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Robust H_∞ Control for a Class of Uncertain Neutral Time-Delay Systems via Dynamic Output Feedback

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***Abstract*— In this paper, the robust H_∞ control problem is considered for uncertain neutral systems with discrete and distributed time delays. By using new Lyapunov-Krasovskii functionals, some sufficient conditions on H_∞ robust performance analysis are given in terms of linear matrix inequalities (LMIs). Based on these conditions the dynamical output feedback controller are designed. Numerical examples are included to illustrate the proposed method.**

I. INTRODUCTION

Recently, the stability analysis and control problem for neutral systems with time delay has received considerable attention and has become one of the most interesting topics in control theory [1]-[12]. In many practical systems, such as distributed networks and chemical reactors, the models of systems should be described by functional differential equations of neutral type, which depend on the delays of state and state derivative [1]-[5]. H_∞ control method for neutral is proposed in [6], [7], while positive real control method in [8]. A guaranteed cost control problem for neutral systems has been investigated in [9] by the linear matrix inequality (LMI) method. A control method is developed with a single input and some restrictions on the system matrices using a differential-difference inequality and the transformation technique [10]. The static and dynamic output feedback controllers to stabilize neutral systems are respectively designed in [11] and [12]. However, few results have been reported for the dynamic output-feedback control problem for uncertain neutral systems with time delay that motivates the present paper.

In this paper, we consider the problem of robust H_∞ control for a class of neutral time-delay systems with parameter uncertainties allowed to be time-varying but norm-bounded. Our goal is to design a full order strictly proper dynamic output-feedback controller such that the closed-loop system is asymptotically stable and guarantees a prescribed H_∞ performance level for all admissible uncertainties. All the conditions are given in terms of LMIs. Two numerical examples illustrate the effectiveness of our solutions as compared to results obtained by other methods.

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For simplification, we define $T_{zw}(s) = C(sI - A)^{-1}B + D$, which is the transfer function from w to z and use the symbol $Sym\{\cdot\}$ to denote $Sym\{X\} \overset{\text{def}}{=} X + X^T$, the symbol $*$ to denote the symmetric part.

II. PROBLEM FORMULATION AND PRELIMINARIES

Consider the following system with discrete and distributed delays and parameter uncertainties:

$$\begin{aligned}\dot{x}(t) &= [A + \Delta A(t)]x(t) + [A_{d1} + \Delta A_{d1}(t)]x(t - \tau_1) \\ &\quad + [A_{d2} + \Delta A_{d2}(t)] \int_{t-\tau_2}^{t} x(s)ds + A_{d3}\dot{x}(t - \tau_3) \\ &\quad + B_1 w(t) \\ z(t) &= C_1 x(t) + D_{11} w(t) \\ x(t) &= \varphi(t), \forall t \in [-\tau, 0]\end{aligned} \tag{1}$$

where $x(t) \in R^n$ is the state, $z(t) \in R^r$ is the controlled output, $w(t) \in R^l$ is the disturbance of finite energy in the space $L_2[0, \infty)$, and A, $Ad_i, i = 1, 2, 3$, B_1, C_1, D_{11} are known constant matrices of appropriate dimensions. $\Delta A(t)$, $\Delta A_{d1}(t)$, $\Delta A_{d2}(t)$ are unknown matrices representing time-varying parameter uncertainties, the scalars $\tau_i > 0, i = 1, 2, 3$ are time delays and τ_2 is known, $\tau = max(\tau_1, \tau_2, \tau_3)$, $\varphi(t)$ is a real-valued continuous initial function on $[-\tau, 0]$. In this paper, the parameter uncertainties are assumed to be of the form

$$\begin{aligned}&\begin{bmatrix} \Delta A(t) & \Delta A_{d1}(t) & \Delta A_{d2}(t) \end{bmatrix} \\ = \; & DF(t) \begin{bmatrix} E_1 & E_{d1} & E_{d2} \end{bmatrix}\end{aligned} \tag{2}$$

where D, E_1, E_{d1}, E_{d2} are known real constant matrices of appropriate dimensions. $F(\cdot) : R \to R^{k \times l}$ is an unknown time-varying matrix function satisfying

$$F^T(t)F(t) \leq I, \forall t \tag{3}$$

Assume that all the elements of $F(t)$ are Lebesgue measurable. The uncertain matrices $\Delta A(t)$, $\Delta A_{d1}(t)$, $\Delta A_{d2}(t)$ are said to be admissible if both (2) and (3) hold.

