)\%$ for the non-reject rate. The boundary between these two modes (reject and non-reject) is defined by the threshold th. In other words, this threshold, deduced from the exponential distribution $\mathscr{E}(\lambda)$, is defined such that: $\int_0^{th} f_{\mathscr{E}(\lambda)}(t)dt = \frac{1}{\lambda}$, with $\lambda > 1$ and where the solution of this equation gives the expression of th:

$$th = -\frac{1}{\lambda} \ln\left(\frac{\lambda - 1}{\lambda}\right) \quad (5)$$

From the exponential distribution $\mathscr{E}(\lambda)$, a random sample, called t_λ, is extracted for each product exiting the station. This sample is compared to the reject threshold th defined above. If the sample t_λ is under the threshold when the product exits the station, then a reject flag is given to the product. The function $\Delta(t)$ is given by:

$$\Delta(t) = \begin{cases} 0 & \text{if } v(t) = 0 \\ 1 & \text{if } v(t) \neq 0 \text{ and } t_\lambda \leq th \end{cases} \quad (6)$$

This type of model can be used to provide high-level abstractions of discrete event systems (DES) models. In a DES (or a timed-DES), the ability to monitor single parts is limited since each action taken on each part must be tracked.

B. Five Station Manufacturing System

As the manufacturing system grows in size, it becomes more complex and modifications to the model are required. Before focusing on another example, some definitions need to be given about categories of stations. It is obvious that each station will have its own properties and so the manufacturing line may be divided into different sets, where the definition of these sets provides better understanding of the manufacturing line modeling.

First, a set I is defined which corresponds to the set of all stations of the manufacturing line. As shown in Figure 2, I can be divided in subsets I^P (set of *processing stations*), I^C (set of *control stations*) and I^Q (set of *quality stations*). The set I^C is decomposed in two parts I^D and its complement $I^C \backslash I^D$, where I^D corresponds to the set of *decision stations*, stations in that set take the decision to send the product to the next section or the quality loop. The set I is also divided as N_Σ subsets (called sections) $I_{\Sigma_q(q=1,\ldots,N_\Sigma)}$ (see Figure 2), in that way $I = \bigcup_{q=1}^{N_\Sigma} I_{\Sigma_q}$ and $I_{\Sigma_q} = I^P_{\Sigma_q} \bigcup I^C_{\Sigma_q} \bigcup I^Q_{\Sigma_q}$. Moreover, sets I^P, I^C, I^D or I^Q can be divided in the same way as N_Σ subsets, e.g.: $I^P = \bigcup_{q=1}^{N_\Sigma} I^P_{\Sigma_q}$.

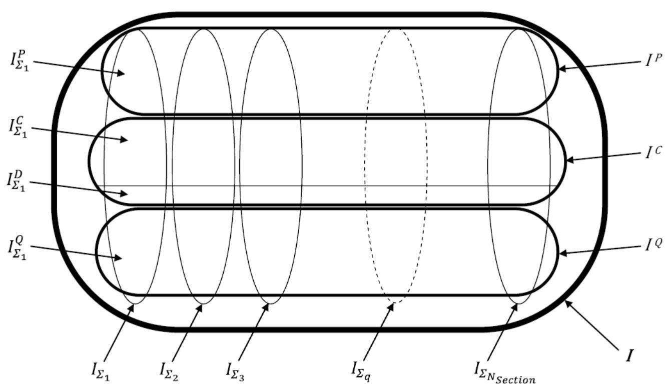

Fig. 2. Station set definition scheme.

In order to illustrate these definitions, a slightly more complex five station system is examined (Figure 3), the modeling is the same as previously seen. Each station is indexed by $i_{(i \in \{1,2,\ldots,5\})}$.

