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# A Geometric Approach to Fault Detection and Isolation of Neutral Time-Delay Systems

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***Abstract*— This paper investigates development of Fault Detection and Isolation (FDI) filters for neutral time-delay systems. A bank of residual generators is designed such that each residual is affected by one fault and is decoupled from the others while the $H_\infty$ norm of the transfer function between the disturbance and the residual signals are less that a prespecified value. Simulation results presented in the paper demonstrate the effectiveness of our proposed FDI algorithm.**

Keywords: Fault detection and isolation (FDI), Neutral time delay systems , Unobservability subspaces, Geometric FDI.

## I. INTRODUCTION

Modern control systems are becoming increasingly more complex and issues of availability, efficiency, reliability, operating safety, and environmental protection concerns are receiving more attention. This requires a fault diagnosis system that is capable of reliably detecting plant, actuator and sensor faults when they occur, and of identifying and isolating the faulty component in the system. In the past three decades, a number of fundamental results on fault detection and isolation (FDI) have been developed [1]–[14]. However, limited results exist on designing FDI strategies for time-delay systems. Time-delay is an inherent characteristic of many physical systems, such as rolling mills, chemical processes, water resources, biological, economical and traffic control systems, to name a few. In this paper, we investigate development and design of a fault detection and isolation scheme for neutral time-delay systems.

In recent years, only a few results on FDI of time-delay systems have been developed. In [15], [16], an unknown input observer (UIO) is designed for FDI and [17] proposed a robust UIO approach for uncertain retarded time-delay systems with bounded uncertainty. In this work, some assumptions on the system structures are considered. Both approaches are based on determining a suitable state transformation and designing a reduced order observer for the transformed system. Parity space approach is also developed in [18] for fault detection of time-delay systems. In [19]–[25], a robust fault detection problem for linear retarded time-delay systems is investigated by solving an $H_\infty$ optimization problem. In this approach one attempts to keep the sensitivity of the residual signal to unknown inputs (disturbances) less than a specific bound while increase the sensitivity of the residual signal to the fault over the frequency range of the fault. In [26]–[28], an adaptive observer approach is developed for estimating the fault signal in retarded time-delay systems. However, FDI problem for the neutral time-delay systems has only been investigated in [29] where the $H_\infty$ filter approach is used for robust FDI of neutral time-delay systems.

In this paper, a set of residuals that are based on the dedicated residual scheme [6], [12] is generated by generalizing the geometric FDI results in [6] to neutral time-delay systems. The notion of a common unobservability subspace is introduced for neutral time-delay systems and an algorithm for constructing the smallest common unobservability subspace containing a given subspace is also proposed. Based on the developed geometric framework, a set of residuals is generated such that each residual is affected by one fault and is decoupled from others. At the same time the effects of disturbances on the residuals are attenuated by using an $H_\infty$ optimization technique and the LMI approach is used for solving this optimization problem. The main contribution of this work is in developing a geometric FDI framework for linear neutral time-delay systems.

The remainder of this paper is organized as follows. In section II, a brief background on geometric properties of linear systems and an $H_\infty$ control for neutral time-delay systems are reviewed. The problem formulation and framework of our proposed fault detection and isolation strategy are presented in section III. In section IV, a robust fault detection and isolation strategy for time-delay systems is presented. In section V, the effectiveness and capabilities of our proposed algorithm are shown through simulation results. Conclusions and future work are presented in section VI.

The following notation is used throughout this paper. Script letters $\mathcal{X}, \mathcal{U}, \mathcal{Y}, ...$, denote real vector spaces. Matrices and linear maps are denoted by capital italic letters $A, B, C, ...$; the same symbol is used both for a matrix and its map; the zero space, zero vector ,..., are denoted by 0. $\mathcal{B} = \text{Im } B$ denotes the image of $B$; Ker $C$ denotes the kernel of $C$. If a map $C$ is epic, then $C^{-r}$ denotes a right inverse of $C$ (i.e., $CC^{-r} = I$)). A subspace $\mathcal{S} \subseteq \mathcal{X}$ is termed $A$-invariant if $A\mathcal{S} \subseteq \mathcal{S}$. For $A$-invariant subspace $\mathcal{S} \subseteq \mathcal{X}$, $A : \mathcal{S}$ denotes the restriction of $A$ to $\mathcal{S}$, and $A : \mathcal{X}/\mathcal{S}$ denotes the map induced by $A$ on the factor space $\mathcal{X}/\mathcal{S}$. For a linear

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system $(C, A, B)$, $<$ Ker $C|A>$ denotes the unobservable subspace of $(C, A)$.