Lemma 1: Let D, S, F be real matrices of appropriate dimensions and F satisfying $F^T F \leq I$. Then the following statements hold:

For any scalar $\varepsilon > 0$ and vectors $x, y \in R^n$,

$$2x^T DFSy \leq \varepsilon^{-1} x^T DD^T x + \varepsilon y^T S^T Sy$$

III. MAIN RESULTS

A. Robust H_∞ Performance Analysis

For simplification, we define the operator $\mathscr{D}$: $\mathscr{C}([-\tau, 0], R^n) \to R^n$ as $\mathscr{D}x_t = x(t) - A_{d3}x(t - \tau_3)$.

Throughout this paper, we assume that

Γ. All the eigenvalues of matrix A_{d3} are inside the unit circle.

Theorem 1: Assume $\tau_2 > 0$, $\gamma > 0$ are given positive scalars. Under Γ, the system (1) is robustly asymptotically stable and satisfies $\|T_{zw}(s)\|_\infty < \gamma$ for all admissible uncertainties, if there exist matrices $P > 0$, $Q_i > 0$, $i = 1, 2, 3$ and scalars $\varepsilon_1 > 0$, $\varepsilon_2 > 0$ (and let $\varepsilon = \varepsilon_1 + \varepsilon_2$) such that the LMI holds, as shown in (4) at the top of the next page.

where

$$(1,1) = A^T P + PA + Q_1 + \tau_2^2 Q_2 + Q_3 + \varepsilon E_1^T E_1$$

Proof: Consider the following Lyapunov-Krasovskii functional candidate of the form

$$V(t) = \mathscr{D}x_t^T P \mathscr{D}x_t + V_1(t) + V_2(t) + V_3(t) + V_4(t)$$

where

$$\begin{aligned}
&\mathscr{D}x_t = x(t) - A_{d3}x(t-\tau_3)\\
&V_1(t) = \int_{t-\tau_1}^t x^T(s)Q_1x(s)ds\\
&V_2(t) = \int_{t-\tau_2}^t [\int_s^t x^T(\theta)d\theta]Q_2[\int_s^t x(\theta)d\theta]ds\\
&V_3(t) = \int_0^{\tau_2} ds \int_{t-s}^t (\theta - t + s)x^T(\theta)Q_2x(\theta)d\theta\\
&V_4(t) = \int_{t-\tau_3}^t x^T(s)Q_3x(s)ds
\end{aligned}$$

The time derivative of $V(t)$ along the trajectory of the system (1) is given by

$$\begin{aligned}
\dot{V}(t) = 2\mathscr{D}x_t^T P &\left[(A_{d2} + \Delta A_{d2}(t)) \int_{t-\tau_2}^t x(s)ds\right.\\
&+(A + \Delta A(t))x(t) + (A_{d1} + \Delta A_{d1}(t))x(t-\tau_1)\\
&\left.+B_1w(t)\right] + \dot{V}_1(t) + \dot{V}_2(t) + \dot{V}_3(t) + \dot{V}_4(t)
\end{aligned} \quad (5)$$

where

$$\dot{V}_1(t) = x^T(t)Q_1x(t) - x^T(t-\tau_1)Q_1x(t-\tau_1) \quad (6)$$

$$\begin{aligned}\dot{V}_2(t) = &\,2\int_{t-\tau_2}^t (\theta - t + \tau_2)x^T(t)Q_2x(\theta)d\theta\\ &- \left[\int_{t-\tau_2}^t x^T(\theta)d\theta\right] Q_2 \left[\int_{t-\tau_2}^t x^T(\theta)d\theta\right]\end{aligned} \quad (7)$$

$$\begin{aligned}\dot{V}_3(t) = &\,\tfrac{1}{2}\tau_2^2 x^T(t)Q_2x(t)\\ &- \int_{t-\tau_2}^t (\theta - t + \tau_2)x^T(\theta)Q_2x(\theta)d\theta\end{aligned} \quad (8)$$

$$\dot{V}_4(t) = x^T(t)Q_3x(t) - x^T(t-\tau_3)Q_3x(t-\tau_3) \quad (9)$$

Now, by lemma 1, it can be shown that

$$2x^T(t)Q_2x(\theta) \le x^T(t)Q_2x(t) + x^T(\theta)Q_2x(\theta)$$

Therefore

$$\begin{aligned}\dot{V}_2(t) \le &\int_{t-\tau_2}^t (\theta - t + \tau_2)x^T(\theta)Q_2x(\theta)d\theta\\ &- \left[\int_{t-\tau_2}^t x^T(\theta)d\theta\right] Q_2 \left[\int_{t-\tau_2}^t x(\theta)d\theta\right]\\ &+\tfrac{1}{2}\tau_2^2 x^T(t)Q_2x(t)\end{aligned}$$