The link between the definitions of sets and figures 2 and 3 can be easily made now. In Figure 3, I is defined by stations S_i (i=1,...,5), stations S_1 to S_4 belong to the first section: I_{Σ_1} and S_5 belongs to the second section: I_{Σ_2}. Stations S_1 and S_5 are defined as *processing stations*: I^P, stations S_2 and S_3 are *control stations* and belong to I^C, station S_3 is a *decision station* and belongs to I^D and finally, station S_4 is a *quality station* Q_1 and belongs to I^Q. Products can move from one station to the next (for example, from S_1 to S_2). But, in some instances, as at the exit of station S_3, two paths are possible. From S_3, products can be directed to S_4 or station S_5. If the product has not triggered a quality rejects before station S_3, this product goes to station S_5. If a reject has occurred, the product must be repaired, and is therefore sent to station S_4. In station S_4, the problem identified at station S_3 on the product is repaired. The product then returns to station S_2 to be processed and diagnosed a second time.

To complete the model, new definitions are required to generalize equations (1) to (6). Now, a set L_i is defined as the set of all stations linked with the input of station i, for example in Figure 3, $L_2 = \{S_1, S_4\}$. The input u_i of the station i can be defined as:

$$u_i(t) = \sum_{l \in L_i} b_{l,i}(t) v_l(t) \tag{7}$$

where $b_{l,i}$ is a boolean function (with the property $\sum_i b_{l,i} = 1$) that determines in which line the product is produced from station l to station i. In Figure 3, after station S_3 two choices exist, if for example the product must be repaired than $b_{3,4}(t) = 1$ and also $b_{3,5}(t) = 0$.

C. Complete Manufacturing System Model

The previous section provides definitions for the modeling of all controlling stations in the complete manufacturing line, however, the product type and the shift is not taken into account. Therefore, before giving a complete model of the entire manufacturing line, further variables need to be defined to represent the dynamics of the system.

A product is defined by a unique number p (such as a unique serial number). This product number belongs to a product type j defined by $\mathbf{J}(p) = j$ where $\mathbf{J}$ is a function that gives the product type j of a product p. A production shift is defined by a number k and defined by $\mathbf{K}(t) = k$ where $\mathbf{K}$ is a function of time. In this section, the model of the entire manufacturing line is developed for all possible configurations (i, j, k), meaning for each station $i \in I$, for each product type $j \in J = \{\text{product type}\}$ and each shift $k \in K = \{\text{production shift}\}$.

The same functions, as defined previously in equations (1) to (6), can be defined for all configurations $(i, j, k)_{\{i \in I, j \in J, k \in K\}}$, functions $s_{i,j,k}(t)$, $v_{i,j,k}(t)$, etc... are also defined. In this paper, the SFM dynamics of the entire manufacturing line are modeled as a hybrid stochastic system. Note that only one engine and only one shift can be available at station i at time t, therefore at a specific station $i \in I$:

$$\begin{aligned} s_i(t) &= s_{i,j,k}(t) \\ &= \begin{cases} 1 & \text{if a product is made in station } i \\ 0 & \text{otherwise} \end{cases} \end{aligned} \tag{8}$$

$$\begin{aligned} v_i(t) &= v_{i,j,k}(t) \\ &= \begin{cases} 0 & \text{if } s_i(t) = 0 \text{ or } x_{b_i}(t) = 0 \\ \rho_{i,j,k}(t) & \text{otherwise} \end{cases} \end{aligned} \tag{9}$$

The buffer dynamic is modeled in the same way as in (3) by the difference between the input and the output flow:

$$\dot{x}_{b_i}(t) = u_i(t) - v_i(t) \tag{10}$$

where u_i is the input of station i, defined by (7).

This first part gives the product flow evolution model. Considering the quality index modeling on the manufacturing