## II. Background

Consider the linear system

$$\Sigma : \begin{cases} \dot{x}(t) = Ax(t) + Bu(t) \\ y(t) = Cx(t) \end{cases}$$

where $x \in \mathcal{X}$ is the state of the system with dimension $n$, $u \in \mathcal{U}, y \in \mathcal{Y}$ are input and output signals with dimensions $m$ and $q$, respectively.

***Definition** 1:* A subspace $\mathcal{S}$ is a $(C, A)$ *unobservability subspace* (u.o.s.) [3] if $\mathcal{S} =<$ Ker $HC|A + DC>$ for some output injection map $D : \mathcal{Y} \rightarrow \mathcal{X}$ and measurement mixing map $H : \mathcal{Y} \rightarrow \mathcal{Y}$.

Given an u.o.s. $\mathcal{S}$, a measurement mixing map $H$ can be computed from $\mathcal{S}$ by solving the equation Ker$HC$ = Ker$C+\mathcal{S}$. Let $\underline{D}(\mathcal{S})$ denote the class of all maps $D : Y \rightarrow X$ such that $(A + DC)\mathcal{S} \subseteq \mathcal{S}$. The notation $\underline{\mathcal{S}}(\mathcal{L})$ refers to the class of $(C, A)$ u.o.s. containing $\mathcal{L} \subseteq \mathcal{X}$. The class of u.o.s. is closed under intersection; therefore, it contains an infimal element $\mathcal{S}^* = \inf \underline{\mathcal{S}}(\mathcal{L})$. In [6] an algorithm for computing $\mathcal{S}^*$ is proposed.

Given the matrices $A_i, i = 0, ..., N$ and $C$, a subspace $\mathcal{S}^{0,...,N}$ is called a common u.o.s. for the pairs $(C, A_i), i = 0, ..., N$ if

$$\mathcal{S}^{0,...,N} =< \text{Ker } HC|A_i + D_iC>, \quad i = 0, ..., N \tag{1}$$

The notation $\mathcal{S}^{0,...,N}(\mathcal{L})$ refers to a common u.o.s. containing $\mathcal{L} \subseteq \mathcal{X}$. The following algorithm can be used for finding the smallest common u.o.s. $\mathcal{S}^{(0,...,N)^*}(\mathcal{L})$ for the pairs $(C, A_i), i = 0, ..., N$ containing $\mathcal{L}$

$$CUOS : \begin{cases} \mathcal{S}_0 = \mathcal{X} \\ \mathcal{S}_k = \mathcal{W}^* + (\cap_{i=0}^N A_i^{-1} S_{k-1}) \cap \text{Ker } C \end{cases}$$

where $\mathcal{S}^{(0,...,N)^*}(\mathcal{L}) = lim\ \mathcal{S}_k$ and $\mathcal{W}^* = lim\ \mathcal{W}_k$ where $\mathcal{W}_k$ can be obtained from the following algorithm

1) $\mathcal{W}_0 = \mathcal{L}$
2) $\mathcal{W}_k = \mathcal{W}_{k-1} + \sum_{i=0}^N A_i(\mathcal{W}_{k-1} \cap \text{Ker } C)$

For details see [3], [30] and [31].

Let $\mathcal{S} \subset \mathcal{X}$ be an u.o.s. , i.e., $\mathcal{S} =<$ Ker $HC|A+D_0C>$, then the factor system of $\Sigma$ which is denoted by $\Sigma : \mathcal{X}/\mathcal{S}$ is defined as

$$\Sigma : \mathcal{X}/\mathcal{S} \begin{cases} \dot{x}(t) = A_\mathcal{S}x(t) + B_\mathcal{S}u(t) \\ y(t) = C_\mathcal{S}x(t) \end{cases}$$

where $A_\mathcal{S} = A + D_0C : \mathcal{X}/\mathcal{S}$, $B_\mathcal{S} = PB$, $C_\mathcal{S}$ is the unique solution of $C_\mathcal{S}P = HC$, $D_0 \in \underline{D}(\mathcal{S})$ and $P : \mathcal{X} \rightarrow \mathcal{X}/\mathcal{S}$ is the canonical projection.