This together with (6), (8) and (9) implies

$$\begin{aligned}
&\dot{V}_1(t) + \dot{V}_2(t) + \dot{V}_3(t) + \dot{V}_4(t)\\
&\le x^T(t)(Q_1 + \tau_2^2 Q_2 + Q_3)x(t) - x^T(t-\tau_1)Q_1x(t-\tau_1)\\
&\quad - \left[\int_{t-\tau_2}^t x^T(\theta)d\theta\right] Q_2 \left[\int_{t-\tau_2}^t x(\theta)d\theta\right]\\
&\quad -x^T(t-\tau_3)Q_3x(t-\tau_3)
\end{aligned} \quad (10)$$

Noting (2) and using lemma 1, we have

$$\begin{aligned}
&2\mathscr{D}x_t^T P \left[\Delta A(t)x(t) + \Delta A_{d1}(t)x(t-\tau_1)\right.\\
&\left.+\Delta A_{d2}(t) \int_{t-\tau_2}^t x(s)ds\right]\\
&\le \varepsilon_1^{-1} x^T(t)PDD^TPx(t) + (\varepsilon_1 + \varepsilon_2)\alpha^T(t)M^TM\alpha(t)\\
&+\varepsilon_2^{-1} x^T(t-\tau_3)A_{d3}^T PDD^T PA_{d3}x(t-\tau_3)
\end{aligned} \quad (11)$$

It then follows from (5), (10) and (11) that

$$\dot{V}(t) \le \alpha^T(t)[\Xi + (\varepsilon_1 + \varepsilon_2)M^TM]\alpha(t) \quad (12)$$

where

$$\alpha(t) = \begin{bmatrix} x(t) & x(t-\tau_1) & \int_{t-\tau_2}^t x(s)ds & x(t-\tau_3) \end{bmatrix}^T$$

$$\Xi = \begin{bmatrix} (1,1) & PA_{d1} & PA_{d2} & A^TPA_{d3} \\ * & -Q_1 & 0 & A_{d1}^TPA_{d3} \\ * & * & -Q_2 & A_{d2}^TPA_{d3} \\ * & * & * & (4,4) \end{bmatrix}$$

$$(1,1) = A^TP + PA + Q_1 + \tau_2^2 Q_2 + Q_3 + \varepsilon_1^{-1}PDD^TP$$

$$(4,4) = -Q_3 + \varepsilon_2^{-1}A_{d3}^T PDD^T PA_{d3}$$

$$M = \begin{bmatrix} E_1 & E_{d1} & E_{d2} & 0 \end{bmatrix}$$

Now, from the LMI in (4), it is easy to see, by the schur complement formula, (4) implies that $\Xi + (\varepsilon_1 + \varepsilon_2)M^TM < 0$. Then we can have $\dot{V}(t) < 0$ for all $\alpha(t) \ne 0$ when $w(t) = 0$. Note that Γ guarantees that the operator $\mathscr{D}x_t$ is stable. Therefore, the system (1) is robustly asymptotically stable.

Next, we shall establish the H_∞ performance of the system (1) under the zero initial condition. To this end, we introduce

$$J(t) = \int_0^t [z^T(s)z(s) - \gamma^2 w^T(s)w(s)]ds$$

Noting the zero initial condition, it can be shown that

$$\begin{aligned}
J(t) &= \int_0^t [z^T(s)z(s) - \gamma^2 w^T(s)w(s) + \dot{V}(s)]ds - V(t)\\
&\le \int_0^t [z^T(s)z(s) - \gamma^2 w^T(s)w(s) + \dot{V}(s)]ds
\end{aligned}$$

Note

$$\begin{aligned}&z^T(t)z(t)\\ &= [x^T(t)C_1^T + w^T(t)D_{11}^T][C_1x(t) + D_{11}w(t)]\end{aligned} \quad (13)$$