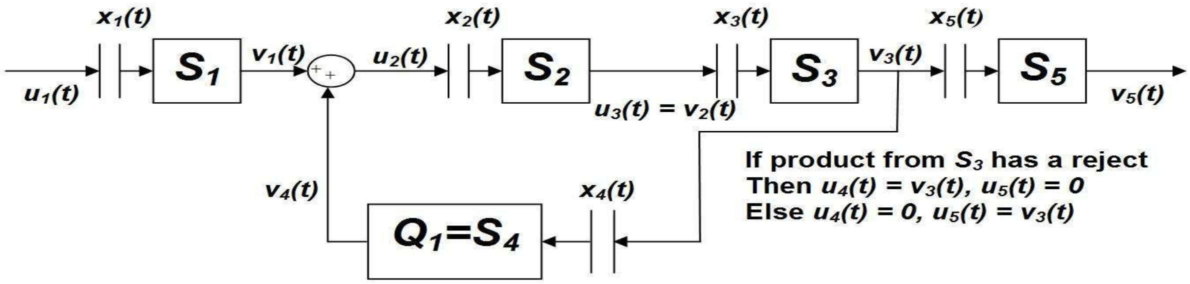

Fig. 3. Simple five-station SFM quality system

line, (4) should be defined as a hybrid flow model and reject functions R_p and r_p should be introduced, but also functions N_C and N_M. N_C corresponds to the reject number of product p at station i. When a product exits the station and when a Bernoulli-type reject occurs, N_C equals 1, but quality-loop rejects could also appear, and the product will loop $p_{\tilde{\lambda}_{i,j,k}}$-times at a station $i \in I \setminus I^P$ in the specific section.

$$N_C(p,i) = \begin{cases} 0 & \text{initial value} \\ 1 & \text{if } \mathfrak{a}_1 \text{ and } \mathfrak{a}_2, \forall i \in I^P \\ p_{\tilde{\lambda}_{i,j,k}} & \text{if } \mathfrak{a}_1 \text{ and } \mathfrak{a}_2, \forall i \in I \setminus I^P \\ \max(0, N_C(p,i)-1) & \text{if } \mathfrak{a}_1 \text{ and not}(\mathfrak{a}_2) \\ N_C(p,i) & \text{otherwise} \end{cases}$$

where $\mathfrak{a}_1 := \{v_i(t) \neq 0\}$, $\mathfrak{a}_2 := \left\{t_{\lambda_{i,j,k}} \leq th_{i,j,k}\right\}$, the random number $p_{\tilde{\lambda}_{i,j,k}}$ follows a Poisson distribution law $\mathscr{P}(\tilde{\lambda}_{i,j,k})$, $t_{\lambda_{i,j,k}}$ follows a exponential distribution $\mathscr{E}(\lambda_{i,j,k})$, $th_{i,j,k}$ is calculated using (5), parameters $\tilde{\lambda}_{i,j,k}$ and $\lambda_{i,j,k}$ are estimated from recorded data using the maximum likelihood for each configuration (i,j,k).

N_M represents the number of consecutive or persistent-type rejects at station i. These rejects are associated with a temporary reject problem located at station i and independent of the product type j and the shift k. Then $p_{\tilde{\mu}_i}$ consecutive rejects could happen at this station independently to j or k.

$$N_M(p,i) = \begin{cases} 0 & \text{initial value} \\ p_{\tilde{\mu}_i} & \text{if } \mathfrak{a}_1 \text{ and } t_{\mu_i} \leq th_i \\ \max(0, N_M(p-1,i)-1) & \text{if } \mathfrak{a}_1 \text{ and } t_{\mu_i} > th_i \\ N_M(p^*,i) & \text{otherwise} \end{cases}$$

where the random number $p_{\tilde{\mu}_i}$ follows a Poisson distribution law $\mathscr{P}(\tilde{\mu}_i)$ t_{μ_i} follows an exponential distribution $\mathscr{E}(\mu_i)$, th_i is calculated using (5), parameters $\tilde{\mu}_i$ and μ_i are estimated from recorded data using the maximum likelihood for each station i and p^* corresponding to the product number at previous time instant in station i (p^* equals p or $p-1$).

Functions N_C and N_M are useful for modeling reject functions r and R which correspond respectively to the label of a reject code on a product p for the complete manufacturing line and for the usual section. The function r is equal to 1 (for a defined product p) when this product has a reject, and the value of r cannot be changed. The function R has the same rules as function r but the value of R is set to zero when the product is repaired.

$$r(p,t) = \begin{cases} 0 & \text{initial value} \\ r(p,t^-) & \text{if } \mathfrak{a}_3 \\ 1 & \text{if not}(\mathfrak{a}_3) \text{ and } \mathfrak{a}_4 \\ 0 & \text{if not}(\mathfrak{a}_3) \text{ and not}(\mathfrak{a}_4) \end{cases} \tag{11}$$