In the following, certain results on $H_\infty$ disturbance attenuation of neutral time-delay systems are presented. Consider a linear time-delay system

$$\begin{aligned} &\dot{x}(t) = A_0x(t) + A_1x(t-h) + A_2\dot{x}(t-d) + Dd(t) \\ &y(t) = Cx(t) \\ &x(\theta) = 0 \quad \forall\theta \in [-\tau, 0] \end{aligned} \tag{2}$$

where $d(t)$ represents the unknown input vector including modeling errors and uncertain disturbances. Without loss of generality, it is assumed that $d$ is $L_2$-norm bounded. The next theorem provides a sufficient condition for asymptotic stability of system (2) while the $H_\infty$ norm of the transfer function between the disturbance $d$ and the output signal $y$ is less that a given positive value $\gamma$.

***Theorem** 2.1:* Given $\gamma > 0$ and the time-delay system (2), if there exist positive-definite matrices $R$, $Q_1$ and $Q_2$ such that the following inequality is satisfied

$$\begin{bmatrix} RA_0 + A_0^TR + A_0^TQ_1A_0 + Q_2 + C^TC & RA_1 + A_0^TQ_1A_1 & RA_2 + A_0^TQ_1A_2 & RD + A_0^TQ_1D \\ * & A_1^TQ_1A_1 - Q_2 & A_1^TQ_1A_2 & A_1^TQ_1D \\ * & * & A_2^TQ_1A_2 - Q_1 & A_2^TQ_1D \\ * & * & * & -\gamma^2I + D^TQ_1D \end{bmatrix} < 0 \tag{3}$$

then system (2) is asymptotically stable and its $L_2$ gain is not greater than $\gamma$, i.e.

$$\int_0^\infty y^T(t)y(t)dt \leq \gamma^2 \int_0^\infty d^T(t)d(t)dt \quad \forall d(t) \in L_2[0, \infty] \tag{4}$$

***Proof:*** Define a difference operator $\mathbb{D}$ as $\mathbb{D}(\phi) = \phi(0) - A_2\phi(-d)$, then according to inequality (3), we have $A_2^TQ_1A_2 - Q_1 < 0$. Hence the operator $\mathbb{D}$ is stable. Let $V(t)$ be a Lyapunov-Krasovskii functional of the form $V(t) = V_1(t) + V_2(t) + V_3(t)$ [32] where

$$\begin{aligned} V_1(t) &= x^T(t)Rx(t) \\ V_2(t) &= \int_{-d}^0 \dot{x}^T(t+s)Q_1\dot{x}(t+s)ds \\ V_3(t) &= \int_{-h}^0 x^T(t+s)Q_2x(t+s)ds \end{aligned}$$

The asymptotical stability can be shown according to Theorem 2 in [32]. Define an associated Hamiltonian $H(x_t, d(t), t)$ as follows:

$$H(x_t, d(t), t) = \dot{V}(x_t) + y^T(t)y(t) - \gamma^2d^T(t)d(t) \tag{5}$$

It is sufficient [33] to show that under zero initial conditions $H(x_t, d(t), t) < 0$. It can be shown through some algebraic manipulations:

$$H(x_t, d(t), t) \leq \eta^T(t)\Phi\eta(t)$$

where $\eta^T(t) = [x(t)^T, x(t-h)^T, \dot{x}^T(t-d), d^T(t)]$ and $\Phi$ is the matrix in inequality (3) (the details are omitted due to space limitations). Therefore, $\Phi < 0$ leads to $H(x_t, d(t), t) < 0$ and hence the inequality (4) holds. ■

Using the Schur complement, the inequality (3) can be rewritten as

$$\left[\begin{array}{ccccc} RA_0+A_0^TR+Q_2+C^TC & RA_1 & RA_2 & RD+A_0^TQ_1D & A_0^TQ_1 \\ * & -Q_2 & 0 & A_1^TQ_1D & A_1^TQ_1 \\ * & * & -Q_1 & A_2^TQ_1D & A_2^TQ_1 \\ * & * & * & -\gamma^2I+D^TQ_1D & 0 \\ * & * & * & * & -Q_1 \end{array}\right] < 0 \quad (6)$$

which is in an LMI form.

## III. Problem Formulation

Consider the following linear neutral time-delay system

$$\begin{aligned} \dot{x}(t) &= A_0x(t)+A_1x(t-h)+A_2\dot{x}(t-d)+B_0u(t) \\ &\quad +\sum_{j=1}^{L}B_ju(t-\tau_{uj})+\sum_{l=1}^{k}L_lm_l(t)+Dd(t) \\ y(t) &= Cx(t) \end{aligned} \quad (7)$$