Then we can get

$$\begin{aligned}&z^T(s)z(s) - \gamma^2 w^T(s)w(s) + \dot{V}(s)\\ &\le \xi^T(s)[\Pi + (\varepsilon_1 + \varepsilon_2)\bar{M}^T\bar{M}]\xi(s)\end{aligned} \quad (14)$$

where

$$\Pi = \begin{bmatrix} (1,1) & PA_{d1} & PA_{d2} & A^TPA_{d3} & (1,5) \\ * & -Q_1 & 0 & A_{d1}^TPA_{d3} & 0 \\ * & * & -Q_2 & A_{d2}^TPA_{d3} & 0 \\ * & * & * & (4,4) & A_{d3}^TPB_1 \\ * & * & * & * & (5,5) \end{bmatrix}$$

$$
\begin{bmatrix}
(1,1) & PA_{d1}+\varepsilon E_1^T E_{d1} & PA_{d2}+\varepsilon E_1^T E_{d2} & A^T PA_{d3} & PB_1 & C_1^T & PD & 0 \\
* & -Q_1+\varepsilon E_{d1}^T E_{d1} & \varepsilon E_{d1}^T E_{d2} & A_{d1}^T PA_{d3} & 0 & 0 & 0 & 0 \\
* & * & -Q_2+\varepsilon E_{d2}^T E_{d2} & A_{d2}^T PA_{d3} & 0 & 0 & 0 & 0 \\
* & * & * & -Q_3 & A_{d3}^T PB_1 & 0 & 0 & A_{d3}^T PD \\
* & * & * & * & -\gamma^2 I & D_{11}^T & 0 & 0 \\
* & * & * & * & * & -I & 0 & 0 \\
* & * & * & * & * & * & -\varepsilon_1 I & 0 \\
* & * & * & * & * & * & * & -\varepsilon_2 I
\end{bmatrix} < 0 \tag{4}
$$

$$
\begin{aligned}
(1,1) &= A^T P + PA + Q_1 + \tau_2^2 Q_2 + Q_3 + \varepsilon_1^{-1} PDD^T P \\
&\quad + C_1^T C_1 \\
(1,5) &= PB_1 + C_1^T D_{11} \\
(4,4) &= -Q_3 + \varepsilon_2^{-1} A_{d3}^T PDD^T PA_{d3} \\
(5,5) &= -\gamma^2 I + D_{11}^T D_{11} \\
\xi(s) &= \begin{bmatrix} \alpha(s) \\ w(s) \end{bmatrix} \\
\bar{M} &= \begin{bmatrix} M & 0 \end{bmatrix}
\end{aligned}
$$

$\Pi + (\varepsilon_1+\varepsilon_2)\bar{M}^T\bar{M} < 0$ implies that $J(t) < 0$. By Schur complement Lemma, the inequality $\Pi + (\varepsilon_1+\varepsilon_2)\bar{M}^T\bar{M} < 0$ can be equivalently changed to (4). This completes the proof.

B. H_∞ Output-Feedback Synthesis

Consider the following system with discrete and distributed delays and parameter uncertainties:

$$
\begin{aligned}
\dot{x}(t) &= [A+\Delta A(t)]x(t) + [A_{d1}+\Delta A_{d1}(t)]x(t-\tau_1) \\
&\quad + [A_{d2}+\Delta A_{d2}(t)]\int_{t-\tau_2}^{t} x(s)ds + A_{d3}\dot{x}(t-\tau_3) \\
&\quad + B_1 w(t) + [B_2+\Delta B_2(t)]u(t) \\
z(t) &= C_1 x(t) + D_{11} w(t) + D_{12} u(t) \\
y(t) &= C_2 x(t) + D_{21} w(t), x(t) = \varphi(t), \forall t \in [-\tau, 0]
\end{aligned} \tag{15}
$$

where $u(t) \in R^p$ is the control input and B_2, C_2, D_{12}, D_{21} are known constant matrices of appropriate dimensions. $\Delta B_2(t)$ is unknown matrix representing time-varying parameter uncertainties with the form $\Delta B_2(t) = DF(t)E_2$, where E_2 is known real constant matrix of appropriate dimension. $\Delta B_2(t)$ is said to be admissible if both (2) and (3) hold. The other signals are the same with the system (1).

Denote the output feedback controller by:

$$
\begin{aligned}
\dot{\hat{x}}(t) &= A_K \hat{x}(t) + B_K y(t), \quad \hat{x}(0) = \hat{x}_0 \\
u(t) &= C_K \hat{x}(t)
\end{aligned} \tag{16}
$$

where $\hat{x}(t) \in R^n$ is the controller state.