$\mathfrak{a}_3 := \{v_i(t)=0 \text{ or } r(p,t^-) \neq 0\}$, $\mathfrak{a}_4 := \{N_C(p,i)>0 \text{ or } N_M(p,i)>0\}$, $\text{not}(\mathfrak{a}_4) := \{N_C(p,i)=0 \text{ and } N_M(p,i)=0\}$ and where t^- stands for the instant before the usual instant t when the update is done ($t^- = \lim_{\varepsilon \to 0, \varepsilon > 0} \ t - \varepsilon$).

$$R(p,t) = \begin{cases} 0 & \text{initial value or } i \in I^Q \\ R(p,t^-) & \text{if } i \notin I^Q \text{ and } \mathfrak{a}_5 \\ 1 & \text{if } i \notin I^Q \text{ and not}(\mathfrak{a}_5) \text{ and } \mathfrak{a}_4 \\ 0 & \text{if } i \notin I^Q \text{ and not}(\mathfrak{a}_5) \text{ and not}(\mathfrak{a}_4) \end{cases} \tag{12}$$

where $\mathfrak{a}_5 := \{v_i(t) = 0 \text{ or } R(p,t^-) \neq 0\}$.

Now, a model should be given to the function $b_{i,l}$ ($i \in I_{\Sigma_q}$) associated to (7):

- if $i \in I^P \bigcup \left\{I^C \backslash I^D\right\}$ then l is the next station and

$$b_{i,l}(t) = \begin{cases} 1 & \text{if } v_i(t) \neq 0 \\ 0 & \text{if } v_i(t) = 0 \end{cases}$$

- if $i \in I^D$ then
  - if $l \in I^Q$

$$b_{i,l}(t) = \begin{cases} 1 & \text{if } v_i(t) \neq 0 \text{ and } R(p,t) \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

  - if $l \in I^P_{\Sigma_{q+1}}$

$$b_{i,l}(t) = \begin{cases} 1 & \text{if } v_i(t) \neq 0 \text{ and } R(p,t) = 0 \\ 0 & \text{otherwise} \end{cases}$$

- if $i \in I^Q$ then $l \in I^C$ (l is the first station of $I^C_{\Sigma_q}$) and

$$b_{i,l}(t) = \begin{cases} 1 & \text{if } v_i(t) \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

When the product p exits the last station of the entire manufacturing line, the reject index function $X_r(t)$ is updated:

$$\dot{X}_r(t) \quad = \quad \begin{cases} \delta(t) & \text{if } r_p(t) \neq 0 \\ 0 & \text{otherwise} \end{cases} \tag{13}$$

with the initial condition $X_r(0) = 0$.

D. Global quality index

Since the purpose of this work is to provide a control scheme that can respond to fluctuations in quality in a manufacturing line, a measurable output variable of system quality needs to be defined. Quality in manufacturing lines is usually monitored by elaborate sensor-based and/or person-based fault detection controls. The accounting of rejects provides an effective measure of the quality.

Therefore, a measure of quality is obtained using the percentage of parts that exit the station without rejects. If $N+1$ corresponds to the size of the set I and if the $(N+1)$-th station is a quality station (as seen on Figure 4), the N-th station is a decision station that represents the last station of the entire manufacturing line, the measure called *global quality index* (GQI) is given by the following equation:

$$\mathrm{GQI}(t) = \frac{\int_0^t v_N(\tau)d\tau - X_r(t)}{\int_0^t v_N(t)d\tau} \times 100 \tag{14}$$

where $\int_0^t v_N(\tau)d\tau$ corresponds to the number of products which have exited the line at time instant t and X_r the number of rejects on the same time horizon.

III. Model Computation

A. Model description

The approach presented in section II stands for a global modeling aspect for manufacturing lines, this new section focuses on a manufacturing line illustrated in Figure 4 and composed of 10 stations and 2 sections. Process stations are represented by $I = \{S_1, S_2, S_3, S_7, S_8\}$, control stations by $I^C = \{S_4, S_5, S_9\}$, and quality stations by $I^Q = \{S_6, S_{10}\}$.