with the continuous initial condition $x(\theta)=\phi(\theta), \theta\in[-\tau,0]$ where $x\in\mathcal{X}$ is the state of the system with dimension $n$, $u\in\mathcal{U}, y\in\mathcal{Y}$ are input and output signals with dimensions $m$ and $q$, respectively, $m_i\in\mathcal{M}_i$ are the fault modes with dimension $k_i$, $L_i$'s are fault signatures and $\tau=\max\{h,d\}$. The fault modes together with the fault signatures may be used to model the effects of actuator faults, sensor faults and system faults on the dynamics of the system. For modeling a fault in the $i$-th actuator, $L_i=[b_{0i},b_{1i},...,b_{Li}]$ and the fault mode $m_i$ is chosen to model the type of a fault where $b_{ji}, j=0,...,L$ denote the $i$-th column of matrices $B_j, j=0,...,L$. For example a complete failure of an actuator can be represented and modeled by $m_i(t)=[-u_i(t),-u_i(t-\tau_{u1}),...,-u_i(t-\tau_{uL})]^T$. A system fault can be represented by a potential variation in the parameters of the $A_0, A_1$ and $A_2$ matrices as shown below:

$$\begin{aligned} \dot{x}(t) =&(A_0+\Delta A_0)x(t)+(A_1+\Delta A_1)x(t-h) \\ &+(A_2+\Delta A_2)\dot{x}(t-d)+B_0u(t)+\sum_{j=1}^{L}B_ju(t-\tau_{uj}) \\ y(t) =&Cx(t) \end{aligned}$$

As an example, a change in the $i$-th row and $j$-th column of matrix $A_2$ can be modeled as $\Delta A_2\dot{x}(t-d)=I_i\Delta a_{2_{ij}}\dot{x}_j(t-d)$ where $x_j$ is the $j$-th element of the vector $x$ and $I_i$ is an n-dimensional vector with all zero elements except one in the $i$-th element. Define the signal $m_i(t)\triangleq\Delta a_{2_{ij}}\dot{x}_j(t-d)$ as an external input and fault signature $L_i=I_i$, then this fault can be modeled as in equation (7).

It should be noted that sensor faults can initially be modeled as additive inputs in the measurement equation $y=Cx+\sum_{j=1}^{q}E_jn_j$ where $E_j$ is an $q\times1$ unit vector with a one at the $j$-th position and $n_j\in\mathbb{R}$ is a sensor fault mode, which corresponds to a fault in the $j$-th sensor. For example, a complete failure of the $j$-th sensor can be represented and modeled by $n_j=-c_jx$ where $c_j$ is the $j$-th row of the matrix $C$. The sensor fault signature can also be modeled as an input to the system [6], [9] and [34]. Following [9], let $f_j$ be the solution to $E_j=Cf_j$. The new states can be defined according to $\bar{x}(t)=x(t)+\sum_{j=1}^{q}f_jn_j(t)$, where the state space representation for the new states can be written as

$$\begin{aligned} \dot{\bar{x}}(t) &= A_0\bar{x}(t)+A_1\bar{x}(t-h)+A_2\bar{x}(t-d)+B_0u(t) \\ &\quad +\sum_{j=1}^{L}B_ju(t-\tau_{uj})+\sum_{j=1}^{q}L_jm_j(t) \\ y(t) &= C\bar{x}(t) \end{aligned} \quad (8)$$

where $L_j=\left[\begin{array}{cccc} f_j & A_0f_j & A_1f_j & A_2f_j \end{array}\right]$ and $m_j(t)=[\dot{n}_j(t),-n_j(t),-n_j(t-h),-\dot{n}_j(t-d)]^T$.

We are now in a position to formally introduce the robust fault detection and isolation problem considered in this paper.

## IV. Robust Fault Detection and Isolation of Neutral Time-Delay Systems

The Robust Extended Fundamental Problem in Residual Generation (REFPRG) for the neutral time-delay system (7) is to design a set of filters that generate $k$ residuals $r_i(t)$ such that a fault in the $i$-th component $L_i$ can only affect the residual $r_i(t)$ and no other residual $r_j(t)$ $(i\neq j)$ and

$$\int_0^\infty r_i^T(t)r_i(t)dt \le \gamma^2\int_0^\infty d^T(t)d(t)dt,\ i=1,...,k \quad (9)$$

Specifically, the residual signals $r_i(t)$ are generated according to the following filters:

$$\begin{aligned} \dot{w}_i(t) =&F_{i0}w_i(t)+F_{i1}w_i(t-h)+F_{i2}\dot{w}_i(t-d) \\ &-E_{i0}y(t)-E_{i1}y_i(t-h)-E_{i2}\dot{y}_i(t-d) \\ &+K_iu(t)+\sum_{j=1}^{L}K_{ij}u(t-\tau_{uj}) \\ r_i(t) =& M_iw_i(t)-H_iy(t) \end{aligned} \quad (10)$$

which has a similar structure as an observer that is considered in [32] and [35].