Then, we can have the closed-loop system:

$$
\begin{aligned}
\dot{\bar{x}}(t) &= \bar{A}(t)\bar{x}(t) + \bar{A}d_1(t)\bar{x}(t-\tau_1) + \bar{A}d_3\dot{x}(t-\tau_3) \\
&\quad + \bar{A}d_2(t)\int_{t-\tau_2}^{t}\bar{x}(s)ds + \bar{B}_1 w(t) \\
\bar{z}(t) &= \bar{C}_1\bar{x}(t) + \bar{D}_{11} w(t)
\end{aligned} \tag{17}
$$

where

$$
\bar{A}(t) = \bar{A} + \Delta\bar{A}(t), \bar{A}d_i(t) = \bar{A}d_i + \Delta\bar{A}d_i(t), i = 1,2 \tag{18}
$$

$$
\bar{A} = \begin{bmatrix} A & B_2 C_K \\ B_K C_2 & A_K \end{bmatrix}, \bar{A}d_i = \begin{bmatrix} A_{di} & 0 \\ 0 & 0 \end{bmatrix}, i = 1,2,3 \tag{19}
$$

$$
\begin{aligned}
\Delta\bar{A}(t) &= \begin{bmatrix} \Delta A(t) & \Delta B_2(t) C_K \\ 0 & 0 \end{bmatrix} \\
&= \begin{bmatrix} D \\ 0 \end{bmatrix} F(t) \begin{bmatrix} E_1 & E_2 C_K \end{bmatrix}
\end{aligned}
$$

$$
\begin{aligned}
&\Delta\bar{A}d_i(t) = \begin{bmatrix} \Delta A_{di}(t) & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} D \\ 0 \end{bmatrix} F(t) \begin{bmatrix} E_{di} & 0 \end{bmatrix} \\
&i = 1,2
\end{aligned} \tag{20}
$$

$$
\bar{B}_1 = \begin{bmatrix} B_1 \\ B_K D_{21} \end{bmatrix}, \bar{C}_1 = \begin{bmatrix} C_1 & D_{12} C_K \end{bmatrix}, \bar{D}_{11} = D_{11} \tag{21}
$$

Define

$$
\begin{aligned}
\bar{D} &= \begin{bmatrix} D \\ 0 \end{bmatrix}, \bar{E}_1 = \begin{bmatrix} E_1 & E_2 C_K \end{bmatrix} \\
\bar{E}d_i &= \begin{bmatrix} E_{di} & 0 \end{bmatrix}, i = 1,2
\end{aligned}
$$

We can get

$$
\Delta\bar{A}(t) = \bar{D}F(t)\bar{E}_1, \Delta\bar{A}d_i(t) = \bar{D}F(t)\bar{E}_{di}, i = 1,2 \tag{22}
$$

Theorem 2: Assume ε_1, ε_2, $\gamma > 0$ are given positive scalars (let $\varepsilon = \varepsilon_1 + \varepsilon_2$). Under Γ, there exists a dynamical output feedback controller such that the closed-loop uncertain system (17) is asymptotically stable and satisfies $\|T_{zw}(s)\|_\infty < \gamma$ for all the admissible uncertainties, if there exist symmetric positive-definite matrices $X > 0$, $Y > 0$, $Q_i > 0$, $i = 1,2,3$, and matrices N, $\hat{A}$, $\hat{B}$, $\hat{C}$, such that $\Psi = \begin{bmatrix} X & I \\ * & Y \end{bmatrix} > 0$ and the LMI (23) holds, as shown at the top of the next page.

where

$$
\begin{aligned}
(1,1) &= \begin{bmatrix} Sym\{AX + B_2\hat{C}\} & \hat{A}^T + A \\ * & Sym\{YA + \hat{B}C_2\} \end{bmatrix} \\
(1,2) &= \begin{bmatrix} A_{d1} + \varepsilon X E_1^T E_{d1} + \varepsilon\hat{C}^T E_2^T E_{d1} & 0 \\ YA_{d1} + \varepsilon E_1^T E_{d1} & 0 \end{bmatrix} \\
(1,3) &= \begin{bmatrix} A_{d2} + \varepsilon X E_1^T E_{d2} + \varepsilon\hat{C}^T E_2^T E_{d2} & 0 \\ YA_{d2} + \varepsilon E_1^T E_{d2} & 0 \end{bmatrix} \\
(1,4) &= \begin{bmatrix} \hat{A}^T A_{d3} & 0 \\ A^T Y A_{d3} + C_2^T \hat{B}^T A_{d3} & 0 \end{bmatrix} \\
(1,5) &= \begin{bmatrix} B_1 & XC_1^T + \hat{C}^T D_{12}^T \\ YB_1 + \hat{B}D_{21} & C_1^T \end{bmatrix} \\
(1,6) &= \begin{bmatrix} XE_1^T + \hat{C}^T E_2^T \\ E_1^T \end{bmatrix} \\
(2,2) &= -Q_1 + \begin{bmatrix} \varepsilon E_{d1}^T E_{d1} & 0 \\ 0 & 0 \end{bmatrix} \\
(3,3) &= -Q_2 + \begin{bmatrix} \varepsilon E_{d2}^T E_{d2} & 0 \\ 0 & 0 \end{bmatrix}
\end{aligned}
$$