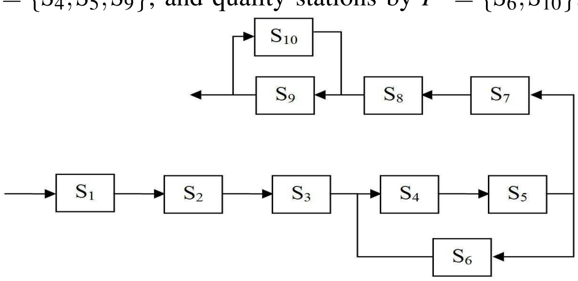

Fig. 4. Layout of the manufacturing line.

Two product types are considered $J = \{P_1, P_2\}$ and the production shift is divided in three parts $K = \{T_1, T_2, T_3\}$ with $T_k = [8*(k-1), 8*k]$ *hours*, $k = 1,2,3$.

B. Distribution parameter estimation

To simulate the entire manufacturing line and calculate the global quality index as defined in (14), the estimation of distribution parameters ($\lambda_{i,j,k}$, $\tilde{\lambda}_{i,j,k}$, μ_i and $\tilde{\mu}_i$) is provided by using the maximum likelihood estimation from recorded data. In this paper, estimated values of these parameters are given in order to strengthen the approach.

The parameter μ_i (reject rate when consecutive rejects occur) is equal to 15000 when $i \in I^P$, 16000 when $i \in I^C$ and 23000 when $i \in I^Q$, for all product type $j \in J$ and all shift $k \in K$. Parameter $\tilde{\lambda}_{i,j,k}$ is equally defined by 1.074 for all configurations ($i \in I \setminus I^P$). Parameter $\tilde{\mu}_i$ is also defined constant for all configurations with the value 2.656.

TABLE I

Numbers of products between two rejects - Inverse of the reject rate $\lambda_{i,j,k}$

Product type		$j=1$		$j=2$	
Shift		$k=1$	$k=2$, $k=3$	$k=1$, $k=2$	$k=3$
$\lambda_{i,j,k}$	$i \in I^P$	500	100	50	200
	$i \in I^C$	100	200	10	20
	$i \in I^Q$	20	150	250	200

C. Simulation description

To start the simulation, products are sent to the first station of the entire line. When a product p associated to a product type j exits a station i at time t (corresponding to a shift k), functions $N_C(p,i)$ and $N_M(p,i)$ are updated. Their values can change if a reject occurs or in other words if the random samples $t_{\lambda_{i,j,k}}$ and t_{μ_i} are under the thresholds $th_{i,j,k}$ and th_i. If the random sample is associated to a reject, the product is labeled with a reject flag otherwise no flag is exhibited. When a consecutive reject occurs at station i occurs ($t_{\mu_i} \leq th_i$), the number of rejects $p_{\tilde{\mu}_i}$ is computed and given to $N_M(p,i)$. When a reject associated to N_C ($t_{\lambda_{i,j,k}} \leq th_{i,j,k}$) occurs, the number of rejects $p_{\tilde{\lambda}_{i,j,k}}$ equals 1 if a process station is concerned ($i \in I^P$), otherwise $p_{\tilde{\lambda}_{i,j,k}}$ follows a Poisson distribution with the parameter $\tilde{\lambda}_{i,j,k}$ ($i \in I \setminus I^P$). Functions r and R are updated.

When a product p exits a decision station ($i \in I^D$), if $R(p,t) = 0$, no reject rejects occurred in the current section then the product is sent to next section or exits the line and (14) is updated, otherwise ($R \neq 0$) the product goes to quality station (I^Q) and R is set to 0 when the product is repaired.

IV. Results: GQI simulation

This section addresses the accuracy of the SFM approach to estimate manufacturing line quality. In the first set of results, validation of quality estimations is determined over shift transitions for products P_1 and P_2, shown respectively in figures 5 and 6. These figures show a set of simulations over one day with error bounds associated to an error of 1% and 5% around the expected value. Several results prove the accuracy of the SFM approach demonstrated with stochastic estimations that are not scattered by more than 5% around the expected value after 5 hours of simulation. The error, shown on figure 5, is less than 1% with the first product type, proving the high accuracy of this approach.