The following theorem summarizes our proposed strategy.

***Theorem 4.1:*** The REFPRG problem defined by expressions (9) and (10) has a solution for the linear neutral time-delay system (7) if there exist the following common unobservability subspaces

$$\mathcal{S}_i=\mathcal{S}^{(0,1,2)^*}(\sum_{j\neq i}\mathcal{L}_j),\quad i=1,...,k \quad (11)$$

such that $\mathcal{L}_i\cap\mathcal{S}_i=0, i=1,...,k$ as well as the matrices $T_{i0}, T_{i1}, T_{i2}, i=1,...,k$ and positive-definite matrices $R_i$, and $Q_i, i=1,...,k$ such that the inequality (12) holds where $P_i$ is the canonical projection of $\mathcal{X}$ on $\mathcal{X}/\mathcal{S}_i$, $D_{\mathcal{S}}=-P_iD$ and the pairs $(M_{\mathcal{S}_i},A_{0_{\mathcal{S}_i}})$, $(M_{\mathcal{S}_i},A_{1_{\mathcal{S}_i}})$, $(M_{\mathcal{S}_i},A_{2_{\mathcal{S}_i}})$ are the factor system of the pairs $(C,A_0)$, $(C,A_1)$ and $(C,A_2)$ on $\mathcal{X}/\mathcal{S}_i$, respectively.

$$\begin{bmatrix} R_iA_{0_{\mathcal{S}_i}}+A_{0_{\mathcal{S}_i}}^TR_i+T_{i0}M_{\mathcal{S}_i}+M_{\mathcal{S}_i}^TT_{i0}^T+M_{\mathcal{S}_i}^TM_{\mathcal{S}_i}+Q_i & R_iA_{1_{\mathcal{S}_i}}+T_{i1}M_{\mathcal{S}_i} & R_iA_{2_{\mathcal{S}_i}}+T_{i2}M_{\mathcal{S}_i} & R_iD_{\mathcal{S}}+A_{0_{\mathcal{S}_i}}^TR_iD_{\mathcal{S}}+M_{\mathcal{S}_i}^TT_{i0}^TD_{\mathcal{S}} & A_{0_{\mathcal{S}_i}}^TR_i+M_{\mathcal{S}_i}^TT_{i0}^T \\ * & -Q_i & 0 & A_{1_{\mathcal{S}_i}}^TR_iD_{\mathcal{S}}+M_{\mathcal{S}_i}^TT_{i1}^TD_{\mathcal{S}} & A_{1_{\mathcal{S}_i}}^TR_i+M_{\mathcal{S}_i}^TT_{i1}^T \\ * & * & -R_i & A_{2_{\mathcal{S}_i}}^TR_iD_{\mathcal{S}}+M_{\mathcal{S}_i}^TT_{i2}^TD_{\mathcal{S}} & A_{2_{\mathcal{S}_i}}^TR_i+M_{\mathcal{S}_i}^TT_{i2}^T \\ * & * & * & -\gamma^2I+D_{\mathcal{S}}^TR_iD_{\mathcal{S}} & 0 \\ * & * & * & * & -R_i \end{bmatrix} < 0 \quad (12)$$

***Proof:*** Given the unobservability subspaces $\mathcal{S}_i$, there exist output map injections $D_{i0}, D_{i1}, D_{i2}$ and measurement mixing map $H_i$ such that

$$\mathcal{S}_i =< \text{Ker } H_iC|A_0 + D_{ij}C >, \quad j = 0, 1, 2$$

where $H_i$ is the solution to Ker $H_iC = \mathcal{S}_i +$ Ker $C$ and is common for all $A_i$'s. Let $M_{\mathcal{S}_i}$ be a unique solution to $M_{\mathcal{S}_i}P_i = H_iC$ and $A_{0_{\mathcal{S}_i}} = (A_0 + D_{i0}C : \mathcal{X}/\mathcal{S}_i)$, $A_{1_{\mathcal{S}_i}} = (A_1 + D_{i1}C : \mathcal{X}/\mathcal{S}_i)$, $A_{2_{\mathcal{S}_i}} = (A_2 + D_{i2}C : \mathcal{X}/\mathcal{S}_i)$ where