$$
\begin{bmatrix}
(1,1) & (1,2) & (1,3) & (1,4) & (1,5) & \begin{bmatrix} D & 0 \\ YD & 0 \end{bmatrix} & \Psi & \Psi & \Psi & (1,6) \\
* & (2,2) & \begin{bmatrix} \varepsilon E_{d1}^T E_{d2} & 0 \\ 0 & 0 \end{bmatrix} & \begin{bmatrix} A_{d1}^T Y A_{d3} & 0 \\ 0 & 0 \end{bmatrix} & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & (3,3) & \begin{bmatrix} A_{d2}^T Y A_{d3} & 0 \\ 0 & 0 \end{bmatrix} & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & -Q_3 & (4,5) & \begin{bmatrix} 0 & A_{d3}^T YD \\ 0 & 0 \end{bmatrix} & 0 & 0 & 0 & 0 \\
* & * & * & * & (5,5) & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & -\begin{bmatrix} \varepsilon_1 I & 0 \\ * & \varepsilon_2 I \end{bmatrix} & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & (7,7) & 0 & 0 & 0 \\
* & * & * & * & * & * & * & (8,8) & 0 & 0 \\
* & * & * & * & * & * & * & * & (9,9) & 0 \\
* & * & * & * & * & * & * & * & * & -\varepsilon^{-1} I
\end{bmatrix} < 0 \qquad (23)
$$

$$
\begin{aligned}
(4,5) &= \begin{bmatrix} A_{d3}^T Y B_1 + A_{d3}^T \hat{B} D_{21} & 0 \\ 0 & 0 \end{bmatrix} \\
(5,5) &= \begin{bmatrix} -\gamma^2 I & D_{11}^T \\ * & -I \end{bmatrix} \\
(7,7) &= \begin{bmatrix} -2I & -Y \\ * & -N - N^T \end{bmatrix} + Q_1 \\
(8,8) &= \tau_2^{-2} \begin{bmatrix} -2I & -Y \\ * & -N - N^T \end{bmatrix} + \tau_2^{-2} Q_2 \\
(9,9) &= \begin{bmatrix} -2I & -Y \\ * & -N - N^T \end{bmatrix} + Q_3
\end{aligned}
$$

Proof: Apply Theorem 1 to the closed-loop system (17), then the system (17) is robustly asymptotically stable and satisfies $\|T_{zw}(s)\|_\infty < \gamma$ for all admissible uncertainties, if the operator $\mathscr{D}x_t$ is stable and there exist matrices $P > 0$, $Q_i > 0$, $i = 1, 2, 3$ and scalars $\varepsilon_1 > 0$, $\varepsilon_2 > 0$ (and let $\varepsilon = \varepsilon_1 + \varepsilon_2$) such that the equation (4) holds, where A, B_1, C_1, D_{11}, A_{di} are substituted with $\bar{A}$, $\bar{B}_1$, $\bar{C}_1$, $\bar{D}_{11}$, $\bar{A}_{di}$, i=1,2,3.