In a second set of results shown in Figure 7, the complete hybrid components are combined. Shift transitions occur at $8h$ and $16h$, product type switches occur at $t = 4h$ (P_1 to P_2), $t = 12h$ (P_2 to P_1), and $t = 20h$ (P_1 to P_2). The results presented in figure 7 show the high accuracy of the SFM approach, results are confined in the 5% error bounds. The results for all simulations are really closed to the expected value.

V. Conclusion and perspectives

In this paper, a manufacturing system model for quality control is developed considering Bernoulli-type rejects, persistent rejects and quality-loop rejects. This model includes

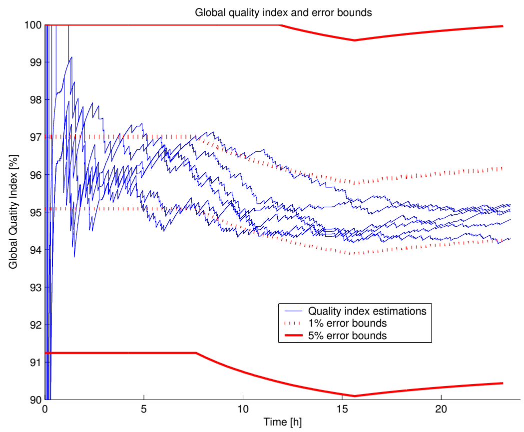

Fig. 5. Comparison for the three shifts of GQI estimated by the SFM approach and the expected value for the first product type.

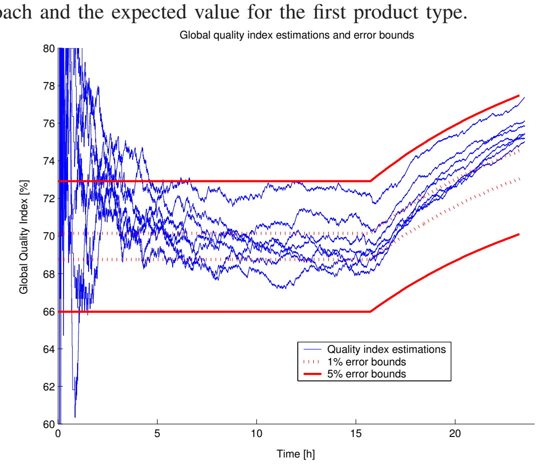

Fig. 6. Comparison for the three shifts of GQI estimated by the SFM approach and the expected value for the second product type.

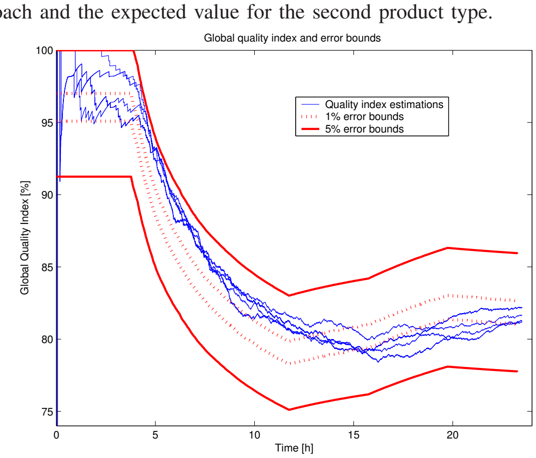

Fig. 7. GQI estimations and error bounds.

hybrid control aspects with different discrete states and also represents decision-making associated to the transfer of rejected products into rework quality loops. The formulation of the model takes also into account the non-stationary behavior of model parameters and a maximum likelihood estimation technique is used for the parameter estimation. The results of the study is focused on a specific case but the global and generic aspect is deduced from the model description. The results demonstrate that the SFM model provides an accurate tool for dynamic modeling of a quality performance metric such as the GQI introduced here. Finally, this model can improve decision-making in quality prediction since an accurate quality indicator is available for an entire manufacturing line. Future work includes a closed-loop estimation of parameters, a modeling of machine degradation linked to maintenances in order to maximize the quality with a minimum maintenance cost.

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