$$P_i(A_j + D_{ij}C) = A_{j_{\mathcal{S}_i}}P_i, \quad j = 0, 1, 2 \quad (13)$$

Define $G_{i0} = R_i^{-1}T_{i0}$, $G_{i1} = R_i^{-1}T_{i1}$ and $G_{i2} = R_i^{-1}T_{i2}$ where $T_{i0}, T_{i1}, T_{i2}$ and $R_i$ are the solution to the inequality (12). Let $F_{i0} = A_{0_{\mathcal{S}_i}} + G_{i0}M_{\mathcal{S}_i}$, $F_{i1} = A_{1_{\mathcal{S}_i}} + G_{i1}M_{\mathcal{S}_i}$, $F_{i2} = A_{2_{\mathcal{S}_i}} + G_{i2}M_{\mathcal{S}_i}$ and $E_{i0} = P_i(D_{i0} + P_i^{-r}G_{i0}H_i)$, $E_{i1} = P_i(D_{i1} + P_i^{-r}G_{i1}H_i)$, $E_{i2} = P_i(D_{i2} + P_i^{-r}G_{i2}H_i)$ and $M_i = M_{\mathcal{S}_i}$, $K_i = P_iB_0$, and $K_{ij} = P_iB_j, j = 1, ..., L$. Define $e_i(t) = w_i(t) - P_ix(t)$, then using (10) we have

$$\begin{aligned}
&\dot{e}_i(t) = F_{i0}w_i(t) + F_{i1}w_i(t-h) + F_{i2}\dot{w}_i(t-d) \\
&- E_{i0}y(t) - E_{i1}y_i(t-h) - E_{i2}\dot{y}_i(t-d) \\
&+ K_iu(t) + \sum_{j=1}^{L} K_{ij}u(t-\tau_{uj}) \\
&- P_i(A_0x(t) + A_1x(t-h) + A_2\dot{x}(t-d) + B_0u(t) \\
&+ \sum_{j=1}^{L} B_ju(t-\tau_{uj}) + \sum_{l=1}^{k} L_lm_l(t) + Dd(t)) \\
&= F_{i0}w_i(t) + F_{i1}w_i(t-h) + F_{i2}\dot{w}_i(t-d) \\
&- P_i(A_0 + D_{i0}C)x(t) - G_{i0}M_iP_ix(t) \\
&- P_i(A_1 + D_{i1}C)x(t-h) - G_{i1}M_iP_ix(t-h) \\
&- P_i(A_2 + D_{i2}C)\dot{x}(t-d) - G_{i2}M_iP_i\dot{x}(t-d) \\
&- P_iL_im_i(t) - P_iDd(t) \\
&= F_{i0}e_i(t) + F_{i1}e_i(t-h) + F_{i2}\dot{e}_i(t-d) \\
&- P_iL_im_i(t) - P_iDd(t)
\end{aligned}$$

Note that $P_iL_j = 0, j \neq i$, since $\mathcal{L}_j \in \mathcal{S}_i, j \neq i$. Also

$$\begin{aligned}
r_i(t) &= M_iw_i(t) - H_iy(t) = M_iw_i(t) - H_iCx(t) \\
&= M_ie_i(t)
\end{aligned}$$

Consequently, the error dynamics can be written as

$$\begin{aligned}
\dot{e}_i(t) =& F_{i0}e_i(t) + F_{i1}e_i(t-h) + F_{i2}\dot{e}_i(t-d) \\
&- P_iL_im_i(t) + D_{\mathcal{S}}d(t) \\
r_i(t) =& M_ie_i(t)
\end{aligned} \quad (14)$$

Using Theorem 2.1 by restricting $Q_1 = R$ and the inequality (12), it follows that the inequality (9) holds and the operator $\mathbb{D} = \phi(0) - F_{i2}\phi(-d)$ is also stable. Moreover, from the error dynamics (14), it follows that $r_i(t)$ is only affected by $L_i$ and is decoupled from other fault signatures. ■

The generic conditions for existence of the unobservability subspaces of Theorem 4.1 can be stated as follows.

***Proposition*** *4.2:* Let $A_i, i = 0, 1, 2, C$ and $L_i$ be arbitrary matrices of dimensions $n \times n$, $q \times n$ and $n \times k_i$, respectively. Let $v = \sum_{i=1}^{k} k_i$. The unobservability subspaces of Theorem 4.1 generically exist if and only if (a) $v \leq n$ and (b) $v - \min\{k_i, i = 1, ..., k\} < q$

***Proof:*** The proof is the same as in the EFPRG problem for linear systems [6] and is omitted due to space limitations. ■