Firstly, partition the matrix P and its inverse as:

$$
P = \begin{bmatrix} Y & N \\ * & W \end{bmatrix}, P^{-1} = \begin{bmatrix} X & M \\ * & Z \end{bmatrix} \qquad (24)
$$

where $Y, X \in R^{n \times n}$ are positive definite matrices, and M and N are invertible matrices. Note that the equality $P^{-1}P = I$ gives that:

$$
MN^T = I - XY \qquad (25)
$$

Define

$$
F_1 = \begin{bmatrix} X & I \\ M^T & 0 \end{bmatrix}, F_2 = \begin{bmatrix} I & Y \\ 0 & N^T \end{bmatrix} \qquad (26)
$$

Then, it follows that

$$
PF_1 = F_2, F_1^T P F_1 = F_2^T F_1 = \begin{bmatrix} X & I \\ * & Y \end{bmatrix} > 0 \qquad (27)
$$

Next, post-multiply and pre-multiply the equation (4) by the matrix $diag\{F_1^T, \underbrace{I, \cdots, I}_{7}\}$ and its transpose, respectively, and then by the Schur complement formula, we can have the LMI (28), as shown at the top of the next page. where

$$
\begin{aligned}
(1,1) &= Sym\{F_1^T \bar{A}^T P F_1\} \\
(1,2) &= F_1^T P \bar{A}_{d1} + \varepsilon F_1^T \bar{E}_1^T \bar{E}_{d1} \\
(1,3) &= F_1^T P \bar{A}_{d2} + \varepsilon F_1^T \bar{E}_1^T \bar{E}_{d2} \\
(2,2) &= -Q_1 + \varepsilon \bar{E}_{d1}^T \bar{E}_{d1} \\
(3,3) &= -Q_2 + \varepsilon \bar{E}_{d2}^T \bar{E}_{d2}
\end{aligned}
$$

Obviously, we can have

$$
-F_2^T Q_i^{-1} F_2 - Q_i \le -F_2^T - F_2, i = 1, 2, 3 \qquad (29)
$$

Post-multiply and pre-multiply the inequality (28) by the matrix $diag\{\underbrace{I, \cdots, I}_{8}, F_2^T, F_2^T, F_2^T, I\}$ and its transpose, respectively and utilize the inequality (29).
Use the equation (29) and denote

$$
\begin{aligned}
\hat{A} &= YAX + NB_K C_2 X + YB_2 C_K M^T + NA_K M^T \\
\hat{B} &= NB_K, \hat{C} = C_K M^T
\end{aligned} \qquad (30)
$$

Theorem 2 can be obtained immediately. This completes the proof.

Remark 1: For given scalars $\varepsilon_1 > 0$, $\varepsilon_2 > 0$, the inequality (23) is an LMI. Therefore, we can utilise the feasp solver of LMI control toolbox to solve the inequality (23).

Remark 2: Given any solution of the LMI (23) in theorem 2, a corresponding controller of the form (16) will be constructed as follows:
1.Using the two positive definite solutions X, Y and the matrix N, computer the invertible matrix M satisfying (25).
2.Using (30) and the matrices M and N obtained above, compute the controller data A_K, B_K and C_K.

IV. ILLUSTRATIVE EXAMPLES

Example 1: Consider the example studied in [12].

$$
\begin{aligned}
\dot{x}(t) &= Ax(t) + A_{d1}x(t-h) + A_{d2}\int_{t-h}^{t} x(s)ds \\
&\quad + A_{d3}\dot{x}(t-h) + Bu(t) \\
y(t) &= Cx(t)
\end{aligned}
$$

$$
\begin{bmatrix}
(1,1) & (1,2) & (1,3) & F_1^T\bar{A}^TP\bar{A}_{d3} & F_1^TP\bar{B}_1 & F_1^T\bar{C}_1^T & F_1^TP\bar{D} & 0 & F_1^T & F_1^T & F_1^T & F_1^T\bar{E}_1^T \\
* & (2,2) & \varepsilon\bar{E}_{d1}^T\bar{E}_{d2} & \bar{A}_{d1}^TP\bar{A}_{d3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & (3,3) & \bar{A}_{d2}^TP\bar{A}_{d3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & -Q_3 & \bar{A}_{d3}^TP\bar{B}_1 & 0 & 0 & \bar{A}_{d3}^TP\bar{D} & 0 & 0 & 0 & 0 \\
* & * & * & * & -\gamma^2I & \bar{D}_{11}^T & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & -I & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & -\varepsilon_1I & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & * & -\varepsilon_2I & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & * & * & -Q_1^{-1} & 0 & 0 & 0 \\
* & * & * & * & * & * & * & * & * & -\tau_2^{-2}Q_2^{-1} & 0 & 0 \\
* & * & * & * & * & * & * & * & * & * & -Q_3^{-1} & 0 \\
* & * & * & * & * & * & * & * & * & * & * & -\varepsilon^{-1}I
\end{bmatrix} < 0 \quad (28)
$$

where

$$A = \begin{bmatrix} 2 & 1 \\ 1 & -1 \end{bmatrix}, A_{d1} = \begin{bmatrix} 0.3 & 0.1 \\ 0.1 & 0.5 \end{bmatrix}$$
$$A_{d2} = \begin{bmatrix} 0.1 & 0.1 \\ 0.1 & 0.2 \end{bmatrix}, A_{d3} = \begin{bmatrix} 0.2 & -0.1 \\ -0.1 & 0.2 \end{bmatrix}$$
$$B = \begin{bmatrix} 2 \\ 1 \end{bmatrix}, C = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