After constructing the residual signals $r_i(t), i = 1, ..., k$, the last step is to determine the threshold $J_{th_i}$ and the evaluation function $J_{r_i}(t)$. In this paper, the following evaluation functions and thresholds are selected

$$J_{r_i}(t) = \int_{t-T_0}^{t} r_i^T(t)r_i(t)dt, \quad i = 1, ..., k \quad (15)$$

$$J_{th_i} = \sup_{d\in L_2, m_j=0, j=1,...,k} (J_{r_i}), \quad i = 1, ..., k \quad (16)$$

where $T_0$ is the length of the evaluation window. Based on the above thresholds and evaluation functions, the occurrence of a fault can be detected and isolated by using the following decision logics

$$J_{r_i}(t) > J_{th_i} \Longrightarrow m_i \neq 0, \quad i = 1, ...., k \quad (17)$$

## V. NUMERICAL EXAMPLE

To illustrate the effectiveness and capabilities of our proposed FDI algorithm, a numerical example is provided in this section. Consider the time-delay system (7) that is specified with parameters

$$A_0 = \begin{bmatrix} 2 & -1.5 & 1 & 1 \\ 1 & -1 & 0.5 & 2 \\ 1 & 2 & -3 & 0 \\ 2 & 0 & -1 & 1 \end{bmatrix}, B_0 = \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 1 \\ 0 & 1 \end{bmatrix}$$

$$A_1 = \begin{bmatrix} -1 & 2 & 0.2 & 0 \\ -0.1 & 1.3 & 0.5 & 1 \\ 0.1 & -1 & 2 & 0.1 \\ 1 & 0.1 & 1 & 2 \end{bmatrix}, C = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0.2 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} 0.1 & 0 & 0.0 & 0.1 \\ 0.2 & -0.1 & 0.2 & 0.3 \\ 0.4 & 0.3 & 0.4 & 0.1 \\ -0.1 & 0 & 0.2 & 0.3 \end{bmatrix}, D = \begin{bmatrix} 0.3 \\ 0.2 \\ 0 \\ 0.6 \end{bmatrix}, B_1 = 0$$

and $h = 2$, $d = 1$ and $L = 0$. The fault signatures $L_1$ and $L_2$ are selected as the first and second columns of the matrix $B_0$, and hence they represent actuator faults for the time-delay system.

The subspaces in Theorem 4.1 for the above time-delay system can be determined using the *CUOS* algorithm and are given by $\mathcal{S}_1 = \mathcal{L}_1, \mathcal{S}_2 = \mathcal{L}_2$. After determining the subspaces $\mathcal{S}_1$ and $\mathcal{S}_2$, the maps $D_{i0}$, $D_{i1}, D_{i2}, H_i, M_i, i = 1, 2$ and matrices $A_{0_{\mathcal{S}_1}}, A_{1_{\mathcal{S}_1}}, A_{2_{\mathcal{S}_1}}$, $A_{0_{\mathcal{S}_2}}, A_{1_{\mathcal{S}_2}}, A_{2_{\mathcal{S}_2}}$ can be found according to Theorem 4.1. Using the LMI tools, the gain matrices $G_{10}, G_{11}, G_{12}, G_{20}, G_{21}$ and $G_{22}$ are computed by solving the LMI inequality (12) for $\gamma = 1$. An $H_\infty$ robust state feedback control $u(t) = Kx(t)$ is also designed for the closed-loop system to ensure its stability.

A disturbance input $d(t)$ is assumed to be a band-limited white-noise with power of 0.2. The thresholds are calculated as $J_{th_1} = 0.06$ and $J_{th_2} = 0.07$ for $T_0 = 5$ seconds. Figure 1 shows the residuals and their evaluation functions corresponding to the healthy operation of the system. As shown in this figure, no false alarm is generated during normal operation of the system. Figure 2 shows the residuals and the evaluation functions corresponding to a fault in the second actuator ($u_2$) of the system where the gain of the actuator is decreased by 55% at $t = 10$ seconds. This type of fault can be modeled as $m_2(t) = -0.55u_2(t)$, where $m_2(t)$ is the fault mode of the second actuator. As shown in this figure, the fault is detected and isolated at $t = 13.3$ seconds and the evaluation function of residual $r_1$ (i.e. $J(r_1)$) remains below its corresponding threshold. Figure 3 shows the residuals and evaluation functions corresponding to a fault in the first actuator where the gain of the actuator is decreased by 40% at $t = 10$ seconds. This fault can be modeled as $m_1(t) = -0.4u_1(t)$, where $m_1(t)$ is the fault mode of the first actuator. As shown in this figure, this fault is detected and isolated at $t = 14$ seconds and the evaluation function of $r_2$ (i.e. $J(r_2)$) remains below its corresponding threshold. Figure 4 shows the residuals and the evaluation functions corresponding to simultaneous faults in both actuators where 40% loss of effectiveness (gain) is occurred in the first actuator at $t = 5$ seconds and 50% loss of effectiveness is occurred in the second actuator at $t = 10$ seconds. According to this figure, the fault in the first actuator is detected at $t = 7.7$ seconds and the fault in the second actuator is detected at $t = 13.5$ seconds. It should be noted that in all above scenarios the time-delay system remains stable and well-behaved, which makes the FDI problem more challenging.

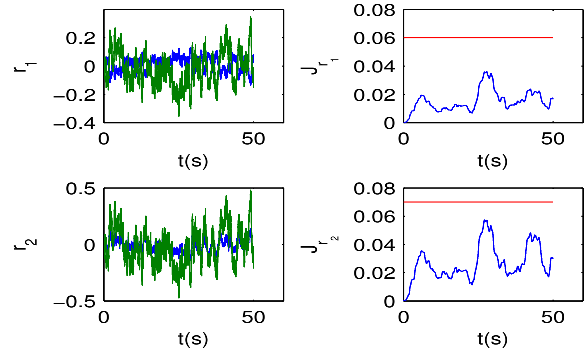


Fig. 1. Residual signals and their evaluation functions corresponding to the normal mode (healthy operation).

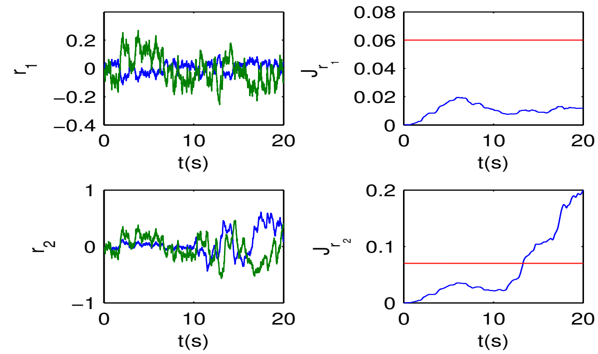


Fig. 2. Residual signals and their evaluation functions corresponding to a fault in the second actuator.

***Remark***: It should be emphasized that the presently available FDI algorithm for neutral time delay systems [29] cannot generate the residual signals with the above decoupling properties. In [29], faults that one needs to be decoupled are considered as unknown inputs and the algorithm seeks to attenuate the effects of faults on the residual. Therefore, those type of algorithms cannot decouple fault effects from the residuals. However, in our proposed approach, the residual signals that can decouple the faults from each other and are robust with respect to disturbances are constructed where one can easily use these residuals for both fault detection and isolation.

## VI. Conclusions

A geometric approach to fault detection and isolation of faults in linear neutral time-delay systems is developed in this paper. The set of residual signals are generated so that each residual is only affected by one fault and is decoupled from the others while the $H_\infty$ norm of the transfer function between the unknown input (disturbances and modeling errors) and residual signals is less than a given positive

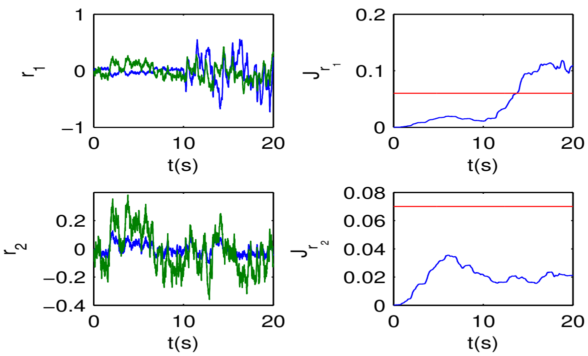


Fig. 3. Residual signals and their evaluation functions corresponding to a fault in the first actuator.

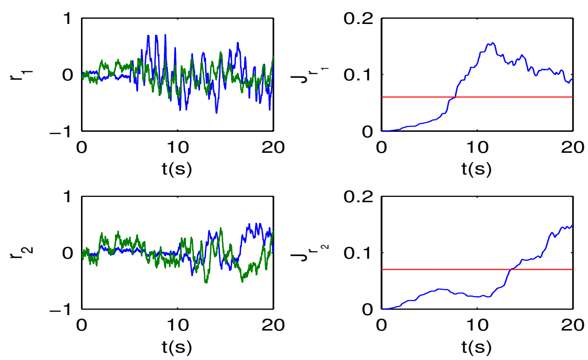


Fig. 4. Residual signals and their evaluation functions corresponding to simultaneous faults in both actuators.

value. Simulation results demonstrate the effectiveness of our proposed method.

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