When taking no account of parameter uncertainties and disturbance, system (1) reduces to the system studied in [12] with $\tau_1 = \tau_2 = \tau_3 = h$. With the method presented in [12] and in this paper, we can obtain the maximum delay 0.99 and 3.54 respectively. And it is noted that the design problem in [12] can not be solved via LMI techniques, which are easy to be calculated. Therefore, the effectiveness of our solutions is illustrated as compared to results obtained by the method in [12].

Example 2: Consider the uncertain delay system (1), where

$$A = \begin{bmatrix} -2.2 & 0.5 \\ -0.2 & -1.8 \end{bmatrix}, A_{d1} = \begin{bmatrix} -0.1 & 0.6 \\ 0.5 & 0.1 \end{bmatrix}$$
$$A_{d2} = \begin{bmatrix} 0.1 & -0.3 \\ 0 & 0.1 \end{bmatrix}, A_{d3} = \begin{bmatrix} 0.1 & 0.3 \\ 0 & 0.1 \end{bmatrix}$$
$$B_1 = \begin{bmatrix} 0.3 \\ 0.8 \end{bmatrix}, B_2 = \begin{bmatrix} 0 \\ 0.3 \end{bmatrix}, D = \begin{bmatrix} 0.1 \\ 0.3 \end{bmatrix}$$
$$C1 = \begin{bmatrix} 1 & 0 \end{bmatrix}, C2 = \begin{bmatrix} 1 & 0 \end{bmatrix}, E_1 = \begin{bmatrix} -0.1 & 0.1 \end{bmatrix}$$
$$E_{d1} = \begin{bmatrix} 0.1 & 0.5 \end{bmatrix}, E_{d2} = \begin{bmatrix} 0.3 & -0.1 \end{bmatrix}$$
$$D_{11} = 0, D_{12} = 0.1, D_{21} = 0.2, E_2 = 0.2, \tau_2 = 0.5$$
$$\varepsilon_1 = 1, \varepsilon_2 = 0.8$$

In this example, the H_∞ performance level γ is specified to be 1.2. We can solve the LMIs in theorem 2, and obtain the solution as follows:

$$X = \begin{bmatrix} 0.9049 & -0.0262 \\ -0.0262 & 1.0462 \end{bmatrix}, Y = \begin{bmatrix} 1.6455 & -0.2931 \\ -0.2931 & 1.8010 \end{bmatrix}$$
$$\hat{A} = \begin{bmatrix} -0.3613 & 1.3631 \\ -0.1871 & -2.1968 \end{bmatrix}, \hat{B} = \begin{bmatrix} -3.1734 \\ -1.4469 \end{bmatrix}$$
$$\hat{C} = \begin{bmatrix} -1.4329 & -2.6207 \end{bmatrix}$$
$$N = \begin{bmatrix} 16.0169 & 1.5459 \\ -2.1557 & 16.4037 \end{bmatrix}$$

Then we can obtain the matrix M satisfying (25)

$$M = \begin{bmatrix} -0.0324 & 0.0148 \\ 0.0267 & -0.0509 \end{bmatrix}$$

Finally, we can get the gain matrices of the stabilizing dynamic output-feedback controller for the system (1):

$$A_K = \begin{bmatrix} -13.2216 & -6.0871 \\ -7.5334 & -7.1245 \end{bmatrix}$$
$$B_K = \begin{bmatrix} -0.1872 \\ -0.1128 \end{bmatrix}$$
$$C_K = \begin{bmatrix} 88.9903 & 98.3247 \end{bmatrix}$$

V. Conclusions

In this paper, we have considered the design problem of robust H_∞ dynamic output-feedback controller for a class of neutral systems with discrete and distributed time delays and time-varying norm-bounded parameter uncertainties. Robust H_∞ performance analysis conditions and output-feedback solutions are given in terms of LMIs. Examples have been provided to illustrate the effectiveness of the proposed approach.

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